Precise predictions on Higgs pair productions for general self-couplings

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Higgs potential 2025
Chengdu, 12.19

Higgs sector in SM

In the Standard Model (SM), the Higgs sector is described by

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 (\phi^\dagger \phi) - \lambda (\phi^\dagger \phi)^2 \,,$$

After symmetry breaking, the Higgs field can be written as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

In the broken phase the Lagrangian takes the form

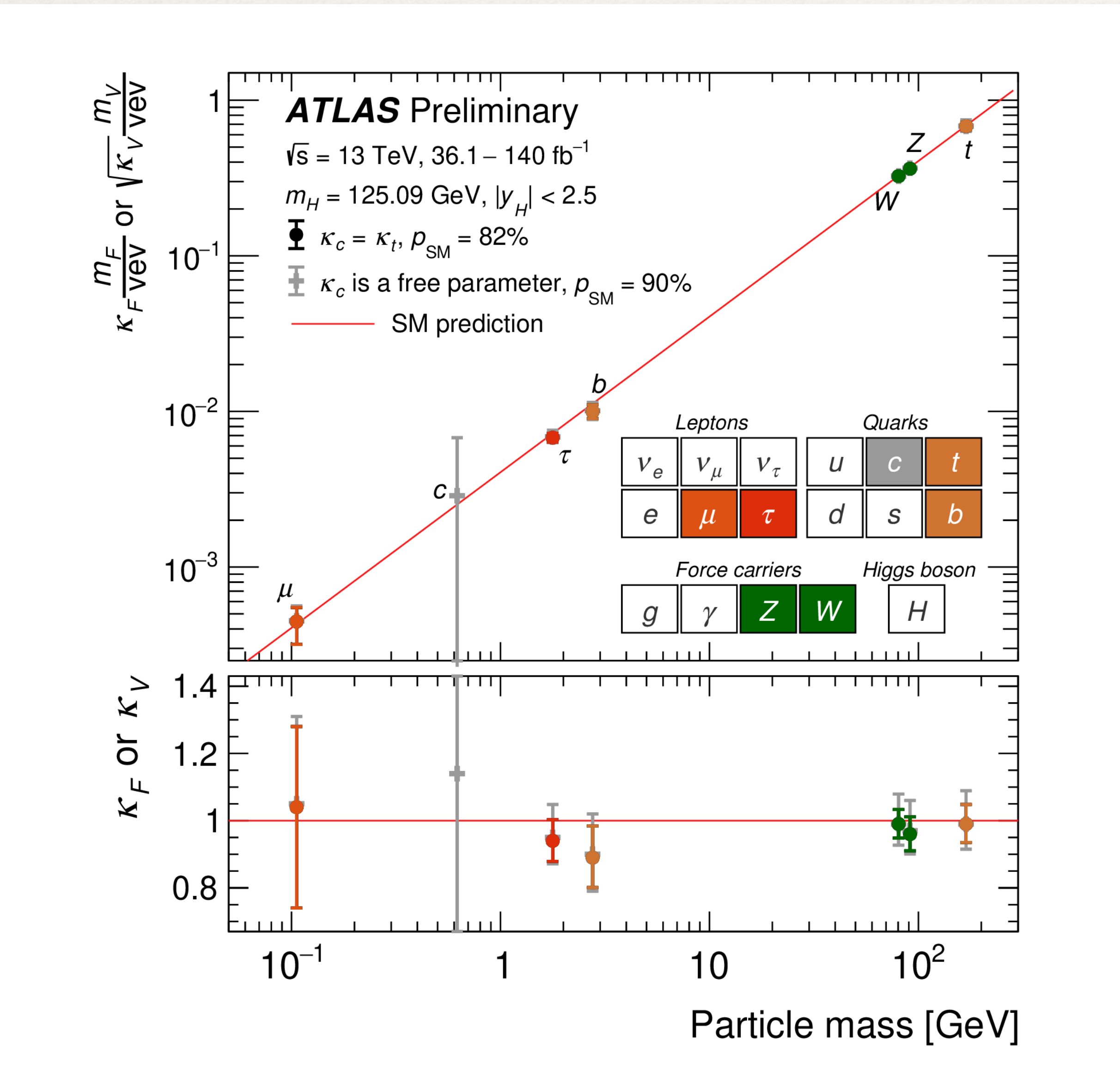
$$\mathcal{L} \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} m_H^2 H^2 - v\lambda H^3 - \frac{\lambda}{4} H^4$$

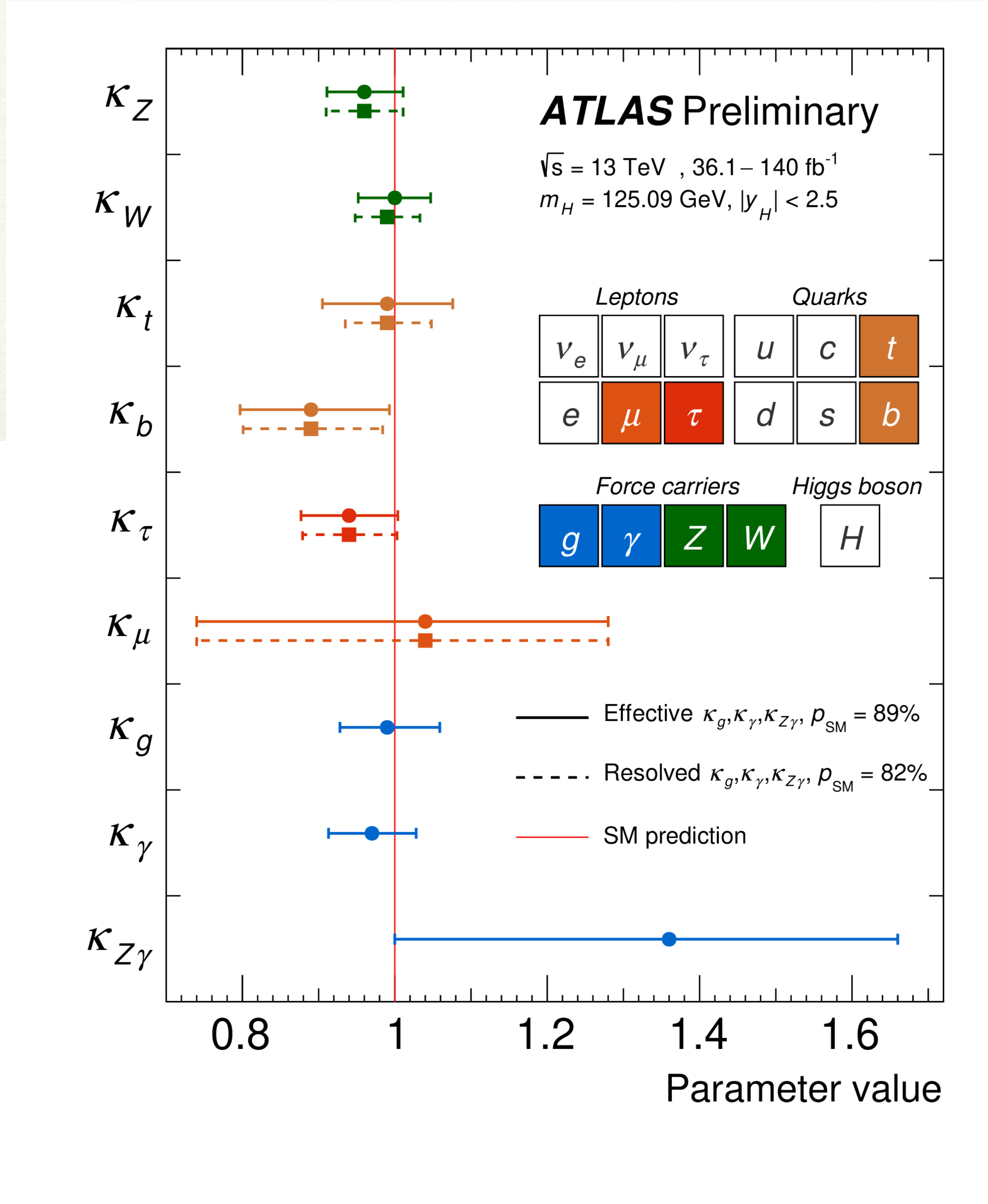
The EW gauge boson and Higgs masses are expressed as

$$m_W = \frac{g_2 v}{2} \,, m_Z = \frac{\sqrt{g_1^2 + g_2^2}\, v}{2} \,, m_H = \sqrt{2\lambda} v$$

So far, the H(125) properties are consistent with the SM expectation!

Its couplings with SM particles are proportional to their masses; <u>Higgs mechanism</u>



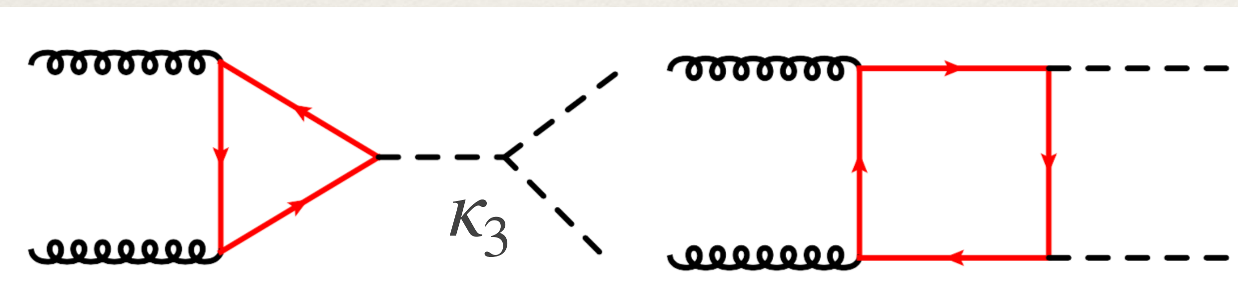


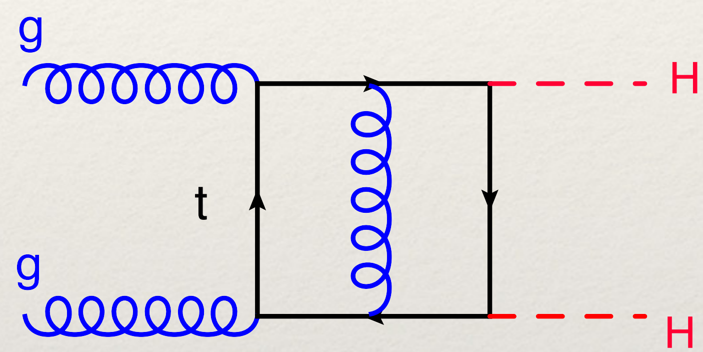
Higgs sector in BSM

In the κ framework

"Handbook of LHC Higgs Cross Sections" 1307.1347

$$\mathcal{L} \supset m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu - \frac{1}{2} m_H^2 H^2 - \kappa_3 v \lambda H^3 - \kappa_4 \frac{\lambda}{4} H^4$$



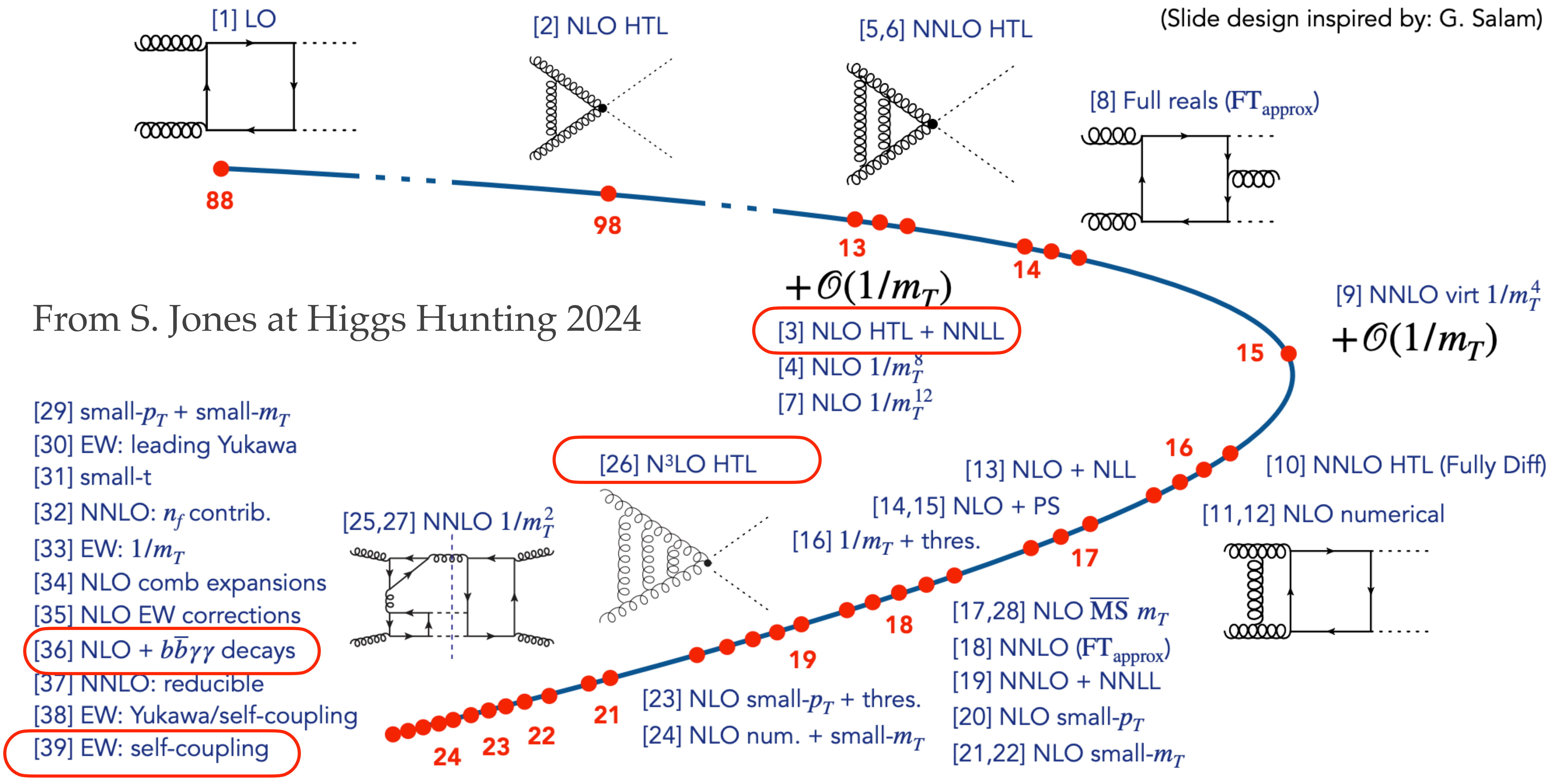


$$\sigma_{HH} = A + B\kappa_3 + C\kappa_3^2$$

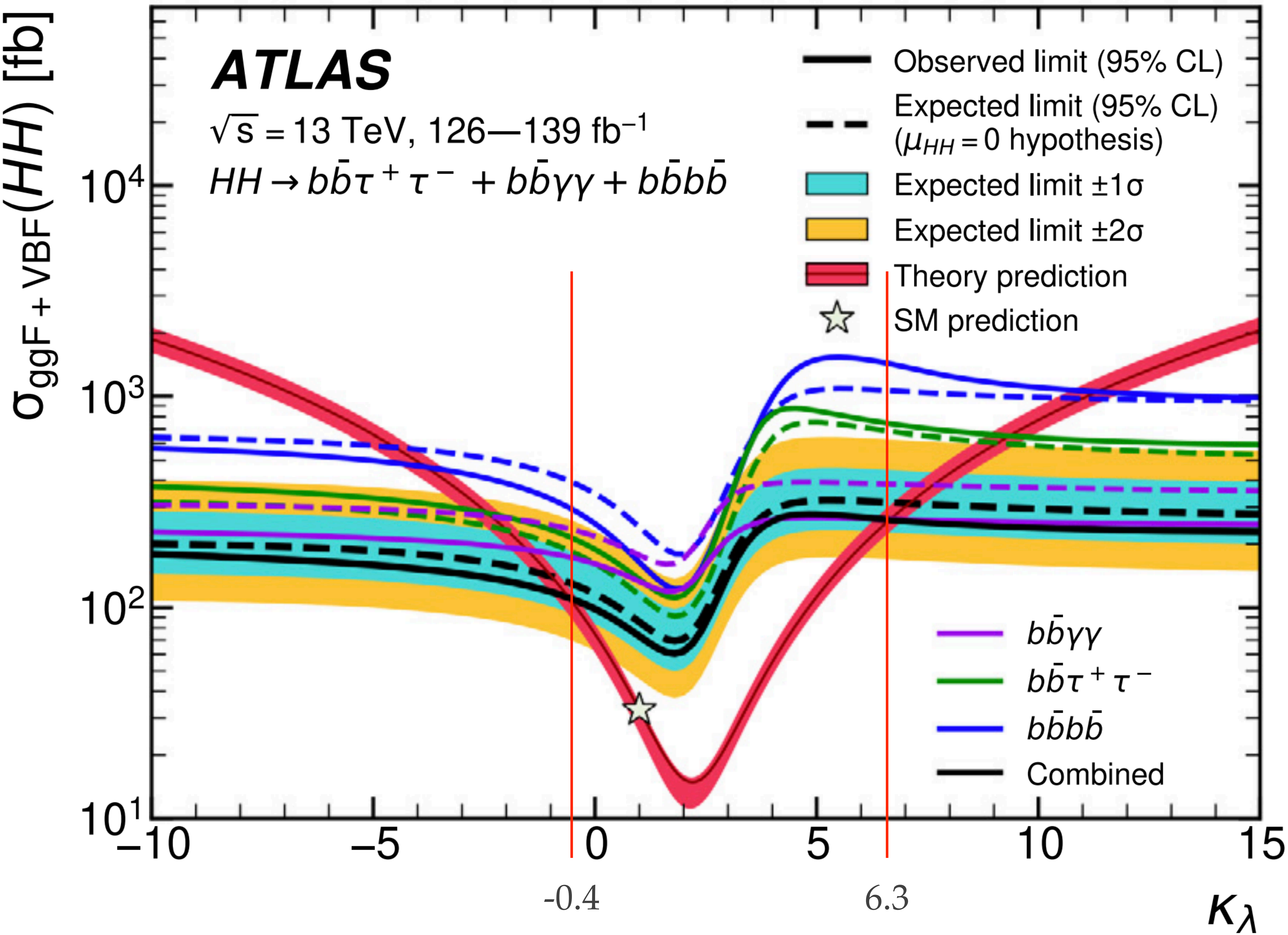
computation	A [fb]	A/A(LO)	B [fb]	B/B(LO)	C [fb]	C/C(LO)
LO m_t fin	35.0		-23.0		4.73	
NLO m_t fin	62.6	1.79	-44.4	1.93	9.64	2.04
NLO m_t fin × NNLO SM FTApprox	70.0	2.00	-49.6	2.16	10.8	2.28
NNLO + NNLL $m_t \to \infty$ × NNLO+NLL SM (partial m_t fin)	71.3	2.04	-47.7	2.08	9.93	2.10

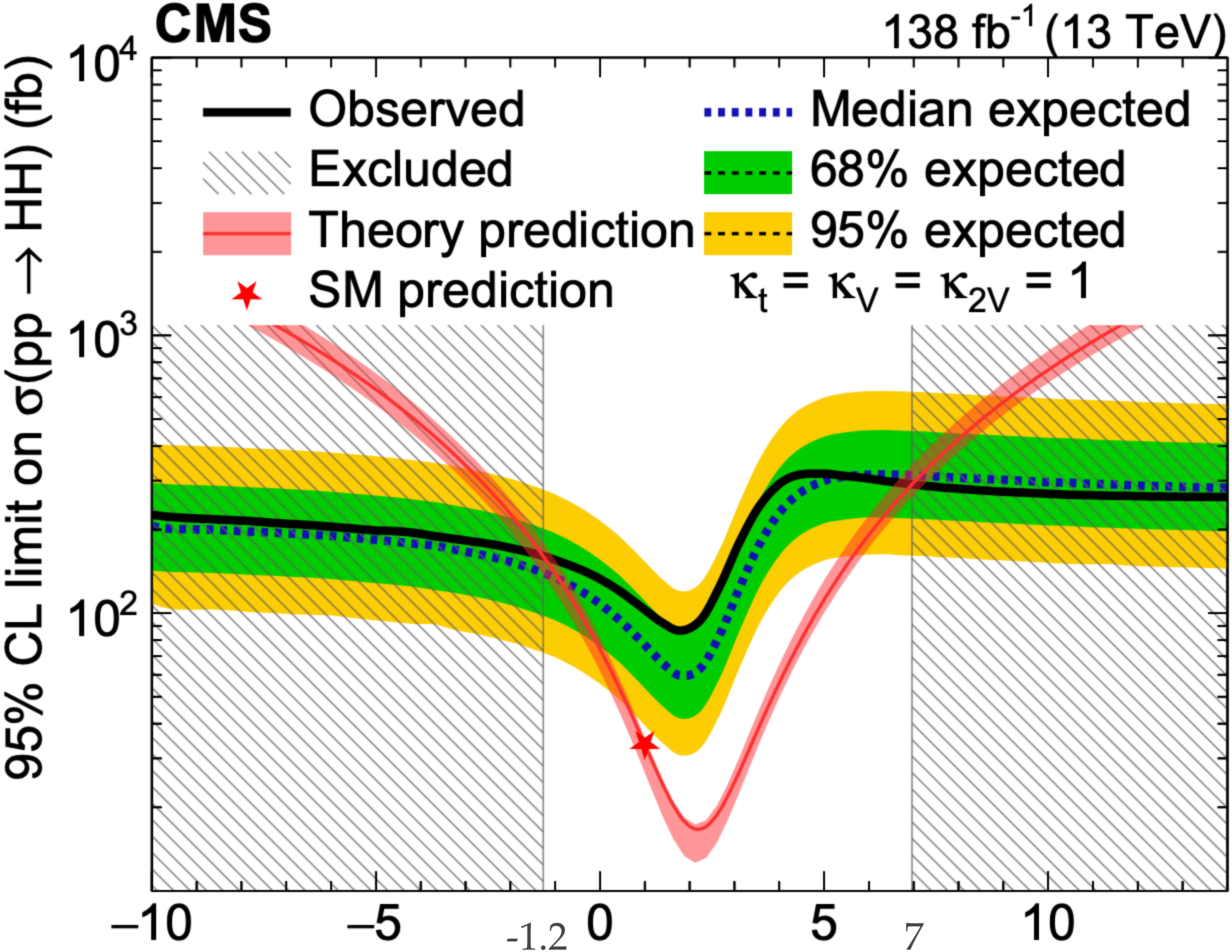
1910.00012

Long history in high precision calculations



[1] Glover, van der Bij 88; [2] Dawson, Dittmaier, Spira 98; [3] Shao, Li, Li, Wang 13; [4] Grigo, Hoff, Melnikov, Steinhauser 13; [5] de Florian, Mazzitelli 13; [6] Grigo, Melnikov, Steinhauser 14; [7] Grigo, Hoff 14; [8] Maltoni, Vryonidou, Zaro 14; [9] Grigo, Hoff, Steinhauser 15; [10] de Florian, Grazzini, Hanga, Kallweit, Lindert, Maierhöfer, Mazzitelli, Rathlev 16; [11] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Schubert, Zirke 16; [12] Borowka, Greiner, Heinrich, SPJ, Kerner, Schlenk, Zirke 16; [13] Ferrera, Pires 16; [14] Heinrich, SPJ, Kerner, Luisoni, Vryonidou 17; [15] SPJ, Kuttimalai 17; [16] Gröber, Maier, Rauh 17; [17] Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 18; [18] Grazzini, Heinrich, SPJ, Kallweit, Kerner, Lindert, Mazzitelli 18; [19] de Florian, Mazzitelli 18; [20] Bonciani, Degrassi, Giardino, Gröber 18; [21] Davies, Mishima, Steinhauser, Wellmann 18, 18; [22] Mishima 18; [23] Gröber, Maier, Rauh 19; [24] Davies, Heinrich, SPJ, Kerner, Mishima, Steinhauser, David Wellmann 19; [25] Davies, Steinhauser 19; [26] Chen, Li, Shao, Wang 19, 19; [27] Davies, Herren, Mishima, Steinhauser 19, 21; [28] Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira 21; [29] Bellafronte, Degrassi, Giardino, Gröber, Vitti 22; [30] Davies, Mishima, Schönwald, Steinhauser, Zhang 22; [31] Davies, Mishima, Schönwald, Steinhauser 23; [32] Davies, Schönwald, Steinhauser 23; [33] Davies, Schönwald, Steinhauser, Zhang 23; [34] Bagnaschi, Degrassi, Gröber 23; [35] Bi, Huang, Huang, Ma Yu 23 [36] Li, Si, Wang, Zhang, Zhao 24; [37] Davies, Schönwald, Steinhauser, Vitti 24; [38] Heinrich, SPJ, Kerner, Stone, Vestner [39] Li, Si, Wang, Zhang, Zhao 24


ATLAS
$\sqrt{s} = 13$ TeV, 126—139 fb^{-1}
$HH \to b\bar{b}\tau^+\tau^- + b\bar{b}\gamma\gamma + b\bar{b}b\bar{b}$
Observed limit (95% CL)
Expected limit (95% CL) ($\mu_{HH} = 0$ hypothesis)
Expected limit ±1σ
Expected limit ±2σ
Theory prediction
SM prediction
$b\bar{b}\gamma\gamma$
$b\bar{b}\tau^+\tau^-$
$b\bar{b}b\bar{b}$
Combined
$\sigma_{ggF + VBF}(HH)$ [fb]
κ_λ
-0.4
6.3

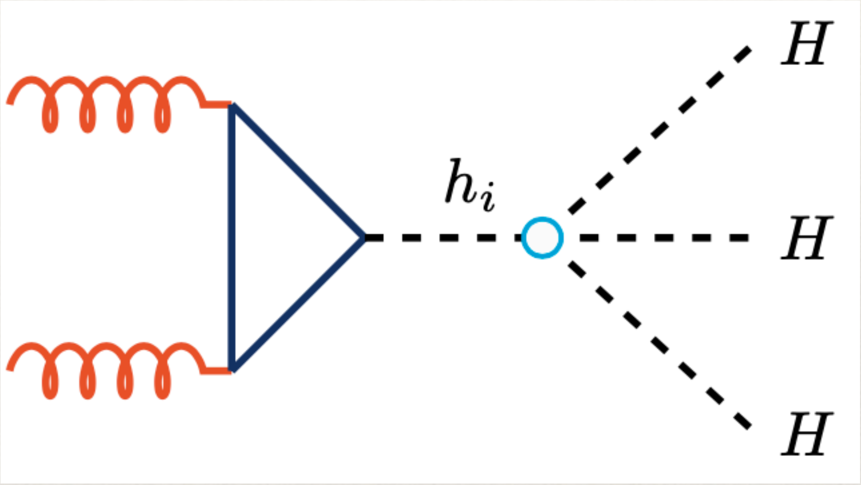


CMS
138 fb-1 (13 TeV)
95% CL limit on σ(pp → HH) (fb)
Observed
Excluded
Theory prediction
SM prediction
Median expected
68% expected
95% expected
κt = κV = κ2V = 1
κλ
-1.2
7

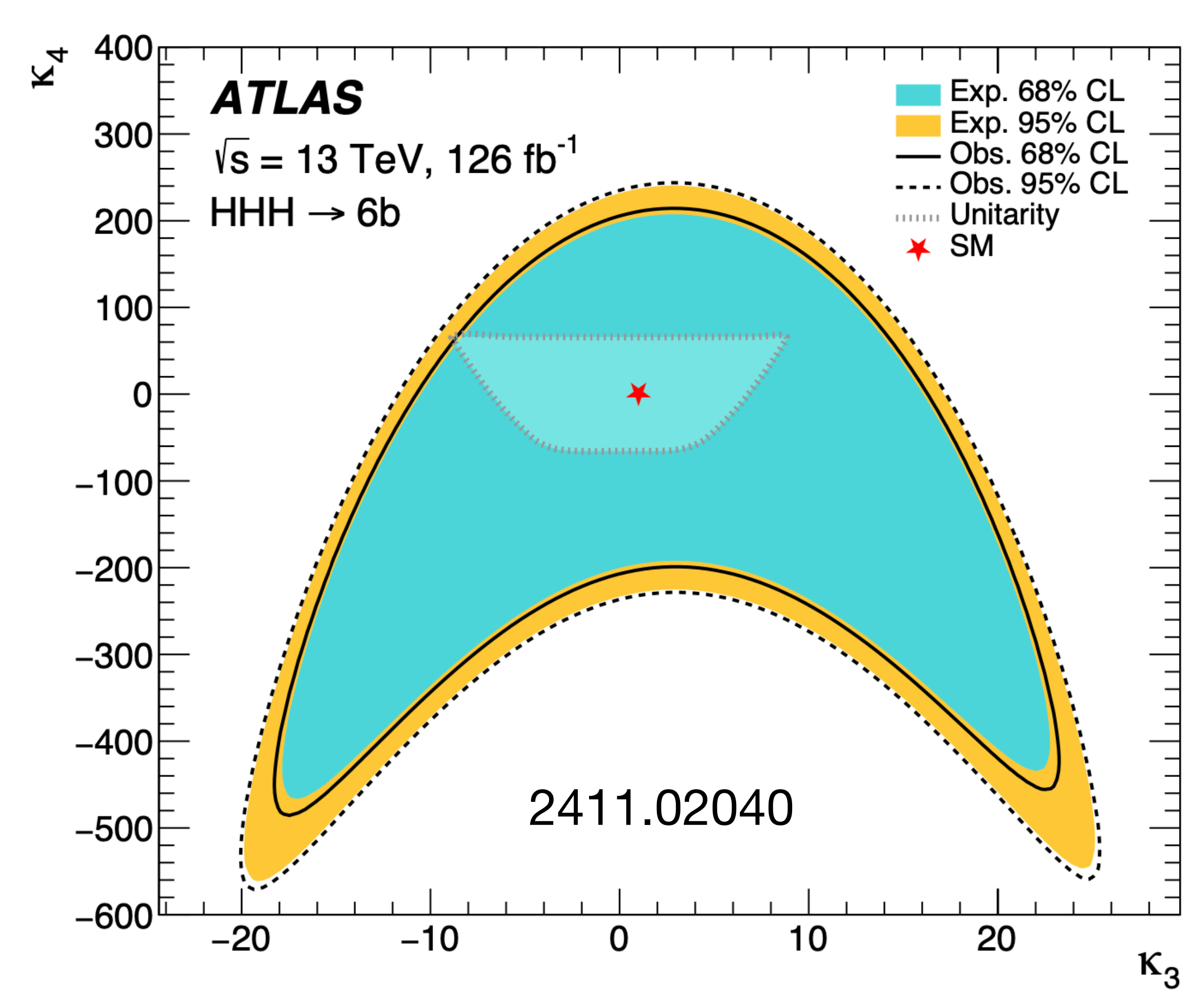
2510.07527

Constraint on Higgs quartic self-coupling

Triple-Higgs production provides a direct probe of the Higgs quartic self-coupling. However, the cross section is 300 times smaller than double-Higgs production cross section.



See Shu Li's talk tomorrow



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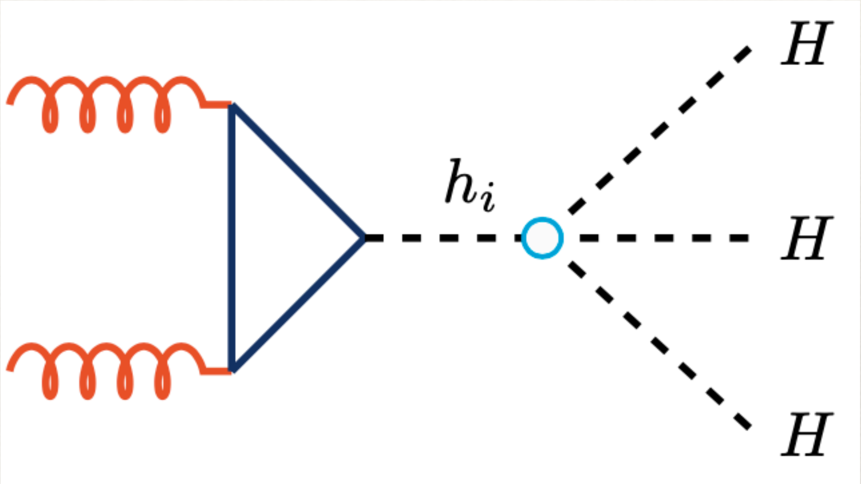

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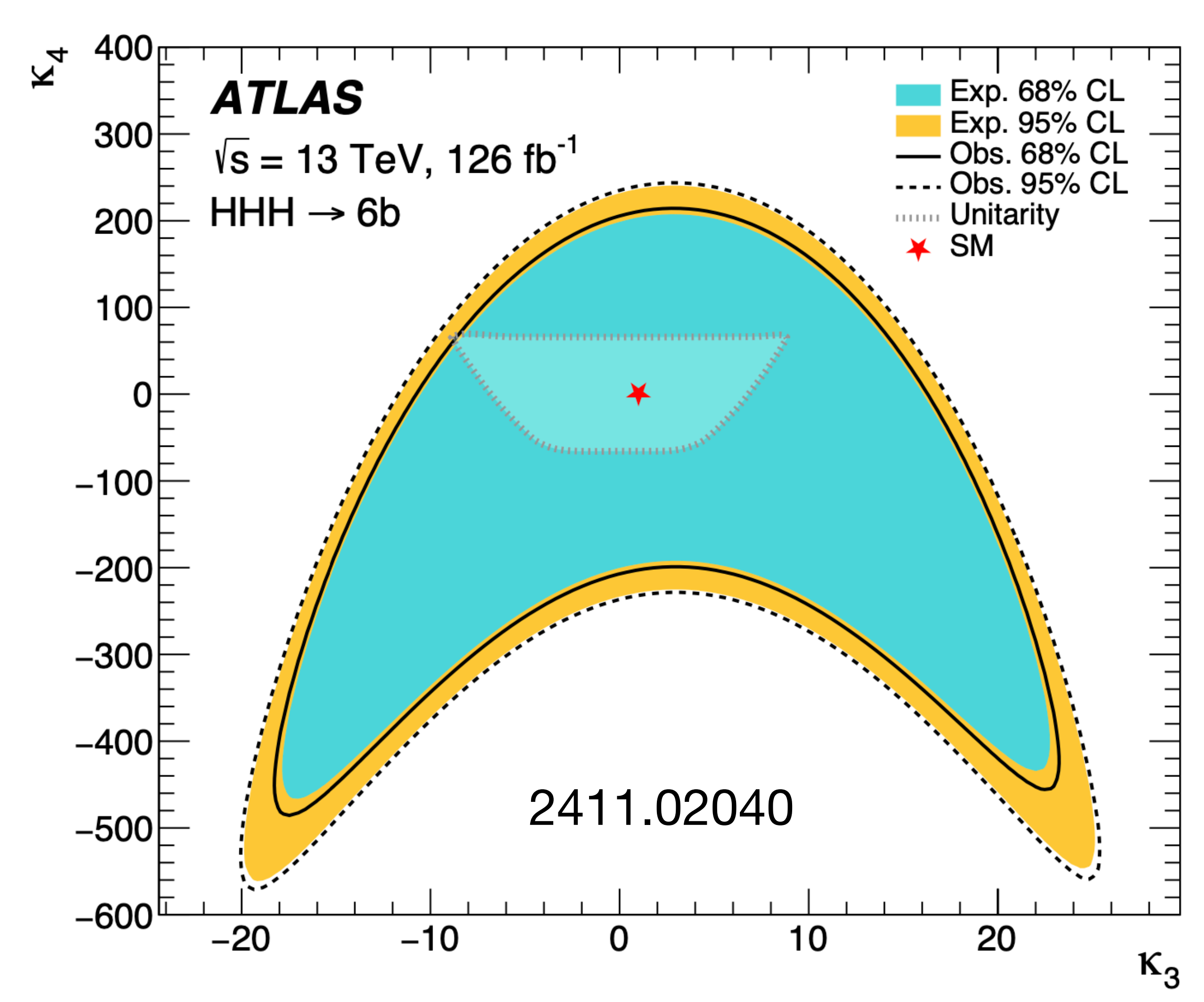
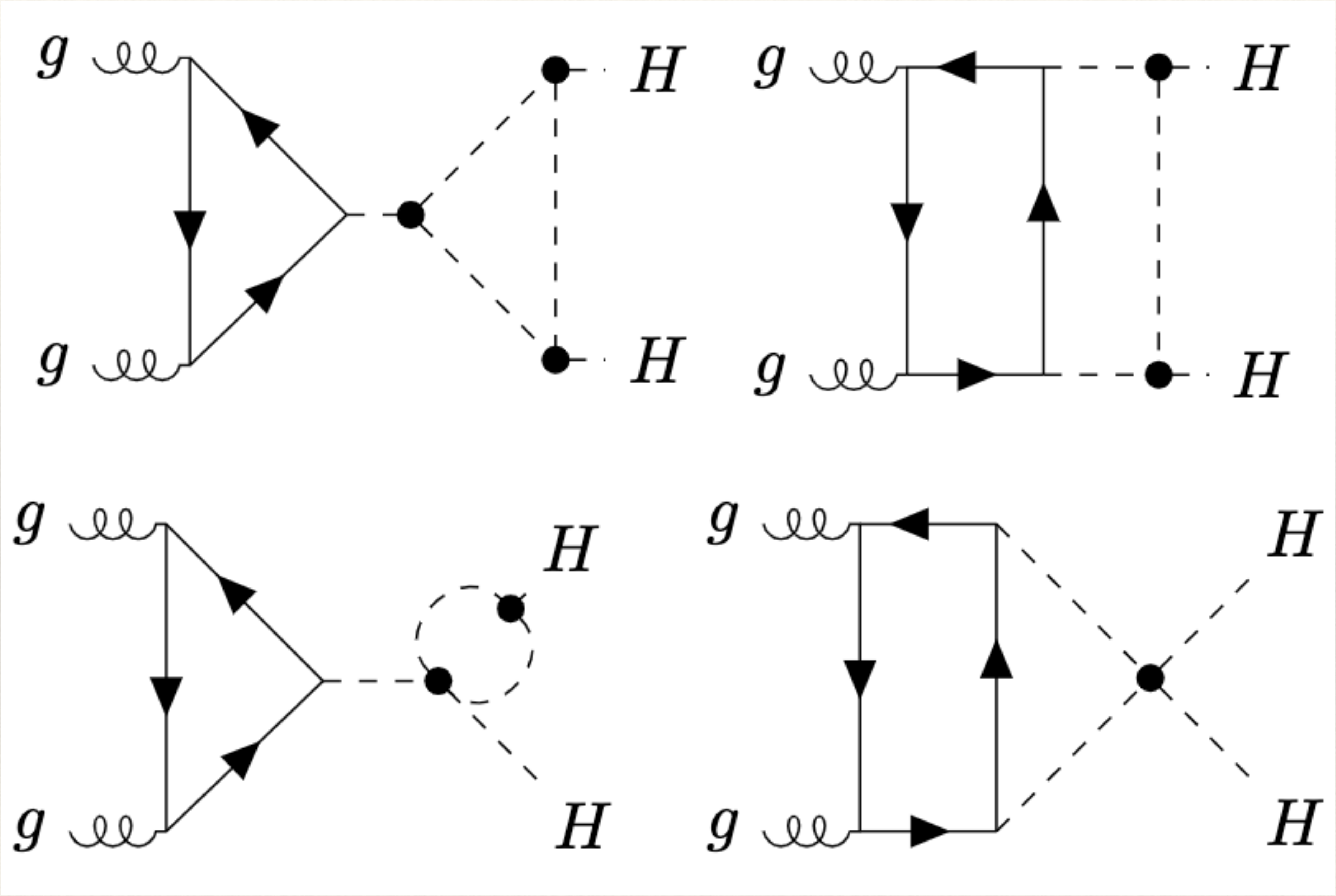


Is it possible to derive the constraint in another way?

A more realistic function form



$$\sigma_{HH} = A + B\kappa_3 + C\kappa_3^2 + D\kappa_3^3 + E\kappa_3^4 + F\kappa_3^2\kappa_4 + G\kappa_3\kappa_4 + H\kappa_4 + \cdots$$

HEFT

- To describe the new physics beyond the SM, we often take a consistent effective field theory framework.
- HEFT: Higgs field as a singlet, non-linear, no explicit constraints on couplings

$$\mathcal{L}_{\mathrm{HEFT}} = \mathcal{L}_2 + \mathcal{L}_4$$

$$\begin{aligned}\mathcal{L}_2 = &\frac{v^2}{4}\left(1 + 2a\frac{H}{v} + b\left(\frac{H}{v}\right)^2 + \cdots\right)\mathrm{Tr}[D_\mu U^\dagger D^\mu U] \\ &+ \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) - \frac{1}{2g^2}\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] \\ &- \frac{1}{2g'^2}\mathrm{Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}.\end{aligned}$$

$$U(\pi^a) = e^{i\pi^a\tau^a/v}$$

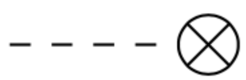
$$\begin{aligned}V(H) = &(-\mu^2 + \lambda v^2)vH + \frac{1}{2}(-\mu^2 + 3\lambda v^2)H^2 \\ &+ \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4.\end{aligned}$$

Buchalla et al, 1307.5017

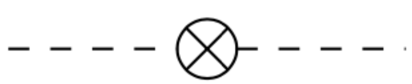
Renormalization

The renormalized Lagrangian in the κ framework after EW gauge symmetry breaking:

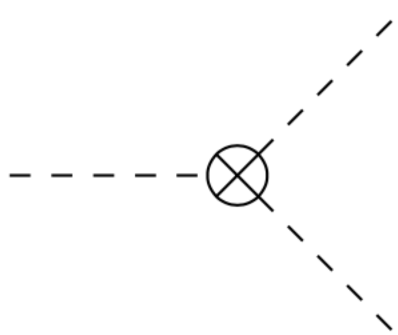
$$\mathscr{L}^{\kappa}_{\rm H} = \frac{1}{2} Z_\phi (\partial_\mu H)^2 - \left(-\frac{1}{2} Z_{\mu^2} Z_\phi Z_v^2 \mu^2 v^2 + \frac{1}{4} Z_\lambda Z_\phi^2 Z_v^4 \lambda v^4 \right) - (Z_\lambda Z_\phi^2 Z_v^3 \lambda v^3 - Z_{\mu^2} Z_\phi Z_v \mu^2 v) H$$
$$- \left(\frac{3}{2} Z_\lambda Z_\phi^2 Z_v^2 \lambda v^2 - \frac{1}{2} Z_{\mu^2} Z_\phi \mu^2 \right) H^2 - Z_{\kappa_{3\rm H}} Z_\lambda Z_\phi^2 Z_v \lambda_{3\rm H} v H^3 - \frac{1}{4} Z_{\kappa_{4\rm H}} Z_\lambda Z_\phi^2 \lambda_{4\rm H} H^4 + \cdots$$



$$= -2i\lambda v^3 \left(\delta Z_{m^2} - 2\delta Z_v - \delta Z_\lambda \right),$$



$$= i \left[p^2 \delta Z_H - (\delta Z_{m^2} + \delta Z_H) m_H^2 \right],$$



$$= -6i\kappa_3 \lambda v \left(\frac{3}{2} \delta Z_H + \delta Z_\lambda + \delta Z_v + \delta Z_{\kappa_3} \right).$$

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

We choose the renormalization scheme in which there is no tadpole contributions.

$m_h^2 = 2\lambda v^2$ and $(\delta Z_{m^2} - \delta Z_\lambda - 2\delta Z_v)\mu^2 v + \delta T = 0$ with δT the one-loop diagrams.

$$\delta T = \frac{3\lambda_{3\mathrm{H}} v}{16\pi^2} m_H^2 \left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 1 \right)$$

We choose the on-shell renormalization scheme for the Higgs mass.

$$\delta Z_{m^2} = \frac{3\lambda_{4\mathrm{H}}}{16\pi^2} \left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 1 \right) + \frac{9\lambda_{3\mathrm{H}}^2 v^2}{m_H^2} \frac{1}{8\pi^2} \left(\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m_H^2} + 2 - \frac{\pi}{\sqrt{3}} \right)$$

$$\delta Z_\phi = \frac{9\lambda_{3\mathrm{H}}^2 v^2}{8\pi^2} \frac{\sqrt{3} - 2\pi/3}{\sqrt{3} m_H^2}$$

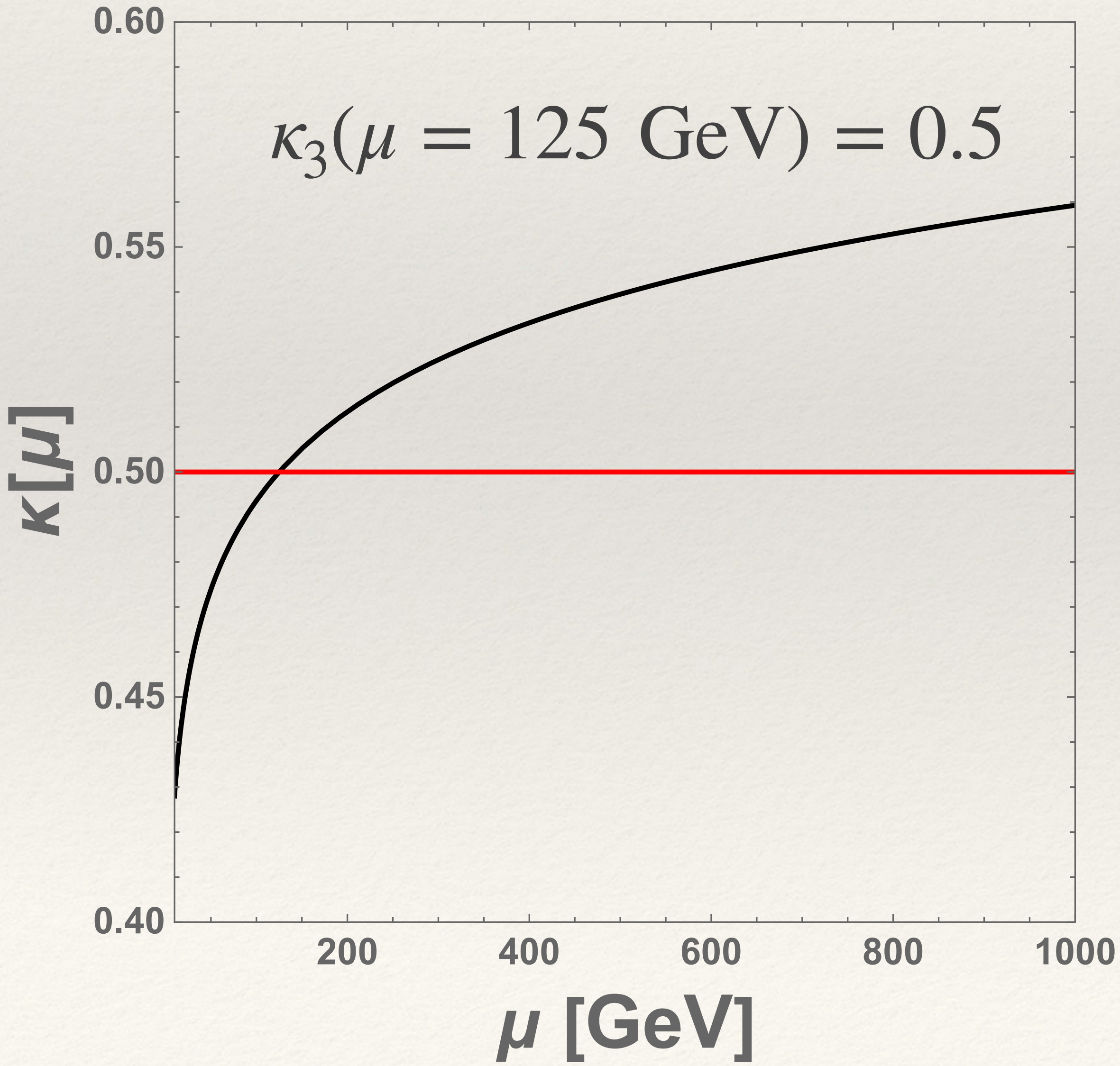
Since W/Z mass corrections are not affected by the Higgs self-couplings at one-loop, we can simply take $\delta Z_v = 0$.

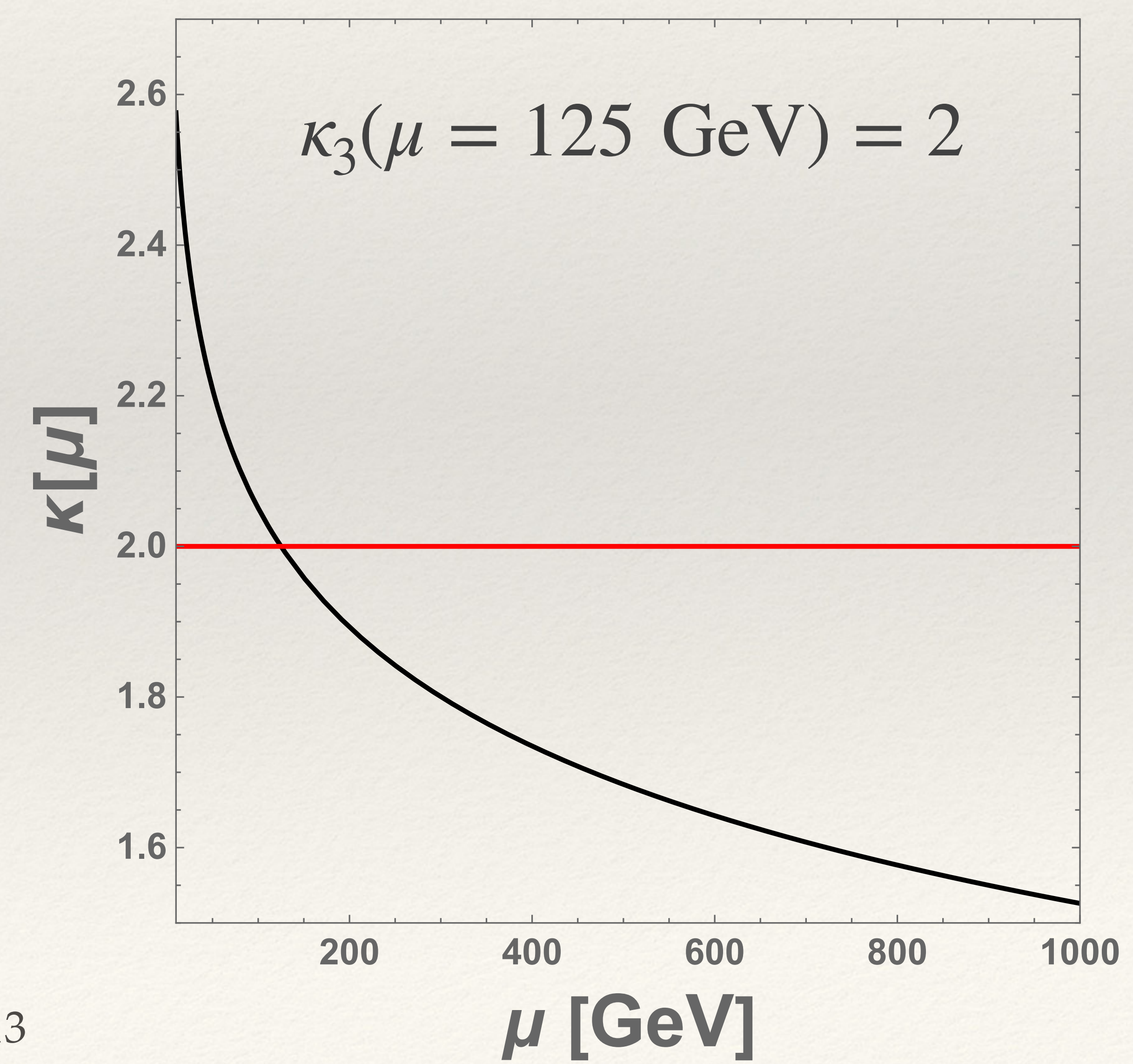
H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Renormalization

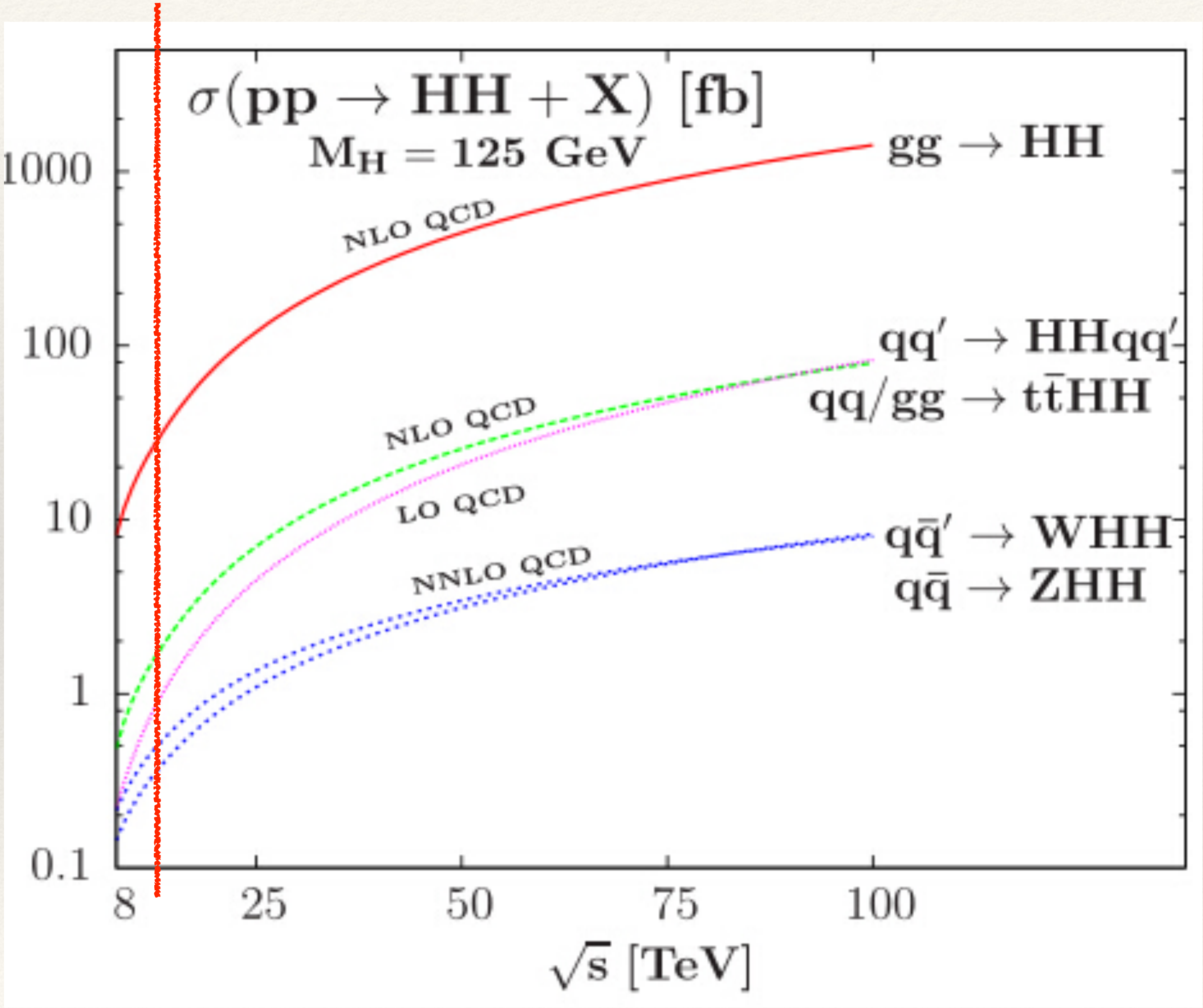
The coupling modifier κ_3 is renormalized in the $\overline{\text{MS}}$ scheme.

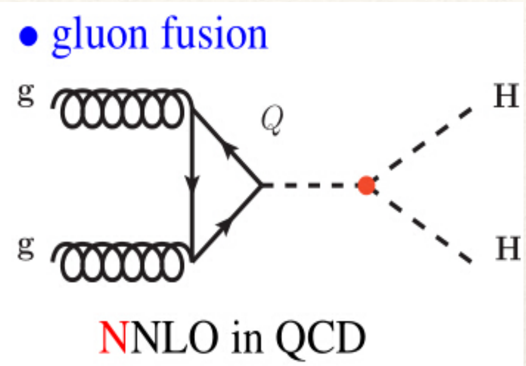
$$\delta Z_{\kappa_3} = \frac{3\lambda}{16\pi^2}\frac{1}{\epsilon}\left(\kappa_3 + 2\kappa_4 - 3\kappa_3^2\right)$$

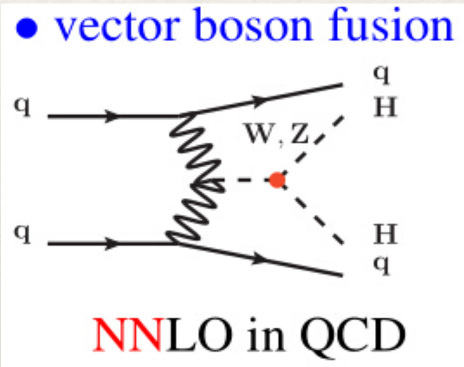


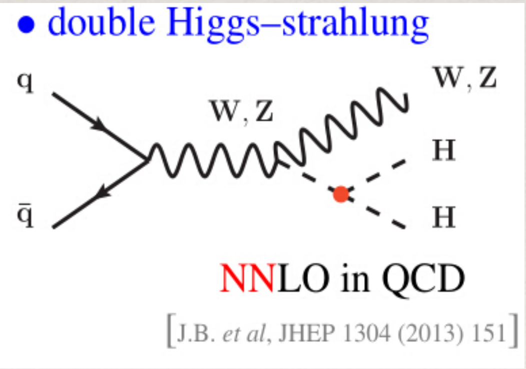


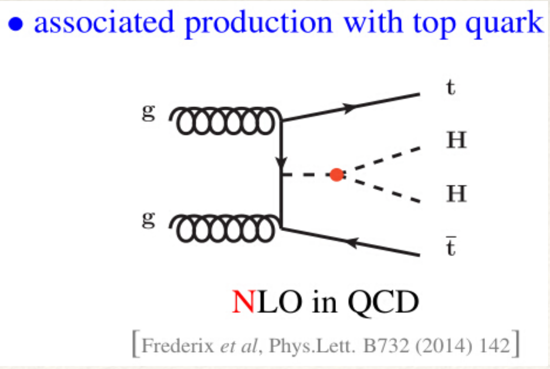
Higgs pair production at the LHC











Updated function forms

The λ dependent correction is

$$\delta\sigma^{\kappa_\lambda}_{\mathrm{ggF,EW}} = (0.075\kappa^4_{\lambda_{3H}} - 0.158\kappa^3_{\lambda_{3H}} - 0.006\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.058\kappa^2_{\lambda_{3H}} + 0.070\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}})\ \ \mathrm{fb}$$

$$\delta\sigma^{\kappa_\lambda}_{\mathrm{VBF,EW}} = (0.0215\kappa^4_{\lambda_{3H}} - 0.0324\kappa^3_{\lambda_{3H}} - 0.0019\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.0043\kappa^2_{\lambda_{3H}} + 0.0151\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.0211\kappa_{\lambda_{4H}})\ \ \mathrm{fb}$$

$\kappa_{\lambda_{3H}}$	$\kappa_{\lambda_{4H}}$	ggF			VBF		
		$\sigma^{\kappa_\lambda}_{\mathrm{LO}}$	$\sigma^{\kappa_\lambda}_{\mathrm{NNLO-FT}}$	$\delta\sigma^{\kappa_\lambda}_{\mathrm{EW}}$	$\sigma^{\kappa_\lambda}_{\mathrm{LO}}$	$\sigma^{\kappa_\lambda}_{\mathrm{NNNLO}}$	$\delta\sigma^{\kappa_\lambda}_{\mathrm{EW}}$
1	1	16.7	31.2	-0.225	1.71	1.69	-2.30×10^{-2}
3	1	8.59	18.4	1.28	3.59	3.53	8.35×10^{-1}
6	1	67.3	161	60.6	25.1	24.6	20.7
1	3	16.7	31.2	-0.393	1.71	1.69	-3.89×10^{-2}
1	6	16.7	31.2	-0.646	1.71	1.69	-6.27×10^{-2}
3	3	8.59	18.4	1.30	3.59	3.53	8.50×10^{-1}
6	6	67.3	161	61.0	25.1	24.6	20.7

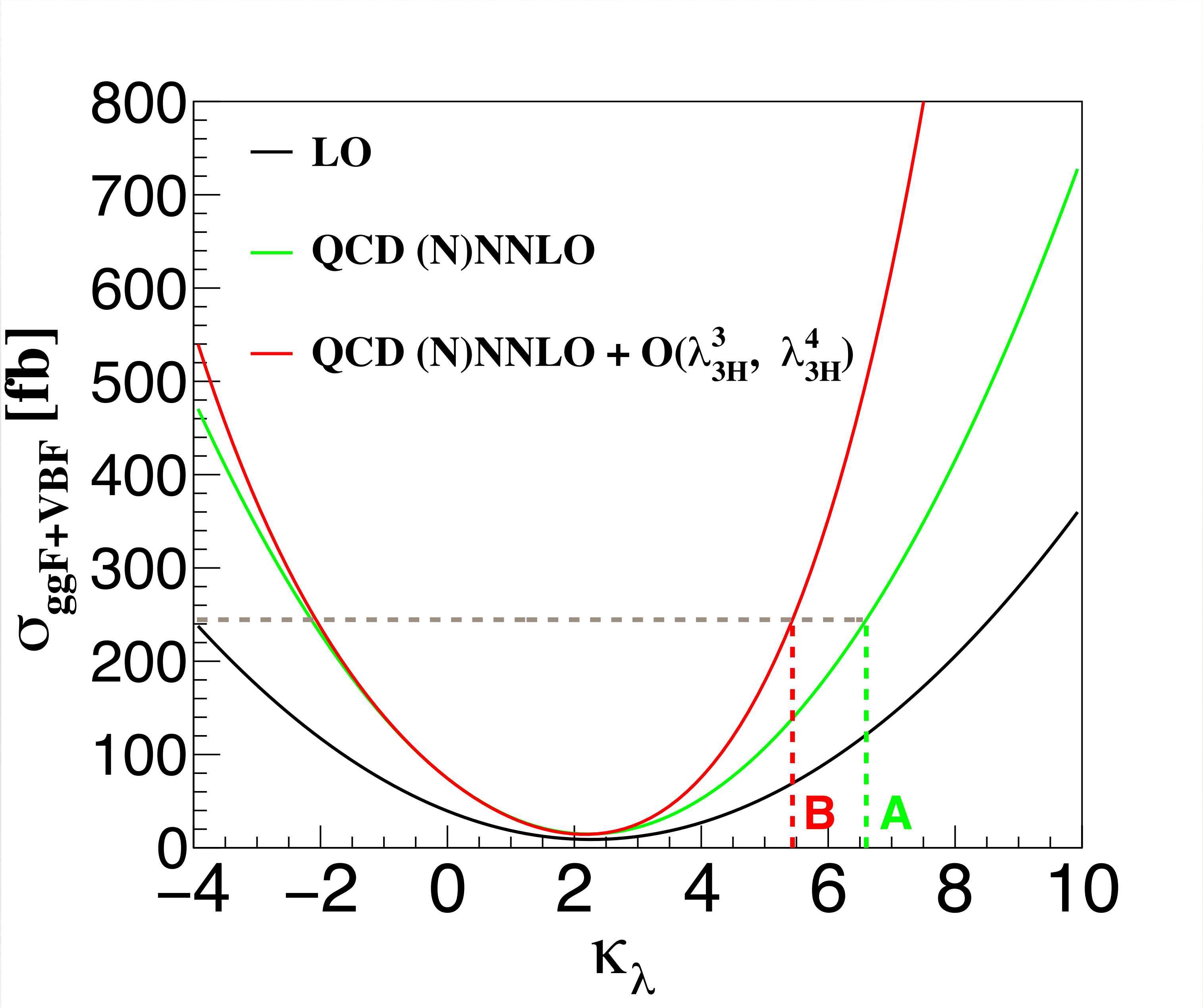
The QCD corrections are significant in ggF, but not sensitive to κ_3.

The EW corrections are 91% (82%) in ggF (VBF) for $\kappa_3 = 6$.

The dependence on κ_4 is weak.

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

More stringent constraint



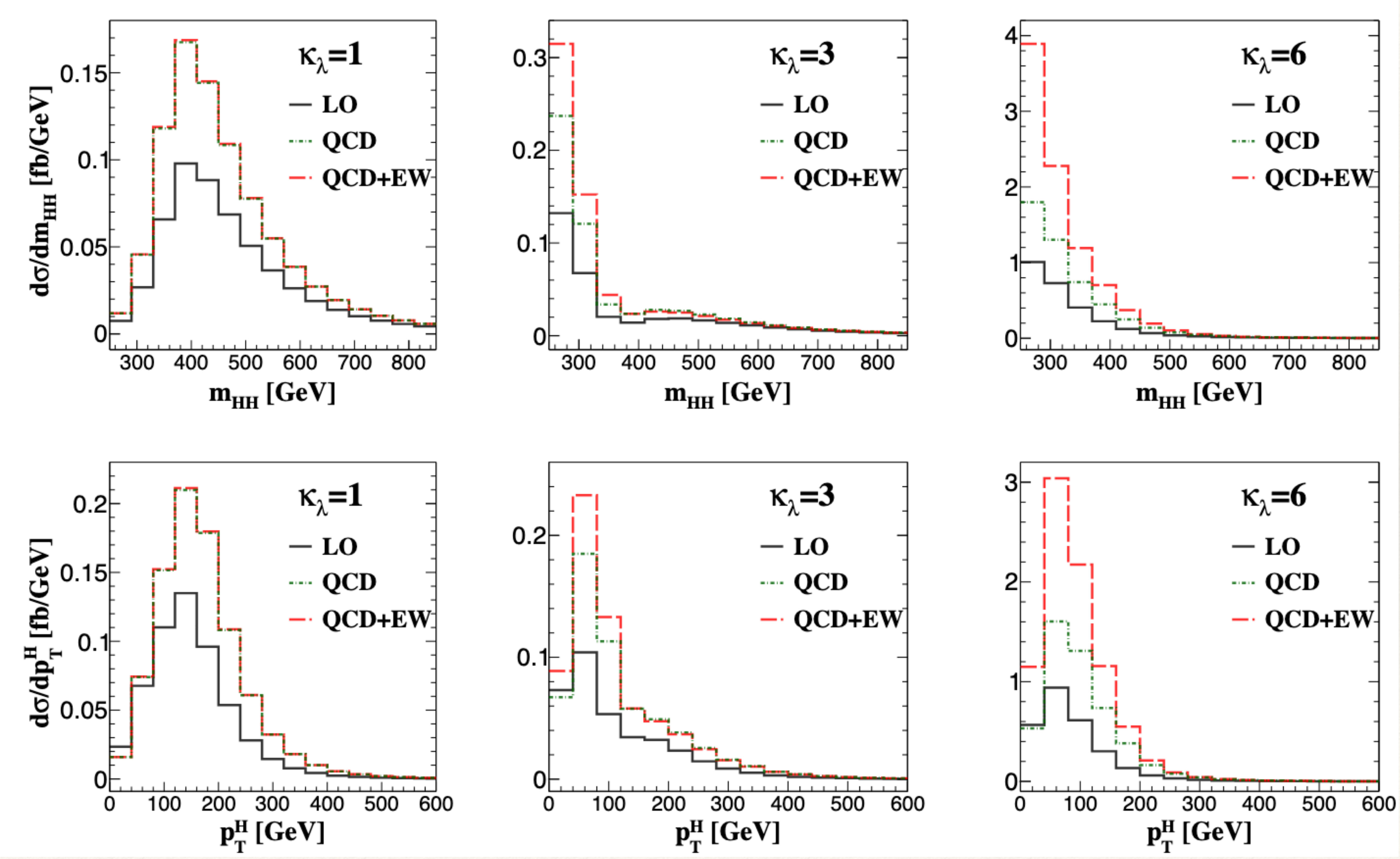
ATLAS (CMS) limit

6.6 (6.49)

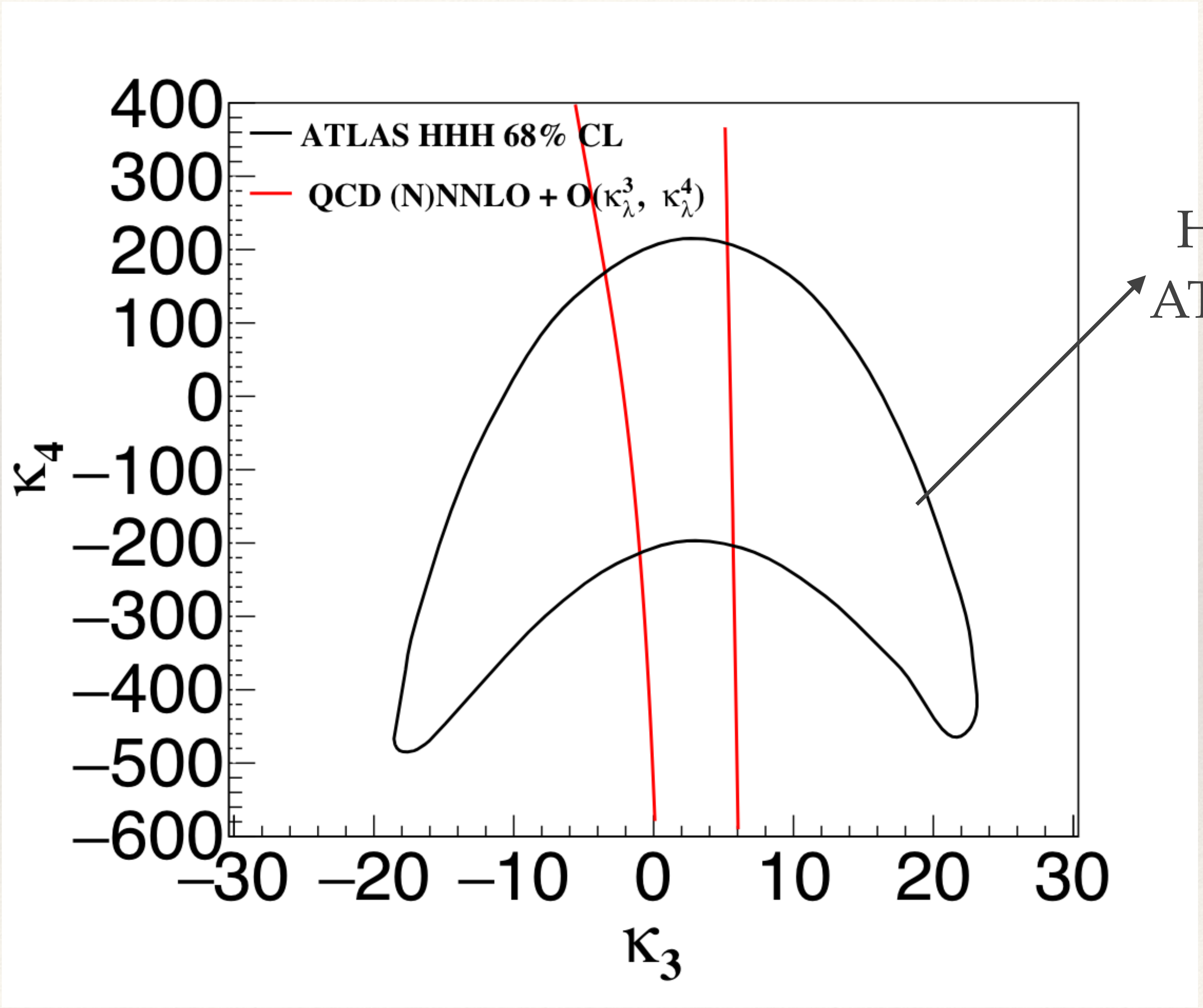
5.4 (5.37)

H.T. Li, Z.G. Si, JW, X. Zhang, D. Zhao, 2407.14716

Differential distributions

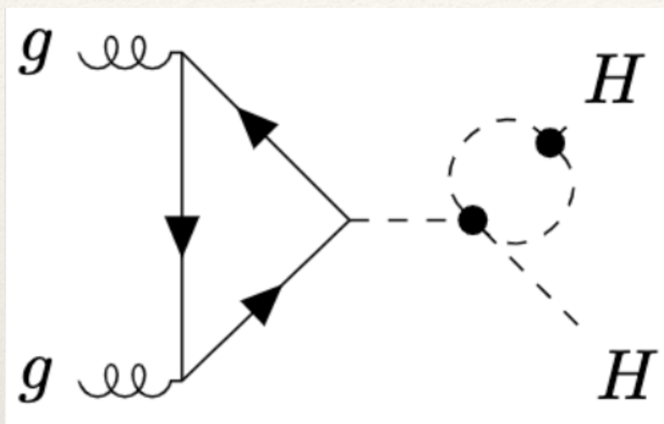


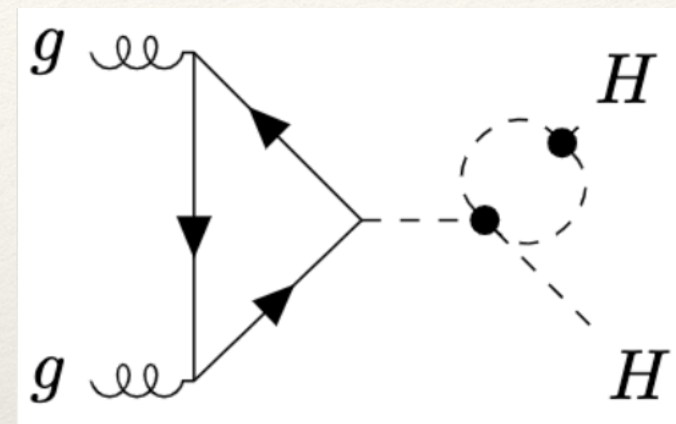
Sensitivity to κ_4



HHH constraint
ATLAS 2411.02040

Sensitivity to κ_4





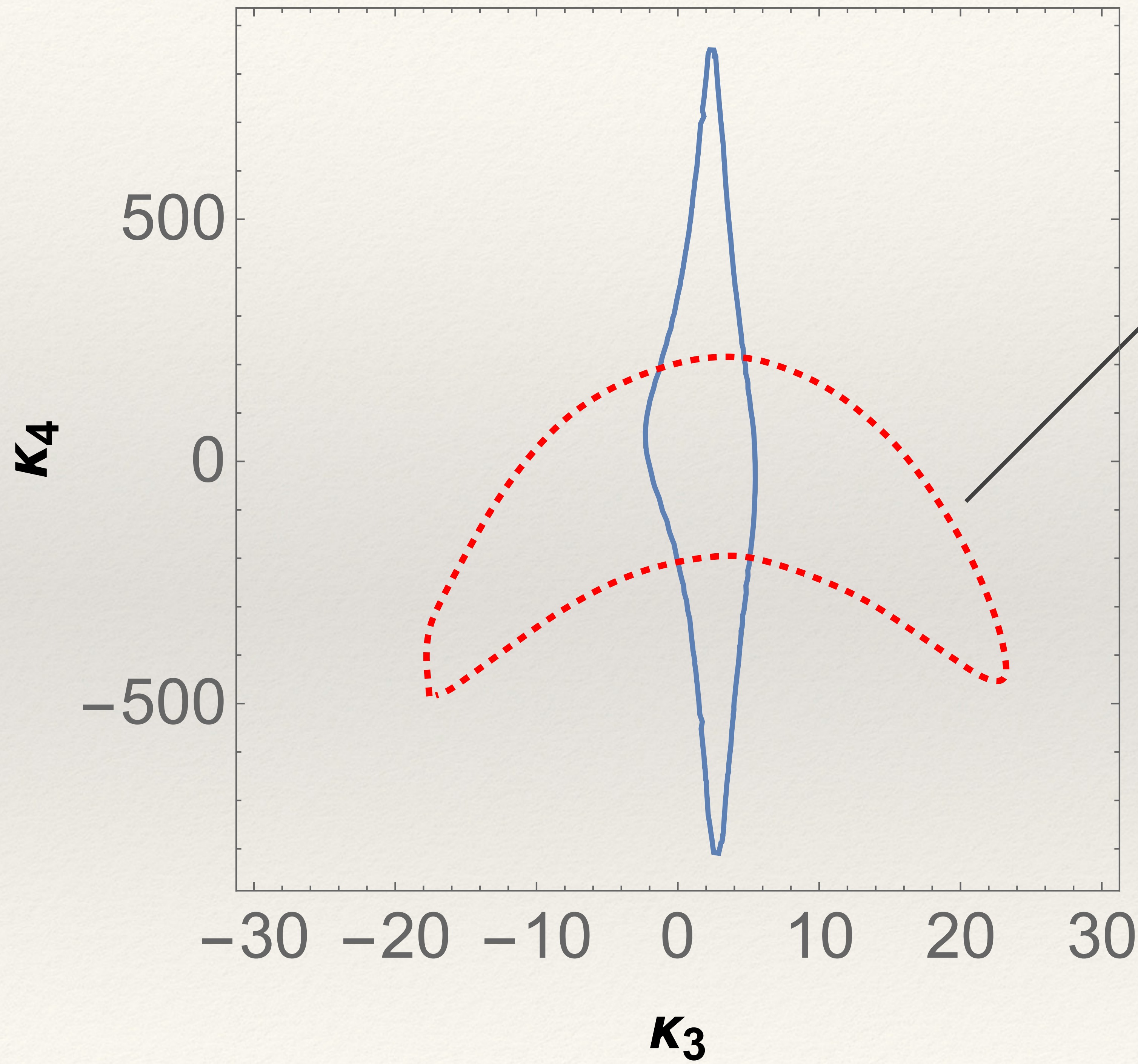
$$\delta\sigma^{\kappa_\lambda}_{\mathrm{ggF,EW}} = (0.075\kappa^4_{\lambda_{3H}} - 0.158\kappa^3_{\lambda_{3H}} - 0.006\kappa^2_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.058\kappa^2_{\lambda_{3H}} + 0.070\kappa_{\lambda_{3H}}\kappa_{\lambda_{4H}} - 0.149\kappa_{\lambda_{4H}}) \quad \mathrm{fb}$$

$$+[0.887\kappa^6_{\lambda_{3H}} - 2.439\kappa^5_{\lambda_{3H}} + 3.200\kappa^3_{\lambda_{3H}} + \kappa_{\lambda_{4H}}(-0.579\kappa^4_{\lambda_{3H}} + 2.821\kappa^3_{\lambda_{3H}} - 2.870\kappa^2_{\lambda_{3H}})$$

$$+\kappa^2_{\lambda_{4H}}(0.135\kappa^2_{\lambda_{3H}} - 0.742\kappa_{\lambda_{3H}} - 1.179)] \times 10^{-3} \quad \mathrm{fb}$$

This is only partial NNLO correction!

Sensitivity to κ_4



HHH constraint
ATLAS 2411.02040

Comparison with SMEFT

- SMEFT: Higgs field as a doublet, Wilson coefficients suppressed by Λ

$$V(\phi_b) = -\mu_b^2(\phi^\dagger\phi)_b + \lambda_b(\phi^\dagger\phi)_b^2 + \frac{d_6^b}{\Lambda^2}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^3 + \frac{d_8^b}{\Lambda^4}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^4 + \frac{d_{10}^b}{\Lambda^6}\left[(\phi^\dagger\phi)_b - \frac{v^2}{2}\right]^5.$$

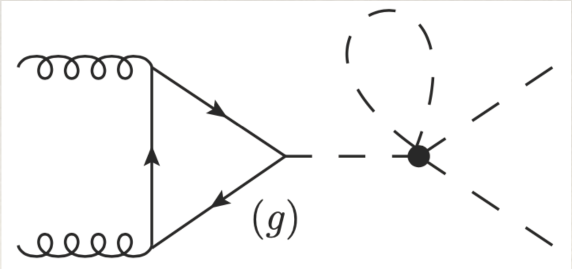
Borowka et al, 1811.12366

$$\mathcal{L}_{3H} = -\kappa_3\lambda v H^3 - (\delta Z_\lambda + \frac{3}{2}\kappa_3\delta Z_\phi + \bar{d}_6\delta Z_{d_6})\lambda v H^3.$$

$$\kappa_3 \equiv 1 + \bar{d}_6$$

$$d_6^b = Z_{d_6}\frac{\lambda\Lambda^2}{v^2}\bar{d}_6$$

$$\mathcal{L}_{4H} + \mathcal{L}_{5H} = -\frac{1}{4}\kappa_4\lambda H^4 - \kappa_5\frac{\lambda}{v}H^5 + \cdots$$



$$\kappa_4 \equiv 1 + 6\bar{d}_6 + \bar{d}_8 \qquad \kappa_5 \equiv \frac{1}{4}(3\bar{d}_6 + 2\bar{d}_8 + \bar{d}_{10})$$

If $\bar{d}_{10} = 0$, $\kappa_5 = (7 - 9\kappa_3 + 2\kappa_4)/4$.

J.L. Ding, H.T. Li, JW, 2512.xxxxx

Comparison with SMEFT

If the new operators are renormalized in the $\overline{\text{MS}}$ scheme. We find that the amplitude of $gg \to H^* \to HH$ is different from that in HEFT.

$$\frac{i\mathcal{M}_{\text{SMEFT2}} - i\mathcal{M}_{\kappa_5}}{i\mathcal{M}_{\text{LO}}} = \frac{3m_H^2(\kappa_3 - 1)\left[-(3-\sqrt{3}\pi)\kappa_3^2 + (\kappa_3 - 3\kappa_3^2 - \kappa_4)(1 + \ln\frac{\mu_R^2}{m_H^2})\right]}{32\pi^2 v^2 \kappa_3}$$

If we require that the amplitude remains the same, we have to change the renormalization condition. The renormalization of the dimension-six operator is not in $\overline{\text{MS}}$ any more.

$$(\delta Z_{d_6})_{\text{finite}} = (\delta Z_\lambda)_{\text{finite}}$$

J.L. Ding, H.T. Li, JW, 2512.xxxxx

Comparison with SMEFT

If the new operators are renormalized in the $\overline{\text{MS}}$ scheme. We find that the amplitude of $gg \to H^* \to HH$ is different from that in HEFT.

$$\frac{i\mathcal{M}_{\text{SMEFT2}} - i\mathcal{M}_{\kappa_5}}{i\mathcal{M}_{\text{LO}}} = \frac{3m_H^2(\kappa_3 - 1)\left[-(3-\sqrt{3}\pi)\kappa_3^2 + (\kappa_3 - 3\kappa_3^2 - \kappa_4)(1 + \ln\frac{\mu_R^2}{m_H^2})\right]}{32\pi^2 v^2 \kappa_3}$$

If we require that the amplitude remains the same, we have to change the renormalization condition. The renormalization of the dimension-six operator is not in $\overline{\text{MS}}$ any more.

$$(\delta Z_{d_6})_{\text{finite}} = (\delta Z_\lambda)_{\text{finite}}$$

$\overline{\text{MS}}$ in HEFT is not $\overline{\text{MS}}$ in SMEFT!

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Summary

Compared with the Higgs couplings to EW gauge bosons or third-generation fermions, the Higgs self-couplings remain only weakly constrained at LHC Run 2.

Probing the Higgs self-couplings at the HL-LHC and future colliders via multi-Higgs production explores largely uncharted model and parameter space and provides genuine discovery potential, even if single-Higgs observables show no anomalies.

Fully exploiting the discovery potential of multi-Higgs production channels requires including higher-order perturbative corrections induced by both QCD and EW interactions in the corresponding theoretical predictions.

Thanks a lot for your attention!

Back-up slides

HEFT

$$
\begin{aligned}
\mathcal{L}_4 = &-a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\partial^\nu H}{v^2}\mathrm{Tr}[\mathcal{V}_\mu\mathcal{V}_\nu] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\partial_\mu H}{v^2}\mathrm{Tr}[\mathcal{V}^\nu\mathcal{V}_\nu] \\
&+\left(a_{11}+a_{H11}\frac{H}{v}+a_{HH11}\frac{H^2}{v^2}\right)\mathrm{Tr}[\mathcal{D}_\mu\mathcal{V}^\mu\mathcal{D}_\nu\mathcal{V}^\nu] - \frac{m_{\mathrm{H}}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v}+a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right)\mathrm{Tr}[\mathcal{V}^\mu\mathcal{V}_\mu] \\
&-\left(a_{HWW}\frac{H}{v}+a_{HHWW}\frac{H^2}{v^2}\right)\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] + i\left(a_{d2}+a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}[\hat{W}_{\mu\nu}\mathcal{V}^\mu] \\
&+\left(a_{\Box\mathcal{V}\mathcal{V}}+a_{H\Box\mathcal{V}\mathcal{V}}\frac{H}{v}\right)\frac{\Box H}{v}\mathrm{Tr}[\mathcal{V}_\mu\mathcal{V}^\mu] + \left(a_{d3}+a_{Hd3}\frac{H}{v}\right)\frac{\partial^\nu H}{v}\mathrm{Tr}[\mathcal{V}_\nu\mathcal{D}_\mu\mathcal{V}^\mu] \\
&+\left(a_{\Box\Box}+a_{H\Box\Box}\frac{H}{v}\right)\frac{\Box H\Box H}{v^2} + a_{dd\Box}\frac{\partial^\mu H\partial_\mu H\Box H}{v^3} \\
&+\left(a_{Hdd}\frac{m_{\mathrm{H}}^2}{v^2}+a_{ddW}\frac{m_{\mathrm{W}}^2}{v^2}+a_{ddZ}\frac{m_{\mathrm{Z}}^2}{v^2}\right)\frac{H}{v}\partial^\mu H\partial_\mu H.
\end{aligned}
$$

Brivio et al, JHEP, 1403, 024
Gavela, Kanshin, Machado, Saa, JHEP, 1503, 043

Renormalization:

- Background field method: Guo, Ruiz-Femenia, Sanz-Cillero, PRD92,074005(2015)
  Buchalla, et al, NPB, 928,93 (2018), PRD104,076005(2021)
- Scattering amplitude: Herrero, Morales, PRD106,073008 (2022)

Comparison with SMEFT

If we require that the amplitude remains the same, we have to change the renormalization condition. The renormalization of the dimension-six operator is not in $\overline{\mathrm{MS}}$ any more.

$$(\delta Z_{d_6})_{\text{finite}} = (\delta Z_\lambda)_{\text{finite}}$$

$$\delta Z_\lambda = -\frac{3\lambda_3}{16\pi^2}\left[\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m^2} + 1\right] + \frac{3\lambda_4}{16\pi^2}\left[\frac{1}{\epsilon} + \ln\frac{\mu_R^2}{m^2} + 1\right] + \frac{3\lambda_3^2 v^2}{8\pi^2 m^2}\left[\frac{3}{\epsilon} + 3\ln\frac{\mu_R^2}{m^2} + 6 - \sqrt{3}\pi\right],$$

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A detailed comparison of the two schemes is on the way.

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