Curvature perturbation and GW induced by U(1) symmetry breaking during inflation

Tingyu Li

Collaborator: Haipeng An, Chen Yang

Tsinghua University, Department of physics

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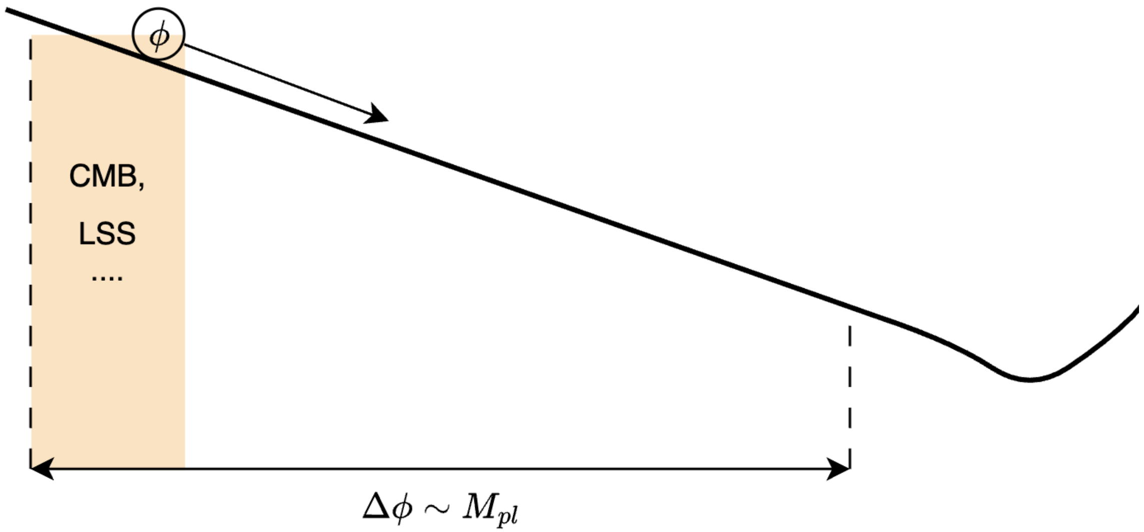
Introduction:

Inflation: the causality problem
the flatness problem
the magnetic monopole problem
the seed of large scale structure…



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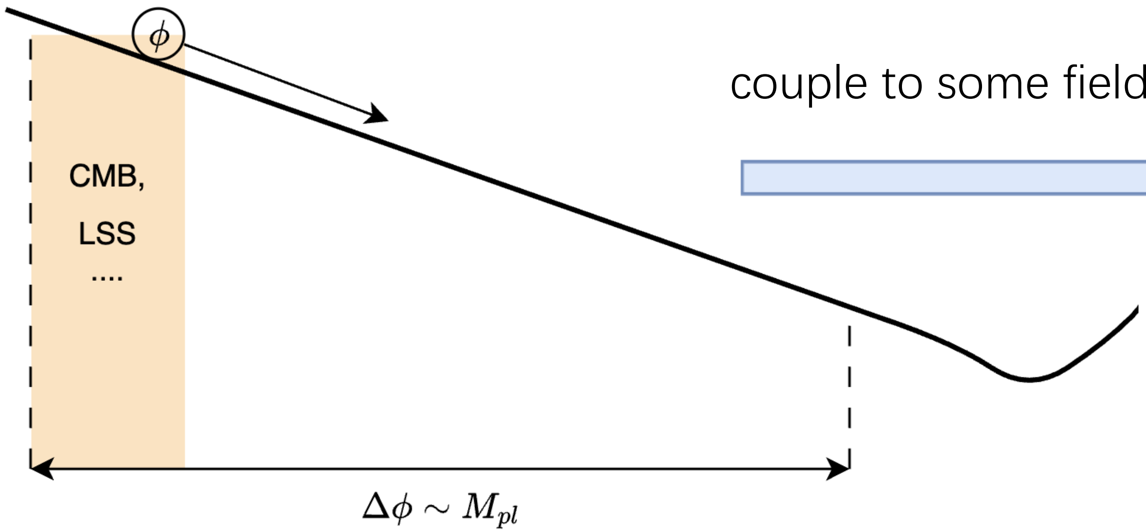
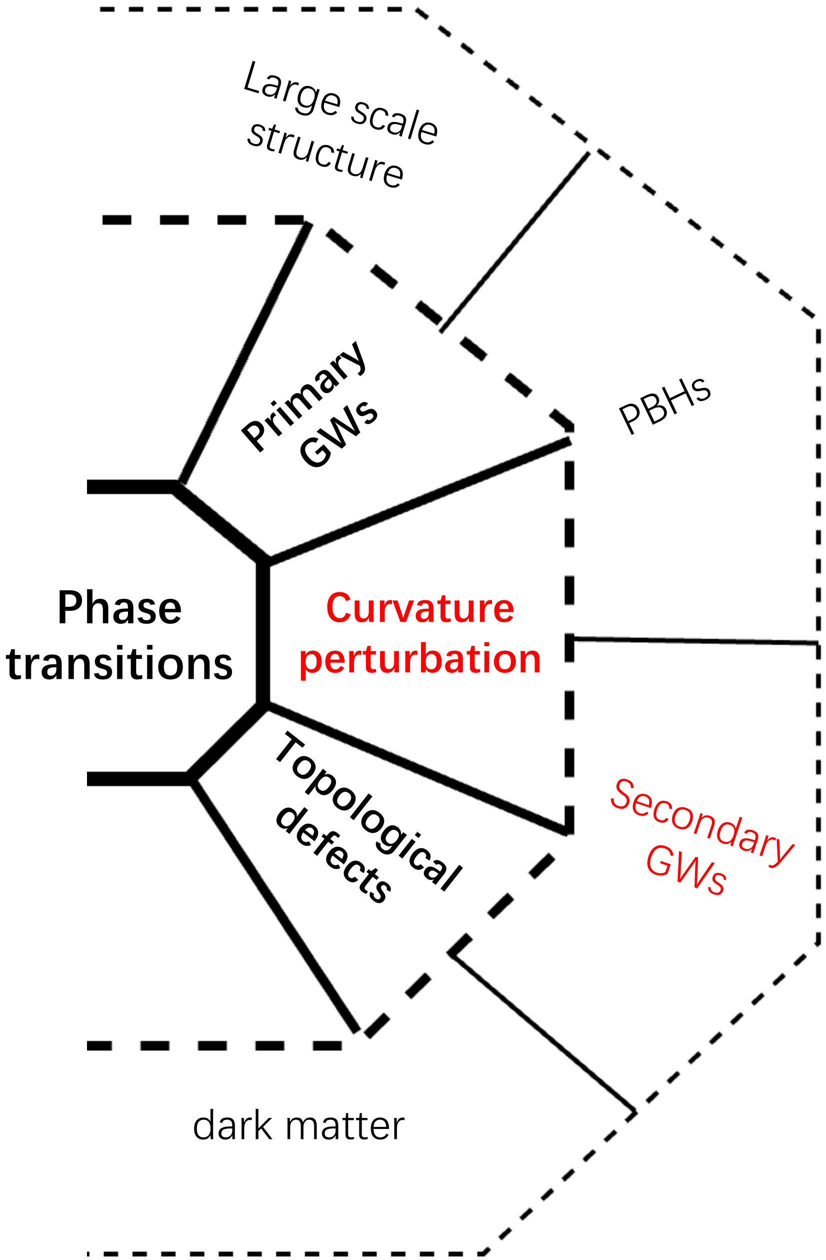


The excursion of the inflaton field must be very large, comparable to the M_{pl}.

couple to some field σ



Inflation triggered phase transition:

We use a toy model with U(1) symmetry

$$V(\phi,\Phi)=\frac{1}{2}\left(g^2\phi^2-m^2\right)|\Phi|^2+\frac{\lambda}{4}|\Phi|^4$$

with

$$\phi(t)=\phi_0-\dot{\phi}t$$

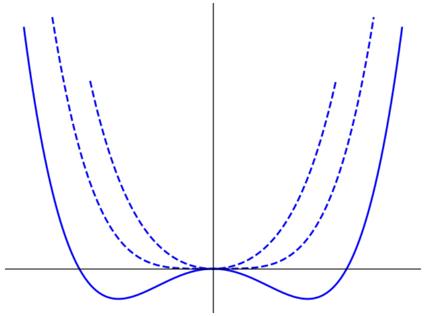
the evolution equation

$$\ddot{\Phi}+3H\dot{\Phi}-a^{-2}\nabla^2\Phi+\left[g^2\left(\phi_0-\dot{\phi}t\right)^2-m^2+\lambda|\Phi|^2\right]\Phi=0$$

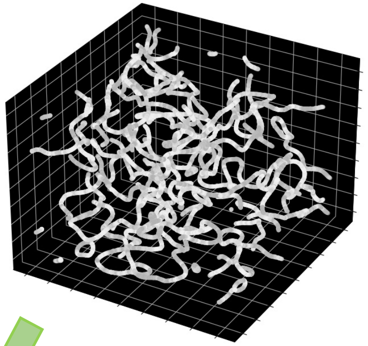
We consider both the global U(1) and gauge U(1) symmetry breaking

$$S_{global}=\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi^*-V(\Phi,\phi)\right]$$

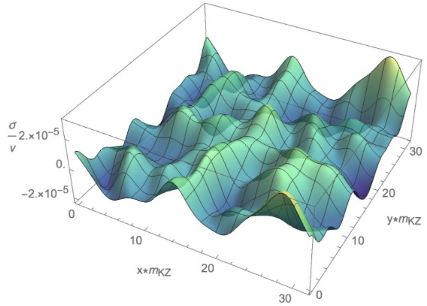
$$S_{gauge}=\int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}D_\mu\Phi D_\nu\Phi^*-\frac{1}{4}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma}F_{\mu\nu}-V(\Phi,\phi)\right]$$



potential



phase transition

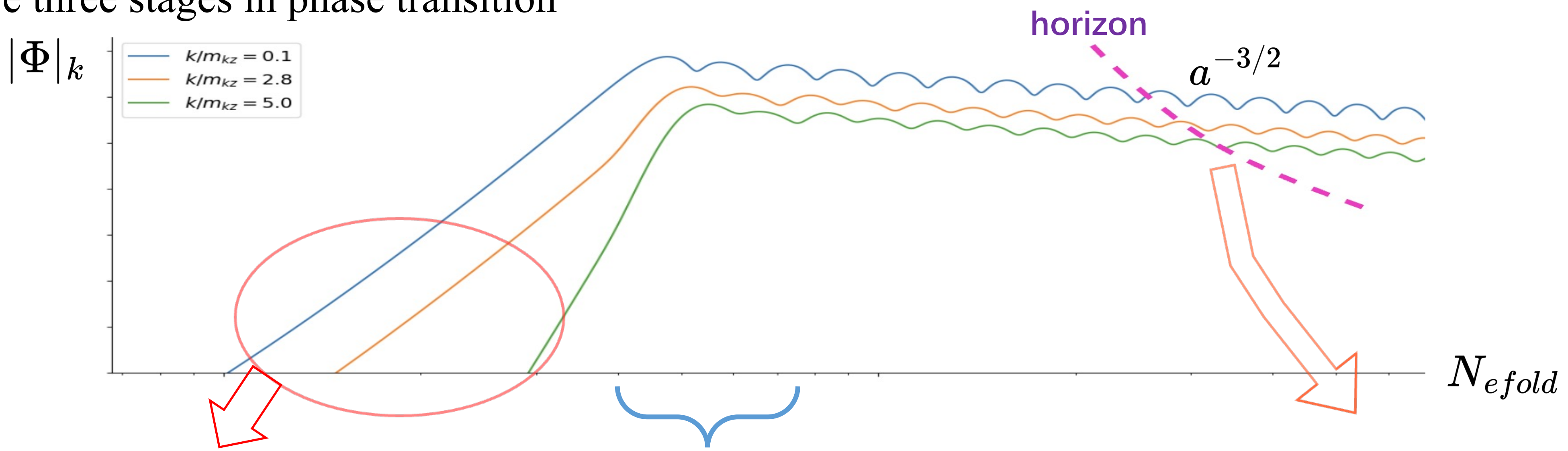


curvature perturbation

Secondary GWs

Phase transition:

There are three stages in phase transition



Tachyonic instability

define:

$$m_{kz} = \sqrt[3]{2m^2\dot{\phi}/\phi_0}$$

ignore nonlinear terms:

$$\ddot{\Phi}_k + 3H\dot{\Phi}_k + \left(\frac{k^2}{a^2} - m_{kz}^3 t\right)\Phi_k = 0$$

modes with

$$k < a\sqrt{{m_{kz}}^3 t}$$

grow exponentially

determines typical scale

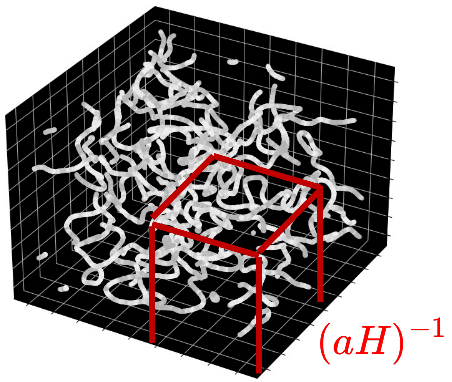
$$l_{string} \sim m_{kz}^{-1}$$

String formation

nonlinearity dominates evolution

the UV modes grow

string forms



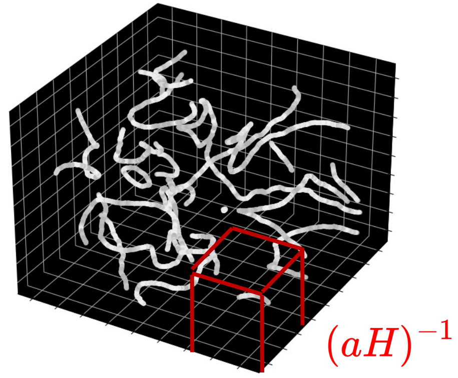
system becomes matter like

Exit the horizon

mode with $k\tau < 1$ then freezes

the string system becomes

comoving static



Tachyonic instability :

In the first stage, the system is around $\Phi = 0$, so the nonlinear term can be neglected.

$$\ddot{\Phi} + 3H\dot{\Phi} - a^{-2}\nabla^2\Phi - \underbrace{2m^2\dot{\phi}/\phi_0 t}_{\text{define } m_{kz}^3}\Phi = 0$$

define m_{kz}^3

Modes with $k < a\sqrt{m_{kz}{}^3 t}$ grow exponentially

We then calculate the mode function of the system

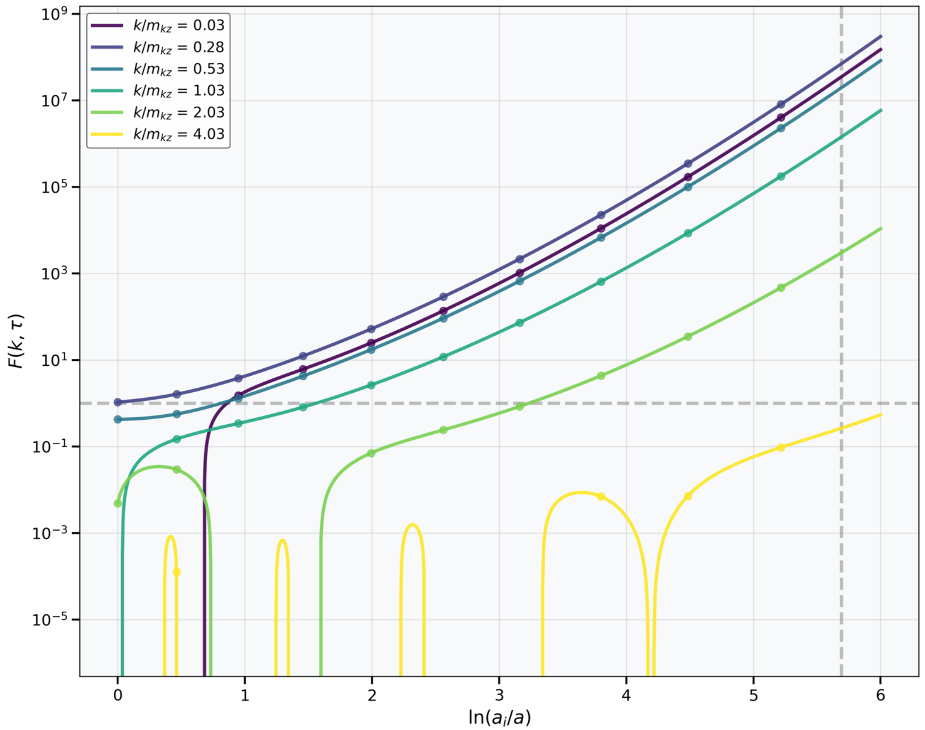
$$f'' - \frac{2f'}{\tau} + \left(k^2 + \frac{m_{\text{eff}}^2}{H^2\tau^2}\right) f = 0$$

The modes become classical when the anticommutation of $\tilde{\sigma}_{\mathbf{k}}$ and $\tilde{\pi}_k$ is significantly larger than their commutation

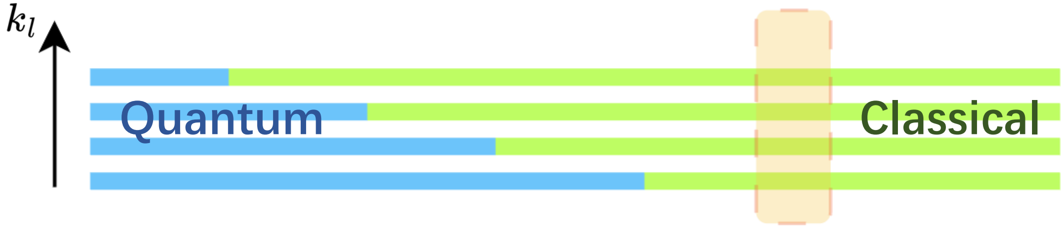
$$\left|\left\langle\left[\hat{\tilde{\sigma}}_{\mathbf{k}}(\tau), \hat{\tilde{\pi}}_{\mathbf{k}}(\tau)\right]_+\right\rangle\right| \gg \left|\left\langle\left[\hat{\tilde{\sigma}}_{\mathbf{k}}(\tau), \hat{\tilde{\pi}}_{\mathbf{k}}(\tau)\right]\right\rangle\right|$$

This is equivalent to require

$$F(k,\tau) = a(\tau)^2\,\mathrm{Re}\left[f'(k,\tau) f^*(k,\tau)\right] \gg 1$$



mode function evolution



initial condition

arXiv:hep-ph/0208228v3

Source of curvature perturbation :

Phase transition can generate curvature perturbation, in Newtonian gauge:

$$\partial_0^2\delta\phi + 2H\partial_0\delta\phi - \partial_i^2\delta\phi + a^2\frac{\partial^2 V}{\partial\phi^2}\delta\phi = -a^2\partial V_1 - 2a^2\Phi_c\frac{\partial V}{\partial\phi} + \phi_0'\partial_0\left(\Phi_c + 3\Psi_c\right)$$

$$\Psi_c' + H\Phi_c = 4\pi G\left[\phi_0'\delta\phi + \frac{\partial_i}{\partial^2}\left(D_i\Phi D_0\Phi^* + g^{\rho\sigma}F_{i\rho}F_{0\sigma}\right)\right]$$

$$\Phi_c - \Psi_c = -12\pi G\partial^{-4}\partial_i\partial_j\left[D_i\Phi D_j\Phi^* + g^{\rho\sigma}F_{i\rho}F_{j\sigma}\right]^{TL}$$

Applying the Green's function method

$$\delta\tilde{\phi}_k = \int_{-\infty}^{\tau}\mathrm{d}\tau' G\left(k,\tau,\tau'\right)\left(\tilde{S}_\Phi + \tilde{S}_{|\Phi|} + \tilde{S}_{T_{ij}}\right)$$

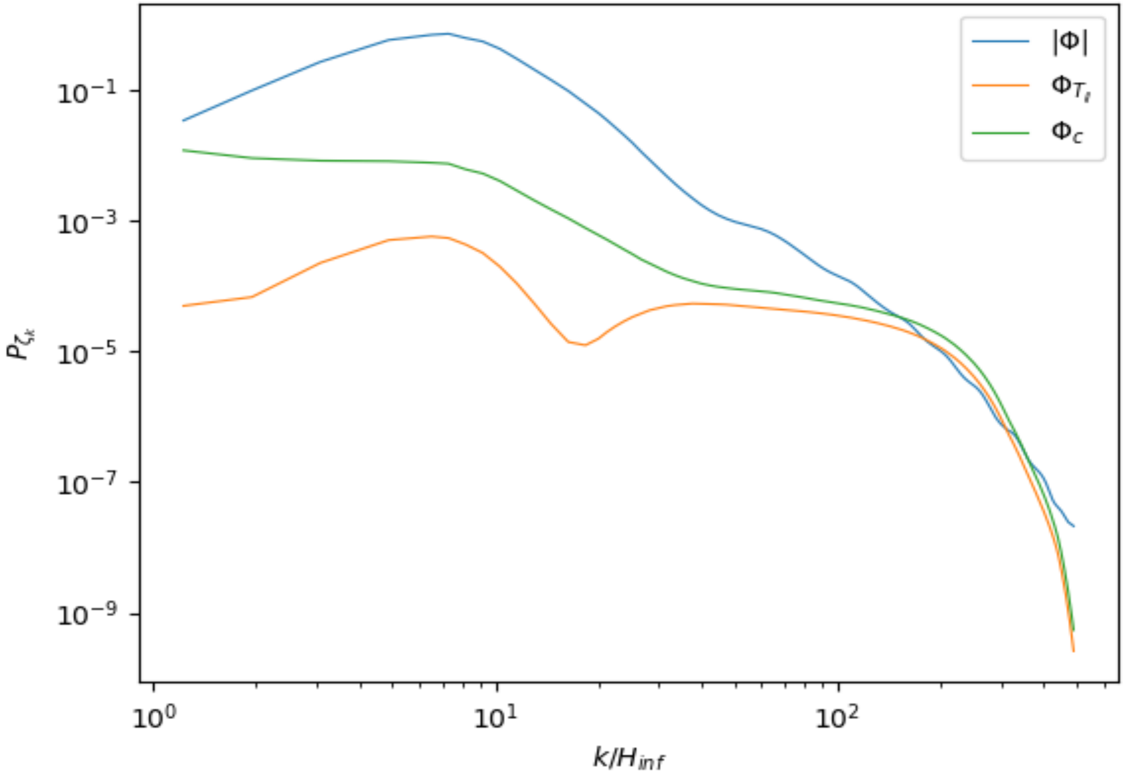
here

$$\tilde{S}_{|\Phi|} = a^2\widetilde{|\Phi|^2}\phi_0$$ dominate

$$S_\Phi = \frac{2\dot{\phi}_0}{H\tau^2}\Phi_c$$

suppressed by ϵ

$$S_{T_{ij}} = 16\phi_0'\pi G\partial^{-4}\partial_i\partial_j T_{ij} - 12\phi_0'\pi G\partial^{-2}\partial_0\partial_i T_{i0}$$



contributions of three components

Source of curvature perturbation :

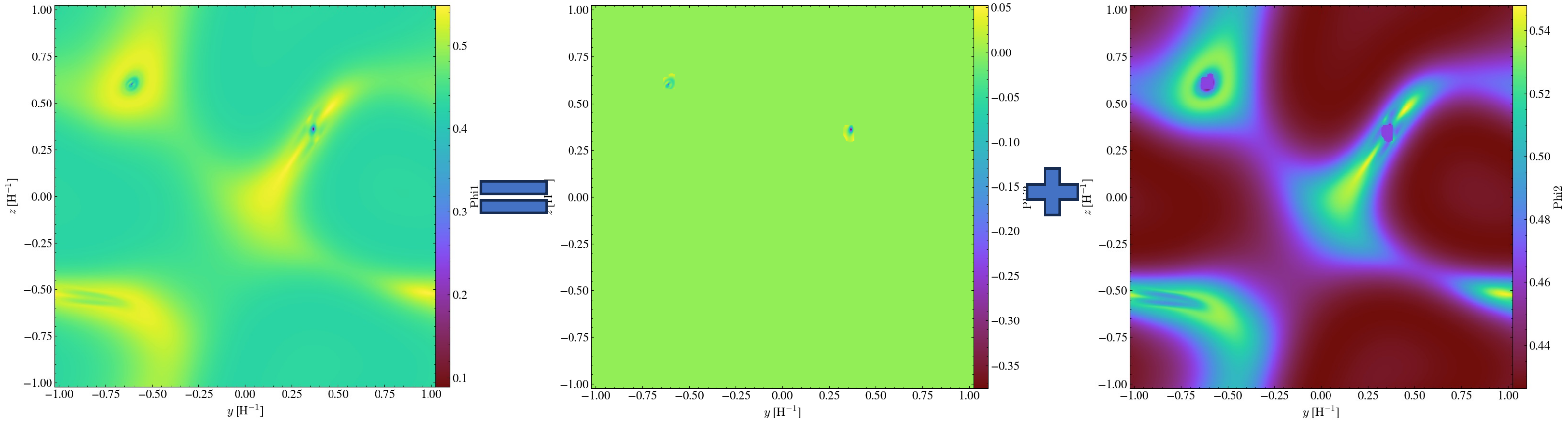
Curvature perturbation is mainly sourced by radial mode:

$$\delta\tilde{\phi}_k = \int_{-\infty}^{\tau} \mathrm{d}\tau' G\left(k,\tau,\tau'\right) a^2 |\Phi|_k^2 \phi_0$$

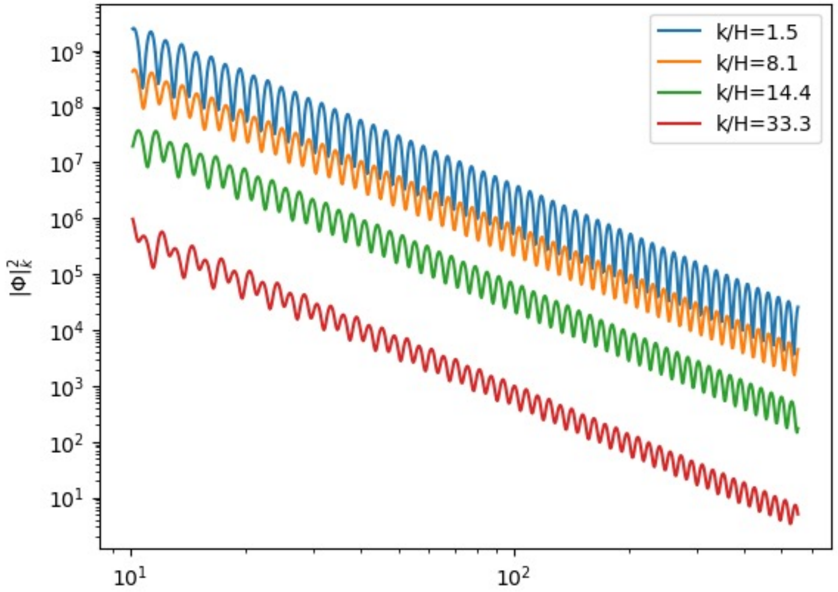
For ground state string

$$|\Phi|_k^2 \sim S_{string}^2 \sim a^{-4}$$

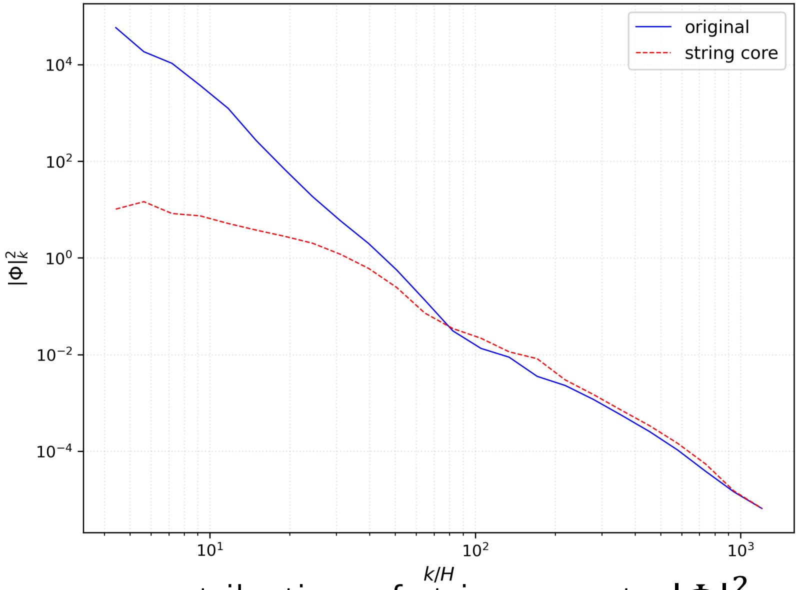
but in simulation $|\Phi|_k^2 \sim a^{-3}$



separate the string core and background



evolution of $|\Phi|_k^2$



contribution of string core to $|\Phi|_k^2$

The radial modes are dominated by field oscillations outside the string core.

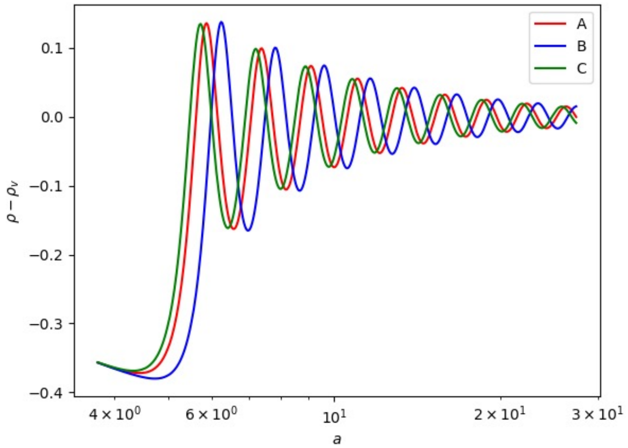
Source of curvature perturbation :

$$\delta\tilde{\phi}_k = \int_{-\infty}^{\tau} \mathrm{d}\tau' G\left(k,\tau,\tau'\right) a^2 |\Phi|_k^2 \phi_0$$

with $|\Phi|_k^2 \sim a^{-3}$

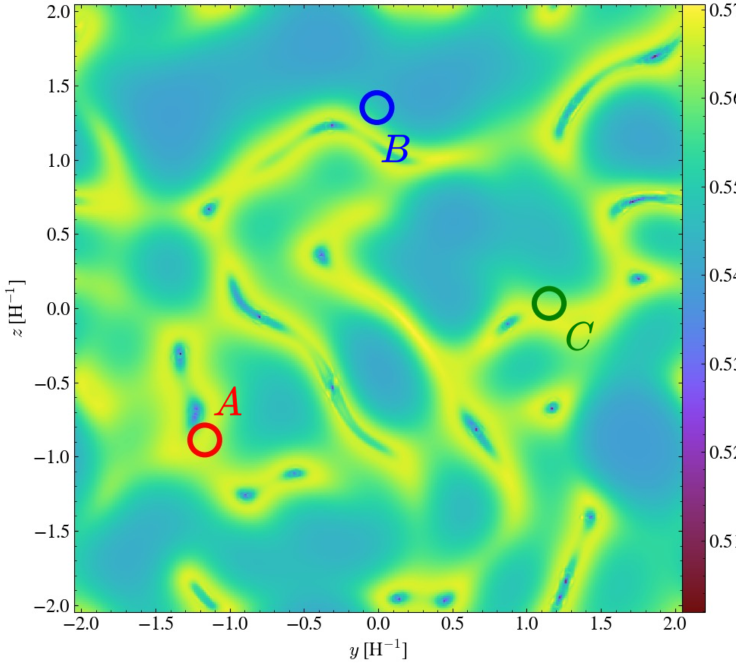
These oscillations trace the string trajectories, thereby defining the spectral characteristics in the Fourier transform of the radial mode.

Therefore, although the direct contribution of defects to curvature perturbation is small, curvature perturbation still contains information about the defects.

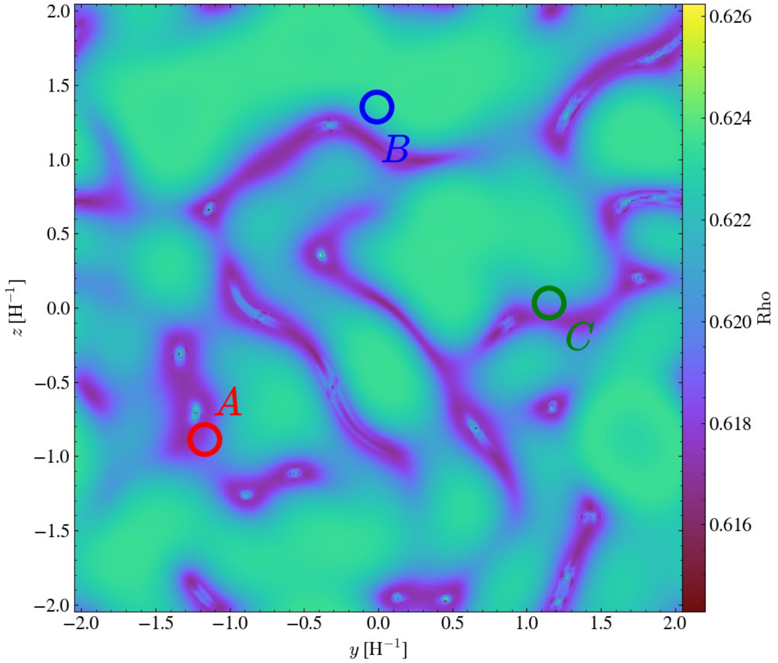


We track three points (A, B, C) during the evolution.

They exhibit similar behavior but with different phases.



radial mode $|\Phi|$ at $N_e = 3$



radial mode $|\Phi|$ at $N_e = 4$

Numerical simulation:

Initial conditions:

$$f'' - \frac{2f'}{\tau} + \left(k^2 + \frac{g^2\phi^2(\tau) - m^2}{H^2\tau^2}\right) f = 0$$

solve mode functions

$$\left|\left\langle \left[\hat{\tilde{\sigma}}_{\mathbf{k}}(\tau), \hat{\tilde{\pi}}_{\mathbf{k}}(\tau)\right]_+ \right\rangle\right| \gg \left|\left\langle \left[\hat{\tilde{\sigma}}_{\mathbf{k}}(\tau), \hat{\tilde{\pi}}_{\mathbf{k}}(\tau)\right] \right\rangle\right|$$

→

long wave-length modes can be treated classically

Generate random field configuration based on mode function as initial condition

Evolution equations:

$$\partial_0 \left(a^2 \partial_0 \Phi\right) = a^2 \left(\nabla^2 \Phi - ig\partial_i A_i \Phi - 2igA_i\partial_i\Phi - A_i^2 g^2 \Phi\right) - a^4 \partial V$$

$$\partial_0 E_i = -\nabla^2 A_i + \partial_i \partial_j A_j - \frac{1}{2} g a^2 i \left(\Phi D_i \Phi^* - \Phi^* D_i \Phi\right)$$

Constraints:

$$\mathcal{G} = \partial_i E_i - g\,\mathrm{Im}\left(\Phi\Phi_p^*\right)$$

How to preserve Gaussian constraints during simulation?

- Solve constraint during simulation
- Use link type variables $U_\mu(n) = e^{iA(n+1/2)}$
- Constraint damping

Numerical simulation :

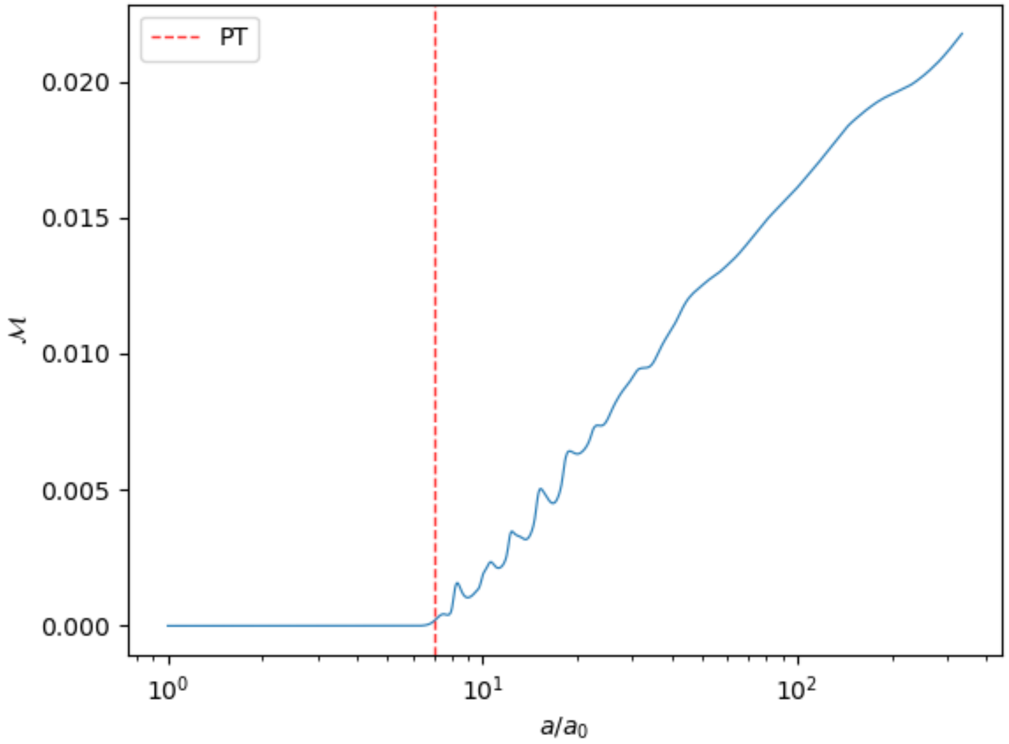
Constraints damping: arXiv:1309.2012

$$\mathcal{G} = \partial_i E_i - g\,\mathrm{Im}\left(\Phi\Phi_p^*\right)$$

In the original numerical form:

$$\partial_0 \mathcal{G} = 0$$ violated by numerical error

$$\mathcal{M} = \frac{\left|\partial_i E_i - g\,\mathrm{Im}\left(\Phi\Phi_p^*\right)\right|}{\left|\partial_i E_i\right| + \left|g\,\mathrm{Im}\left(\Phi\Phi_p^*\right)\right|}$$



Reformulate the evolution equations into a more stable form

define $Z = \partial_i A_i$

$$\partial_0\left(a^2\partial_0\Phi\right) = a^2\left(\nabla^2\Phi - igZ\Phi - 2igA_i\partial_i\Phi - A_i^2 g^2\Phi\right) - a^4\partial V$$

$$\partial_0 E_i = -\nabla^2 A_i + \partial_i Z - \frac{1}{2} g a^2 i\left(\Phi D_i\Phi^* - \Phi^* D_i\Phi\right)$$

$$\partial_0 Z = -\partial_i E_i + c_g^2\left[\partial_i E_i + \frac{1}{2} g a^2 i\left(\Phi\partial_0\Phi^* - \Phi^*\partial_0\Phi\right)\right]$$

Now the constraint satisfies $\left(-\partial_0^2 + c_g^2\nabla^2\right)\mathcal{G} = 0$

Results:

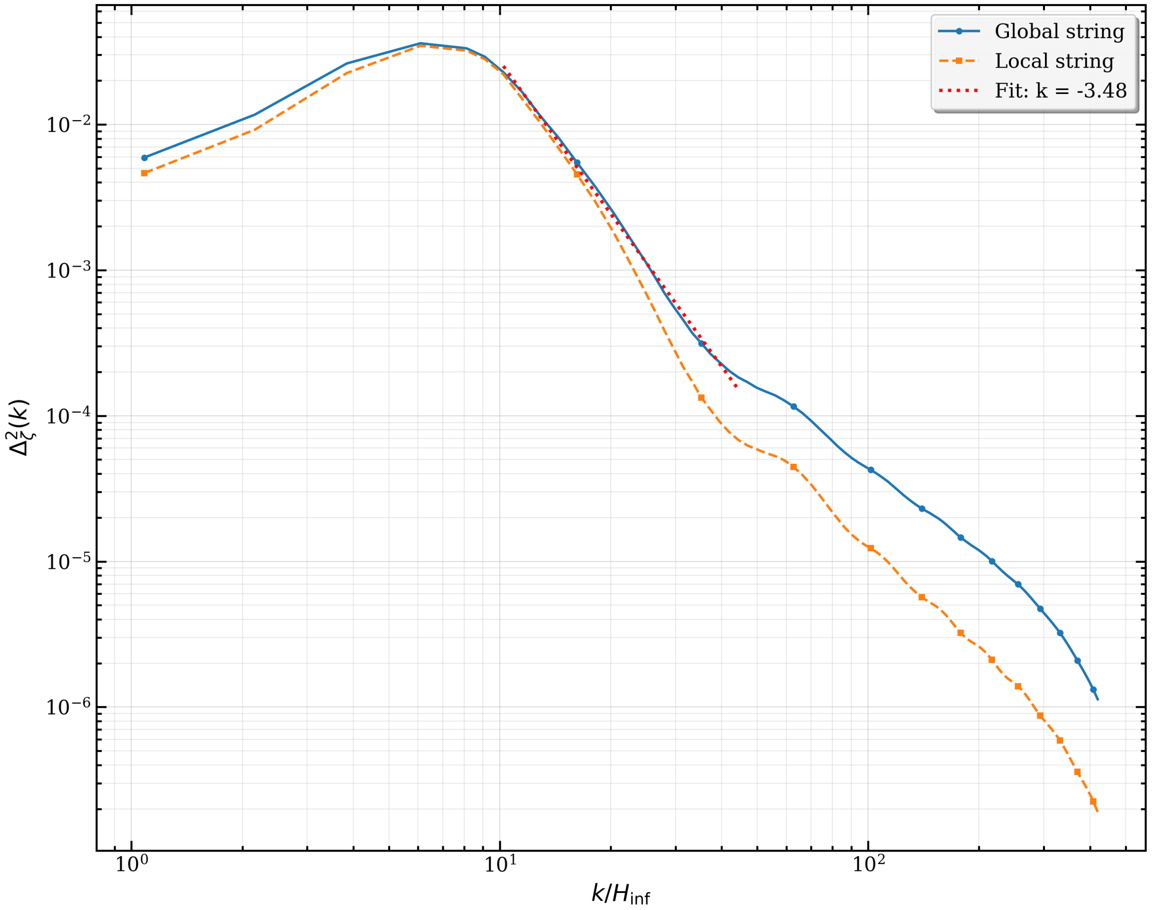
Define gauge-invariant quantity: $\zeta_k = -\Phi_{c,k} - \frac{H}{\dot{\phi}_0}\delta\tilde{\phi}_k$

The power spectrum: $\Delta^2_\zeta(q) = \frac{q^3}{2\pi^2}P_\zeta(q) = \frac{q^3}{2\pi^2}\langle\zeta_{\mathbf{q}}\zeta^*_{\mathbf{q}'}\rangle'$

The power spectra of curvature perturbations from both global and local strings exhibit identical IR behavior and peak positions.

In the UV regime, $P_\zeta(k) \sim k^{-3.5}$

However, in the deep UV region, local strings show suppression compared to global strings



curvature perturbation

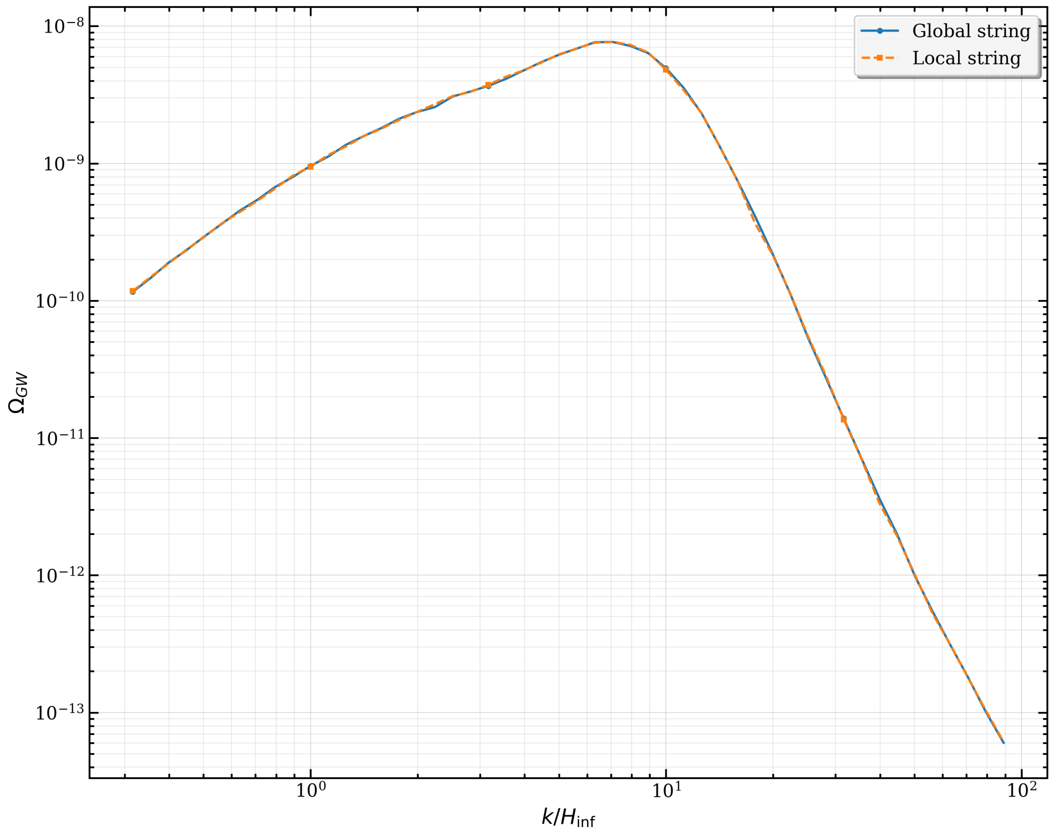
Secondary GW:

From curvature perturbation to secondary GW:

$$\Omega_{\rm GW}(k) = \Omega_{\rm rad}\frac{k^3}{6}\int_0^\infty dk_1 \int_{-1}^{1} d\mu \frac{k_1^3}{k_2^3}\left(1-\mu^2\right)^2 \cdot \overline{I^2}\left(k, k_1, k_2\right)\mathcal{P}_\zeta\left(k_1\right)\mathcal{P}_\zeta\left(k_2\right)$$

The power spectra of curvature perturbations from both case are almost the same.

In the UV regime, $\Omega_{GW} \sim k^{-5}$



power spectrum of secondary GW

Summary:

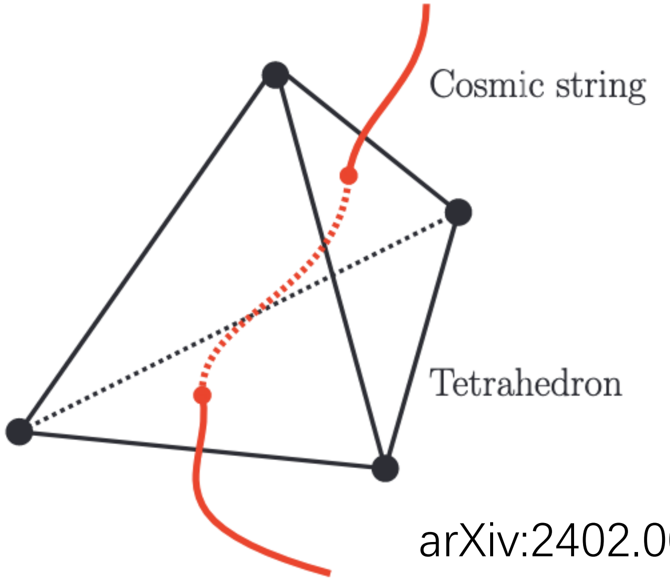
- Phase transitions triggered by inflation can generate considerable curvature perturbations.
- The radial mode of the complex field is the dominant source of these curvature perturbations.
- Our simulations employ constraint damping methods to effectively control Gaussian constraint violation.
- Global and local strings produce power spectra with similar IR behavior and peak features, but diverge significantly in the deep UV regime.

Outlook & Future Work

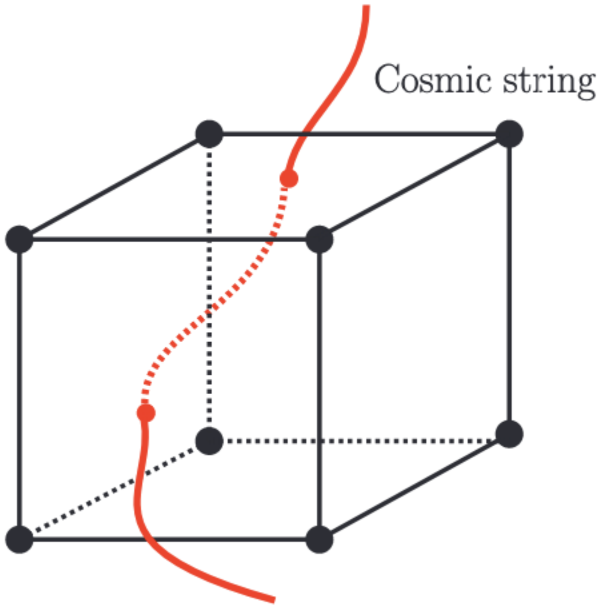
- Systematically explore the parameter dependence of the power spectrum to build a predictive framework.
- Investigate other types of defects to determine if spectral signatures can uniquely identify phase transition mechanisms.

Thank you

Appendix A: string identification



arXiv:2402.00741v3



In each triangular face, define:

$$\omega_{ij} = \Re\phi_i \Im\phi_j - \Im\phi_i \Re\phi_j$$

$$\omega_{123} \equiv \omega_{12} + \omega_{23} + \omega_{31}$$

string go through this face if and only if:

$$\omega_{ij} \geq 0 \quad \text{and} \quad \omega_{123} > 0$$

$$\omega_{ij} \leq 0 \quad \text{and} \quad \omega_{123} < 0$$

String configuration:

$$\Phi(\rho, \theta) = \eta f\left(m_s \rho\right) e^{in\theta}$$

$$f\left(m_s \rho\right) \propto \rho^{|n|}$$

For n=1, near the core:

$$\Phi \sim \rho e^{i\theta} = x + iy$$