Unitarity of Higgs inflation and R^2 correction

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2025/09/26@TDLI

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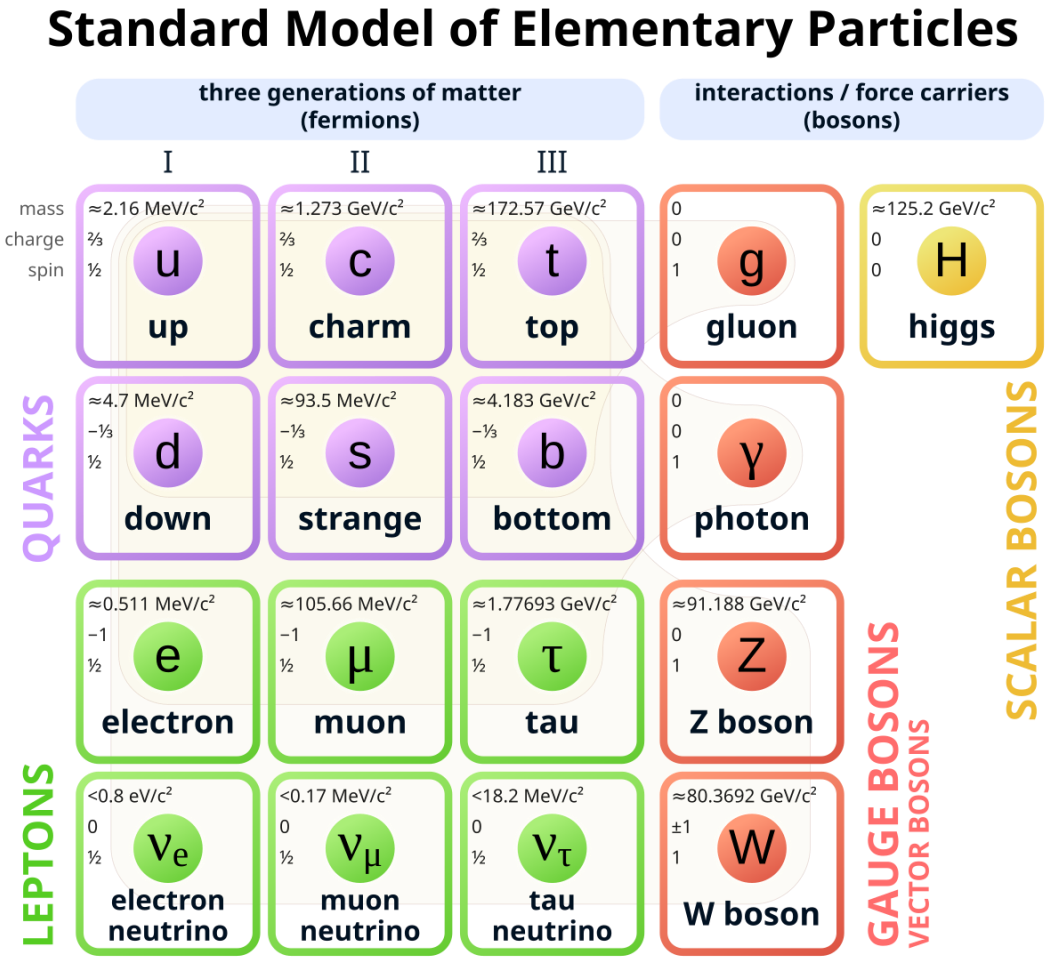
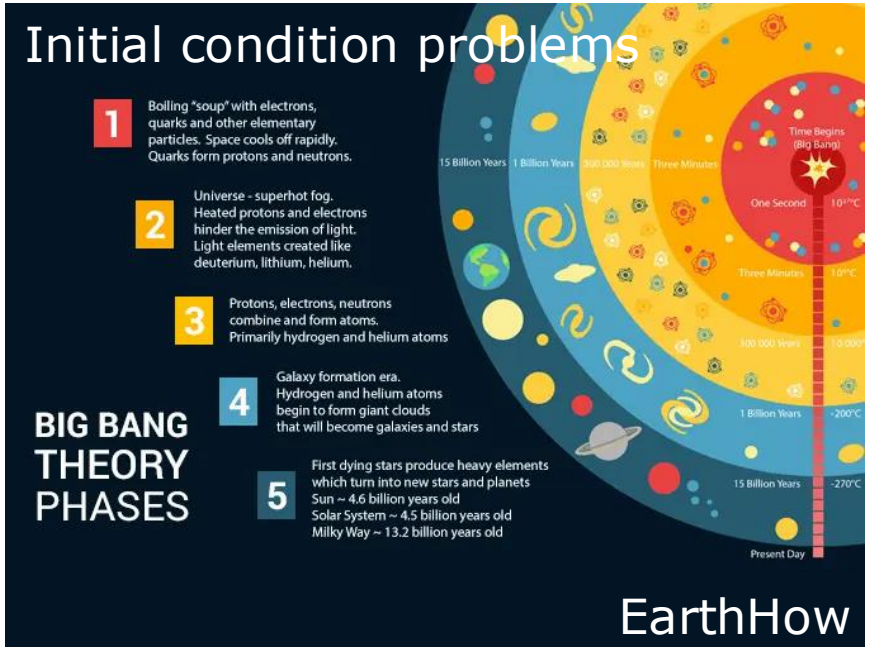
1804.00409 (JCAP), 1812.10099 (PLB), 2308.14398 (JHEP),
2308.15420 (PRL).

Standard Model Higgs field

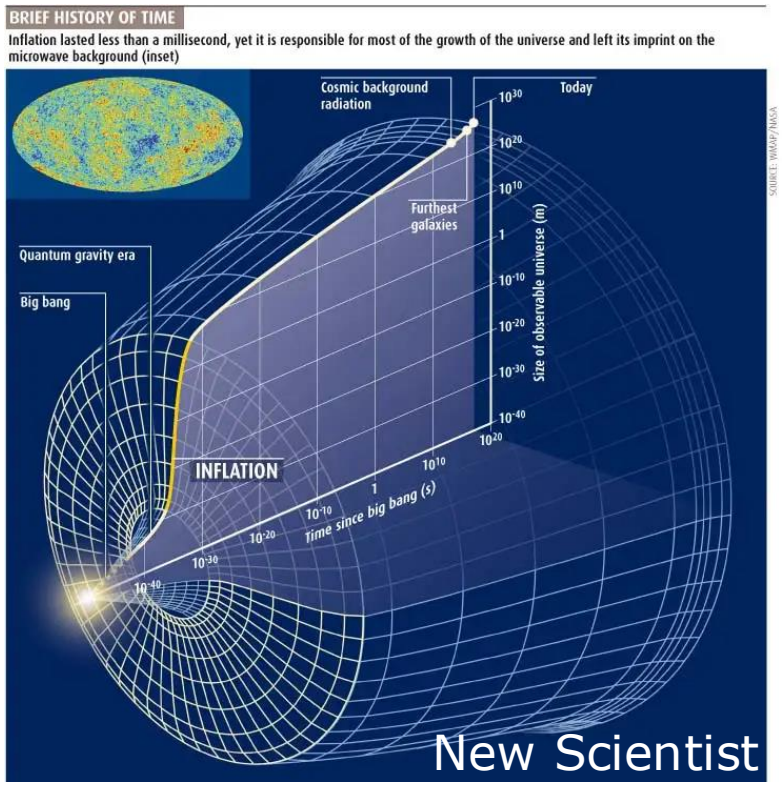


Wikipedia

Inflationary cosmology



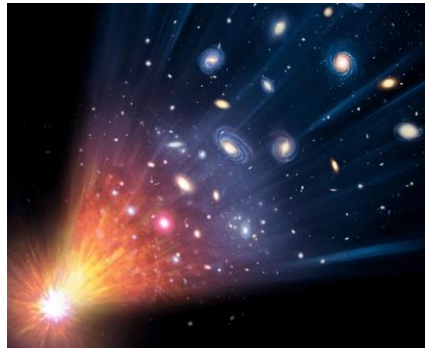
Inflation as a solution
to big bang puzzles:
Why homogeneous?
Why isotropic?
Why flat?



Question 1 Who blows the Universe?

Question 2 How to connect to hot big bang?

Two major questions
to be answered



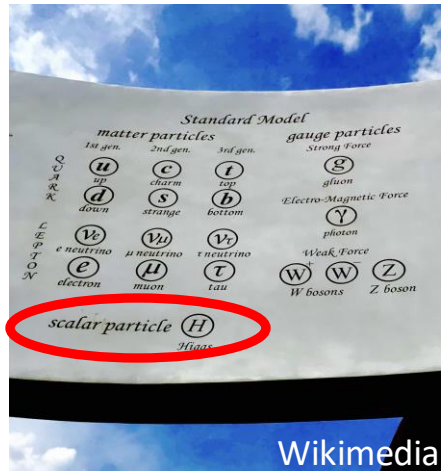
Higgs inflation

Matter field (inflaton) → Inflation (accelerated expansion) → Inflaton decay (reheating, hot big bang)

Gravity $G_{\mu\nu} = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu}$ Matter

Higgs inflation Cervantes-Cota et al 1995, Bezrukov et al 2008, Barvinsky et al 2008

SM Higgs: $\mathcal{H} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$

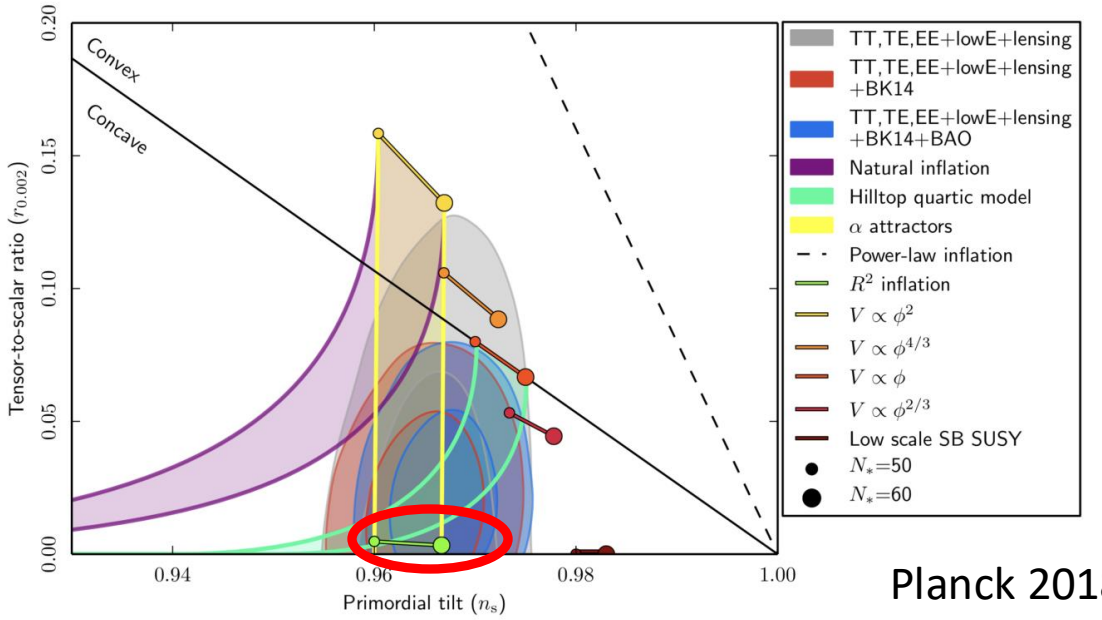


Higgs inflation

- Higgs field as inflaton (with large ξ)

Cervantes-Cota et al 1995,
Bezrukov et al 2008,
Barvinsky et al 2008

$$S = \int \mathrm{d}^4x\sqrt{-g_\mathrm{J}}\left(\frac{M_\mathrm{Pl}^2 + \xi\phi^2}{2}R_\mathrm{J} - \frac{g_\mathrm{J}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V\right) \qquad V = \frac{\lambda}{4}\phi^4$$



Planck 2018

$$\frac{\xi}{\sqrt{\lambda}} \sim O(10^4)$$

Higgs inflation

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- Valid as an effective field theory (EFT)?
  - Potential: ϕ^4, small λ
  - Kinetic term: (non-)linear sigma model
  - Non-minimal coupling with gravity

Burgess, Lee, Trott 2009, 2010, Barbón et al 2009, Lerner et al 2010, Hertzberg 2010, Bezrukov et al 2010, Bezrukov et al 2011, Ren et al 2014, Ema et al 2017, Ito et al 2022, Karananas et al 2022, Mikura et al 2022…

Contents

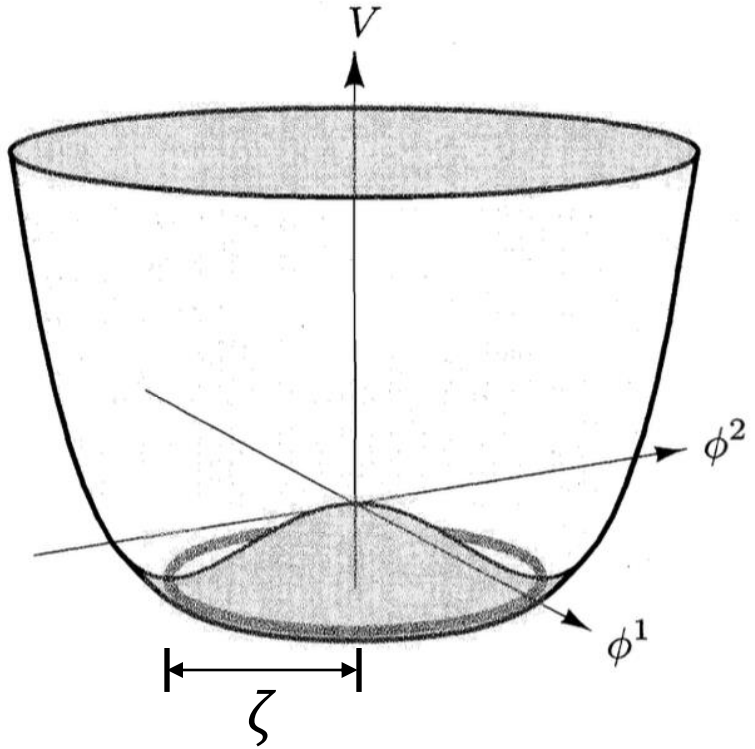
- Calculation of UV cutoff in non-linear sigma model with gravity
- Application to Higgs inflation and unitarity violation
- UV extension with R^2

Calculation of UV cutoff

Non-linear sigma model (NLSM)

Gell-Mann, Lévy 1960

- Examples of space metric in particle physics:



Peskin and Schroeder

$$\mathcal{L} \supset -\frac{1}{2}\delta_{ij}\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi)$$

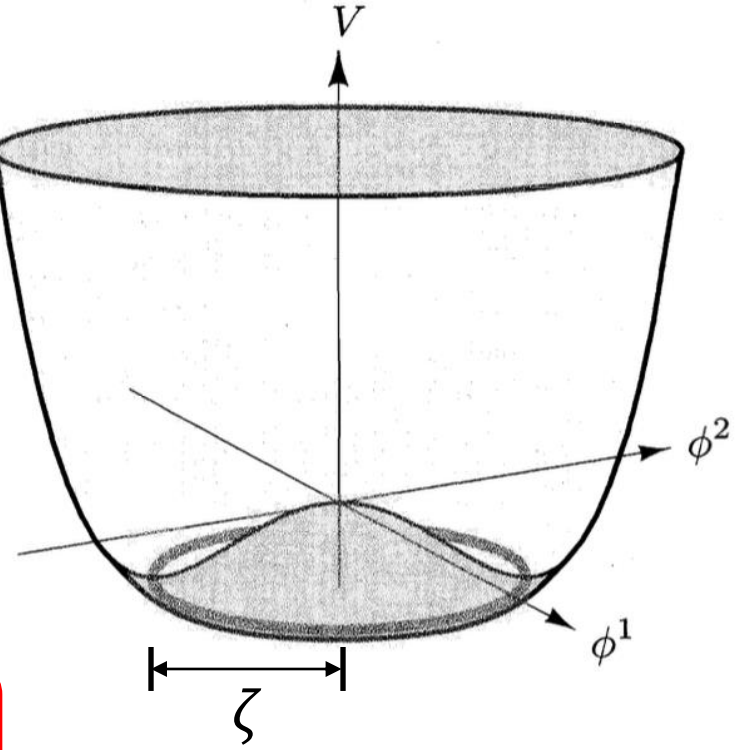
$$\zeta^2 = \phi^i\phi_i$$

$$i = 1,2,3,\dots,N$$
$$\phi^i = (\rho, \pi^1, \dots, \pi^{N-1})$$

$$\mathcal{L} \propto \left(\partial_\mu\pi^i\right)^2 + \frac{(\pi_i\partial_\mu\pi^i)^2}{\zeta^2 - \pi^i\pi_i}$$

Geometry of target space

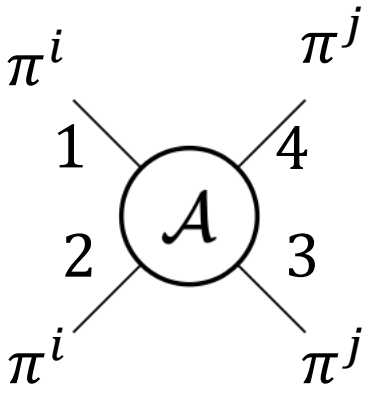
$$\mathcal{L} \supset -\frac{1}{2}\eta^{\mu\nu}\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial_\nu\phi^j - V(\phi)$$



Target space metric

$$\mathcal{G}_{ij}(\pi) = \delta_{ij} + \frac{\pi_i\pi_j}{\zeta^2 - \pi^k\pi_k}$$

At $\pi^i = (0,0,\ldots,0)$



$$\mathcal{M}_{\pi^i\pi^i\to\pi^j\pi^j} = \frac{2}{3}\left(s_{12}\bar{R}_{ijji} + \frac{1}{2}s_{13}\bar{R}_{ijij} + \frac{1}{2}s_{14}\bar{R}_{ijij}\right)$$

Alonso et al 2016A,B, Nagai et al 2019, Cohen et al 2021, Cheung et al 2022

Geometry of target space

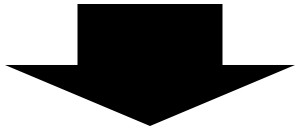
Gell-Mann, Lévy 1960

- In general,

$$\mathcal{L} \supset -\frac{1}{2}\eta^{\mu\nu}\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial_\nu\phi^j - V(\phi)$$

Geometry (e.g. curvature)

with O(N) symmetry



- Key information of the system, e.g. UV cutoff $\propto \bar{R}^{-1/2}$
- Uniqueness: independence of coordinate transformation (field redefinition)

Effect of gravity

- Example: Higgs inflation

Jordan frame $$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}\left(\frac{M_{\mathrm{Pl}}^2+\xi\phi^2}{2}R_{\mathrm{J}}-\frac{g_{\mathrm{J}}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-V\right)$$

$$F(\phi)=1+\xi\frac{\phi^2}{M_{\mathrm{Pl}}^2}$$

$$g_{\mathrm{E}\mu\nu}=\Omega^2 g_{\mathrm{J}\mu\nu},\qquad \Omega^2\equiv 1+\frac{\xi\phi^2}{M_{\mathrm{Pl}}^2},$$

Maeda 1988, 1989

Einstein frame $$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{E}}}\left(\frac{M_{\mathrm{Pl}}^2}{2}R_{\mathrm{E}}-\frac{g_{\mathrm{E}}^{\mu\nu}}{2}\mathcal{G}_{ij}^{\mathrm{E}}\partial_\mu\phi^i\partial_\nu\phi^j-\frac{V}{\Omega^4}\right)$$

$$\mathcal{G}_{ij}^{\mathrm{E}}\equiv\frac{1}{\Omega^2}\left(\delta_{ij}+\frac{6\xi^2}{\Omega^2}\frac{\phi_i\phi_j}{M_{\mathrm{Pl}}^2}\right)$$

Geometry not unique?

Extended target space

- Weyl transformation as coordinate transformation

$$g_{\mathrm{E}\mu\nu} = \Omega^2 g_{\mathrm{J}\mu\nu}$$

  - Conformal mode

Ema, Mukaida, van de Vis 2020, 2021

$$g_{\bullet\mu\nu} = \frac{\Phi_\bullet^2}{6M_{\mathrm{Pl}}^2}\tilde{g}_{\mu\nu}, \qquad \det\left(\tilde{g}_{\mu\nu}\right) = -1$$

in any frame

$$\Phi_{\mathrm{E}}^2 = \Omega^2 \Phi_{\mathrm{J}}^2$$

  - Extend the target space by including the conformal mode

Extended target space

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}\left(\frac{M_{\mathrm{Pl}}^2}{2}fR_{\mathrm{J}}-\frac{g_{\mathrm{J}}^{\mu\nu}}{2}\mathcal{G}_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-V\right)$$

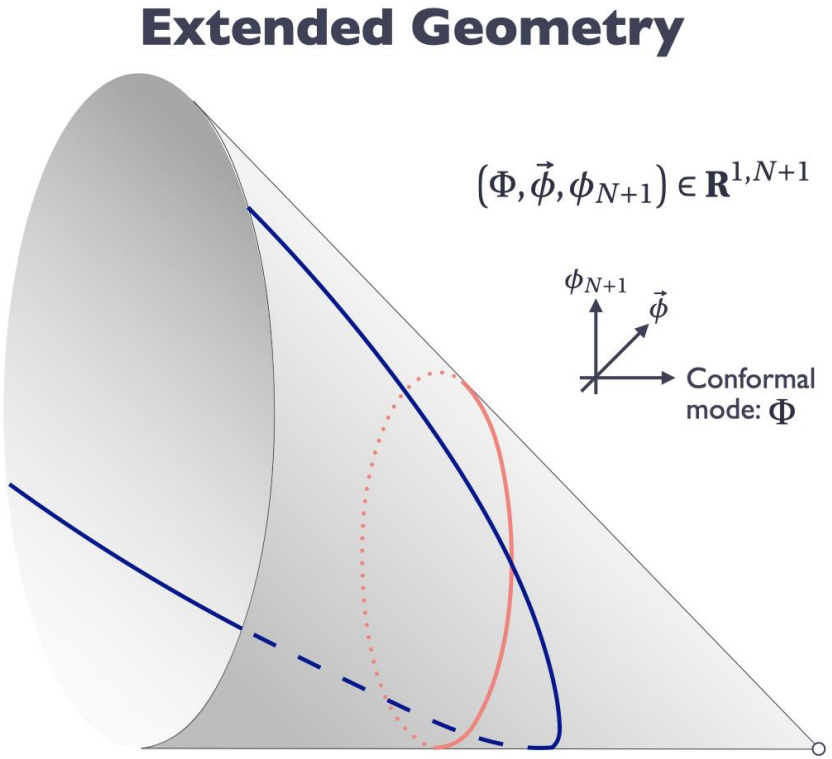
$$g_{\bullet\mu\nu}=\frac{\Phi_\bullet^2}{6M_{\mathrm{Pl}}^2}\tilde{g}_{\mu\nu},\qquad \det\left(\tilde{g}_{\mu\nu}\right)=-1$$

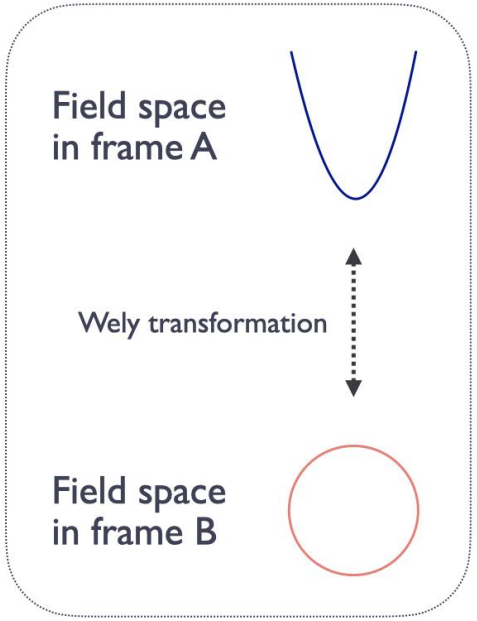
$$S=\int \mathrm{d}^4x\left(\frac{\Phi_{\mathrm{J}}^2}{12}f\tilde{R}-\frac{\tilde{g}^{\mu\nu}}{2}\mathcal{G}^{\mathrm{J}}_{ab}\partial_\mu\varphi^a_{\mathrm{J}}\partial_\nu\varphi^b_{\mathrm{J}}-\frac{\Phi_{\mathrm{J}}^4}{36M_{\mathrm{Pl}}^4}V\right)$$

$$\left(\varphi^a_{\mathrm{J}}\right)=\left(\Phi_{\mathrm{J}},\phi^i\right)$$

$$\left(\mathcal{G}^{\mathrm{J}}_{ab}\right)=\begin{pmatrix}-f & -\frac{\Phi_{\mathrm{J}}}{2}\frac{\partial f}{\partial\phi^j}\\ -\frac{\Phi_{\mathrm{J}}}{2}\frac{\partial f}{\partial\phi^i} & \frac{\Phi_{\mathrm{J}}^2}{6M_{\mathrm{Pl}}^2}\mathcal{G}_{ij}\end{pmatrix}$$

Extended target space





Credit: Kyohei Mukaida

Application to Higgs inflation

UV cutoff of Higgs inflation

- Higgs inflation

$$\Omega^2 = 1 + \xi \frac{\phi^2}{M_{\rm Pl}^2}$$

$$S = \int \mathrm{d}^4x\sqrt{-g_{\rm J}}\left(\frac{M_{\rm Pl}^2 + \xi\phi^2}{2}R_{\rm J} - \frac{g_{\rm J}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V\right)$$

Extended target space ➡ $$\left(\mathcal{G}^{\rm J}_{ab}\right) \equiv \begin{pmatrix} -\Omega^2 & -\xi\Phi_{\rm J}\phi_j/M_{\rm Pl}^2 \\ -\xi\Phi_{\rm J}\phi_i/M_{\rm Pl}^2 & \frac{\Phi_{\rm J}^2}{6M_{\rm Pl}^2}\delta_{ij} \end{pmatrix}$$

$$\frac{\Lambda}{\Lambda_{\rm G}} \propto \frac{\bar{R}^{-1/2}}{\Lambda_{\rm G}} \begin{cases} \sim \begin{cases} 1/\xi & \text{for} \quad v \lesssim M_{\rm Pl}/\xi, \\ v/M_{\rm Pl} & \text{for} \quad M_{\rm Pl}/\xi \lesssim v \lesssim M_{\rm Pl}/\sqrt{\xi}, \\ 1/\sqrt{\xi} & \text{for} \quad M_{\rm Pl}/\sqrt{\xi} \lesssim v, \end{cases} & N > 2 \\ \sim \begin{cases} 1/\xi & \text{for} \quad v \lesssim M_{\rm Pl}/\xi, \\ \xi v^2/M_{\rm Pl}^2 & \text{for} \quad M_{\rm Pl}/\xi \lesssim v \lesssim M_{\rm Pl}/\sqrt{\xi}, \\ 1 & \text{for} \quad M_{\rm Pl}/\sqrt{\xi} \lesssim v, \end{cases} & N = 2 \end{cases}$$

UV cutoff of Higgs inflation

- Higgs inflation

$$\Omega^2 = 1 + \xi \frac{\phi^2}{M_{\rm Pl}^2}$$

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Extended target space ➡ $\left(\mathcal{G}^{\rm J}_{ab}\right)$

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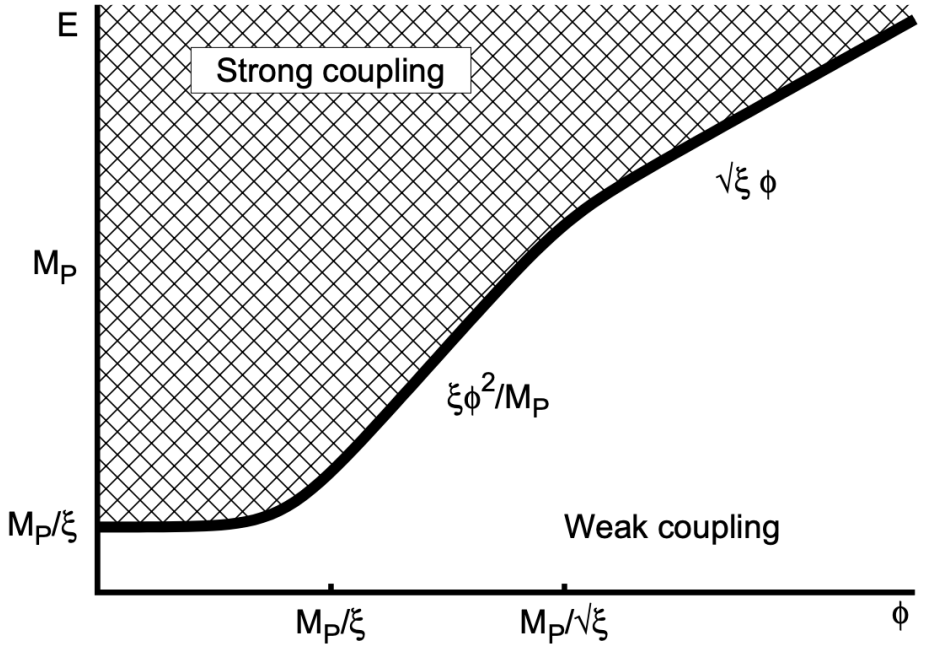


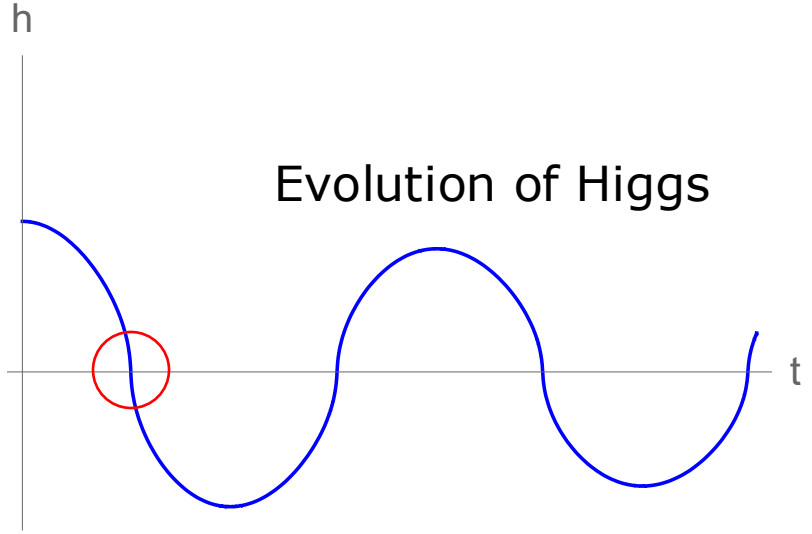
Bezrukov, Magnin, Shaposhnikov, Sibiryakov, 2011

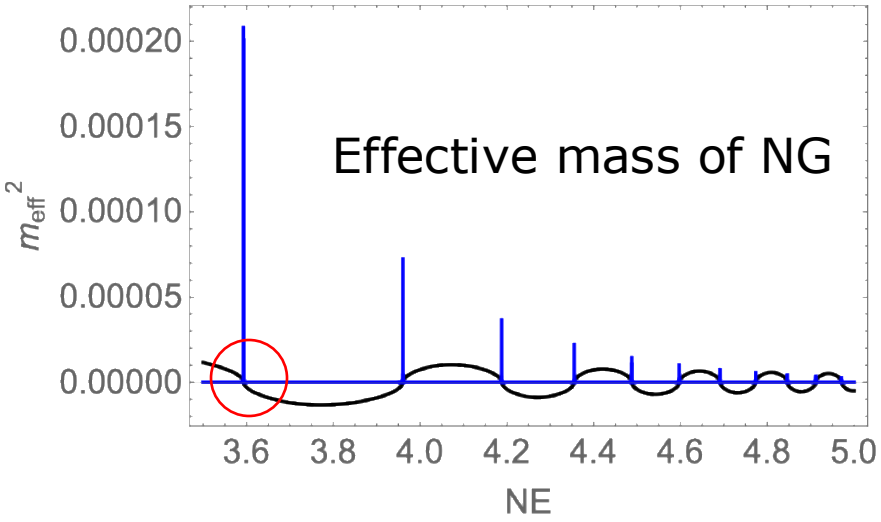
Unitarity violation in Higgs inflation

- Cutoff scale
  - During inflation $\Lambda/\Lambda_G \sim 1/\sqrt{\xi}$
  - During reheating $\Lambda/\Lambda_G \sim 1/\xi$
- During preheating, NG mode production

$$\frac{\xi}{\sqrt{\lambda}} \sim O(10^4)$$

Ema, Jinno, Mukaida, Nakayama 2017





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$$\frac{\xi}{\sqrt{\lambda}} \sim O(10^4)$$

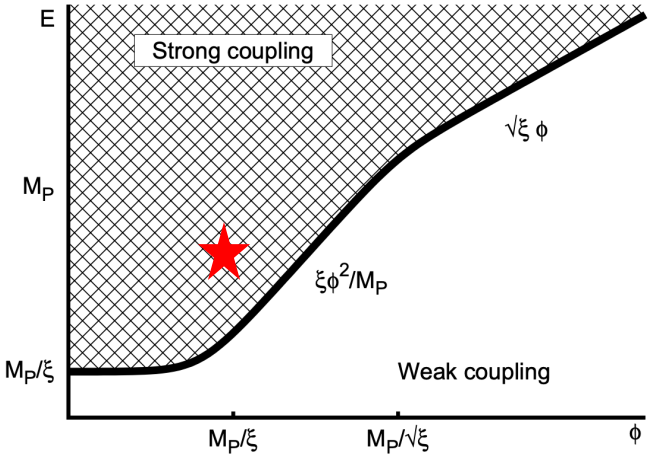
- During preheating, NG mode production

$$\Delta t^{-2} \sim m_{\rm eff}^2 \sim \frac{\xi^2}{M_{\rm Pl}^2} \times V_{\rm end} \sim \lambda M_{\rm Pl}^2 > \Lambda^2$$

Target space curvature R $\quad \dot{\phi}^2 \sim$kinetic$\sim$potential$\sim V_{\rm end} \sim \frac{\lambda M_{\rm Pl}^4}{\xi^2}$

Need UV extension!

Ema, Jinno, Mukaida, Nakayama 2017
Ema, Jinno, Nakayama, van de Vis 2021



Bezrukov, Magnin, Shaposhnikov, Sibiryakov, 2011

UV extension with R^2

Mixed Higgs-R^2 inflation

- UV extended Higgs inflation Ema 2017

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}\left(\frac{M_{\mathrm{Pl}}^2+\xi\phi^2}{2}R_{\mathrm{J}}-\frac{g_{\mathrm{J}}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-V\right)+\alpha R_{\mathrm{J}}^2$$

Extract the new DoF σ from R^2

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{E}}}\left[\frac{M_{\mathrm{Pl}}^2}{2}R_{\mathrm{E}}-\frac{1}{2}g_{\mathrm{E}}^{\mu\nu}\left(\partial_\mu\sigma_{\mathrm{E}}\partial_\nu\sigma_{\mathrm{E}}+e^{-\sqrt{\frac{2}{3}}\frac{\sigma_{\mathrm{E}}}{M_{\mathrm{Pl}}}}\partial_\mu\phi_i\partial_\nu\phi^i\right)-U(\phi,\sigma_{\mathrm{E}})\right]$$

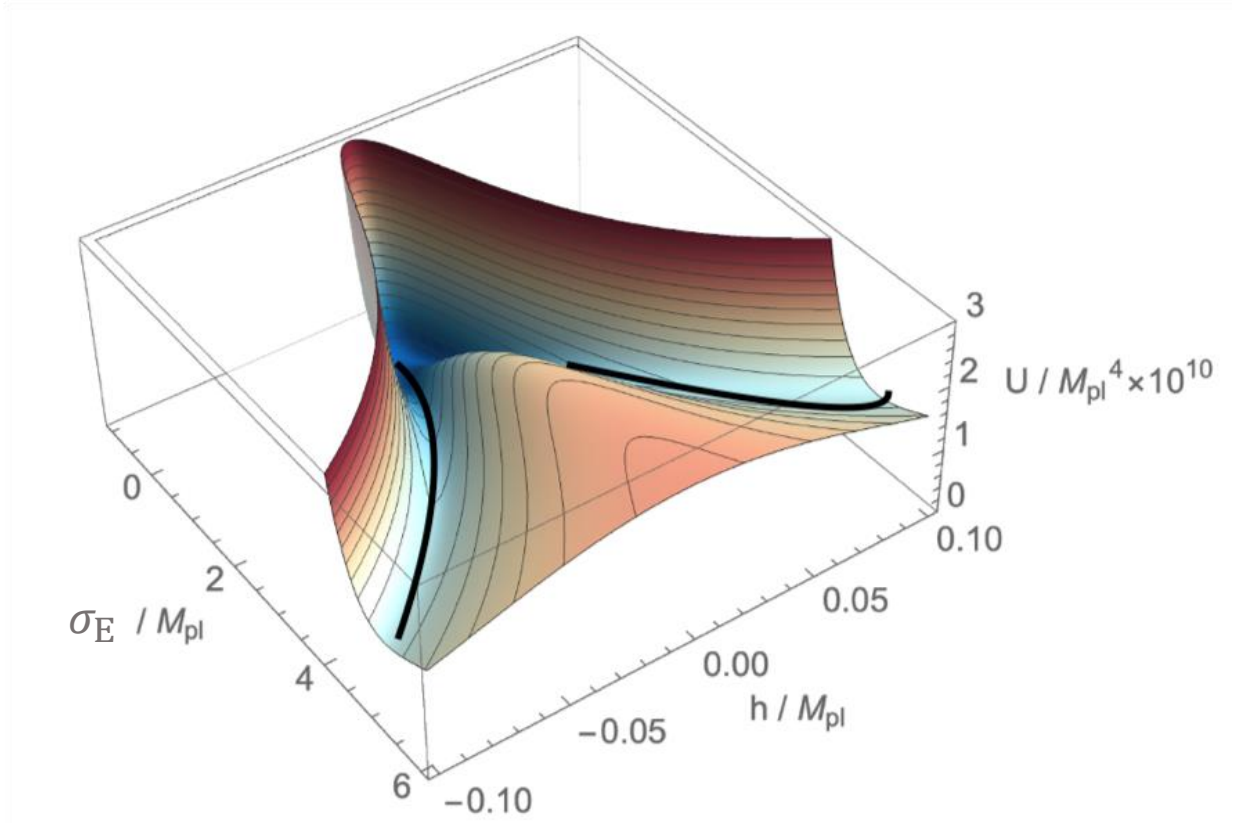
Flat extended target space!

- No cutoff from the scalar sector
- UV cutoff from graviton, M_{Pl}

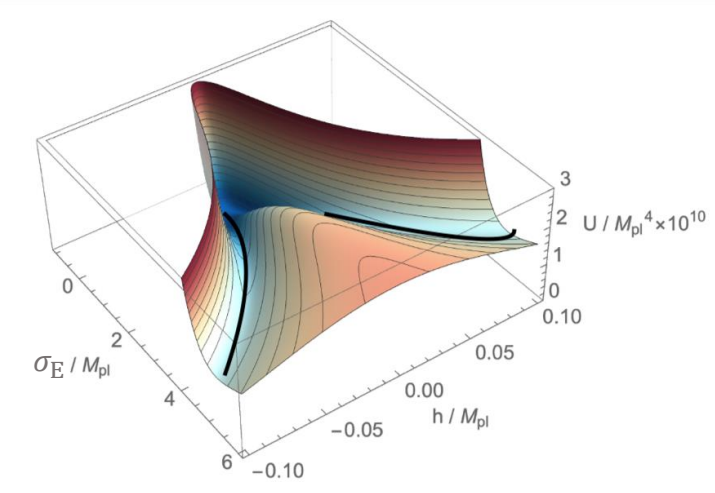
Ema 2017
Gorbunov, Tokareva 2018
Ema, Mukaida, van de Vis 2020,2021

Mixed Higgs-R^2 inflation

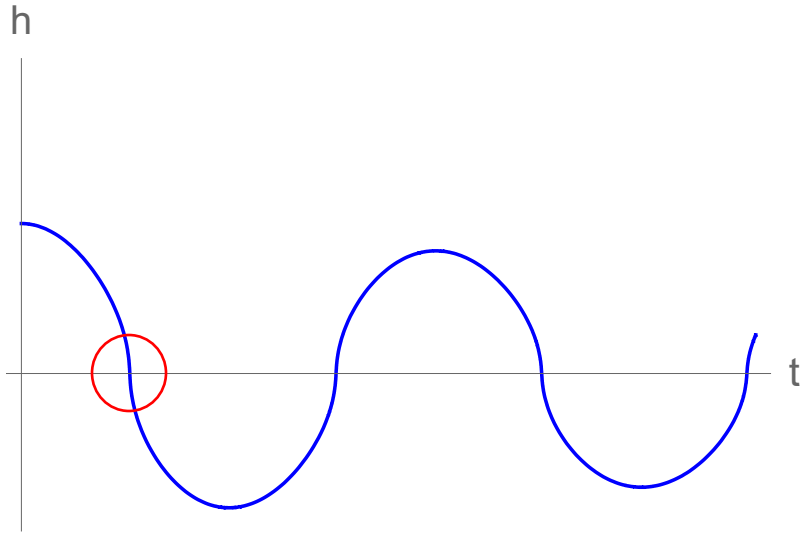
- Potential and dynamics



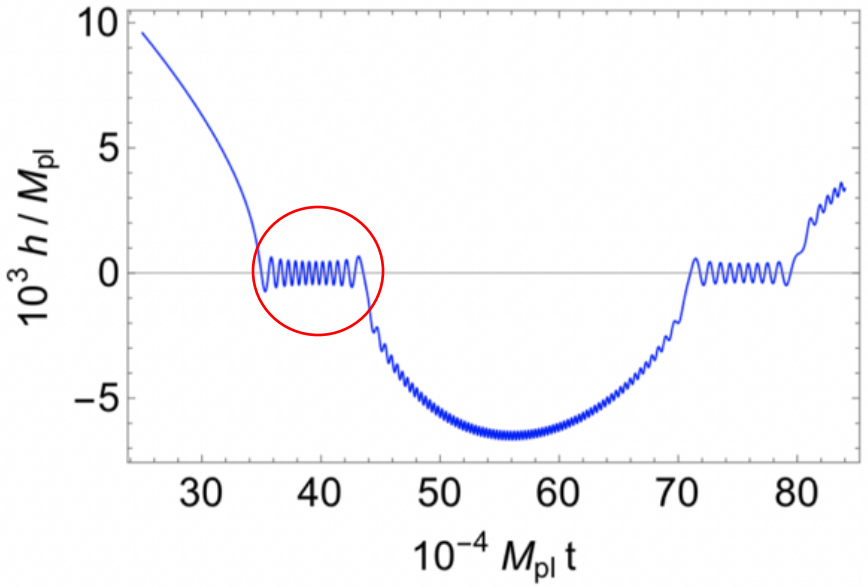
Mixed Higgs-R^2 inflation



- Preheating is not violating unitarity



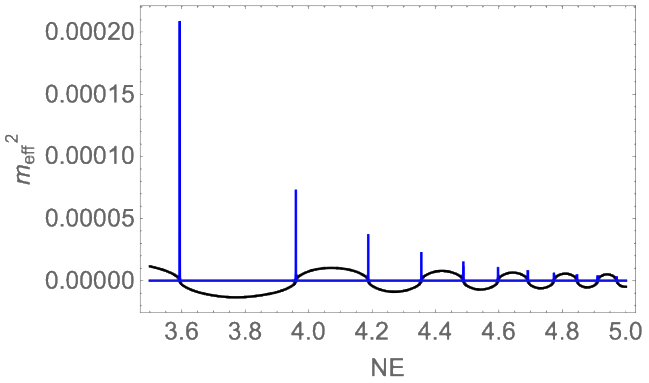
Without R^2



With R^2

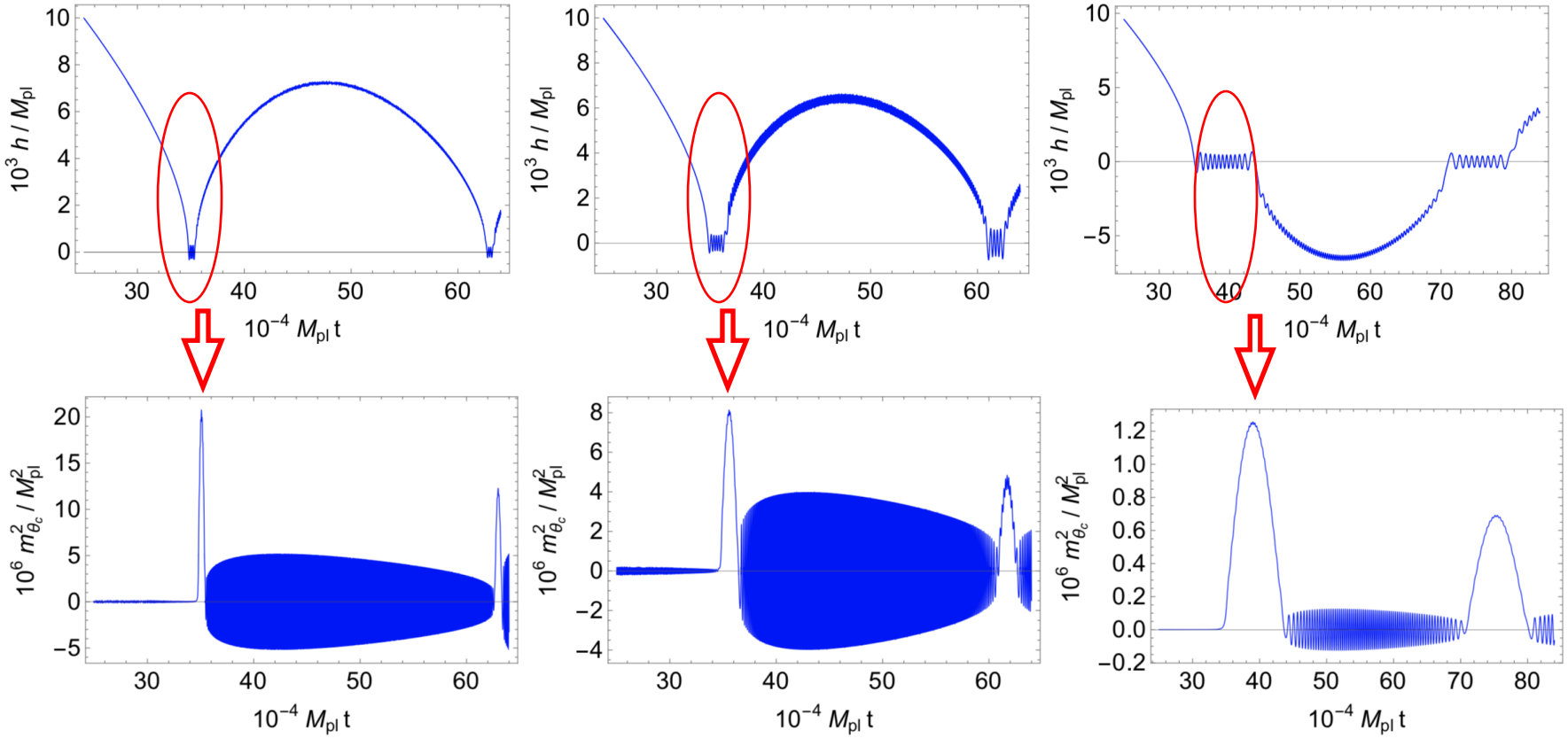
Mixed Higgs-R^2 inflation

c.f.



- Preheating is not violating unitarity

Larger ξ



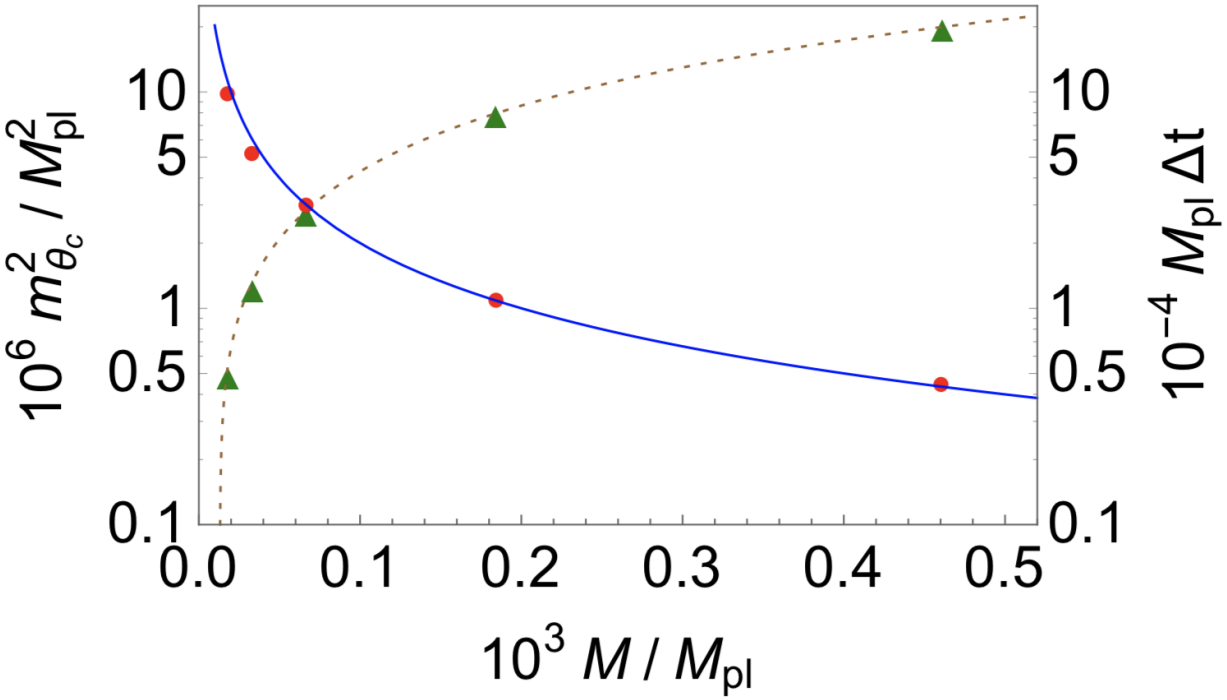
Mixed Higgs-R^2 inflation

- Preheating is not violating unitarity

$$\left(m_{\theta_c}^{\rm sp}\right)^2 \approx C_{\rm m}\sqrt{3\lambda(M^2-M_c^2)}M_{\rm pl}$$

$$\Delta t_{\rm sp} = C_{\rm t} M^{-1}$$

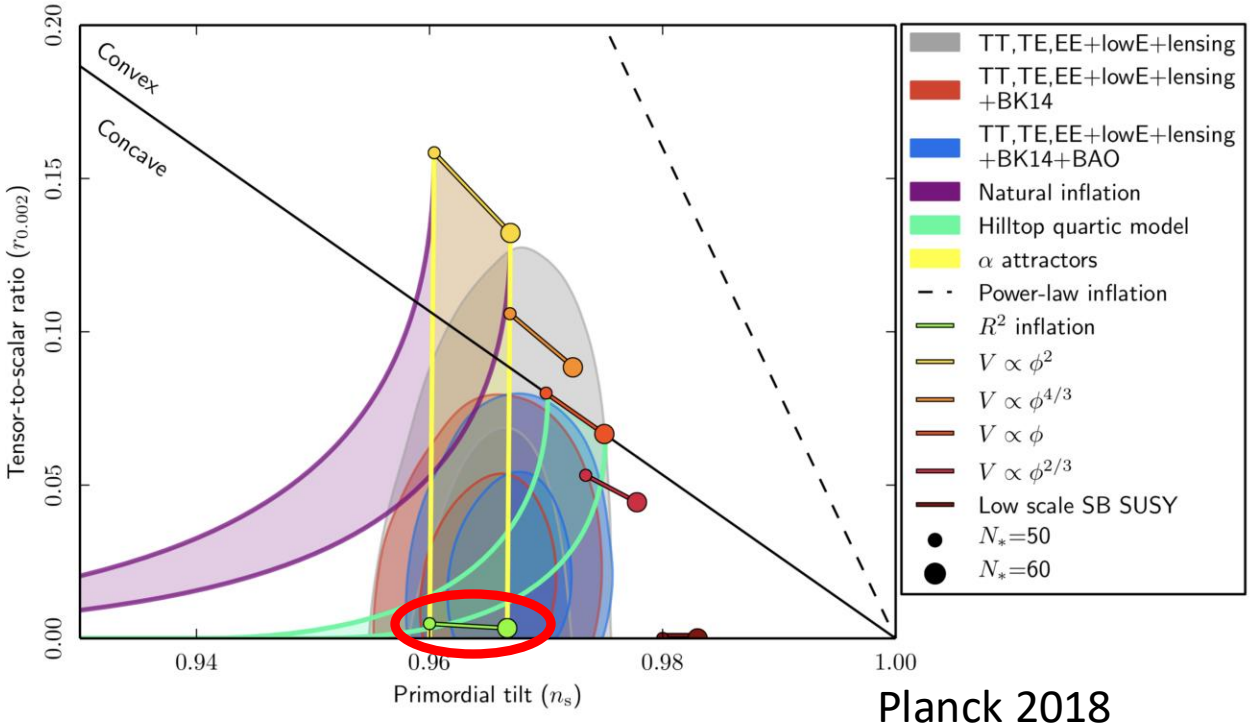
$$C_m, C_t \sim \mathcal{O}(1)$$



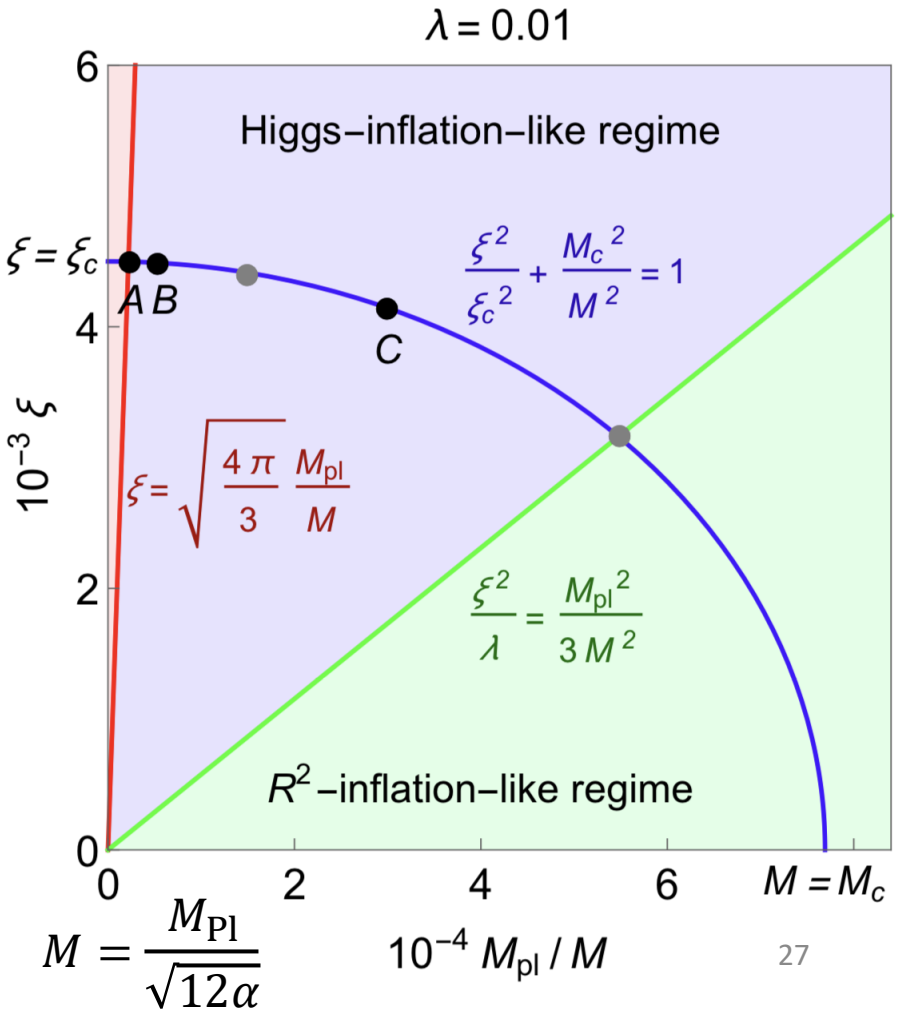
These scales are much smaller than cutoff

Mixed Higgs-R^2 inflation

- Observation and parameter space



Planck 2018



Summary

- EFT in the early Universe
  - Gravity and extended target space
- Higgs inflation and unitarity violation
- UV extension with R^2 term

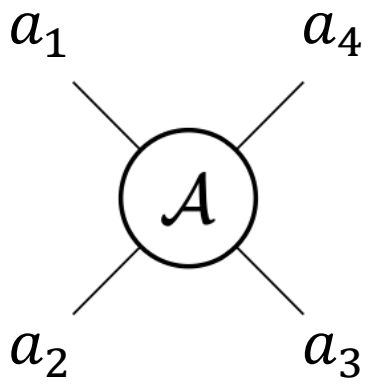
Backup

Geometry of target space

$$\mathcal{L} = -\frac{1}{2}\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j$$

$$\mathrm{d}s^2_\phi = \mathcal{G}_{ij}(\phi)\mathrm{d}\phi^i\mathrm{d}\phi^j$$

- Example: Riemann normal coordinates
- Well-established in SMEFT/HEFT



$$\left(\prod_{i=1}^{4}\bar{\mathcal{G}}^{1/2}_{a_i a_i}\right)\mathcal{M}_{a_1 a_2\leftrightarrow a_3 a_4} = \frac{2}{3}\left[s_{12}\bar{R}_{a_1(a_3 a_4)a_2} + s_{13}\bar{R}_{a_1(a_2 a_4)a_3} + s_{14}\bar{R}_{a_1(a_2 a_3)a_4}\right]$$

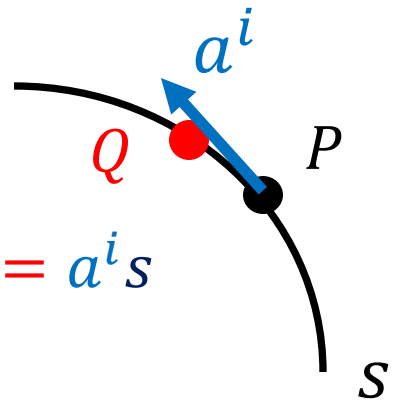
Diagonal (can be removed with vielbein)

Alonso et al 2016A,B, Nagai et al 2019, Cohen et al 2021, Cheung et al 2022

Geometry of target space

$$\mathcal{L} = -\frac{1}{2}\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j$$

$$\boxed{\mathrm{d}s_\phi^2 = \mathcal{G}_{ij}(\phi)\mathrm{d}\phi^i\mathrm{d}\phi^j}$$



- Example: Riemann normal coordinates

$$-\frac{1}{2}\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j = -\frac{1}{2}\underline{\eta_{ij}}\partial_\mu\phi^i\partial^\mu\phi^j + \frac{1}{6}\underline{\bar{R}_{ikjl}}\phi^k\phi^l\partial_\mu\phi^i\partial^\mu\phi^j + \cdots$$

Evaluated at the particular point

Geometry of target space

$$\left(\prod_{i=1}^{4}\bar{G}^{1/2}_{a_i a_i}\right)\mathcal{M}_{a_1 a_2\leftrightarrow a_3 a_4}=\frac{2}{3}\left[s_{12}\bar{R}_{a_1(a_3 a_4)a_2}+s_{13}\bar{R}_{a_1(a_2 a_4)a_3}+s_{14}\bar{R}_{a_1(a_2 a_3)a_4}\right]$$

But $\bar{G}^{\mathrm{J}}_{ab}$ is generally not diagonal

$$\bar{G}^{\mathrm{J}}_{ab}=\bar{e}_a^{\ A}\bar{e}_b^{\ B}\eta_{AB}$$

$$\varphi^A=\bar{e}^A_a\varphi^a_{\mathrm{J}} \qquad \bar{R}_{ABCD}=\bar{e}_A^{\ a}\bar{e}_B^{\ b}\bar{e}_C^{\ c}\bar{e}_D^{\ d}\bar{R}_{abcd}$$

$$\mathcal{M}_{IJ\leftrightarrow KL}=\frac{2}{3}\left[s_{IJ}\bar{R}_{I(KL)J}+s_{IK}\bar{R}_{I(JL)K}+s_{IL}\bar{R}_{I(JK)L}\right]$$

Cheung et al 2022

Geometry of target space

Example: $\mathcal{L} \propto \left(\partial_\mu \pi^i\right)^2 + \frac{(\pi_i \partial_\mu \pi^i)^2}{\zeta^2 - \pi^i \pi_i}$

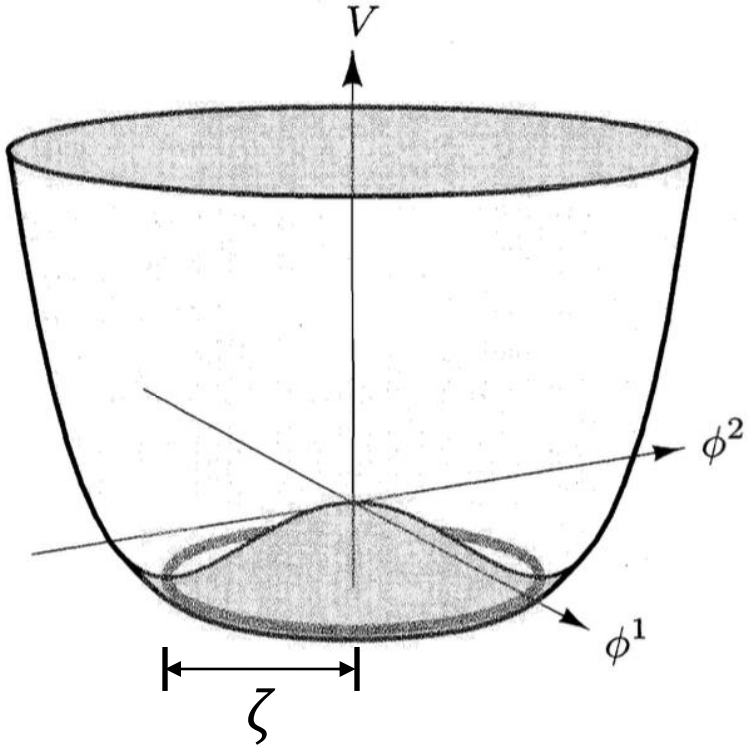
➡ $\mathcal{G}_{ij}(\pi) = \delta_{ij} + \frac{\pi_i \pi_j}{\zeta^2 - \pi^k \pi_k}$

At $\pi^i = (0,0,\ldots,0)$

$$\bar{\mathcal{G}}_{ij} = \delta_{ij}$$

$$\bar{R}_{ikjl} = \bar{R}_{ik}(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk})$$

$$\bar{R}_{ik} = 1/\zeta^2$$



Peskin and Schroeder

Effect of gravity

- With gravity $\mathcal{L} \supset -\frac{1}{2}\eta^{\mu\nu}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V(\phi)$

$\mathcal{L}/\sqrt{-g}$ $\quad$ $g^{\mu\nu}$

$$\mathcal{L} = \sqrt{-g}\frac{M_{\rm Pl}^2}{2}F(\phi)R + \cdots$$

- Weyl (conformal) transformation Maeda 1988, 1989

$$g_{{\rm E}\mu\nu} = \Omega^2 g_{{\rm J}\mu\nu}$$

$$\sqrt{-g} \to \sqrt{-\tilde{g}} = \Omega^4\sqrt{-g}$$

$$R \to \tilde{R} = \frac{R}{\Omega^2} - \frac{(D-1)(D-4)}{\Omega^4}\nabla_\mu\Omega\nabla^\mu\Omega - \frac{2(D-1)}{\Omega^3}\Box\Omega$$

e.g. $\Omega^2 = F(\phi)$ can remove the non-minimal coupling

Extended target space

- Weyl transformation as coordinate transformation

$$g_{\mathrm{E}\mu\nu} = \Omega^2 g_{\mathrm{J}\mu\nu}$$

  - Conformal mode

Ema, Mukaida, van de Vis 2020, 2021

$$g_{\bullet\mu\nu} = \frac{\Phi_\bullet^2}{6M_{\mathrm{Pl}}^2}\tilde{g}_{\mu\nu}, \qquad \det\left(\tilde{g}_{\mu\nu}\right) = -1$$

in any frame

$$\Rightarrow \quad R = \frac{6M_{\mathrm{Pl}}^2}{\Phi^2}\tilde{R} - \frac{36M_{\mathrm{Pl}}^2}{\Phi^3}\tilde{\nabla}^2\Phi$$

"Kinetic term" for conformal mode

Extended target space

- Assume O(N) symmetry ($N \geq 2$), spherical coordinate $(\phi^i) = (h, \vec{\pi})$

$$\mathcal{G}_{ij}\mathrm{d}\phi^i\mathrm{d}\phi^j = G(h^2)\mathrm{d}h^2 + \frac{h^2}{v^2}\left[\mathrm{d}\vec{\pi}^2 + \frac{(\vec{\pi}\cdot\mathrm{d}\vec{\pi})^2}{v^2 - \vec{\pi}^2}\right]$$

$$\Rightarrow \quad \left(\mathcal{G}^{\mathrm{J}}_{ab}\right) \equiv \left(\begin{array}{cc|c} -f & -f'\Phi_{\mathrm{J}}h & \\ -f'\Phi_{\mathrm{J}}h & \frac{\Phi_{\mathrm{J}}^2}{6M_{\mathrm{Pl}}^2}\mathcal{G}_{hh} & \mathbf{0} \\ \hline \multicolumn{2}{c|}{\mathbf{0}} & \frac{\Phi_{\mathrm{J}}^2}{6M_{\mathrm{Pl}}^2}\left(\mathcal{G}_{\pi^i\pi^j}\right) \end{array}\right) \qquad f' = \frac{\partial f}{\partial h^2}$$

$(\bar{\varphi}^a_{\mathrm{J}}) = (\sqrt{6}\Lambda_{\mathrm{G}}/\bar{f}, v, 0, \ldots, 0)$

$\bar{\tilde{g}}_{\mu\nu} = \eta_{\mu\nu}$

$$\bar{R}_{ikjl} = \bar{R}_{ik}\left(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk}\right)$$

$$\bar{R}_{\mathrm{others}} = 0$$

"Others" means at least one index is for Φ.

Extended target space

- Assume O(N) symmetry ($N \geq 2$), spherical coordinate $(\phi^i) = (h, \vec{\pi})$

$$\mathcal{G}_{ij}\mathrm{d}\phi^i\mathrm{d}\phi^j = G(h^2)\mathrm{d}h^2 + \frac{h^2}{v^2}\left[\mathrm{d}\vec{\pi}^2 + \frac{(\vec{\pi}\cdot\mathrm{d}\vec{\pi})^2}{v^2 - \vec{\pi}^2}\right]$$

$$\Rightarrow \quad \left(\mathcal{G}^{\mathrm{J}}_{ab}\right) \equiv \left(\begin{array}{cc|c} -f & -f'\Phi_{\mathrm{J}}h & \\ -f'\Phi_{\mathrm{J}}h & \frac{\Phi_{\mathrm{J}}^2}{6M_{\mathrm{Pl}}^2}\mathcal{G}_{hh} & \mathbf{0} \\ \hline \mathbf{0} & & \frac{\Phi_{\mathrm{J}}^2}{6M_{\mathrm{Pl}}^2}\left(\mathcal{G}_{\pi^i\pi^j}\right) \end{array}\right) \qquad f' = \frac{\partial f}{\partial h^2}$$

Remark: it is always possible to choose a frame such that the target space of ϕ^i is completely flat.

Extended target space

$$\bar{\mathcal{G}}^{\mathrm{J}}_{ab} = \bar{e}_a^{\ A}\bar{e}_b^{\ B}\eta_{AB}$$

$$\varphi^A = \bar{e}^A_a\varphi^a_{\mathrm{J}} \qquad \bar{R}_{ABCD} = \bar{e}_A^{\ a}\bar{e}_B^{\ b}\bar{e}_C^{\ c}\bar{e}_D^{\ d}\bar{R}_{abcd}$$

$$\mathcal{M}_{IJ\leftrightarrow KL} = \frac{2}{3}\left[s_{IJ}\bar{R}_{I(KL)J} + s_{IK}\bar{R}_{I(JL)K} + s_{IL}\bar{R}_{I(JK)L}\right]$$

$$\bar{R} = \bar{\mathcal{G}}^{ab}\bar{\mathcal{G}}^{cd}\bar{R}_{acbd}$$

- Invariant under frame transformation/field-redefinition
- Summing up all the four-leg diagram contributions

Cutoff $\dfrac{\Lambda}{\Lambda_{\mathrm{G}}} \sim \sqrt{\dfrac{N^2}{\bar{R}\Lambda_{\mathrm{G}}^2}}$ Independent of

- Field-redefinition
- Frame transformation

Extended target space

Example: $(\bar{\varphi}^a_{\rm J}) = (\sqrt{6}\Lambda_{\rm G}/\bar{f}, v = 0,0,\dots,0)$ ➡ Vacuum respects O(N)

$$\bar{\tilde{g}}_{\mu\nu} = \eta_{\mu\nu}$$

$$\frac{\Lambda_{\rm NLSM}}{\Lambda_{\rm G}} \sim \sqrt{\frac{N^2}{\bar{R}\Lambda_{\rm G}^2}} \sim \sqrt{\frac{N}{N-1}}\left(M_{\rm Pl}^2\overline{G_0'} + \frac{(1+6M_{\rm Pl}^2\overline{f_0'})^2}{6}\right)^{-1/2}$$

Without gravity

Vanishi for conformal coupling $M_{\rm Pl}^2\overline{f_0'} = -1/6$

Independent of
- Field-redefinition
- Frame transformation

Higgs inflation

	Metric HI
$\bar{R}_{h\pi^k}$	$\dfrac{(1+6\xi)^2\Lambda_{\rm G}^2}{6(M_{\rm Pl}^2+\xi v^2)[M_{\rm Pl}^2+(1+6\xi)\xi v^2]}$
$\bar{R}_{\pi^i\pi^k}$	$\dfrac{(1+6\xi)^2\Lambda_{\rm G}^2}{6(M_{\rm Pl}^2+\xi v^2)[M_{\rm Pl}^2+(1+6\xi)\xi v^2]}$
$\bar{R}_{\rm others}$	0

Scattering amplitudes

$$\left(\bar{e}_A^{\ a}\right) = \frac{M_{\rm Pl}\bar{\Omega}}{\Lambda_{\rm G}} \left(\begin{array}{cc|c} \dfrac{\Lambda_{\rm G}}{M_{\rm Pl}\bar{\Omega}^2} & -\dfrac{\sqrt{6}\xi\Lambda_G v/M_{\rm Pl}^2}{\bar{\Omega}^3\sqrt{1+6\xi^2 v^2/(M_{\rm Pl}^2\bar{\Omega}^2)}} & \mathbf{0} \\ 0 & \dfrac{1}{\sqrt{1+6\xi^2 v^2/(M_{\rm Pl}^2\bar{\Omega}^2)}} & \\ \hline \mathbf{0} & & \mathbb{1} \end{array}\right)$$

$$\mathcal{M}_{\Pi^i\Pi^i\leftrightarrow\Pi^j\Pi^j} = -\frac{s_{12}}{6\Lambda_{\rm G}^2}\frac{(1+6\xi)^2(M_{\rm Pl}^2+\xi v^2)}{M_{\rm Pl}^2+(1+6\xi)\xi v^2} \quad \text{for } i\neq j,$$

$$\mathcal{M}_{HH\leftrightarrow\Pi^i\Pi^i} = -\frac{s_{12}}{6\Lambda_{\rm G}^2}\frac{(1+6\xi)^2(M_{\rm Pl}^2+\xi v^2)^2}{\left[M_{\rm Pl}^2+(1+6\xi)\xi v^2\right]^2}.$$

UV cutoff of Higgs inflation

- Metric Higgs inflation

$$\Omega^2 = 1 + \xi \frac{\phi^2}{M_{\rm Pl}^2}$$

$$S = \int \mathrm{d}^4x\sqrt{-g_{\rm J}}\left(\frac{M_{\rm Pl}^2+\xi\phi^2}{2}R_{\rm J} - \frac{g_{\rm J}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V\right)$$

Extended target space ➡

$$\left(\mathcal{G}^{\rm J}_{ab}\right) \equiv \begin{pmatrix} -\Omega^2 & -\xi\Phi_{\rm J}\phi_j/M_{\rm Pl}^2 \\ -\xi\Phi_{\rm J}\phi_i/M_{\rm Pl}^2 & \dfrac{\Phi_{\rm J}^2}{6M_{\rm Pl}^2}\delta_{ij} \end{pmatrix}$$

$$(\bar{\varphi}^a_{\rm J}) = (\sqrt{6}\Lambda_{\rm G}/\bar{f}, v, 0, \ldots, 0)$$

$$\bar{\tilde{g}}_{\mu\nu} = \eta_{\mu\nu}$$

$$\bar{R}_{ikjl} = \bar{R}_{ik}\left(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{jk}\right)$$

➡

	Metric HI
$\bar{R}_{h\pi^k}$	$\dfrac{(1+6\xi)^2\Lambda_{\rm G}^2}{6(M_{\rm Pl}^2+\xi v^2)[M_{\rm Pl}^2+(1+6\xi)\xi v^2]}$
$\bar{R}_{\pi^i\pi^k}$	$\dfrac{(1+6\xi)^2\Lambda_{\rm G}^2}{6(M_{\rm Pl}^2+\xi v^2)[M_{\rm Pl}^2+(1+6\xi)\xi v^2]}$
$\bar{R}_{\rm others}$	0

UV cutoff of Higgs inflation

	Metric HI
$\bar{R}_{h\pi^k}$	$\frac{(1+6\xi)^2\Lambda_{\rm G}^2}{6(M_{\rm Pl}^2+\xi v^2)[M_{\rm Pl}^2+(1+6\xi)\xi v^2]}$
$\bar{R}_{\pi^i\pi^k}$	$\frac{(1+6\xi)^2\Lambda_{\rm G}^2}{6(M_{\rm Pl}^2+\xi v^2)[M_{\rm Pl}^2+(1+6\xi)\xi v^2]}$
$\bar{R}_{\rm others}$	0

- Cutoff scale

$$\bar{R} = 2(N-1)\left(\bar{R}_{H\Pi^k} + \frac{N-2}{2}\bar{R}_{\Pi^i\Pi^k}\right) \quad \Rightarrow \quad \frac{\Lambda}{\Lambda_{\rm G}} \propto \frac{\bar{R}^{-1/2}}{\Lambda_{\rm G}}$$

$$\frac{\Lambda}{\Lambda_{\rm G}} \propto \frac{\bar{R}^{-1/2}}{\Lambda_{\rm G}} \begin{cases} \sim \begin{cases} 1/\xi & \text{for} \quad v \lesssim M_{\rm Pl}/\xi, \\ v/M_{\rm Pl} & \text{for} \quad M_{\rm Pl}/\xi \lesssim v \lesssim M_{\rm Pl}/\sqrt{\xi}, \\ 1/\sqrt{\xi} & \text{for} \quad M_{\rm Pl}/\sqrt{\xi} \lesssim v, \end{cases} & N > 2 \\ \sim \begin{cases} 1/\xi & \text{for} \quad v \lesssim M_{\rm Pl}/\xi, \\ \xi v^2/M_{\rm Pl}^2 & \text{for} \quad M_{\rm Pl}/\xi \lesssim v \lesssim M_{\rm Pl}/\sqrt{\xi}, \\ 1 & \text{for} \quad M_{\rm Pl}/\sqrt{\xi} \lesssim v, \end{cases} & N = 2 \end{cases}$$

Consistent with known results

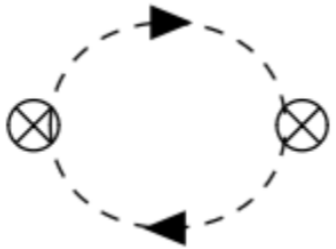
Mixed Higgs-R^2 inflation

- UV extended Higgs inflation Ema 2017

$$S = \int \mathrm{d}^4x\sqrt{-g_\mathrm{J}}\left(\frac{M_\mathrm{Pl}^2+\xi\phi^2}{2}R_\mathrm{J} - \frac{g_\mathrm{J}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V\right) + \alpha R_\mathrm{J}^2$$

- Large-N limit $\mathcal{L} \supset \frac{\xi}{2}\phi^i\phi_i R$

Calmet, Kuntz 2016
Ghilencea 2018
Ema 2019



Counter term $= \alpha R^2$ $\alpha \sim N\xi^2$

- Large-ξ suppresses other terms such as $R^{\mu\nu}R_{\mu\nu}$