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# Global Analysis of Fragmentation Functions to Light Neutral Hadrons

based on arXiv:2503.21311

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# 1. Introduction

# 1.1 Introduction to FFs

In its simplest form, fragmentation functions (FFs) describe number density of the identified hadron wrt the fraction of momentum of the initial parton it carries, as measured in single inclusive hadron production, e.g., from single-inclusive annihilation (SIA), semi-inclusive DIS (SIDIS), *pp* collisions

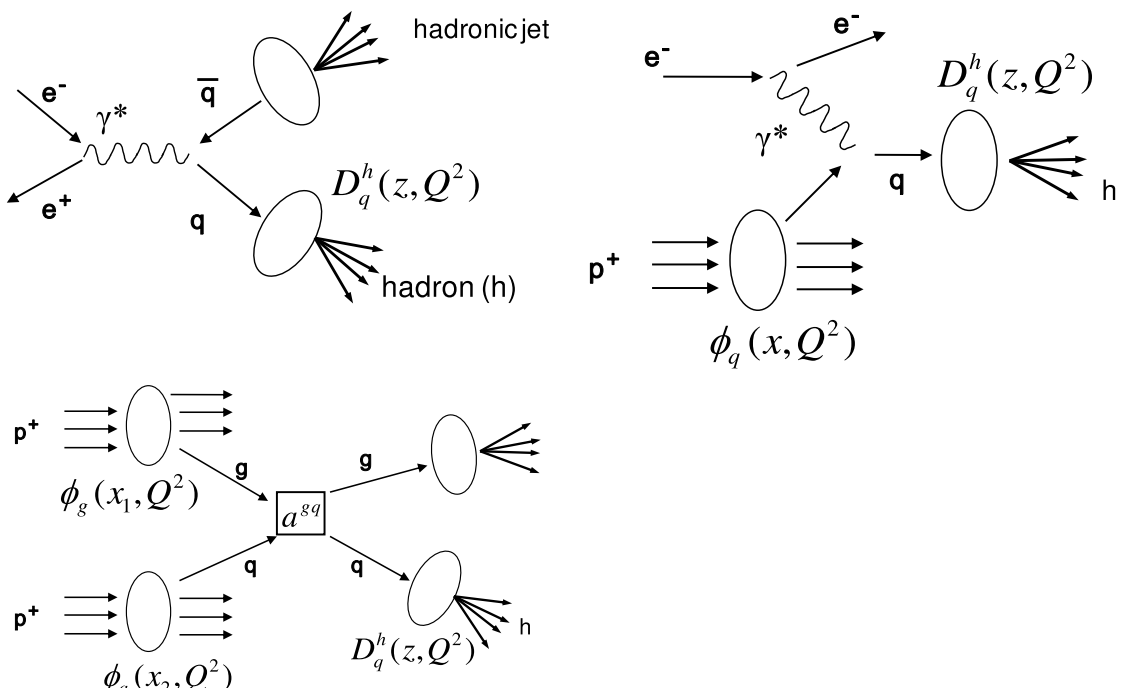
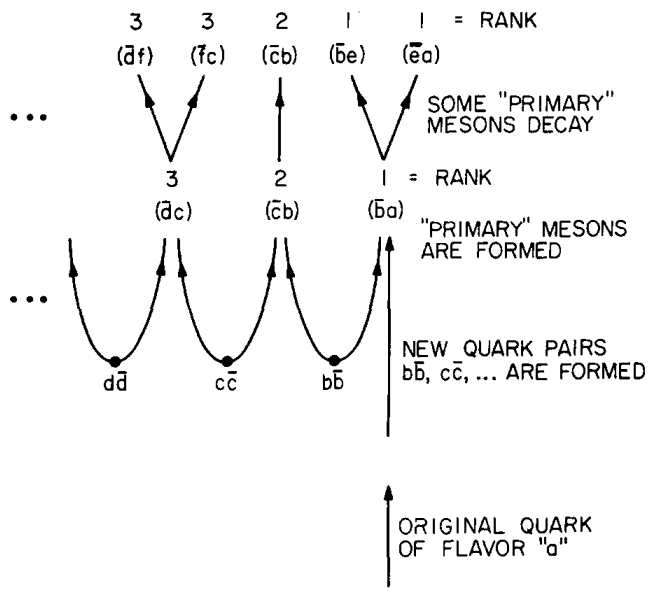


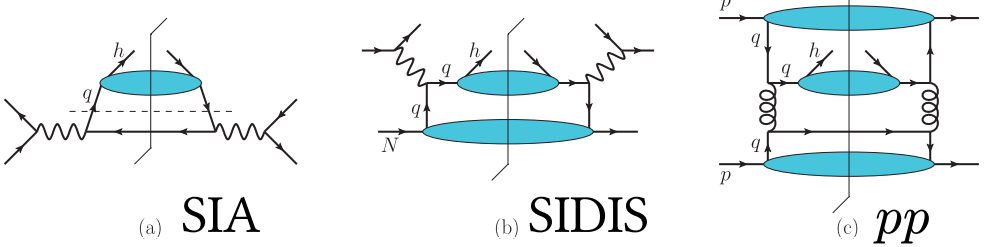

Illustration of hadron production in SIA, SIDIS and *pp* processes




Cascade decay ansatz (R.D. Field & R.P. Feynman)

# 1.2 QCD Factorization Theorem

- QCD factorization theorem enables the separation of the perturbatively calculable part of the cross section from the non-perturbative part which involes initial and final state hadrons. [J. C. Collins, D. E. Soper, G. Sterman]




(a) SIA (b) SIDIS (c) $pp$

$$\sigma^{pp\to hX} = f_{i/p} \otimes f_{j/p} \otimes \hat{\sigma}_{ij\to k} \otimes D_k^h$$

$\sigma^{pp\to hX}$ ⇓ experimental measurements

$\hat{\sigma}_{ij\to k}$ ⇓ pQCD

$D_k^h$ ⇓ global extraction

- Hard scattering processes are independent of hadrons involved. Coefficient functions are perturbatively calculable.
- PDFs/FFs are non-perturbative part that describe the inner structure of hadrons or parton-hadron transition. They are universal and can be fitted from data.
- Evolution of FFs w.r.t. scale is governed by DGLAP equation.

$$\frac{\partial}{\partial \ln Q^2} D_i^h(z,Q) = \sum_j P_{ji}(z,\alpha_s(Q)) \otimes D_j^h(z,Q), i,j = q,\overline{q},g$$

- Operator definition:

$$D_1^{h/q}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^+}{2\pi} e^{ik^-\xi^+} \mathrm{Tr}\Big(\big[\langle 0 \mid \mathcal{W}(\infty^+,\xi^+)\psi_q\big(\xi^+,0^-,\vec{0}_T\big) \mid P_h,S_h;X\rangle \times \langle P_h,S_h;X \mid \overline{\psi}_q\big(0^+,0^-,\vec{0}_T\big)\mathcal{W}(0^+,\infty^+) \mid 0\rangle\gamma^-\big]\Big)$$

# 2. Overview of NPC FFs

# 2.1 Previous Work: NPC FFs for Charged Hadrons

LHAPDF 6.5.5

Main page | PDF sets | Class hierarchy | Examples | More...

| | | | | |
|---|---|---|---|---|
| 2070000 | NPC23_PIp_nlo | (tarball) (info file) | 127 | 1 |
| 2070200 | NPC23_KAp_nlo | (tarball) (info file) | 127 | 1 |
| 2070400 | NPC23_PRp_nlo | (tarball) (info file) | 127 | 1 |
| 2070600 | NPC23_PIm_nlo | (tarball) (info file) | 127 | 1 |
| 2070800 | NPC23_KAm_nlo | (tarball) (info file) | 127 | 1 |
| 2071000 | NPC23_PRm_nlo | (tarball) (info file) | 127 | 1 |
| 2071200 | NPC23_PIsum_nlo | (tarball) (info file) | 127 | 1 |
| 2071400 | NPC23_KAsum_nlo | (tarball) (info file) | 127 | 1 |
| 2071600 | NPC23_PRsum_nlo | (tarball) (info file) | 127 | 1 |
| 2071800 | NPC23_CHHAp_nlo | (tarball) (info file) | 127 | 1 |
| 2072000 | NPC23_CHHAm_nlo | (tarball) (info file) | 127 | 1 |
| 2072200 | NPC23_CHHAsum_nlo | (tarball) (info file) | 127 | 1 |

Figure 1: From https://lhapdf.hepforge.org/pdfsets.html

- First time including jet fragmentation data.
- Joint determination of FFs to charged pion, kaon, proton at NLO including estimation of uncertainties with Hessian sets
- Strong selection criteria to ensure validity of leading twist factorization ($E_h/p_{T,h} > 4$ GeV, $z > 0.01$)
- We arrive at a best-fit of the charged pion, kaon and proton FFs ($\chi^2/N_{pt} = 0.90$ for 1370 points)together with 126 Hessian error FFs

# 2.1 Previous Work: NPC FFs for Charged Hadrons

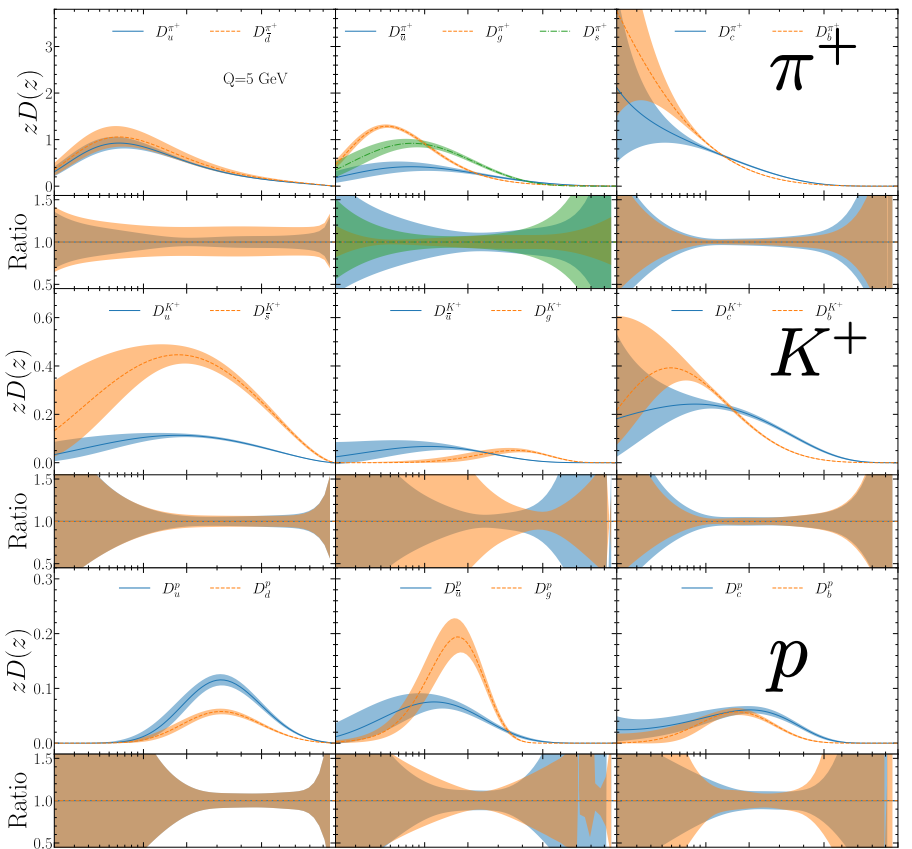



Figure 2: NPC23 FF for $\pi^+, K^+, p$
Phys.Rev.D 110 (2024) 11, 114019

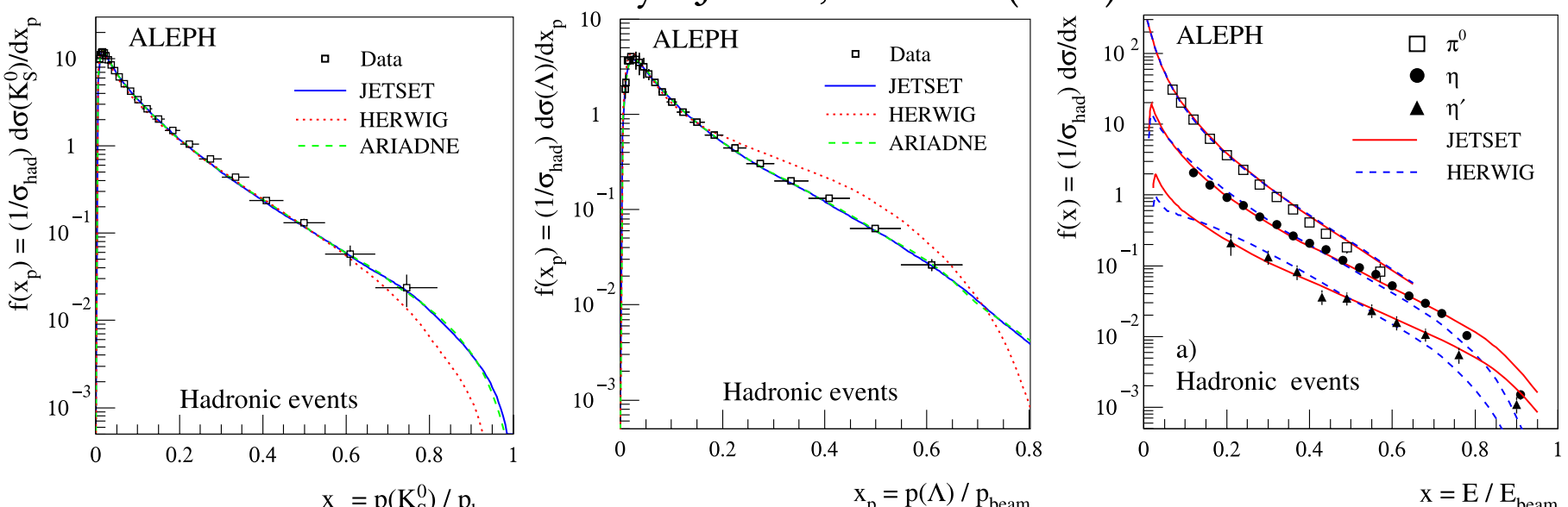
- High precision determination of FFs for charged hadrons
- 4%, 4% and 7% uncertainty for $u$-quark to $\pi^+, K^+, p$ at $z = 0.3$, respectively
- 3%, 4% and 8% uncertainty for gluon to pion at $z = 0.05, 0.1, 0.3$, respectively, mainly due to the jet fragmentation data from LHC
- Heavy quark fragmentation are well constrained in $z = 0.1 \sim 0.5$

# 3. NPC FFs for Neutral Hadrons

# 3.1 Neutral Light Hadrons

| Particle | Quark Composition | Mass (MeV/$c^2$) | Lifetime (s) | Spin |
|---|---|---|---|---|
| $K_S^0$ | $\frac{1}{\sqrt{2}}\left(\lvert d\bar{s}\rangle - \lvert s\bar{d}\rangle\right)$ | 497.611 ± 0.013 | $0.8954 \times 10^{-10}$ | 0 |
| $\Lambda$ | $\lvert uds\rangle$ | 1115.683 ± 0.006 | $2.632 \times 10^{-10}$ | 1/2 |
| $\eta$ | $\frac{1}{\sqrt{6}}\left(\lvert u\bar{u}\rangle - \lvert d\bar{d}\rangle - 2\lvert s\bar{s}\rangle\right)$ | 547.862 ± 0.017 | $5.0 \times 10^{-19}$ | 0 |
| $\pi^0$ | $\frac{1}{\sqrt{2}}\left(\lvert u\bar{u}\rangle - \lvert d\bar{d}\rangle\right)$ | 134.977 ± 0.0005 | $8.52 \times 10^{-17}$ | 0 |

Eur. Phys. J. C 16, 613–634 (2000)



# 3.2 Datasets

We include a comprehensive set of data: SIA below $Z$-pole energy (9.46 GeV $\sim$ 58 GeV), SIA at $Z$-pole inclusive, jet fragmentation and tagged data. $pp$ data and SIDIS data ($K^0_S$, $\Lambda$ only). Jet fragmentation and SIDIS data are incorporated for the first time.

| collaboration | year | $\sqrt{s}$[GeV] | final state | observable | $N_{\rm pt}$ |
|---|---|---|---|---|---|
| TASSO | 1985 | 14, 22 & 34 | $K^0_S$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 9, 6 & 13 |
| TASSO | 1990 | 14.8, 21.5, 34.5, 35 & 42.6 | $K^0_S$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 9, 6, 13, 13 & 13 |
| TPC | 1984 | 29 | $K^0_S$ | $\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$ | 8 |
| MARK II | 1985 | 29 | $K^0_S$ | $\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$ | 17 |
| HRS | 1987 | 29 | $K^0_S$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 12 |
| CELLO | 1990 | 35 | $K^0_S$ | $\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$ | 9 |
| TOPAZ | 1995 | 58 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$ | 4 |
| OPAL | 1991 | 91.2 | $K^0_S$ | $\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$ | 7 |
| OPAL | 1995 | 91.2 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 16 |
| OPAL | 2000 | 91.2 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 16 |
| ALEPH | 1998 | 91.2 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 16 |
| ALEPH | 2000 | 91.2 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$ | 14 |
| ALEPH | 2000 | 91.2 | $K^0_S$ (jet) | $\frac{1}{\sigma_{\rm jet}}\frac{d\sigma}{d\xi}$ | 12, 13 & 11 |
| DELPHI | 1995 | 91.2 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 13 |
| SLD | 1999 | 91.2 | $K^0_S$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 9 |
| SLD (tagged) | 1999 | 91.2 | $K^0_S$ (tagged) | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 9 & 9 |
| ZEUS | 2012 | 318 | $K^0_S$ | $\frac{1}{N_{\rm DIS}}\frac{dN_{K^0_S}}{dz_p}$ | 5, 5 & 2 |
| ALICE | 2021 | 13000 & 7000 | $K^0_S$ | $\frac{dN^{13\,\rm TeV}_{K^0_S}}{dp_T}$ / $\frac{dN^{7\,\rm TeV}_{K^0_S}}{dp_T}$ | 10 |
| ALICE | 2021 | 13000 | $K^0_S$ & $\pi^\pm$ | $\frac{dN_{K^0_S}}{dp_T}$ / $\frac{dN_{\pi^\pm}}{dp_T}$ | 15 |

Table 2: Data table for $K^0_S$

| collaboration | year | $\sqrt{s}$[GeV] | final state | observable | $N_{\rm pt}$ |
|---|---|---|---|---|---|
| TASSO | 1985 | 14, 22 & 34 | $\Lambda$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 3, 4 & 7 |
| SLAC | 1985 | 29 | $\Lambda$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 15 |
| HRS | 1987 | 29 | $\Lambda$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 8 |
| CELLO | 1990 | 35 | $\Lambda$ | $\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$ | 7 |
| DELPHI | 1993 | 91.2 | $\Lambda$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 7 |
| ALEPH | 1994 | 91.2 | $\Lambda$ | $\frac{1}{\sigma_h}\frac{d\sigma}{d\xi}$ | 14 |
| ALEPH | 1998 | 91.2 | $\Lambda$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 16 |
| ALEPH | 2000 | 91.2 | $\Lambda$(jet) | $\frac{1}{\sigma_{\rm jet}}\frac{d\sigma}{d\xi}$ | 13, 12 & 9 |
| OPAL | 1997 | 91.2 | $\Lambda$ | $\frac{1}{\beta\sigma_h}\frac{d\sigma}{dx_h}$ | 12 |
| SLD | 1999 | 91.2 | $\Lambda$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 9 |
| SLD (tagged) | 1999 | 91.2 | $\Lambda$(tagged) | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 4 & 4 |
| ZEUS | 2012 | 318 | $\Lambda$ | $\frac{1}{N_{\rm DIS}}\frac{dN_\Lambda}{dz_p}$ | 5, 3 & 1 |
| CMS | 2011 | 900 | $\Lambda$ & $K^0_S$ | $\frac{dN_\Lambda}{dp_T}$ / $\frac{dN_{K^0_S}}{dp_T}$ | 4 |
| ALICE | 2021 | 13000 & 7000 | $\Lambda$ | $\frac{dN^{13\,\rm TeV}_\Lambda}{dp_T}$ / $\frac{dN^{7\,\rm TeV}_\Lambda}{dp_T}$ | 7 |

Table 3: Data table for $\Lambda$

# 3.2 Datasets

| collaboration | year | $\sqrt{s}$[GeV] | final state | observable | $N_{\mathrm{pt}}$ |
|---|---|---|---|---|---|
| ARGUS | 1990 | 9.46 | $\eta$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 6 |
| HRS | 1988 | 29 | $\eta$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 13 |
| JADE | 1985 | 34.4 | $\eta$ | $\frac{s}{\beta}\frac{d\sigma}{x_h}$ | 2 |
| JADE | 1990 | 35 | $\eta$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 3 |
| CELLO | 1990 | 35 | $\eta$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 5 |
| L3 | 1992 | 91.2 | $\eta$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 4 |
| L3 | 1994 | 91.2 | $\eta$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 10 |
| ALEPH | 1992 | 91.2 | $\eta$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 8 |
| ALEPH | 2000 | 91.2 | $\eta$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 18 |
| ALEPH | 2000 | 91.2 | $\eta$(jet) | $\frac{1}{\sigma_{\mathrm{jet}}}\frac{d\sigma}{dx_h}$ | 7, 6 & 4 |
| ALEPH | 2002 | 91.2 | $\eta$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 5 |
| OPAL | 1998 | 91.2 | $\eta$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 11 |
| PHENIX | 2011 | 200 | $\eta$ & $\pi^0$ | $\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$ | 14 |
| ALICE | 1207 | 2760 | $\eta$ & $\pi^0$ | $\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$ | 6 |
| ALICE | 1202 | 7000 | $\eta$ & $\pi^0$ | $\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$ | 4 |
| ALICE | 1208 | 8000 | $\eta$ & $\pi^0$ | $\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$ | 13 |
| ALICE | 2204 | 13000 | $\eta$ & $\pi^0$ | $\frac{Ed^3\sigma_\eta}{dp^3}/\frac{Ed^3\sigma_{\pi^0}}{dp^3}$ | 14 |

Table 4: Data table for $\eta$

| collaboration | year | $\sqrt{s}$[GeV] | final state | observable | $N_{\mathrm{pt}}$ |
|---|---|---|---|---|---|
| ARGUS | 1990 | 9.46 | $\pi^0$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 14 |
| JADE | 1985 | 34.4 | $\pi^0$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 10 |
| JADE | 1990 | 35 | $\pi^0$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 9 |
| JADE | 1990 | 44 | $\pi^0$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 6 |
| CELLO | 1990 | 35 | $\pi^0$ | $\frac{s}{\beta}\frac{d\sigma}{dx_h}$ | 15 |
| TASSO | 1989 | 44 | $\pi^0$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 6 |
| ALEPH | 1997 | 91.2 | $\pi^0$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 20 |
| ALEPH | 2000 | 91.2 | $\pi^0$(jet) | $\frac{1}{\sigma_{\mathrm{jet}}}\frac{d\sigma}{dx_h}$ | 8, 8 & 6 |
| DELPHI | 1996 | 91.2 | $\pi^0$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 17 |
| DELPHI | 1996 | 91.2 | $\pi^0$($b$-tagged) | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 15 |
| L3 | 1994 | 91.2 | $\pi^0$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_p}$ | 12 |
| OPAL | 1998 | 91.2 | $\pi^0$ | $\frac{1}{\sigma_h}\frac{d\sigma}{dx_h}$ | 10 |
| STAR | 2010 | 200 | $\pi^0$ | $\frac{Ed^3\sigma}{dp^3}$ | 8 |
| PHENIX | 2007 | 200 | $\pi^0$ | $\frac{Ed^3\sigma}{dp^3}$ | 17 |
| PHENIX | 2016 | 510 | $\pi^0$ | $\frac{Ed^3\sigma}{dp^3}$ | 22 |
| ALICE | 2017 | 2760 | $\pi^0$ | $\frac{Ed^3\sigma}{dp^3}$ | 16 |
| ALICE | 2012 | 7000 | $\pi^0$ | $\frac{Ed^3\sigma}{dp^3}$ | 13 |
| ALICE | 2018 | 8000 | $\pi^0$ | $\frac{Ed^3\sigma}{dp^3}$ | 24 |

Table 5: Data table for $\pi^0$

# 3.3 Parametrization

$$zD_i^h(z,Q_0) = z^{\alpha_i^h}(1-z)^{\beta_i^h}\exp\left(\sum_{n=0}^{m}\alpha_{i,n}^h z^{n/2}\right)$$

| parton-to-$K_S^0$ | favored | $a_0$ | $\alpha$ | $\beta$ | $a_1$ |
|---|---|---|---|---|---|
| $u=\overline{u}$ | ✘ | ✓ | ✓ | ✓ | ✓ |
| $d=\overline{d}$ | ✓ | ✓ | $=\alpha_u$ | ✓ | ✓ |
| $s=\overline{s}$ | ✓ | ✓ | $=\alpha_u$ | $=\beta_d$ | $=a_{1,d}$ |
| $c=\overline{c}$ | ✘ | ✓ | ✓ | ✓ | ✓ |
| $b=\overline{b}$ | ✘ | ✓ | ✓ | ✓ | ✓ |
| $g$ | ✘ | ✓ | ✓ | ✓ | ✓ |
| parton-to-$\Lambda$ | favored | $a_0$ | $\alpha$ | $\beta$ | $a_1$ |
| $u=\overline{u}=d=\overline{d}$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $s=\overline{s}$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $c=\overline{c}$ | ✘ | ✓ | ✓ | ✓ | ✘ |
| $b=\overline{b}$ | ✘ | ✓ | ✓ | ✓ | ✘ |
| $g$ | ✘ | ✓ | ✓ | ✓ | ✓ |
| parton-to-$\eta$ | favored | $a_0$ | $\alpha$ | $\beta$ | $a_1$ |
| $u=\overline{u}=d=\overline{d}$ | ✓ | ✓ | ✓ | ✓ | ✓ |
| $s=\overline{s}$ | ✓ | ✓ | $=\alpha_u$ | $=\beta_u$ | $=a_{1,u}$ |
| $c=\overline{c}=b=\overline{b}$ | ✘ | ✓ | ✓ | ✓ | ✘ |
| $g$ | ✘ | ✓ | ✓ | ✓ | ✓ |

Table 6: Parameterization for $K_S^0, \Lambda, \eta$

- $\pi^0$ FFs are constructed from $\pi^\pm$ FFs by

$$D_q^{\pi^0} = \frac{1}{4}\left(D_q^{\pi^+} + D_q^{\pi^-} + D_{q'}^{\pi^+} + D_{q'}^{\pi^-}\right), q(q') = u(d) \text{ or } d(u)$$

$$D_i^{\pi^0} = \frac{1}{2}\left(D_i^{\pi^+} + D_i^{\pi^-}\right), i = g, s, c, b$$

  and are tested by $\pi^0$ production datasets.
- $K_S^0, \Lambda, \eta$ fits are performed independently.
- Theoretical calculations are performed using the FMNLO framework at NLO accuracy. Uncertainties are estimated using Hessian methods [hep-ph/0101032] and various combinations of fragmentation and factorization scales.
- For $pp$ process $p_{T,h} > 4\text{GeV}$, and for SIA and SIDIS process $z > 0.05$

# 3.4 Fit Quality

$K_S^0$

| collaboration | year | $\sqrt{s}$[GeV] | $\chi^2$ | $N_{\text{pt}}$ | $\chi^2/N_{\text{pt}}$ |
|---|---|---|---|---|---|
| TASSO | 1985 | 14 | 5.65 | 9 | 0.63 |
| TASSO | 1985 | 22 | 5.87 | 6 | 0.98 |
| TASSO | 1985 | 34 | 16.03 | 13 | 1.23 |
| TASSO | 1990 | 14.8 | 12.56 | 9 | 1.40 |
| TASSO | 1990 | 21.5 | 3.78 | 6 | 0.63 |
| TASSO | 1990 | 34.5 | 17.51 | 13 | 1.35 |
| TASSO | 1990 | 35 | 14.76 | 13 | 1.14 |
| TASSO | 1990 | 42.6 | 33.60 | 13 | 2.58 |
| TPC | 1984 | 29 | 2.75 | 8 | 0.34 |
| MARK II | 1985 | 29 | 12.65 | 17 | 0.74 |
| HRS | 1987 | 29 | 33.16 | 12 | 2.76 |
| CELLO | 1990 | 35 | 2.71 | 9 | 0.30 |
| TOPAZ | 1995 | 58 | 0.29 | 4 | 0.07 |
| OPAL | 1991 | 91.2 | 7.75 | 7 | 1.11 |
| OPAL | 1995 | 91.2 | 13.63 | 16 | 0.85 |
| OPAL | 2000 | 91.2 | 8.62 | 16 | 0.54 |
| ALEPH | 1998 | 91.2 | 6.39 | 16 | 0.40 |
| ALEPH | 2000 | 91.2 | 12.72 | 14 | 0.91 |
| ALEPH jet 1 | 2000 | 91.2 | 14.91 | 12 | 1.24 |
| ALEPH jet 2 | 2000 | 91.2 | 8.21 | 13 | 0.63 |
| ALEPH jet 3 | 2000 | 91.2 | 8.55 | 11 | 0.78 |
| DELPHI | 1995 | 91.2 | 7.55 | 13 | 0.58 |
| SLD | 1999 | 91.2 | 7.39 | 9 | 0.82 |
| SLD $c$-tagged | 1999 | 91.2 | 17.44 | 9 | 1.94 |
| SLD $b$-tagged | 1999 | 91.2 | 11.12 | 9 | 1.24 |
| SIA sum | | | 285.60 | 277 | 1.03 |
| ZEUS $Q^2 \in 160, 640\,\text{GeV}^2$ | 2012 | 318 | 4.41 | 5 | 0.88 |
| ZEUS $Q^2 \in 640, 2560\,\text{GeV}^2$ | 2012 | 318 | 3.26 | 5 | 0.65 |
| ZEUS $Q^2 \in 2560, 10240\,\text{GeV}^2$ | 2012 | 318 | 2.74 | 2 | 1.37 |
| SIDIS sum | | | 10.41 | 12 | 0.87 |
| ALICE $N_{K_S^0}^{13\,\text{TeV}}/N_{K_S^0}^{7\,\text{TeV}}$ | 2021 | 13000 & 7000 | 2.88 | 10 | 0.29 |
| ALICE $N_{K_S^0}/N_{\pi^\pm}$ | 2021 | 13000 | 5.79 | 15 | 0.39 |
| $pp$ sum | | | 8.67 | 25 | 0.35 |
| **total sum** | | | **304.68** | **314** | **0.97** |

$\Lambda$

| collaboration | year | $\sqrt{s}$ [GeV] | $\chi^2$ | $N_{\text{pt}}$ | $\chi^2/N_{\text{pt}}$ |
|---|---|---|---|---|---|
| TASSO | 1985 | 14 | 1.45 | 3 | 0.48 |
| TASSO | 1985 | 22 | 2.93 | 4 | 0.73 |
| TASSO | 1985 | 34 | 3.95 | 7 | 0.56 |
| SLAC | 1985 | 29 | 19.16 | 15 | 1.28 |
| HRS | 1987 | 29 | 5.57 | 8 | 0.70 |
| CELLO | 1990 | 35 | 3.49 | 7 | 0.50 |
| DELPHI | 1993 | 91.2 | 9.58 | 7 | 1.37 |
| ALEPH | 1994 | 91.2 | 7.40 | 14 | 0.53 |
| ALEPH | 1998 | 91.2 | 4.43 | 16 | 0.28 |
| ALEPH jet 1 | 2000 | 91.2 | 16.50 | 13 | 1.27 |
| ALEPH jet 2 | 2000 | 91.2 | 3.08 | 12 | 0.26 |
| ALEPH jet 3 | 2000 | 91.2 | 3.72 | 9 | 0.41 |
| OPAL | 1997 | 91.2 | 5.83 | 12 | 0.48 |
| SLD | 1999 | 91.2 | 8.08 | 9 | 0.90 |
| SLD $c$-tagged | 1999 | 91.2 | 13.96 | 4 | 3.49 |
| SLD $b$-tagged | 1999 | 91.2 | 0.75 | 4 | 0.19 |
| SIA sum | | | 109.88 | 144 | 0.76 |
| ZEUS $Q^2 \in (160, 640)\text{GeV}^2$ | 2012 | 318 | 14.44 | 5 | 2.89 |
| ZEUS $Q^2 \in (640, 2560)\text{GeV}^2$ | 2012 | 318 | 1.26 | 3 | 0.42 |
| ZEUS $Q^2 \in (2560, 10240)\text{GeV}^2$ | 2012 | 318 | 0.01 | 1 | 0.01 |
| SIDIS sum | | | 15.70 | 9 | 1.74 |
| CMS $N_\Lambda/N_{K_S^0}$ | 2011 | 900 | 2.99 | 4 | 0.75 |
| ALICE $N_\Lambda^{13\text{TeV}}/N_\Lambda^{7\text{TeV}}$ | 2021 | 13000 & 7000 | 1.59 | 7 | 0.23 |
| $pp$ sum | | | 4.58 | 11 | 0.42 |
| **total sum** | | | **130.16** | **164** | **0.79** |

# 3.4 Fit Quality

| collaboration | year | $\sqrt{s}$[GeV] | $\chi^2$ | $N_{\text{pt}}$ | $\chi^2/N_{\text{pt}}$ |
|---|---|---|---|---|---|
| ARGUS | 1990 | 9.46 | 5.94 | 6 | 0.99 |
| HRS | 1988 | 29 | 18.07 | 13 | 1.39 |
| JADE | 1985 | 34.4 | 2.29 | 2 | 1.14 |
| JADE | 1990 | 35 | 3.29 | 3 | 1.09 |
| CELLO | 1990 | 35 | 3.47 | 5 | 0.69 |
| L3 | 1992 | 91.2 | 5.83 | 4 | 1.46 |
| L3 | 1994 | 91.2 | 10.46 | 10 | 1.05 |
| ALEPH | 1992 | 91.2 | 1.48 | 8 | 0.18 |
| ALEPH | 2000 | 91.2 | 18.39 | 18 | 1.02 |
| ALEPH jet 1 | 2000 | 91.2 | 11.26 | 7 | 1.61 |
| ALEPH jet 2 | 2000 | 91.2 | 1.95 | 6 | 0.33 |
| ALEPH jet 3 | 2000 | 91.2 | 10.49 | 4 | 2.62 |
| ALEPH | 2002 | 91.2 | 17.18 | 5 | 3.44 |
| OPAL | 1998 | 91.2 | 7.12 | 11 | 0.65 |
| SIA sum | | | 117.20 | 102 | 1.15 |
| PHENIX | 2011 | 200 | 7.61 | 14 | 0.51 |
| ALICE | 2017 | 2760 | 5.37 | 6 | 0.90 |
| ALICE | 2012 | 7000 | 1.26 | 4 | 0.32 |
| ALICE | 2018 | 8000 | 12.64 | 13 | 0.97 |
| ALICE | 2024 | 13000 | 7.11 | 13 | 0.51 |
| *pp* sum | | | 34.00 | 50 | 0.68 |
| **total sum** | | | **151.21** | **152** | **0.99** |

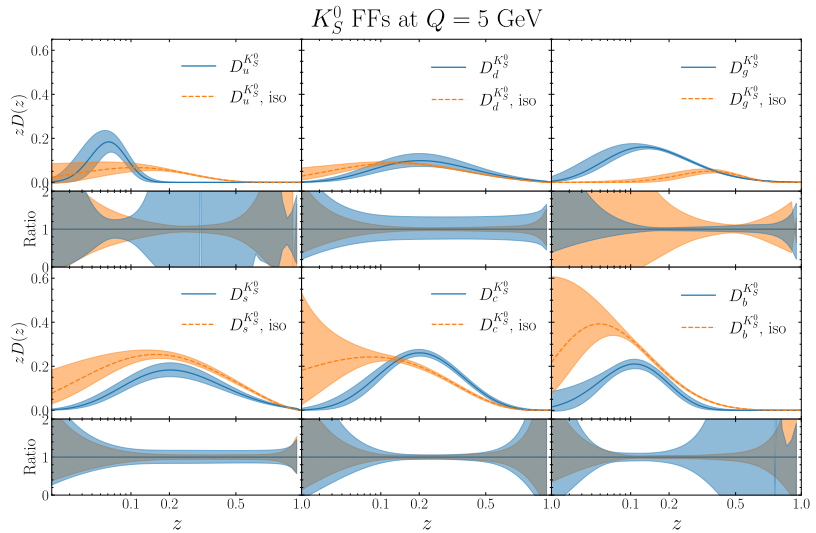
$\eta$

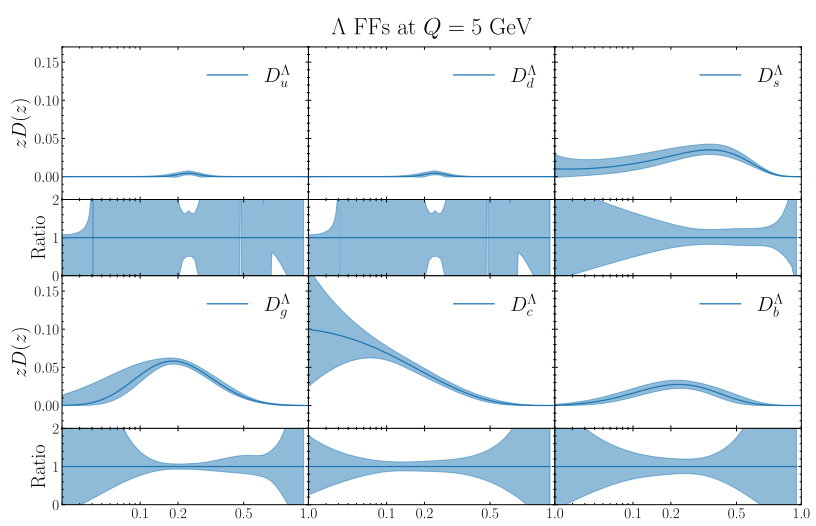
| collaboration | year | $\sqrt{s}$[GeV] | $\chi^2$ | $N_{\text{pt}}$ | $\chi^2/N_{\text{pt}}$ |
|---|---|---|---|---|---|
| ARGUS | 1990 | 9.46 | 20.12 | 14 | 1.44 |
| JADE | 1985 | 34.4 | 7.92 | 10 | 0.79 |
| JADE | 1990 | 35 | 8.66 | 9 | 0.96 |
| JADE | 1990 | 44 | 7.30 | 6 | 1.22 |
| CELLO | 1990 | 35 | 31.31 | 15 | 2.09 |
| TASSO | 1989 | 44 | 1.66 | 6 | 0.28 |
| ALEPH | 1997 | 91.2 | 14.85 | 20 | 0.74 |
| ALEPH jet 1 | 2000 | 91.2 | 2.43 | 8 | 0.30 |
| ALEPH jet 2 | 2000 | 91.2 | 11.69 | 8 | 1.46 |
| ALEPH jet 3 | 2000 | 91.2 | 34.65 | 6 | 5.77 |
| DELPHI | 1996 | 91.2 | 4.86 | 17 | 0.29 |
| DELPHI *b*-tagged | 1996 | 91.2 | 9.12 | 15 | 0.61 |
| L3 | 1994 | 91.2 | 27.34 | 12 | 2.28 |
| OPAL | 1998 | 91.2 | 2.73 | 10 | 0.27 |
| SIA sum | | | 184.64 | 156 | 1.18 |
| STAR | 2009 | 200 | 6.09 | 8 | 0.76 |
| PHENIX | 2007 | 200 | 45.76 | 17 | 2.69 |
| PHENIX | 2016 | 510 | 66.95 | 22 | 3.04 |
| ALICE | 2017 | 2760 | 13.98 | 16 | 0.87 |
| ALICE | 2012 | 7000 | 29.27 | 13 | 2.25 |
| ALICE | 2017 | 8000 | 46.42 | 24 | 1.93 |
| *pp* sum | | | 208.47 | 100 | 2.08 |
| **total sum** | | | **393.11** | **256** | **1.54** |

$\pi^0$

- We arrive at $\chi^2/N_{pt} = 0.97, 0.79, 0.99$ for $K^0_S, \Lambda, \eta$ respectively, indicating a successful fit.
- For each category, $\chi^2/N_{pt}$ is around 1 as well.
- $\pi^0$ data is tested against constructed FFs from $\pi^{\pm}$ FFs. The deviation is primarily from the few subsets with large discrepancies.

# 3.5 Delivered FFs



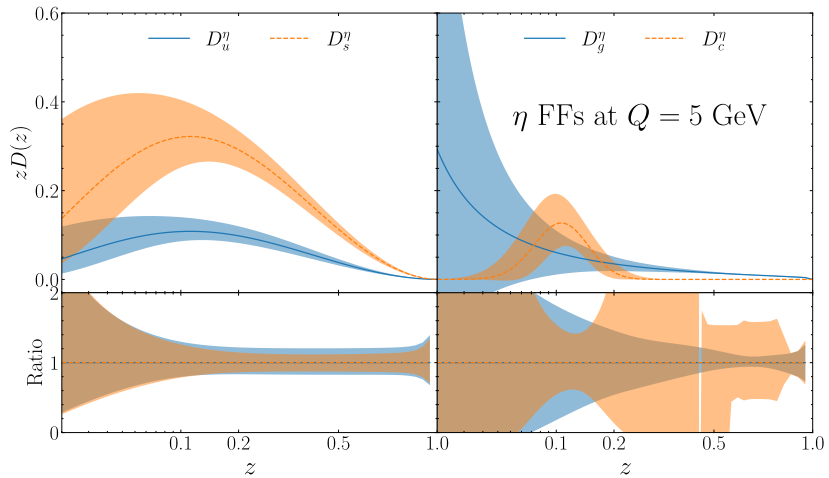



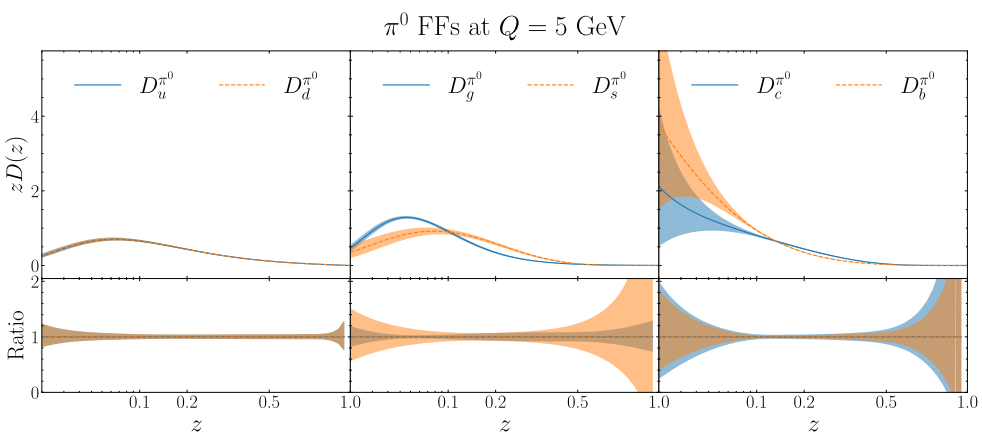

- $K_S^0$ FFs are generally well constrained in $z = 0.1 \sim 0.5$, In general, the fitted and constructed FFs of $K_S^0$ do not coincide with each other. But $s$-quark fragmentation is more favored in both cases. Due to the lack of data, flavor separation bwtween $d$, $s$ is made by theoretical assumptions.
- In $\Lambda$ FFs strange quark is clearly more favored compared to $u$ and $d$ quarks, despite the fact that they are all constituent quarks.

$$D_q^{K_S^0}, \text{iso} = \frac{1}{2}\left(D_{q'}^{K^+} + D_{q'}^{K^-}\right), q(q') = u(d) \text{ or } d(u)$$

$$D_i^{K_S^0}, \text{iso} = \frac{1}{2}\left(D_i^{K^+} + D_i^{K^-}\right), i = g, s, c, b$$

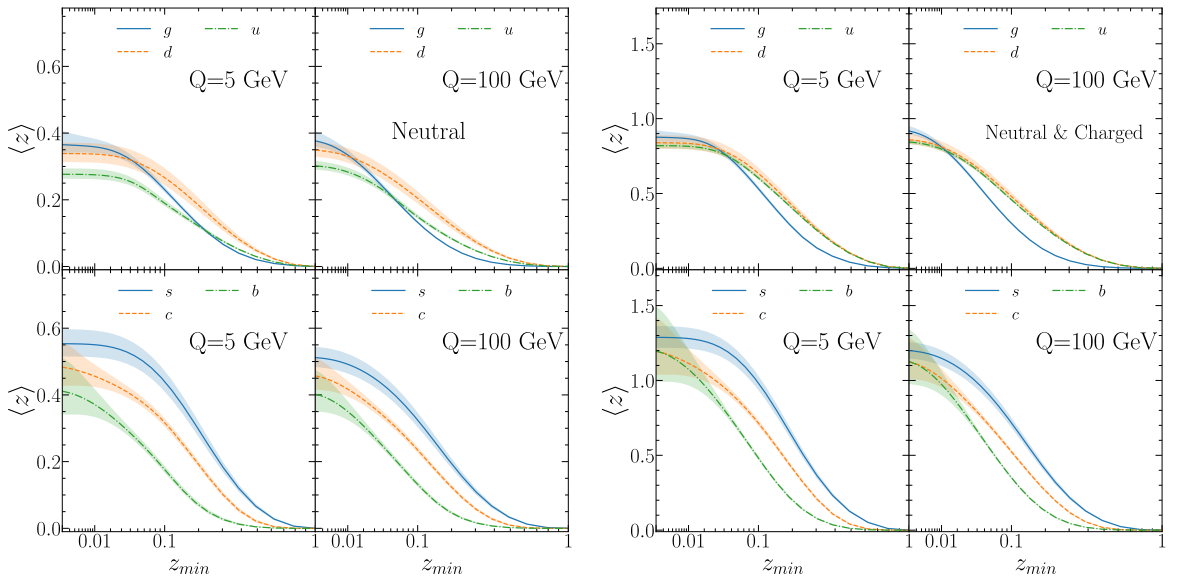
# 3.5 Delivered FFs




- In $\eta$ FFs, $s$-quark fragmentaion is more favored than $u$, $d$, consistent with composition. FFs from heavy quarks are much less than those of the favored quarks, except at $z \sim 0.1$. FFs from gluons remain poorly constrained in the low $z$ region.
- $\pi^0$ FFs are well-constrained due to the comprehensive datasets from charged hadron production data. In our formalism $D_u^{\pi^0} = D_{\overline{u}}^{\pi^0} = D_d^{\pi^0} = D_{\overline{d}}^{\pi^0}$, the FFs for remaining partons are essentially the same as those of $\pi^\pm$

# 3.6 Applications: Momentum Sum Rule




| hadron | $g\ (z_{\min}=0.01)$ | $d\ (z_{\min}=0.01)$ | $u\ (z_{\min}=0.01)$ | $s\ (z_{\min}=0.01)$ | $c\ (z_{\min}=0.01)$ | $b\ (z_{\min}=0.01)$ |
|---|---|---|---|---|---|---|
| neutral | $0.3587^{+0.0243}_{-0.0184}$ | $0.3370^{+0.0322}_{-0.0256}$ | $0.2749^{+0.0173}_{-0.0133}$ | $0.5504^{+0.0424}_{-0.0377}$ | $0.4561^{+0.0350}_{-0.0325}$ | $0.3711^{+0.0465}_{-0.0376}$ |
| charged | $0.5068^{+0.0145}_{-0.0133}$ | $0.5066^{+0.0420}_{-0.0367}$ | $0.5306^{+0.0150}_{-0.0135}$ | $0.7293^{+0.0440}_{-0.0416}$ | $0.6581^{+0.0655}_{-0.0572}$ | $0.7033^{+0.0903}_{-0.0723}$ |
| total | $0.8655^{+0.0306}_{-0.0251}$ | $0.8436^{+0.0591}_{-0.0500}$ | $0.8056^{+0.0214}_{-0.0171}$ | $1.2797^{+0.0728}_{-0.0678}$ | $1.1142^{+0.0974}_{-0.0868}$ | $1.0744^{+0.1350}_{-0.1080}$ |

Figure 3: Momentum fraction carried by each parton from hadrons

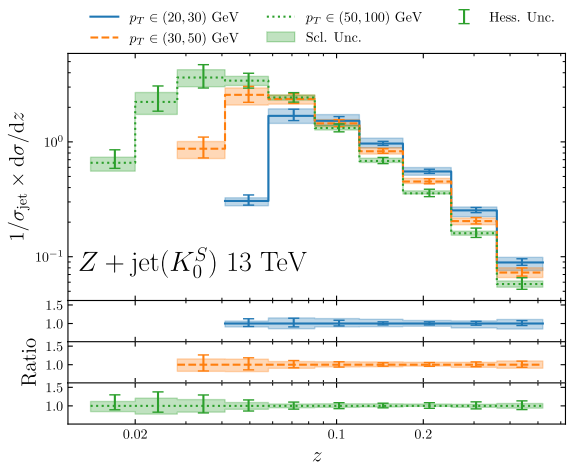
Sum rule: $\sum_h \int_0^1 dzzD_i^h(z) = 1$

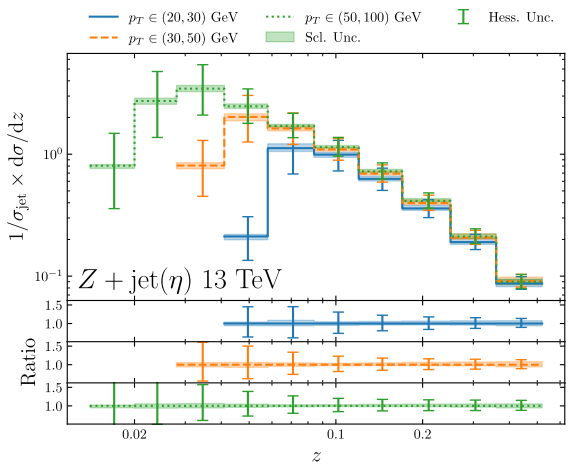
- Momentum fraction carried by parton $i$ from hadron $h$ with cutoff $z_{\min}$ is defined as:

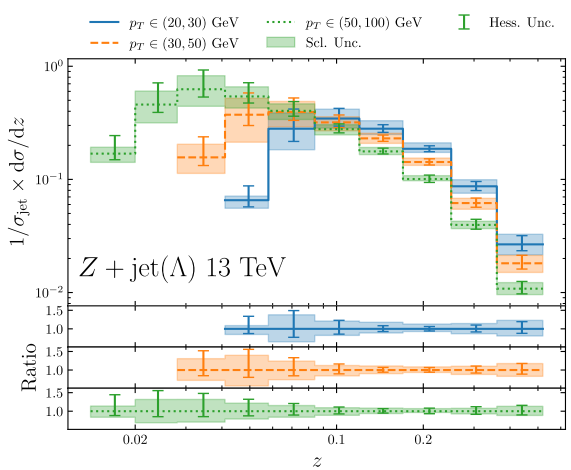
$$\langle z \rangle_i^h = \int_{z_{\min}}^1 dzzD_i^h(z)$$

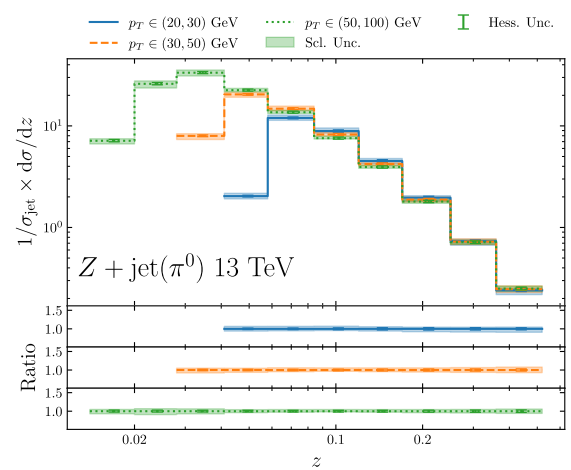
- Definition of FFs will lead to violation of sum rule. [J. Collins and T. Rogers, (2023), arXiv:2309.03346]
- First data-driven test is reported in NPC23 charged fit and rescaled by PYTHIA8. [Gao, **C.Liu**, Shen, Xing, Zhao, PRL, 2024]
- It is tested again with direct fit results of neutral hadrons.

# 3.7 Applications: Predictions on LHCb Hadron Production



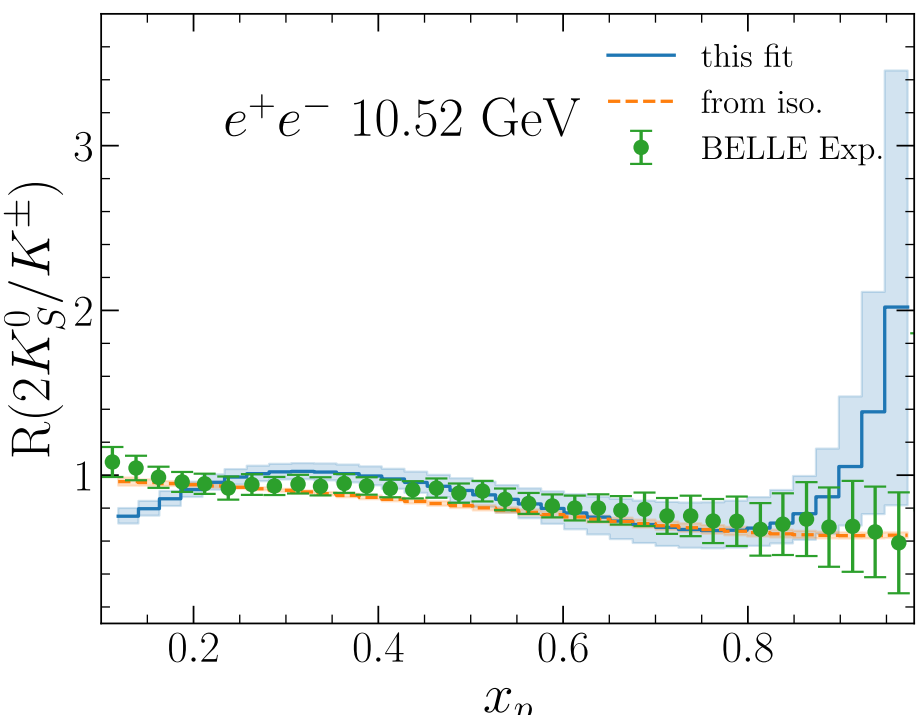



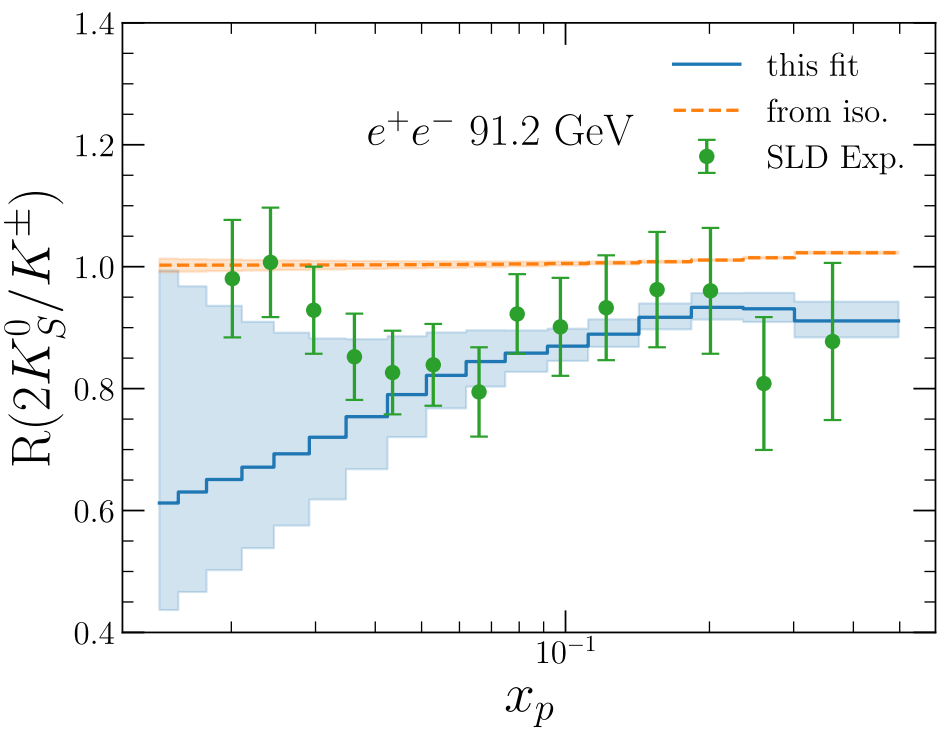




- Predictions of hadron in jet production on LHCb $Z +$ jet process at 13 TeV for 3 different jet $p_T$ regions and 4 neutral hadrons are presented.
- Scale uncertainties, Hessian uncertainties and their relative uncertainties are shown.
- Kinematic bins are taken from previous LHCb measurements.

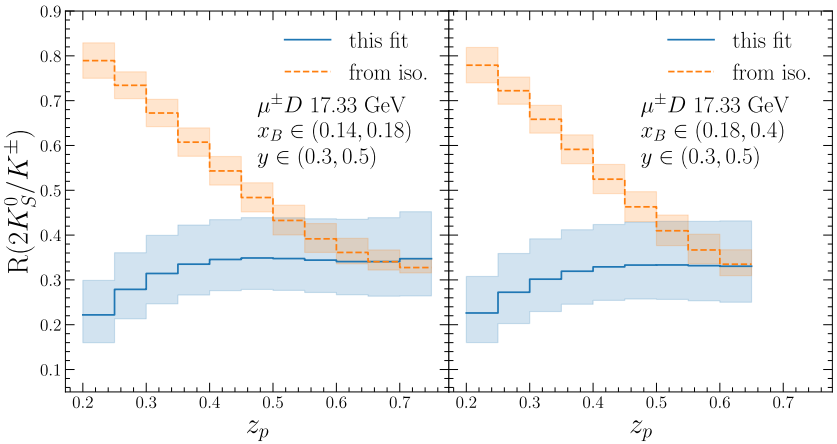
# 3.8 Applications: Production Ratio of $K_S^0$ to $K^\pm$

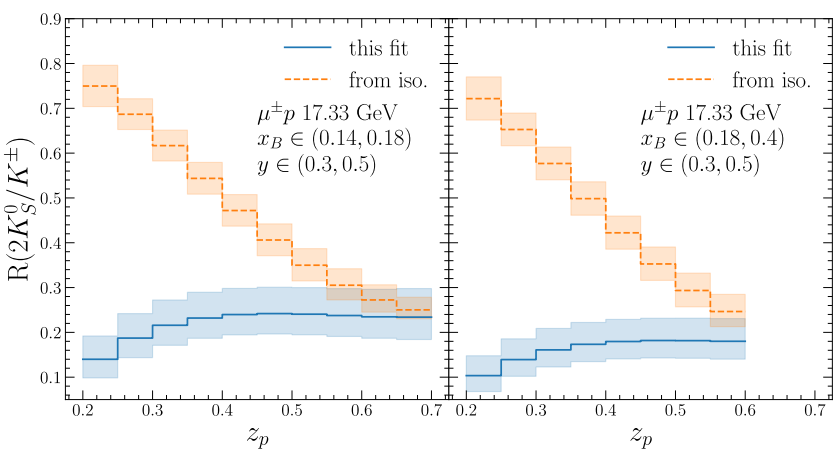
- In $\sqrt{s} = 10.52\text{GeV}$ $K_S^0$ prediction of FFs from iso-spin symmetry is better aligned with experiment. Prediction of FFs from fit aligns in intermediate region.
- In $\sqrt{s} = 91.2\text{GeV}$, FFs from iso-spin symmetry predict neary unity. FFs from fit give better prediction.



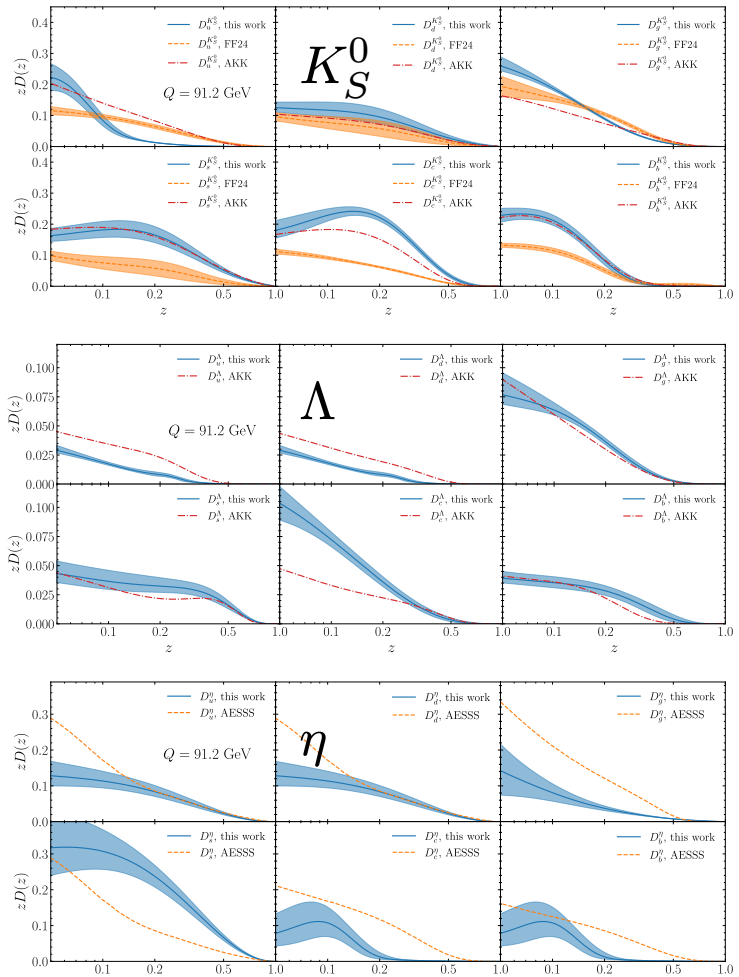
# 3.8 Applications: Production Ratio of $K_S^0$ to $K^{\pm}$




- Predictions of $R\left(2K_S^0/K^{\pm}\right)$ is presented following COMPASS kinematic bins
- The two curves diverge in the low $z$ region. SIDIS process has the advantage of flavor separation
- Future measurements could help better contrain the $K_S^0$ FFs and test isospin symmetry

# 3.9 Comparison with Other Groups




- FF24 provides Monte Carlo uncertainty, AKK & AESSS uncertainties unavailable.
- $K_S^0$: NPC23 and AKK are close in $d, s, b$, NPC23 deviates from FF24 except $d$ after considering uncertainty.
- $\Lambda$: NPC23 and AKK FFs from $g, s, b$ agree in low an high $z$ region, $c$ agree in large $z$ region,AKK is greater in $u, d$ fragmentation
- $\eta$: AESS and NPC FFs from $u, d$ agree in intermidiate and high $z$ region, but differ significantly for other partons.

# 4. Summary

# 4.1 Summary

- In this talk, the previous work of NPC for charged hadrons is introduced.
- Then we present the FFs for $K_S^0, \Lambda, \eta, \pi^0$ in the context of global QCD analysis.
- Data from SIA, SIDIS and $pp$ collisions is incorporated in the analysis. Data from SIA hadron-in-jet production and SIDIS process is included for the first time, which gives better gluon fragmentation constraints and flavor separation.
- With the comprehensive species of FFs extracted within the NPC framework, a test on the momentum sum rule with the light-flavor charged and neutral hadrons is performed.
- The extracted $K_S^0$ fragmentation functions, together with the $K_S^0$ FFs constructed from $K^{\pm}$ FFs via isospin symmetry, are used to test isospin symmetry in kaon fragmentation.
- Comparison of light neutral hadron FFs with other groups is also presented.