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Bias with a Timer: Axion Dark Matter and Domain Wall

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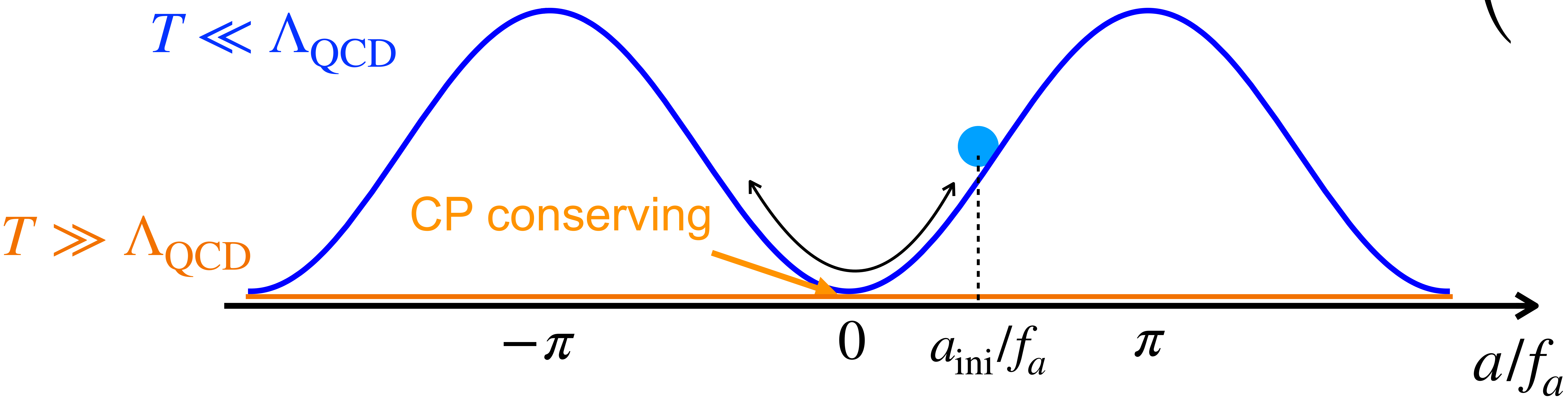
arXiv:2507.12268 with S. Y. Hao, Y. Nakai, and M. Suzuki

1. Introduction

QCD axion can explain the strong CP problem and dark matter (DM) simultaneously.

Peccei and Quinn (1977)
Weinberg (1978), Wilczek (1978)

$$V_{\rm QCD} = m_a^2 f_a^2 \left(1 - \cos\frac{a}{f_a}\right)$$



Preskill, Wise, Wilczek `83, Abbott, Sikivie, `83,
Dine, Fischler, `83

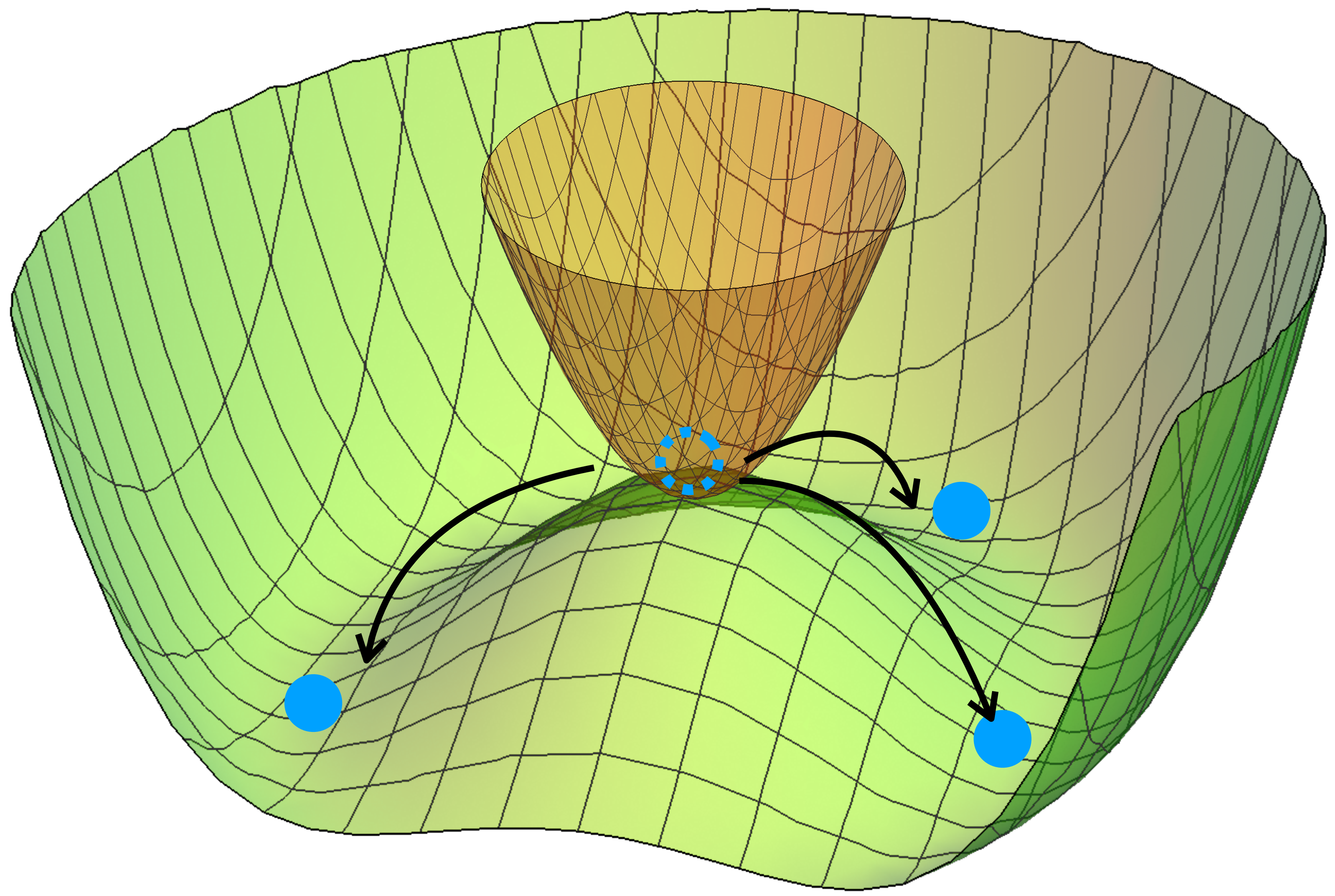
When the Peccei-Quinn (PQ) symmetry is spontaneously broken after inflation, topological defects appear.

Vilenkin, Shellard (2000) for review

$\underline{T \sim v_{\rm PQ}}$ $\quad v_{\rm PQ} \equiv N_{\rm DW} f_a$

- The axion fields are distributed randomly.
- Cosmic string appears.

Kibble (1976), Vilenkin (1981)



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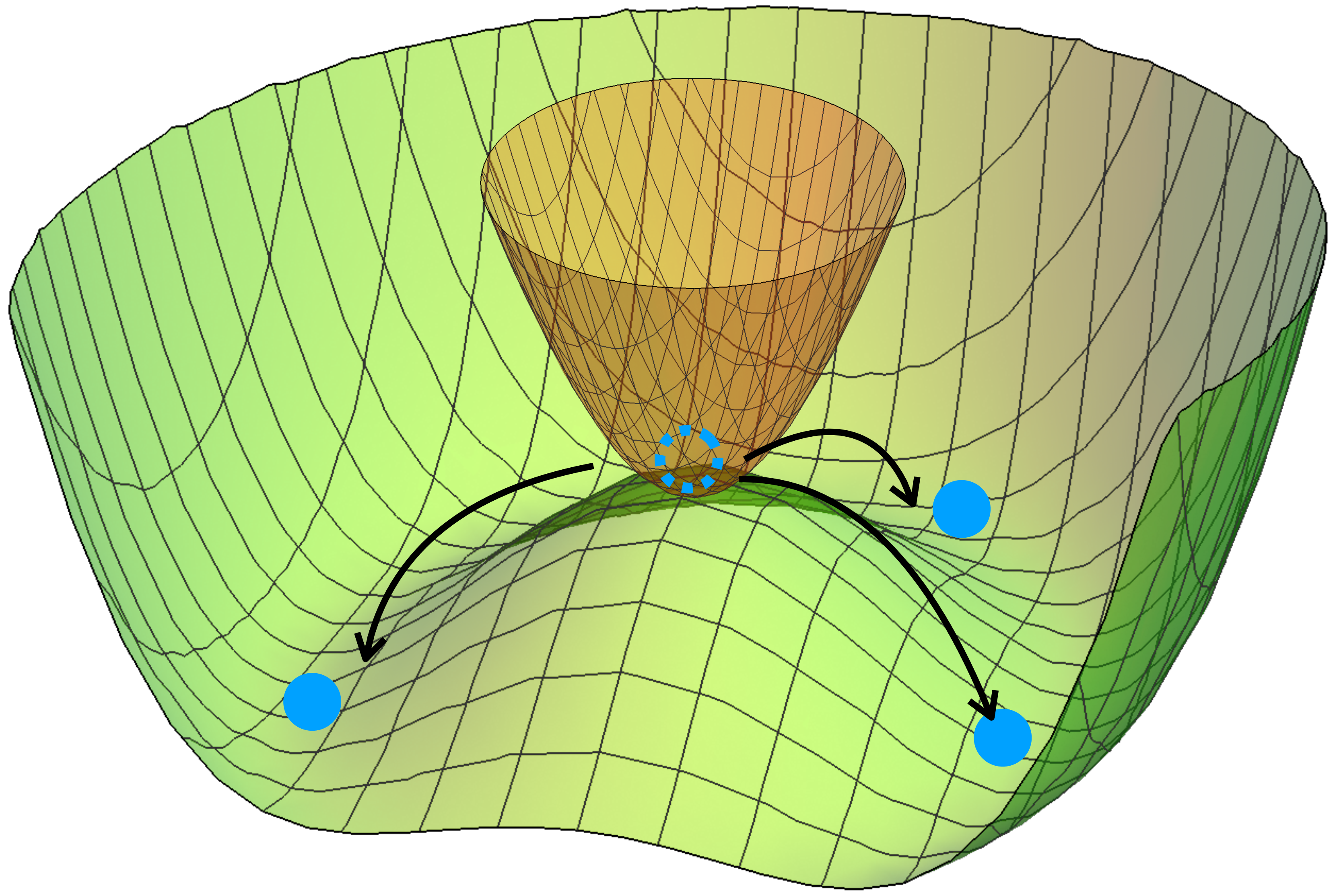
Vilenkin, Shellard (2000) for review



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- The axion fields are distributed randomly.
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Kibble (1976), Vilenkin (1981)

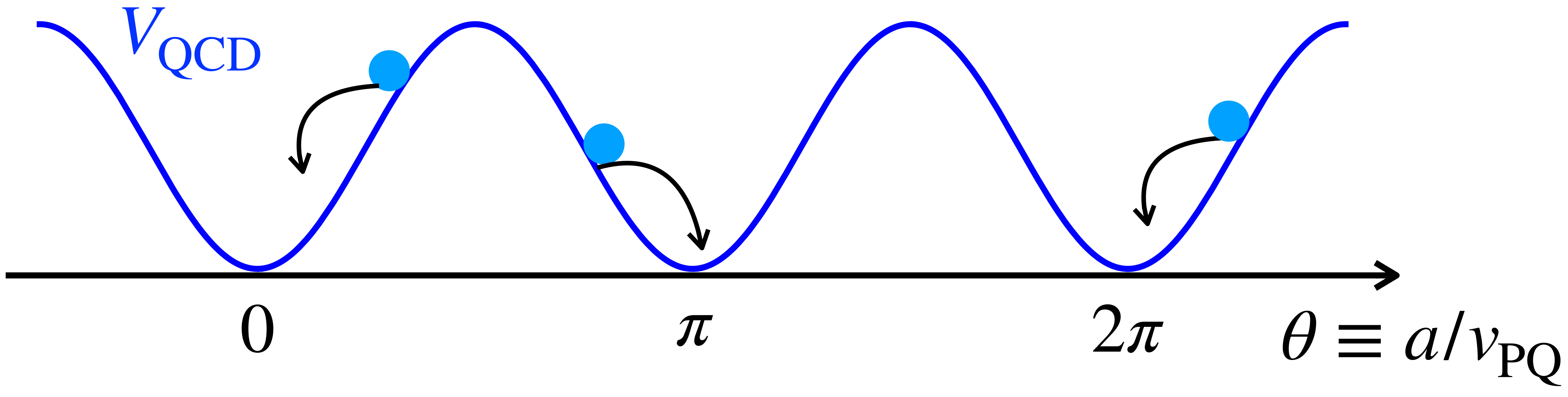
$\underline{T \sim \Lambda_{\rm QCD}}$

- Domain wall (DW) appears.

Vilenkin (1981)

- For $N_{\rm DW} > 1$, it is stable and dominates the Universe.

Domain wall problem



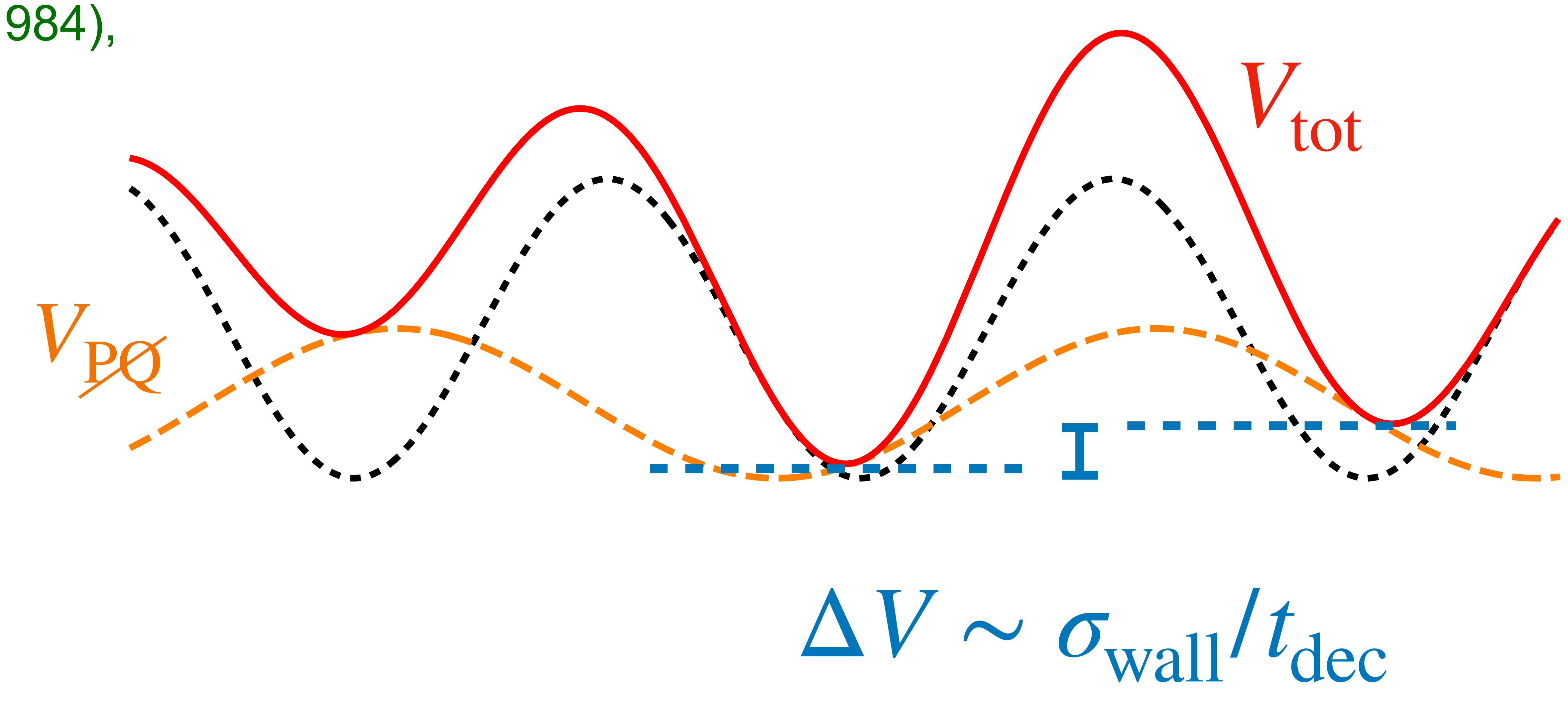
The case for $N_{\rm DW} = 2$

Zeldovich, Kobzarev (1976), Vilenkin (1985)

Possible solutions

- Lazarides-Shafi mechanism   Lazarides, Shafi (1982)
- Biased potential   Sikivie (1972), Mohanty, Stecker (1984), Gelmini, Gleiser, Kolb (1989)
- Biased population   Lalak, Thomas (1993), Lalak, Ovrut, Thomas (1995), Lalak, Lola, Ovrut, Ross (1995), Coulson, Lalak, Ovrut (1996), Larsson, Sarkar, White (1997)

We focus



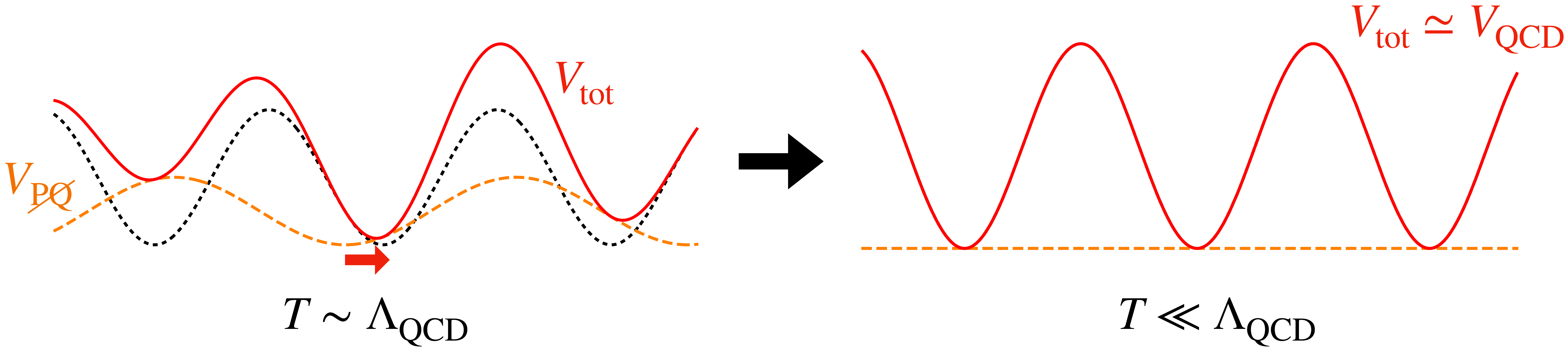
However, such explicit breaking potentials misalign the potential minimum from the CP conserving one.

DW-quality tension   Ringwald, Saikawa (2016)

Our idea is as follows:

Ibe, Kobayashi, Suzuki, Yanagida (2020),
Lee, Murai, Takahashi, Yin (2023)

What if the bias potential is time-dependent?



We consider a mixing coupling between the PQ scalar and a light scalar field, which induces such a bias term.
We discuss the production and fate of domain walls.

Contents

2. PQ mechanism with a light scalar

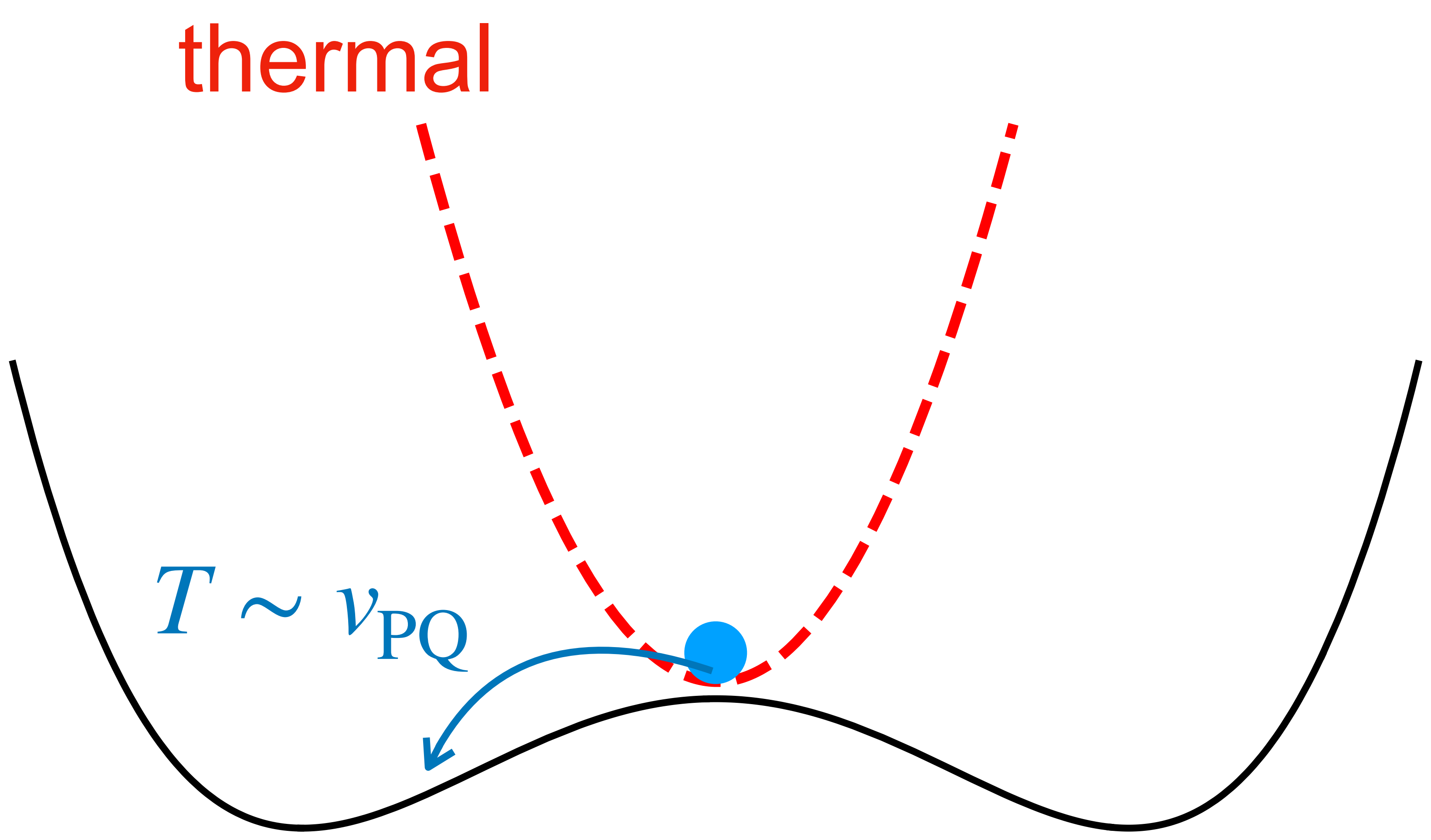
Hao, **SN**, Nakai, Suzuki (2025)

We introduce a light complex scalar field S mixed with the PQ scalar P:

Ibe, Kobayashi, Suzuki, Yanagida (2020)

$$V_{\rm PQ}(P,S) \supset \lambda_P \left(|P|^2 - \frac{v_{\rm PQ}^2}{2}\right)^2 + \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\rm Pl}^{2n-4}}|S|^{2n} + m_S^2|S|^2 + \left(\frac{\lambda}{m!\ell!M_{\rm Pl}^{m+\ell-4}}S^m P^\ell + \text{h.c.}\right)$$

l, m, n: integers



The PQ scalar is back to the symmetric phase after inflation by the thermal effect.

→ string-DW network appears.

The mixing term is essential for string-DW network decay.

$$V_{\rm PQ}(P,S) \supset \lambda_P \left(|P|^2 - \frac{v_{\rm PQ}^2}{2}\right)^2 + \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\rm Pl}^{2n-4}}|S|^{2n}$$

$$+ m_S^2|S|^2 + \left(\frac{\lambda}{m!\ell! M_{\rm Pl}^{m+\ell-4}} S^m P^\ell + \text{h.c.}\right)$$

$$P = \frac{v_{\rm PQ}}{\sqrt{2}} e^{ia/v_{\rm PQ}}$$

$$S = \frac{\chi}{\sqrt{2}} e^{ib/\chi}$$

Nonzero value of S induces an effective PQ breaking potential.

$$V_{\cancel{\rm PQ}} \simeq -\frac{1}{\ell^2} m_{\cancel{\rm PQ}}^2 v_{\rm PQ}^2 \cos\left(\ell \frac{a}{v_{\rm PQ}} + m\frac{b}{\chi} + \delta\right)$$

$\equiv \delta'$ ($\because$ b doesn't move.)

→

$$m_{\cancel{\rm PQ}}^2(T) \simeq \frac{|\lambda|\ell^2}{2^{\ell/2-1} m!\ell!}\frac{\langle S\rangle^m v_{\rm PQ}^{\ell-2}}{M_{\rm Pl}^{m+\ell-4}}$$

We need behavior of S.

Evolution of S

Hariagaya, Ibe, Kawasaki, Yanagida (2015),
Ibe, Kobayashi, Suzuki, Yanagida (2020)

$$V_{\rm PQ}(P,S) \supset m_S^2|S|^2 + \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\rm Pl}^{2n-4}}|S|^{2n} + \left(\frac{\lambda}{m!\ell! M_{\rm Pl}^{m+\ell-4}} S^m P^\ell + {\rm h.c.}\right)$$

This term could backreact the evolution.

$$|\ddot{S}| + 3H|\dot{S}| + \frac{n\lambda_S^2}{(n!)^2 M_{\rm Pl}^{2n-4}}|S|^{2n-1} = 0$$

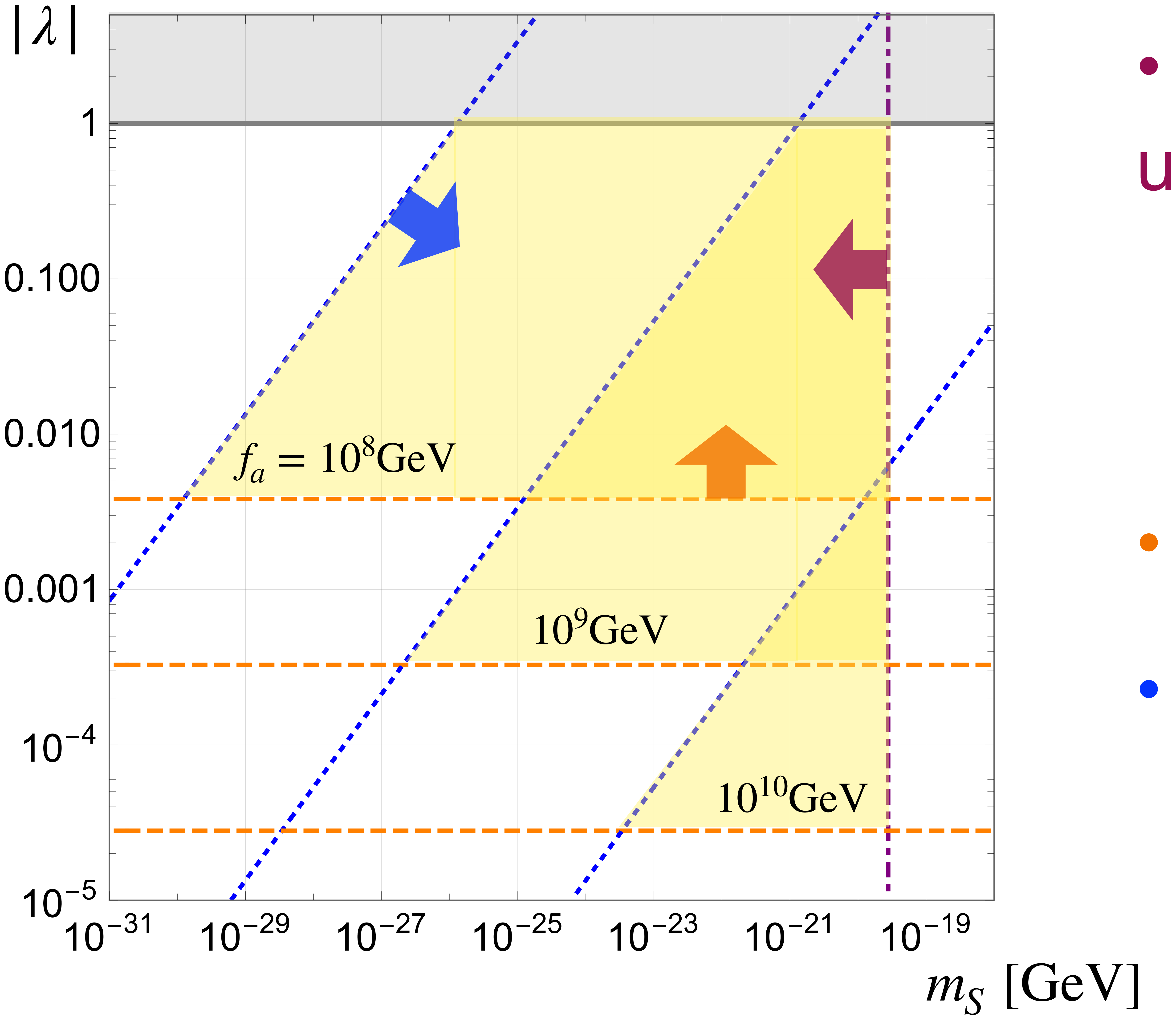
→ $$\langle |S| \rangle \simeq \left[\frac{2(n-3)(n!)^2}{n(n-1)^2\lambda_S^2}\right]^{\frac{1}{2(n-1)}} \left(\frac{H}{M_{\rm Pl}}\right)^{\frac{1}{n-1}} M_{\rm Pl} \quad \text{for } n \geq 6$$

$\therefore V_{\rm PQ} \propto H^{m/(2n-2)}$ slowly decreases

When $H \sim m_S$, S starts to oscillate at the origin ($S \sim 0$), and the effective potential $V_{\rm PQ}$ disappears. → **Bias with a timer!**

Our focus in parameter spaces

Hao, **SN**, Nakai, Suzuki (2025)



$(N_{\mathrm{DW}}, l, m, n) = (2,3,9,6)$ Fix this set in this talk

- We assume $V_{\not{PQ}}$ remains at least until QCD scale.

$$m_S \lesssim \sqrt{\frac{\pi^2 g_*}{90}} \frac{\Lambda_{\mathrm{QCD}}^2}{M_{\mathrm{Pl}}} \simeq 3 \times 10^{-11} \mathrm{eV}$$

- $T_{\mathrm{osc}} > T_{\mathrm{osc}}^{(\mathrm{conv})}$
- To avoid backreaction

$$\frac{1}{(n!)^2} \frac{\lambda_S^2}{M_{\mathrm{Pl}}^{2n-4}} |S|^{2n} > \frac{|\lambda|}{m!l!M_{\mathrm{Pl}}^{m+l-4}} |S|^m v_{\mathrm{PQ}}^l$$

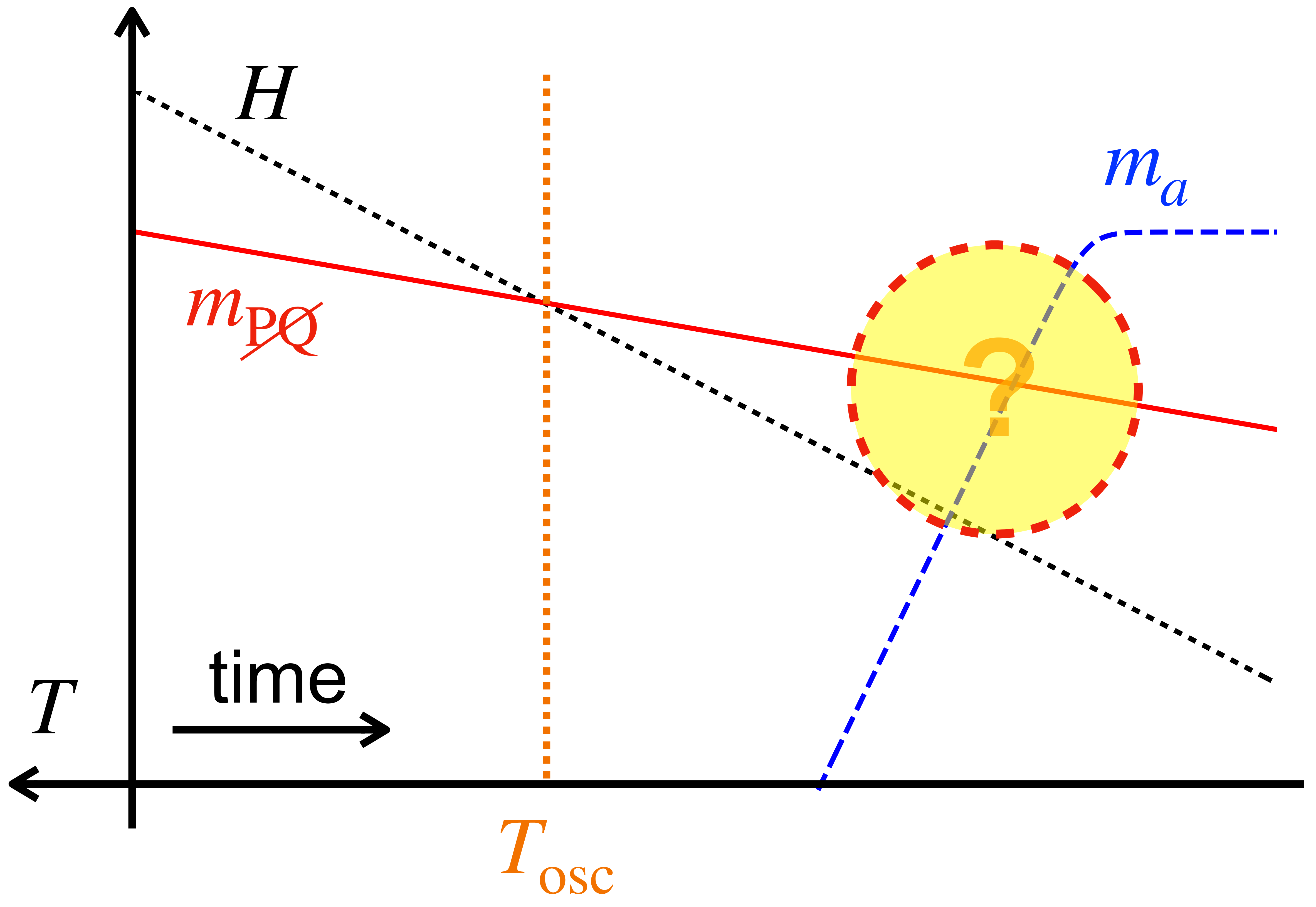
3. Evolution of string-wall system

Hao, **SN**, Nakai, Suzuki (2025)

At $T < T_{\rm osc}$, the l walls are attached with the string.

$$V_{\cancel{\rm PQ}} = -\frac{1}{\ell^2} m_{\cancel{\rm PQ}}^2 v_{\rm PQ}^2 \cos\left(\ell \frac{a}{v_{\rm PQ}} + \delta'\right)$$

\# DW for $V_{\cancel{\rm PQ}}$



Consider how this system can collapse from the following aspects:

(i) Volume pressure

(ii) Structural instability

(i) Volume pressure

The potential difference induces the volume pressure on the domain wall, which makes the system unstable when $p_V \sim p_T$.

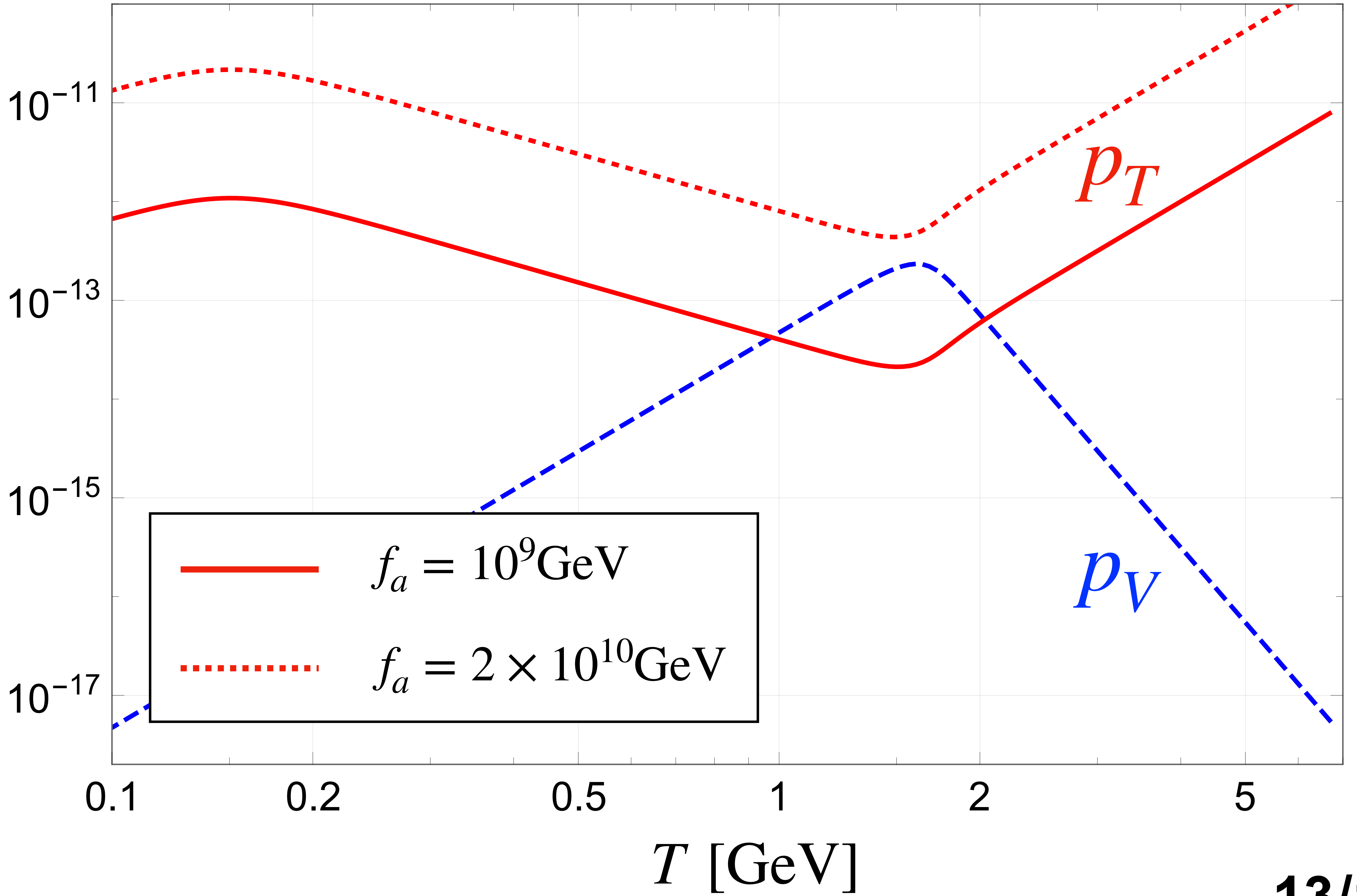
<u>Volume pressure</u>

$$p_V \sim \Delta V$$

<u>Tension force</u>

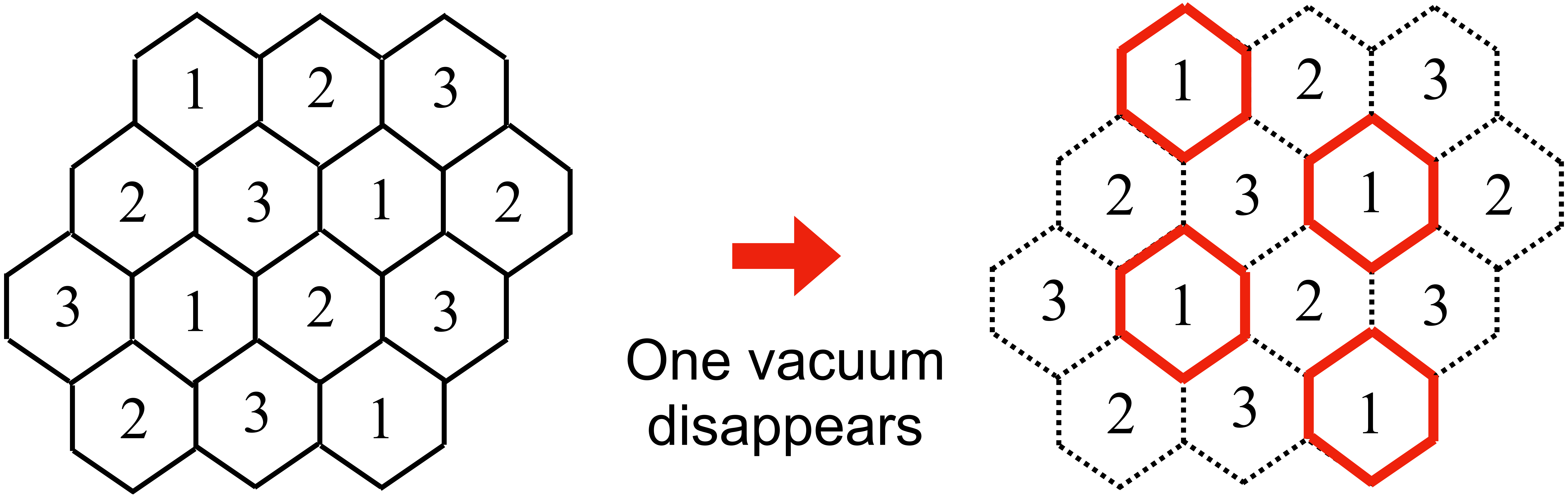
$$p_T \sim \sigma_{\mathrm{wall}} H$$

$f_a \lesssim 10^9\mathrm{GeV}$ is required for the system collapse due to p_V.



(ii) Structural instability

Consider $(N_{\rm DW}, l) = (2{,}3)$.



- The closed domain walls collapse soon due to the wall tension.
- In addition, the axion distribution is biased at the QCD scale.

As a result, such systems may be broken soon.

Kitajima, Lee, Takahashi, Yin (2023)

Axion DM from defects

The annihilation temperature $T_{\rm ann}$ is an important factor for the axion abundance from the defect decay.

When $|V_{\cancel{PQ}}| \sim |V_{\rm QCD}|$, the system seems to be most unstable.

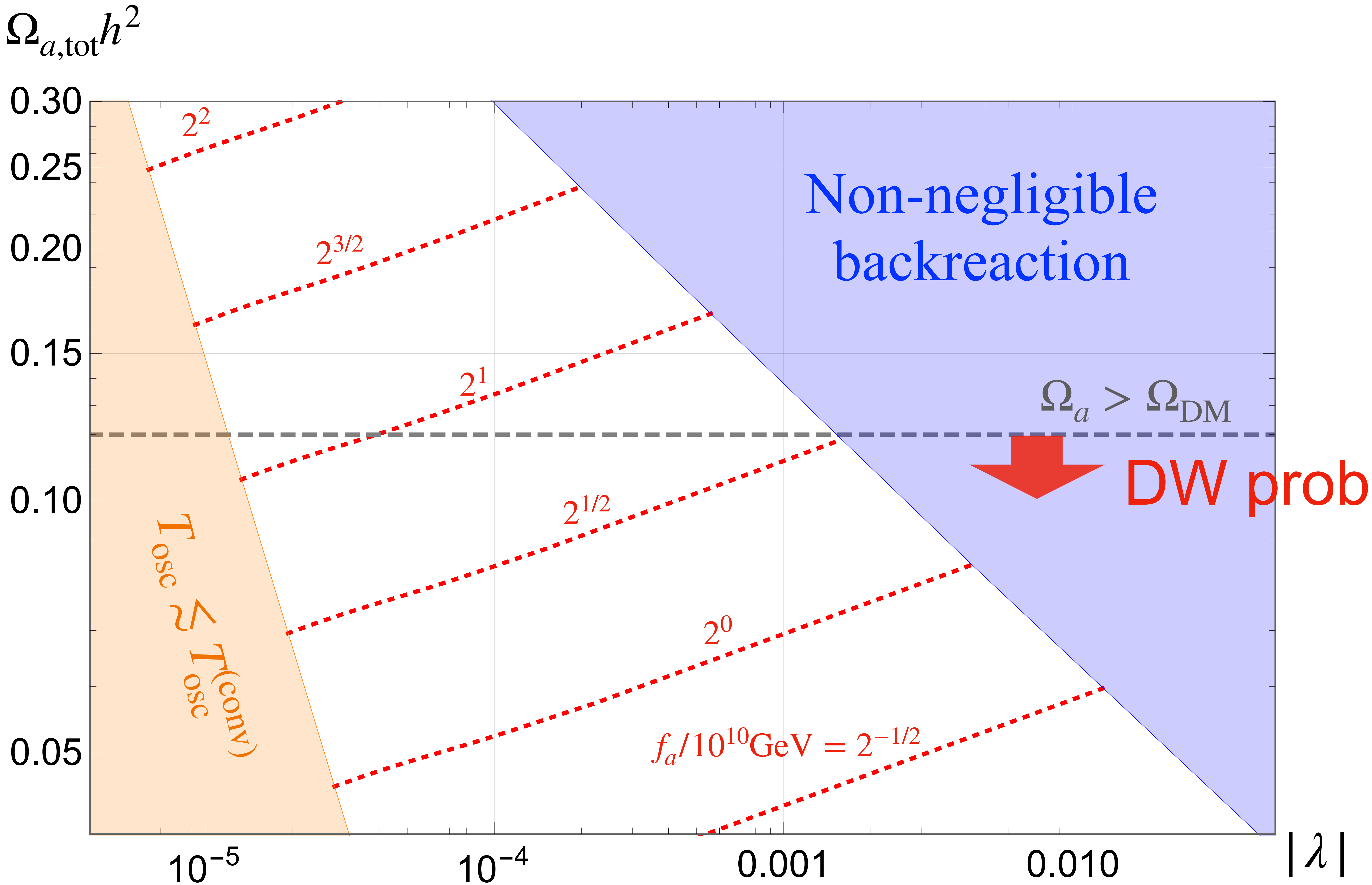
$$m_{\cancel{PQ}}(T_{\rm tr}) \sim \frac{\ell}{N_{\rm DW}} m_a(T_{\rm tr}) \;\leftrightarrow\; T_{\rm tr} \simeq 1.6\ {\rm GeV} \left(\frac{|\lambda|}{0.01}\right)^{-\alpha} \left(\frac{v_{PQ}}{2\times 10^9\ {\rm GeV}}\right)^{-\ell\alpha}$$

We assume that the annihilation occurs at $T_{\rm ann} = \kappa T_{\rm tr}$. $(\kappa < 1)$

$$\Omega_{a,{\rm dec}} h^2 \simeq 0.12 \frac{1}{\sqrt{1+\epsilon_a^2}} \left(\frac{\kappa}{0.1}\right)^{-1} \left(\frac{|\lambda|}{2\times 10^{-4}}\right)^{\alpha} \left(\frac{N_{\rm DW}}{2}\right)^{\ell\alpha} \left(\frac{f_a}{2.4\times 10^{10}\ {\rm GeV}}\right)^{1+\ell\alpha}$$

4. Viable parameter spaces

Hao, **SN**, Nakai, Suzuki (2025)



$$\Omega_{a,\mathrm{tot}} = \Omega_{a,\mathrm{dec}} + \Omega_{a,\mathrm{mis}}$$

DW problem can be solved.

$$f_a \lesssim 2 \times 10^{10}\mathrm{GeV}$$

$(N_{\mathrm{DW}}, l, m, n) = (2,3,9,6)$ $\delta' = 1$ $\kappa = 0.1$ $(T_{\mathrm{ann}} = \kappa T_{\mathrm{tr}})$

Summary

- We consider the DW problem by introducing a mixing coupling between the PQ scalar and a light scalar.

- The mixing coupling induces a time-dependent bias potential, which makes the string-DW system unstable even in the presence of small volume pressure.

- In addition of misalignment contribution, we show that the overproduction can be avoided for $f_a \lesssim 10^{10}\text{GeV}$.

Thanks!

Back up

Possible solutions

- Lazarides-Shafi mechanism Lazarides, Shafi (1982)
- Biased potential Sikivie (1972), Mohanty, Stecker (1984), Gelmini, Gleiser, Kolb (1989)
- Biased population

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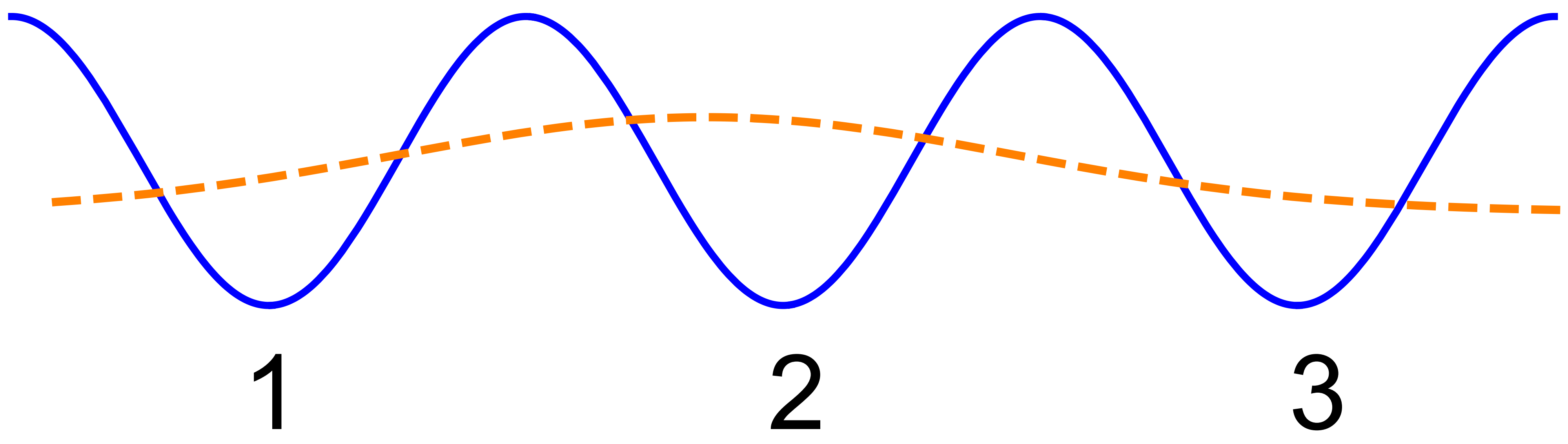
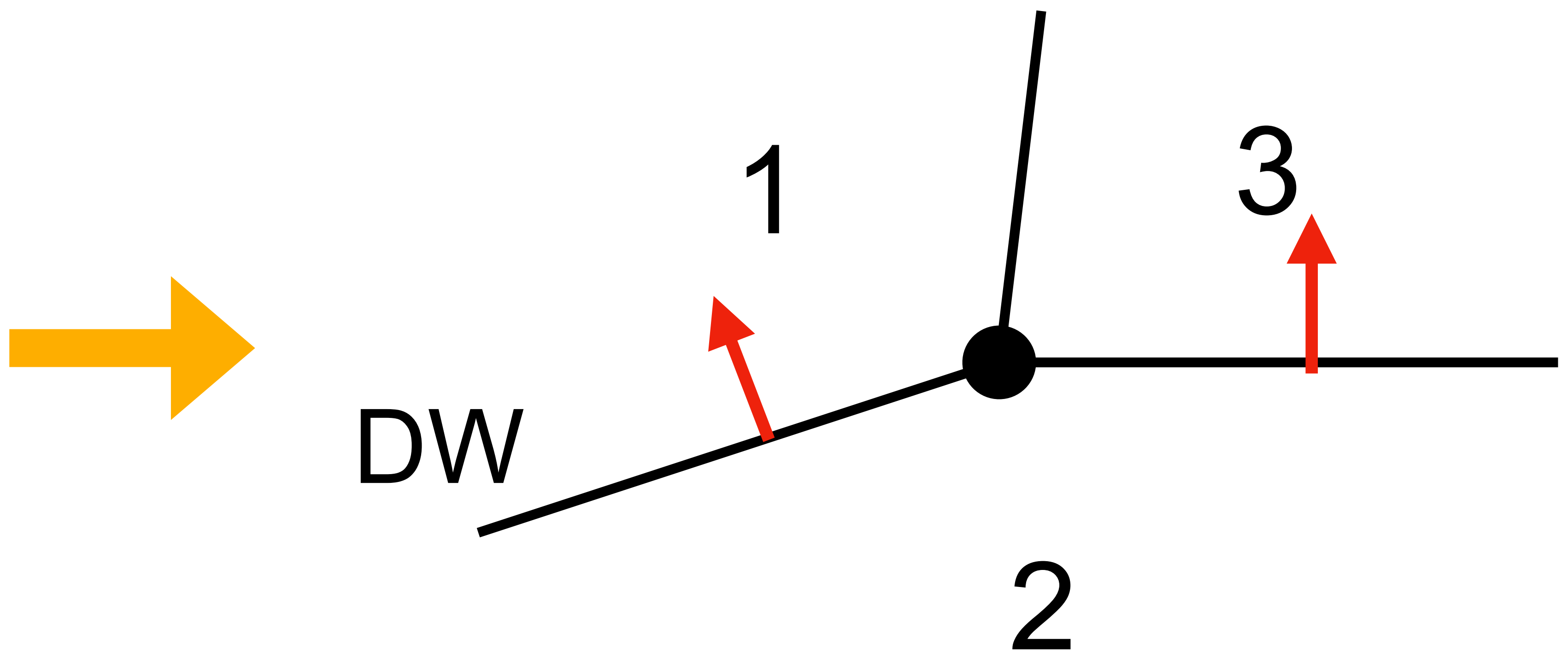




However, such explicit breaking potentials misalign the potential minimum from the CP conserving one.

DW-quality tension Ringwald, Saikawa (2016)

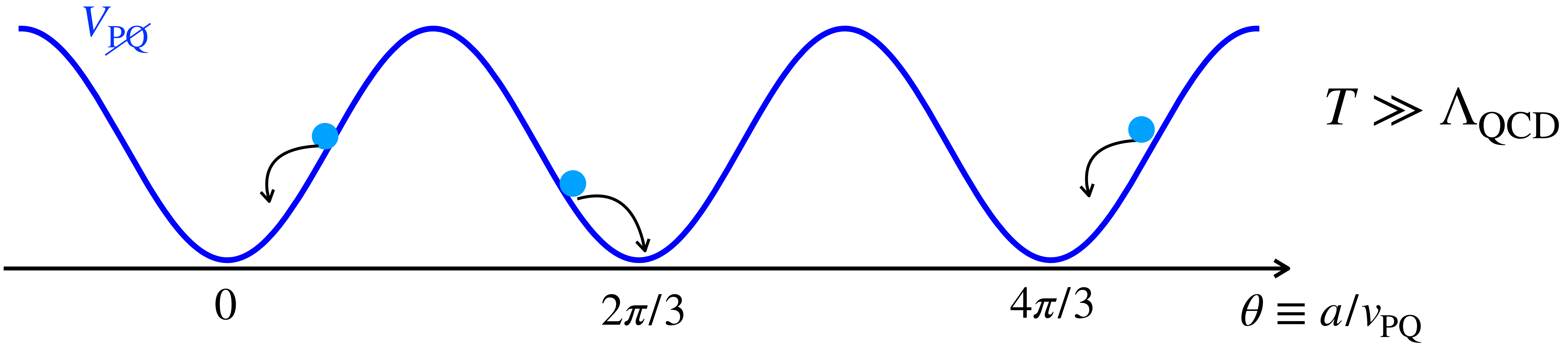
Misalignment contribution

The number density can be estimated as the average,

$$n_a \simeq \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta_{\rm ini} n_a(\theta_{\rm ini}) \simeq \frac{1}{l}\sum_{r=1}^{l} n_a^{(r)}$$

Here anharmonic effect is ignored.

Roughly, the oscillation amplitude at $T \sim \Lambda_{\rm QCD}$ can be determined by which minimum the axion drops.



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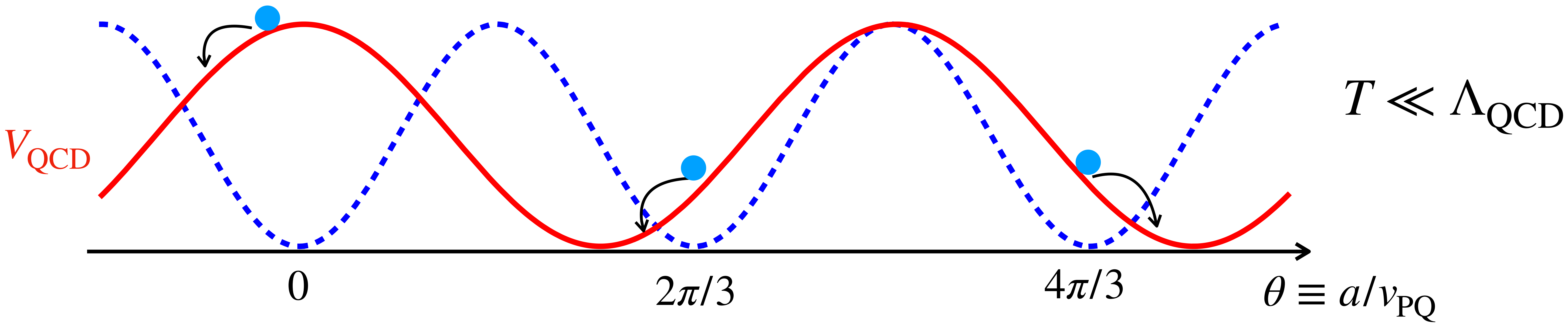
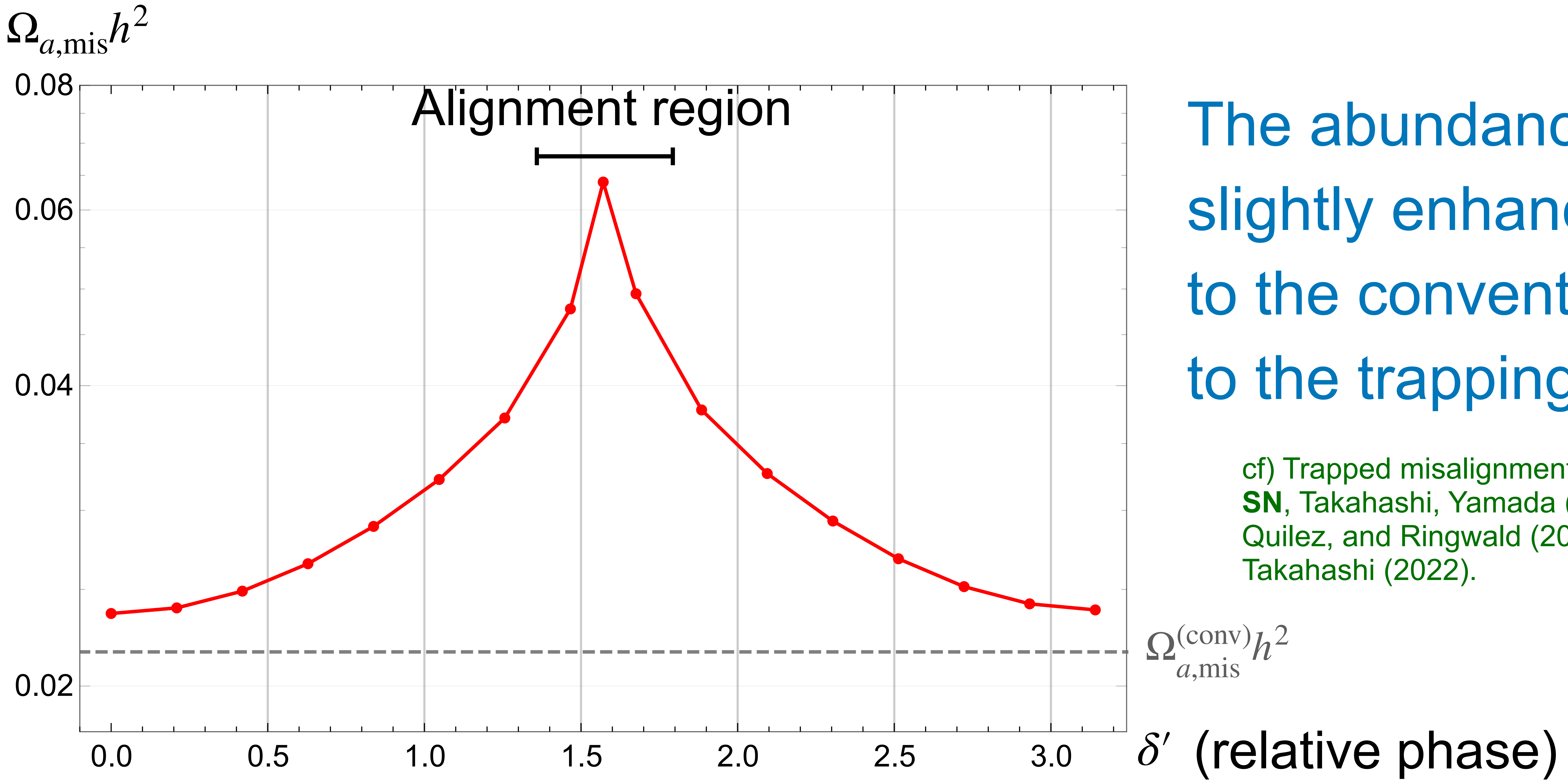


Numerical results

Hao, **SN**, Nakai, Suzuki (2025)



The abundance can be slightly enhanced compared to the conventional one due to the trapping effect.

cf) Trapped misalignment
SN, Takahashi, Yamada (2020). Di Luzio, Gavela, Quilez, and Ringwald (2021). Jeong, Matsukawa, **SN**, Takahashi (2022).

$f_a = 10^{10}\mathrm{GeV}$ is taken.

Isocurvature problem

S field breaks the PQ symmetry during inflation, and the phase acquires quantum fluctuations.

$$V_{\cancel{\rm PQ}} \simeq -\frac{1}{\ell^2} m_{\cancel{\rm PQ}}^2 v_{\rm PQ}^2 \cos\left(\ell \frac{a}{v_{\rm PQ}} + m\frac{b}{\chi} + \delta\right)$$

$$\delta b \to \delta\rho_a$$

The density perturbation has a peculiar footprint on CMB anisotropy spectrum.

$$\frac{\mathcal{P}_{\rm iso}}{\mathcal{P}_\zeta} < 0.038$$

Planck collaboration

Evolution during inflation

Thanks to the dynamics of S, our setup can implement a mechanism for suppressing the isocurvature.

$$V(S) \simeq \frac{1}{(n!)^2} \frac{\lambda_S^2}{M_{\rm Pl}^{2n-4}} |S|^{2n} - c_S H_{\rm inf}^2 |S|^2 \qquad c_S > 0$$

$$\rightarrow \quad \langle S_{\rm inf} \rangle \simeq \left(\sqrt{\frac{c_S}{n}} \frac{n!}{\lambda_S} \right)^{\frac{1}{n-1}} \left(\frac{H_{\rm inf}}{M_{\rm Pl}} \right)^{\frac{1}{n-1}} M_{\rm Pl}$$

e.g. $\langle S_{\rm inf} \rangle \simeq M_{\rm Pl}$ for $H_{\rm inf} = 10^{12}{\rm GeV}$ and $\lambda_S = 10^{-4}$

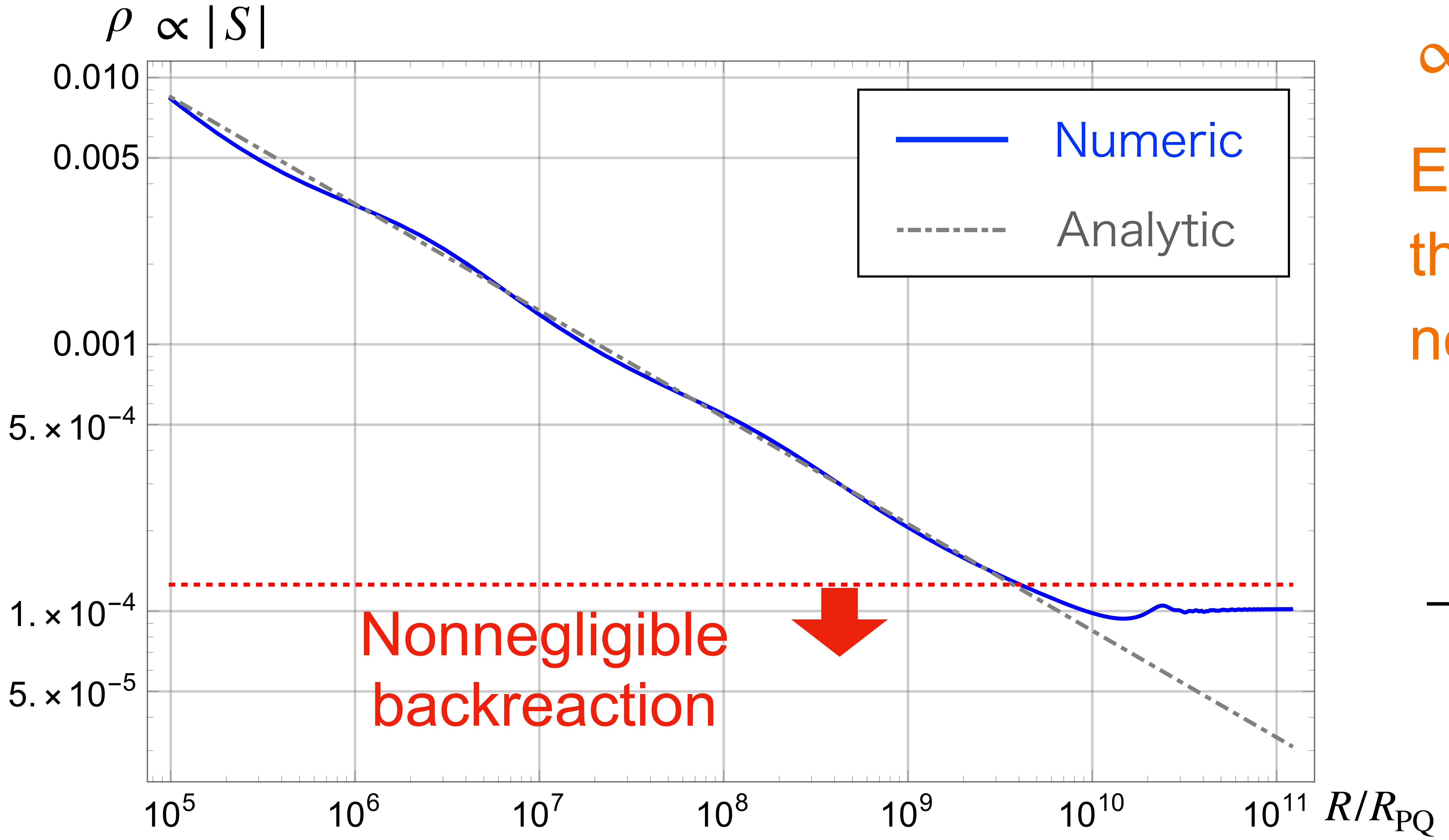
$$\frac{\delta a}{f_a} \sim \frac{\delta b}{\chi} \sim \frac{H_{\rm inf}}{2\pi \langle S_{\rm inf} \rangle}$$

Significantly suppressed

Linde (1991)

What is Backreaction

$$V_{\mathrm{PQ}}(P,S) \supset m_S^2|S|^2 + \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\mathrm{Pl}}^{2n-4}}|S|^{2n} + \left(\frac{\lambda}{m!\ell!M_{\mathrm{Pl}}^{m+\ell-4}} S^m P^\ell + \mathrm{h.c.}\right)$$



$\propto \cos(l\theta + m\theta_b + \delta)$

Energy is minimized, so that the sign is flipped to negative.