Self consistent thermal resummation:

a case study of the phase transition in 2HDM

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with Carlos E. M. Wagner and Subhojit Roy

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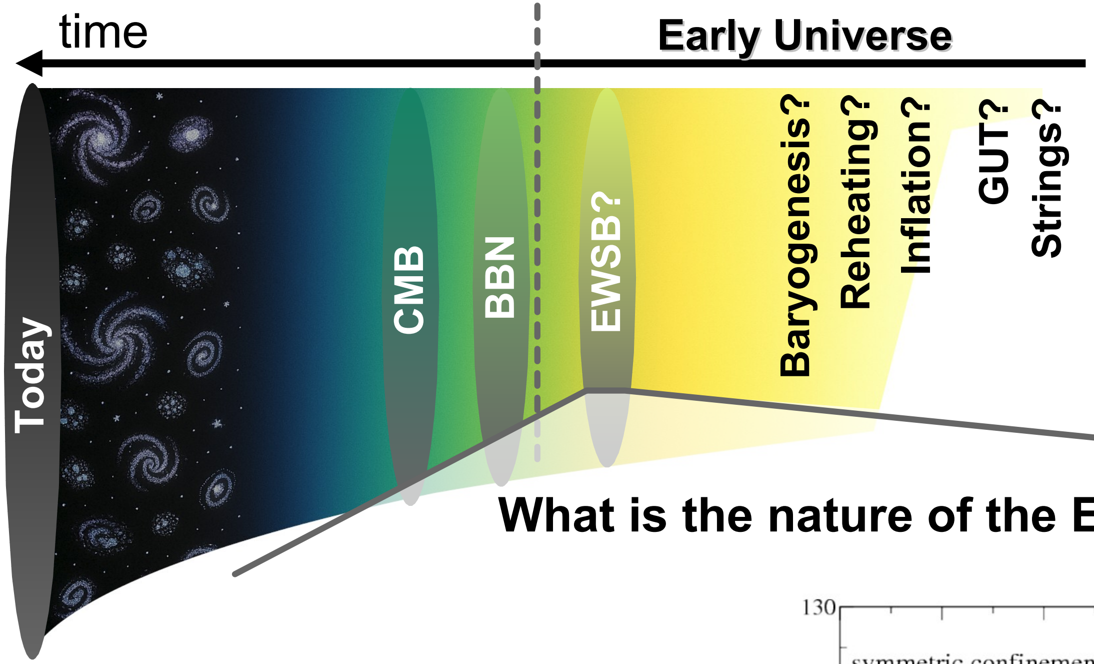
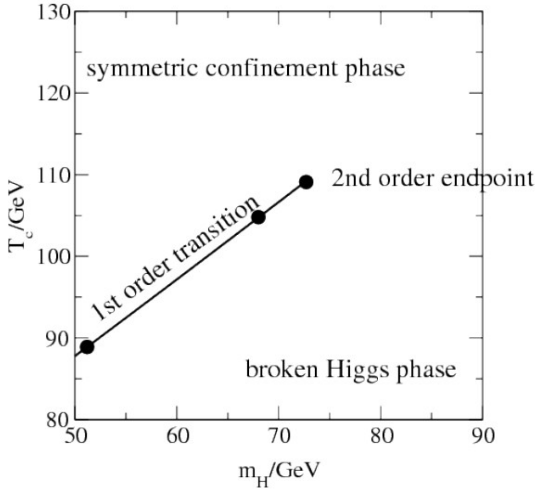


EW Symmetry Breaking and the EW Phase Transition

What is the nature of the EWPT?

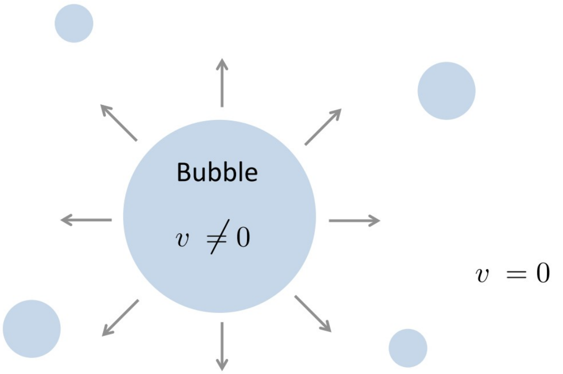


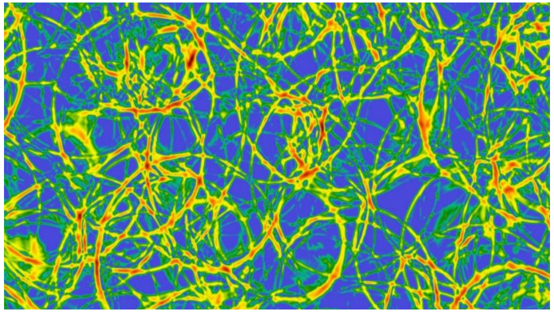
→ Standard model expectation

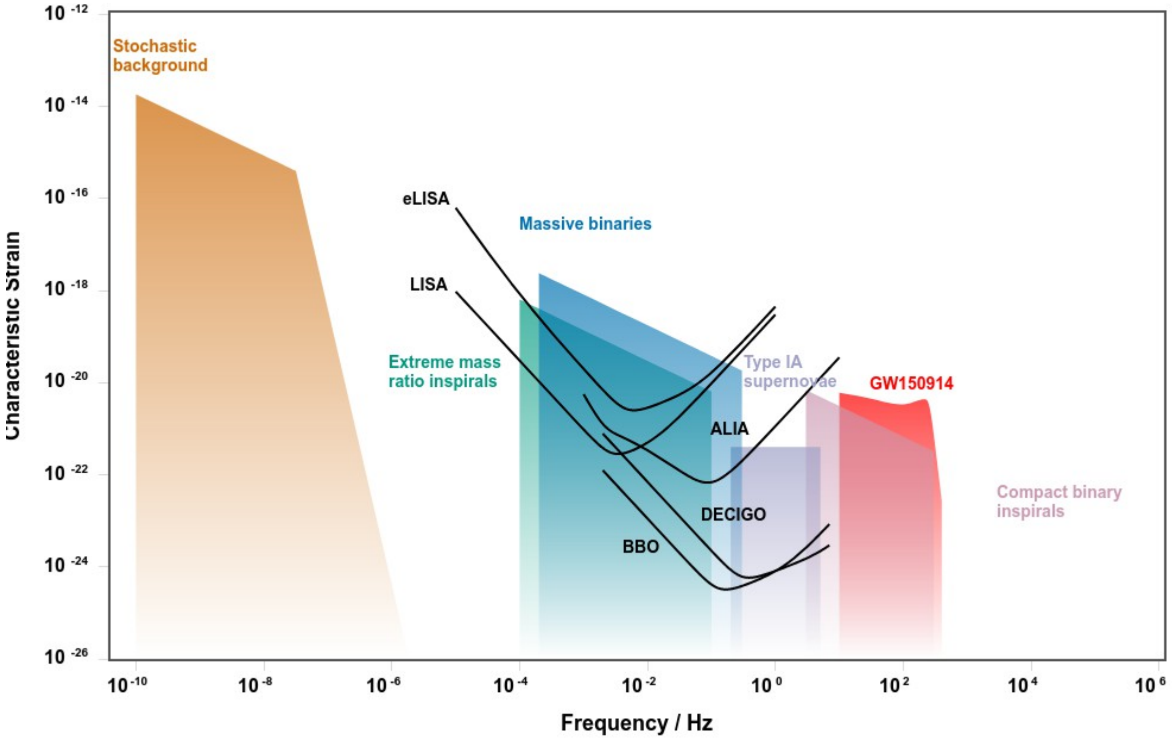
Why would the EW phase transition be first order?

- If it is, we might see gravitational waves $\rightarrow$ Signal!
- EW baryogenesis
- Connections to BSM: Dark matter, extended Higgs sectors
- We would like to know the nature of EWSB in any case

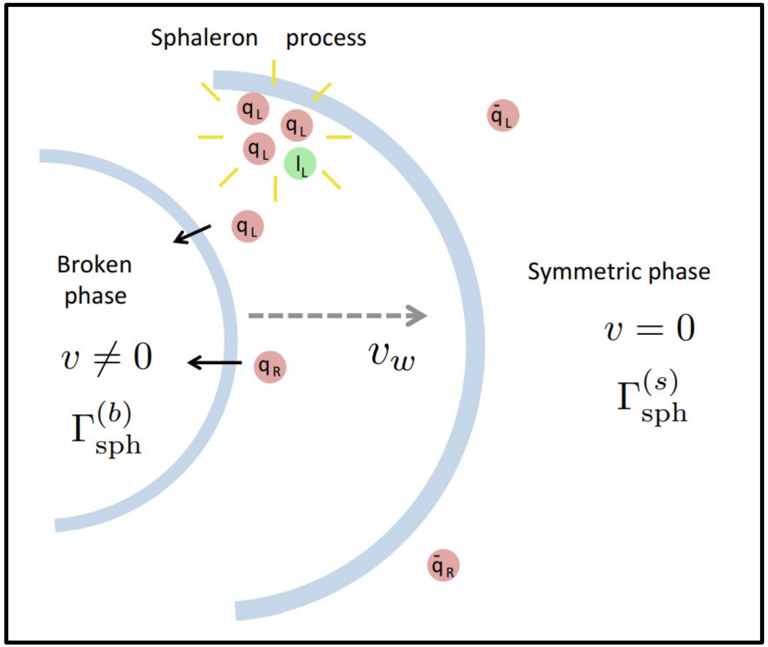
First order phase transitions gained a lot of interest because of **stochastic gravitational wave production**







Why Electroweak Baryogenesis is a great idea



EW baryogenesis is a testable model

- Baryon number violation from Sphalerons
- Needs extra CP violation
- **Needs a strong first order phase transition**

$$\eta_B \simeq \Gamma_{\rm Sph} \frac{\delta_{\rm CP}}{2\pi} \frac{1}{L_{\rm wall}} \frac{1}{T^2} \sim 10^{-10}$$

If one has the right ingredients it gives the expected asymmetry!

Electroweak Baryogenesis is a great idea

But it is in trouble...

Extra sources for CP violation are constrained by EDM of the electron, neutron and some atomic elements (i.e. Hg)

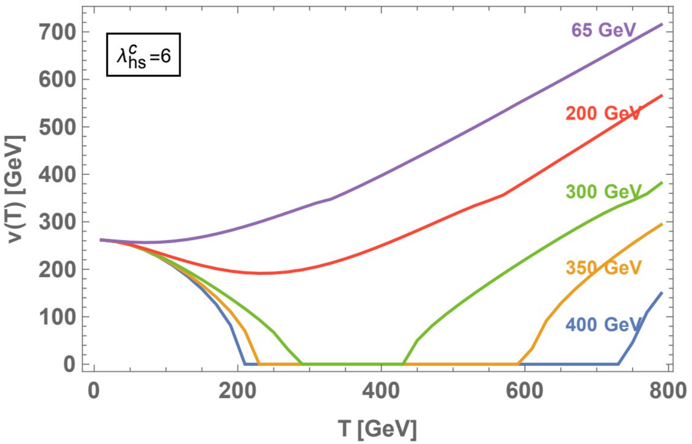
$$|d_e| < 1.1 \times 10^{-29} e\,\mathrm{cm}$$

$$d_{\mathrm{n}} = (0.0 \pm 1.1_{\mathrm{stat}} \pm 0.2_{\mathrm{sys}}) \times 10^{-26}\ e{\cdot}\mathrm{cm}.$$

$$d(^{199}\mathrm{Hg}) = (0.49 \pm 1.29_{stat} \pm 0.76_{syst}) \times 10^{-29} e\ \mathrm{cm},$$

Changing the phase transition dynamics

Symmetry Non-Restoration



[Meade, Ramani, 2018]

Inverse Symmetry Breaking

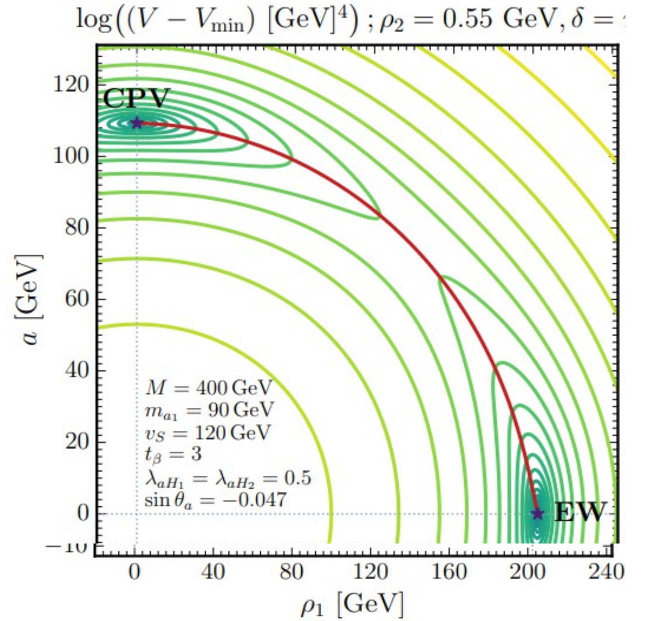
$U(1)_{EM}$, $U(1)_{B\text{-}L}$, ...

T=0 unbroken

↓

$T>T_c$ broken

Multi-step Phase Transition



[Huber, Mimasu, No, 2022]

Electroweak Symmetry Non-Restoration

Usually, symmetry non-restoration occurs where there are **negative thermal masses**

$$m^2(T) \simeq \left(\frac{y_t}{4} + \cdots + N_s \frac{\lambda_{sh}}{12} \right) T^2$$

Number of extra scalars ~100!

Electroweak Symmetry Non-Restoration

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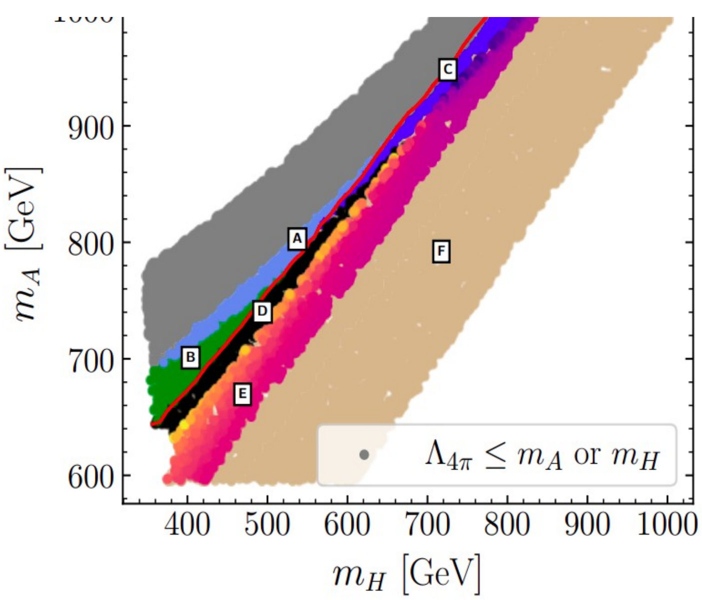
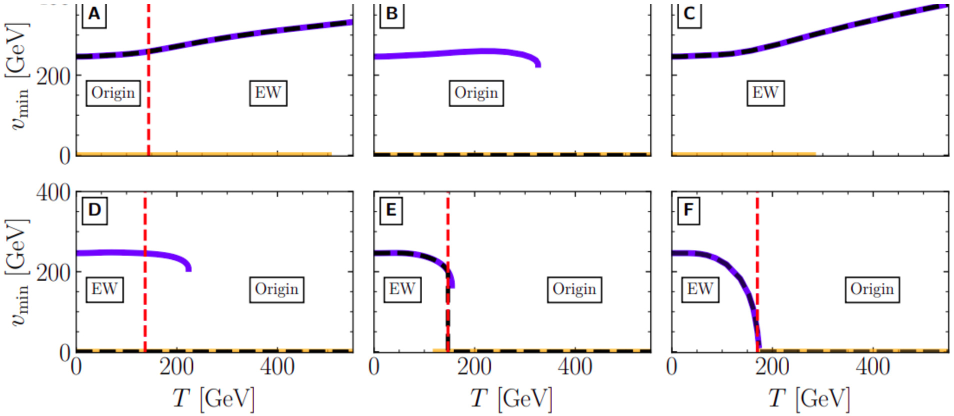
Number of extra scalars ~100!

But, symmetry **non-restoration was found** in a simple CP even 2HDM!

[Biekötter et al 2023]





Questions we had

- Is there really Electroweak Symmetry non-restoration in the 2HDM?

  We found strong disagreement between different resummation methods

- So, what causes the observed symmetry non-restoration?

- **More generally, how reliable are our methods for calculating the thermal effective potential?**

Given a tree-level potential $V_0(\phi)$ **, we can obtain the mass matrix**

$$\mathbb{M}^2_{ij}(\phi) = \frac{\partial^2 V_0(\phi)}{\partial \phi_i \partial \phi_j}, \qquad i = (\text{all scalar fields}).$$

Get field dependent masses:

$$m_k^2(\phi) = U_{ki}(\theta)\mathbb{M}^2_{ij}(\phi)U^\dagger_{jk}(\theta)$$

Zero temperature 1-loop potential

$$V_{\rm CW}(m_i^2(\phi)) = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i} n_i |m_i^2(\phi)|^2 \left[\log\left(\frac{m_i^2(\phi)}{\mu^2}\right) - k_i\right].$$

Static equilibrium thermal field theory

Introduce a thermal plasma from a compact Euclidean time

$$p_\mu = (i\omega_n, \vec{k}) \qquad \omega_n = \begin{cases} 2\pi n/\beta & \text{for bosons} \\ (2n+1)\pi/\beta & \text{for fermions} \end{cases} \qquad \int \frac{dp^0}{2\pi} \to \frac{1}{\beta}\sum_n$$

1-loop thermal potential

$$V_T(m^2(\phi), T) = \frac{T^4}{2\pi^2}\left[\sum_k n_k J_B\left(\frac{m_k^2(\phi)}{T^2}\right) - \sum_{k=\mathrm{F}} n_k J_F\left(\frac{m_k^2(\phi)}{T^2}\right)\right]$$

$$J_{B,F}(y) = \int_0^\infty dk\, k^2 \log\left[1 \mp e^{-\sqrt{k^2+y}}\right]$$

High temperature limit:

$$J_B(y)\big|_{HT} \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12} y - \frac{\pi}{6} y^{3/2} - \frac{1}{32} y^2 \log\left(\frac{y}{a_B}\right) + \mathcal{O}\left(y^3\right) ,$$

$$J_F(y)\big|_{HT} \approx \frac{7\pi^4}{360} - \frac{\pi^2}{24} y - \frac{1}{32} y^2 \log\left(\frac{y}{a_F}\right) + \mathcal{O}\left(y^3\right) ,$$

$y=m^2/T^2$

Low temperature limit:

$$J_{B,F}(y)\big|_{LT} \approx -\left(\frac{\pi}{2}\right)^{1/2} y^{3/4} e^{-\sqrt{y}} \left(1 + \frac{15}{8} y^{-1/2}\right)$$

Propagators *see* thermal bath

$$G(\omega, \mathbf{p}) = \frac{1}{\omega^2 + |\mathbf{p}|^2 - m_0^2(\phi) - \Pi(\omega, \mathbf{p}; T)}$$

Gap equation

$$M_k^2(\phi, T) = m_{0,k}^2(\phi) + \Pi_k(M^2(\phi, T), 0; T)$$

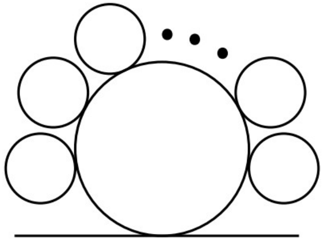
Truncated high temperature thermal mass:

$$M_{ij}^2(\phi, T) \simeq m_{0,ij}^2(\phi) + \Pi_{ij}(m_0^2(\phi), T) \qquad \text{(Truncated gap eq.)}$$

$$m_{\text{eff}}^2(T) \simeq m_0^2 + \Pi(T) \simeq m_0^2 + \lambda T^2$$

The problem is, **finite temperature QFT loses perturbativity** *(near the critical temperature)*

Finite temperature QFT: loss of perturbativity



$$(\text{daisy}) \sim \frac{\lambda^2 T^3}{\bar{m}} \left(\frac{\lambda T^2}{\bar{m}^2} \right)^{n-2} .$$

When $\lambda T^2 > m^2$

Theory becomes non-perturbative!

Daisy diagrams require a reorganization of the perturbative expansion

Dealing with perturbativity breakdown: Resummation

- Arnold-Espinosa scheme
- Parwani scheme

Pros: Easy to implement

Cons: Only reliable at T>>m

Inconsistent higher order corrections

- Full-dressing

With full gap-equation

Pros: Self-consistent at all temperatures

Cons: Harder to implement than AE & Parw

Miscounts Daisy/Superdaisy diagrams

- Partial-dressing

Pros: **Self-consistent at all temperatures**

Correct higher-loop daisy contributions

Cons: **Harder to implement than AE & Parw**

Arnold-Espinosa scheme:

$$V_{\text{eff}}^{\text{AE}}(\phi, T) = V_0(\phi) + V_{\text{CW}}(m_i^2(\phi)) + V_{\text{CT}}(\phi) + V_T(m_i^2(\phi), T) + V_{\text{Daisy}}^{\text{AE}}(\phi, T).$$

$$V_{\text{Daisy}}^{\text{AE}}(\phi, T) = -\frac{T}{12\pi}\sum_i \left((m_{T,i}^2(\phi, T))^{3/2} - (m_i^2(\phi))^{3/2} \right)$$

Resums only the daisy diagrams!

$$V_N^{\text{daisy}} = -\frac{T}{12\pi}\frac{1}{N!}\left(\frac{\lambda T^2}{4}\right)^N \left(\frac{d}{dm^2}\right)^N m^3$$

$$\sum_{N=0}^{\infty} V_N^{\text{daisy}} = -\frac{T}{12\pi}\sum_{N=0}^{\infty}\frac{1}{N!}\left(\frac{\lambda T^2}{4}\frac{d}{dm^2}\right)^N m^3$$

$$= -\frac{T}{12\pi}\exp\left(\frac{\lambda T^2}{4}\frac{d}{dm^2}\right) m^3 .$$

$$\sum_{N=0}^{\infty} V_N^{\text{daisy}} = -\frac{T}{12\pi}\left(m^2 + \frac{\lambda}{4}T^2\right)^{3/2} .$$

Parwani scheme:

$$V_{\text{eff}}^{\text{Par}}(m_{T,i}^2, T) = V_0(\phi) + V_{\text{CW}}(m_{T,i}^2(\phi, T)) + V_{\text{CT}}(\phi) + V_T(m_{T,i}^2(\phi, T)).$$

Add and subtract the thermal mass to the lagrangian

Then, redefine the pole mass to include thermal effects.

In Parwani all modes are resummed, not only the problematic zero modes

<u>Full dressing:</u> Replace thermal mass in the effective potential

Both Arnold-Espinosa and Parwani are examples of **truncated full-dressing**

$$m_{\text{eff}}^2(T) \simeq m_0^2 + \Pi(T) \simeq m_0^2 + \lambda T^2$$

Easy to evaluate analytically in the high-temperature regime.

But, near the critical temperature this approximation is not reliable.

→ One needs to solve the full gap equation self-consistently

→ Then FD takes the thermal mass and replaces it in $V_{\text{eff}}(T)$

$$V_{\text{eff}}(M^2(\phi,T)) = V_0(\phi) + V_{\text{CT}}(\phi) + V_{\text{CW}}(M^2(\phi,T)) + V_T(M^2(\phi,T),T)$$

Partial Dressing (Tadpole resummation)

Resummation Methods at Finite Temperature: The Tadpole Way[1]

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Caltech 452-48, Pasadena CA 91125

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yman Laboratory of Physics, Harvard University, Cambridge MA 0213

Thermal Resummation and Phase Transitions

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Gravitational waves and tadpole resummation: Efficient and easy convergence of finite temperature QFT

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Partial Dressing

1. Solve the full gap-equation for the thermal mass.
2. Insert the obtained thermal mass into **the first derivative of the potential.**
3. Integrate the tadpole to get the resummed potential.

$$M^2(\phi,T) = m_0^2(\phi) + \Pi(M^2(\phi,T),T),$$

$$V_{\text{eff}}^{\text{PD}} = \int d\phi \left(\frac{\partial V_{\text{eff}}(m_i^2(\phi),T)}{\partial \phi} \right)_{m_i^2(\phi)\to M_i^2(\phi,T)}$$

Full vs Partial-Dressing

- The **truncated methods** have several issues:

→ Rely on the high-temperature regime
→ Inconsistent higher-order effects

For strong first order phase transitions they can be very unreliable

(Field excursions of order of the temperature)

- Boyd (1993) showed that full-dressing miscounts 2-loop daisy and superdaisy diagrams

Partial dressing correctly incorporates daisy and superdaisy and allows to go to intermediate temperature regime

Case study: The two-Higgs Doublet Model

$$V_0 = m_{11}^2 \left|\Phi_1\right|^2 + m_{22}^2 \left|\Phi_2\right|^2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \text{h.c.}\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right],$$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \left(v_1 + h_1 + \mathrm{i}a_1\right)/\sqrt{2} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \left(v_2 + h_2 + \mathrm{i}a_2\right)/\sqrt{2} \end{pmatrix}$$

CP-even

$$\Phi_1 \to \Phi_1$$
$$\Phi_2 \to -\Phi_2$$

$$\text{“alignment limit”} \ \cos(\beta - \alpha) = 0$$

Light SM-like Higgs

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$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right],$$

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$$\Phi_1 \to \Phi_1$$
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Light SM-like Higgs

Partial dressing in practice

$$M_k^2(\phi,T) = U_{ki}(\theta_T)\left[\frac{\partial^2 V_0}{\partial\phi_i\partial\phi_j} + \frac{\partial^2 V_{CT}}{\partial\phi_i\phi_j} + \frac{\partial^2 V_{CW}}{\partial\phi_i\phi_j} + \frac{\partial^2 V_T}{\partial\phi_i\phi_j}\right] U^\dagger_{jk}(\theta_T)$$

Strategy: Write the self-energy in terms of

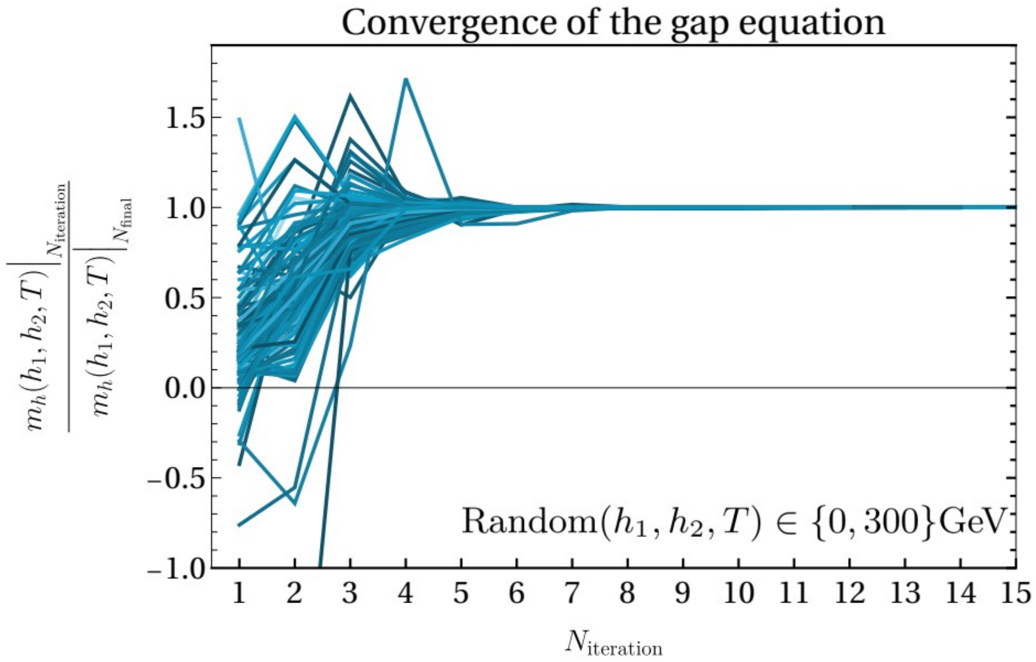
- $m^2_{0,k}(h_1,h_2)$, $\quad k = h, H, G_0, A, G^\pm, H^\pm$ $\quad x_i \equiv M^2(h_1,h_2,T)\big|_{\text{iteration}=i}$
- $\frac{dm^2_{0,k}}{d\phi_a}(h_1,h_2)$, $\quad \phi_a = h_1, h_2, a_1, a_2, \phi_1^\pm, \phi_2^\pm$ $\quad x_{i,a} = x_{i,a}(x_i,\theta_T) = \frac{dx_i}{d\phi_a}$
- $\frac{d^2 m^2_{0,k}}{d\phi_a d\phi_b}(h_1,h_2)$, $\quad \phi_{a,b} = h_1, h_2, a_1, a_2, \phi_1^\pm, \phi_2^\pm$ $\quad x_{i,ab} = x_{i,ab}(x_i,\theta_T) = \frac{d^2 x_i}{d\phi_a d\phi_b}$
- $\theta_T(h_1,h_2,T)$.

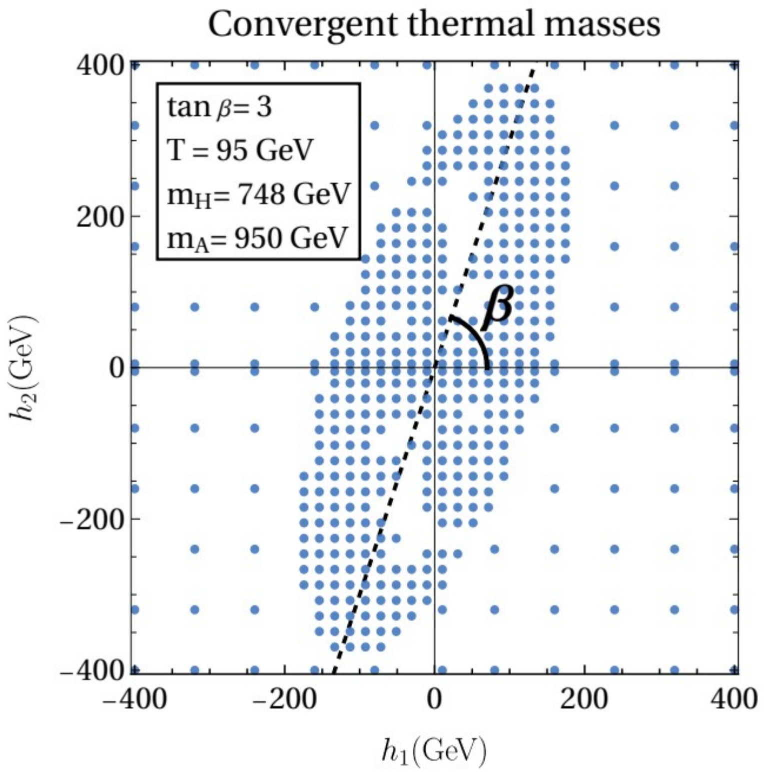
$$x_1 = m_0^2(h_1,h_2)\ ,$$

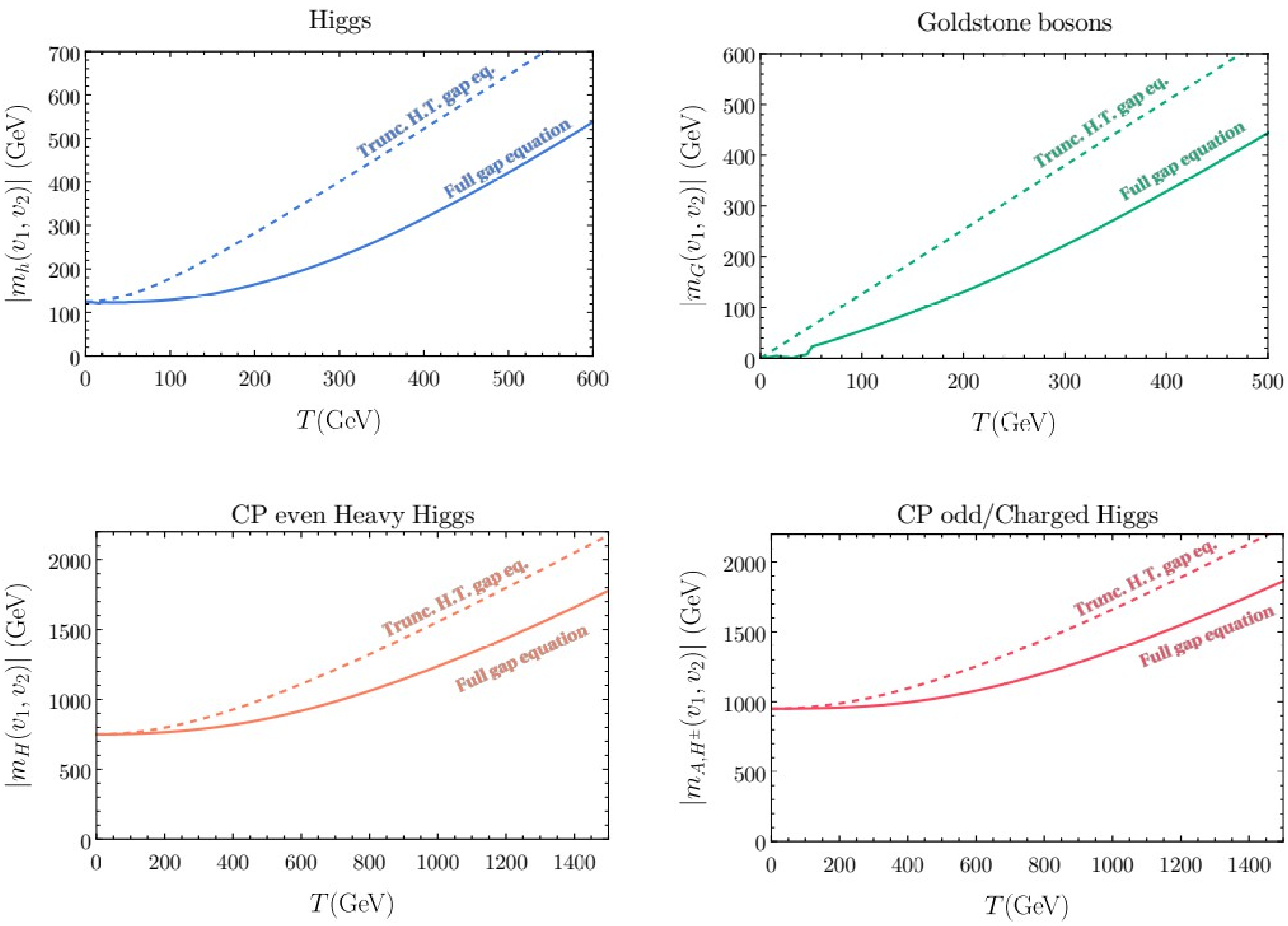
$$x_{i+1} = x_i + U^{-1}(\theta_T)\ \Pi(x_i, x_{i,a}, x_{i,ab})\ U(\theta_T)$$

Gap equation can be unstable in MSbar!

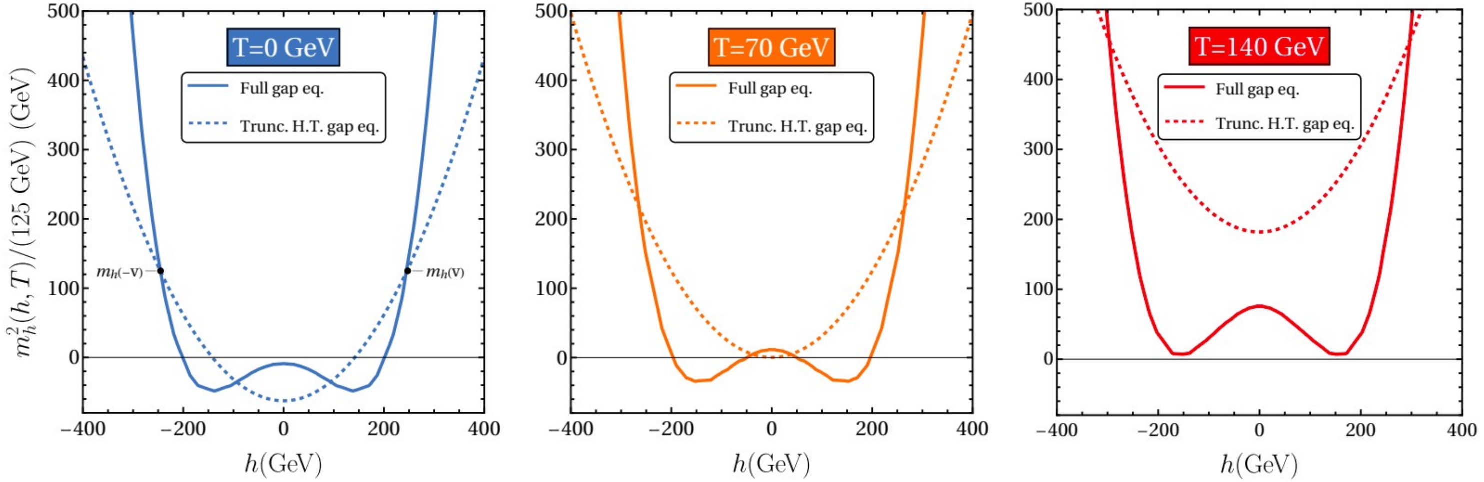
→ On-shell renormalization scheme helps



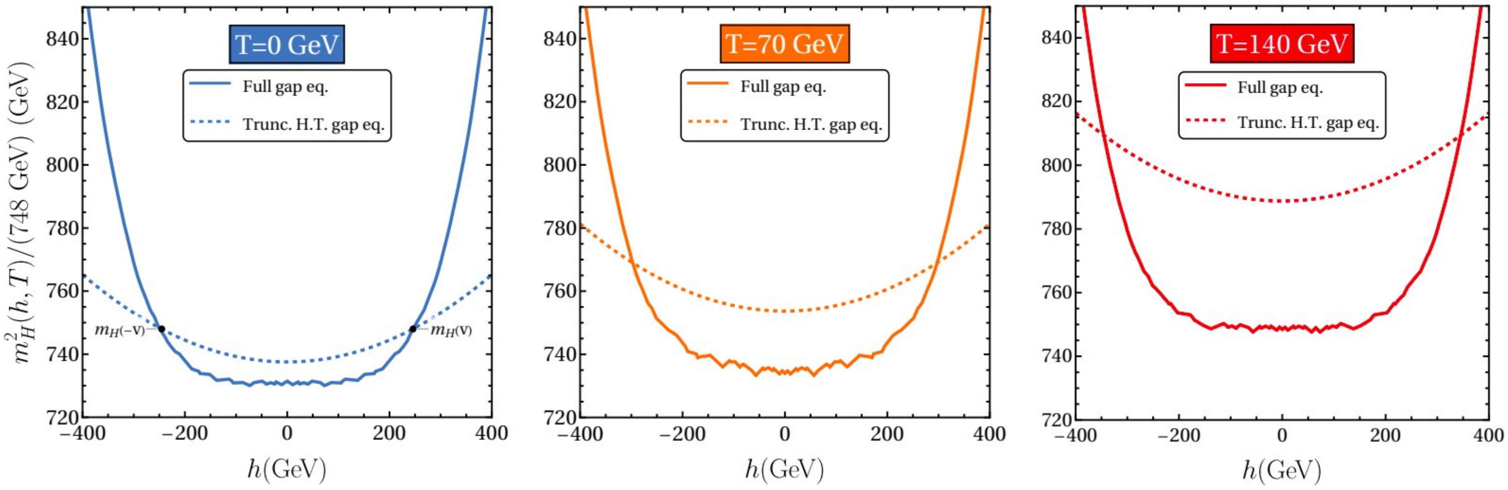


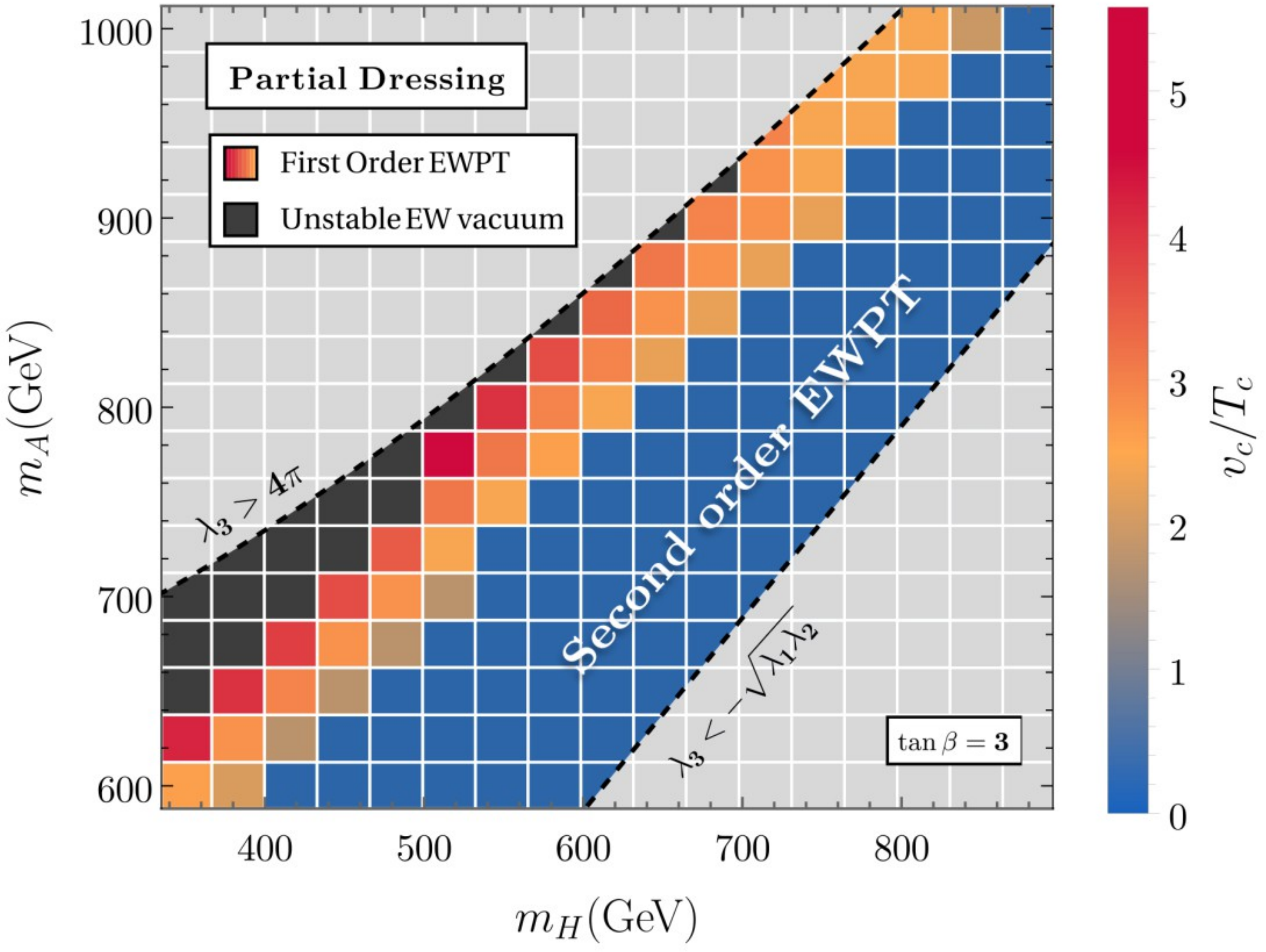

Higgs
Trunc. H.T. gap eq.
Full gap equation
$|m_h(v_1, v_2)|$ (GeV)
T(GeV)
Goldstone bosons
Trunc. H.T. gap eq.
Full gap equation
$|m_G(v_1, v_2)|$ (GeV)
T(GeV)
CP even Heavy Higgs
Trunc. H.T. gap eq.
Full gap equation
$|m_H(v_1, v_2)|$ (GeV)
T(GeV)
CP odd/Charged Higgs
Trunc. H.T. gap eq.
Full gap equation
$|m_{A,H^\pm}(v_1, v_2)|$ (GeV)
T(GeV)

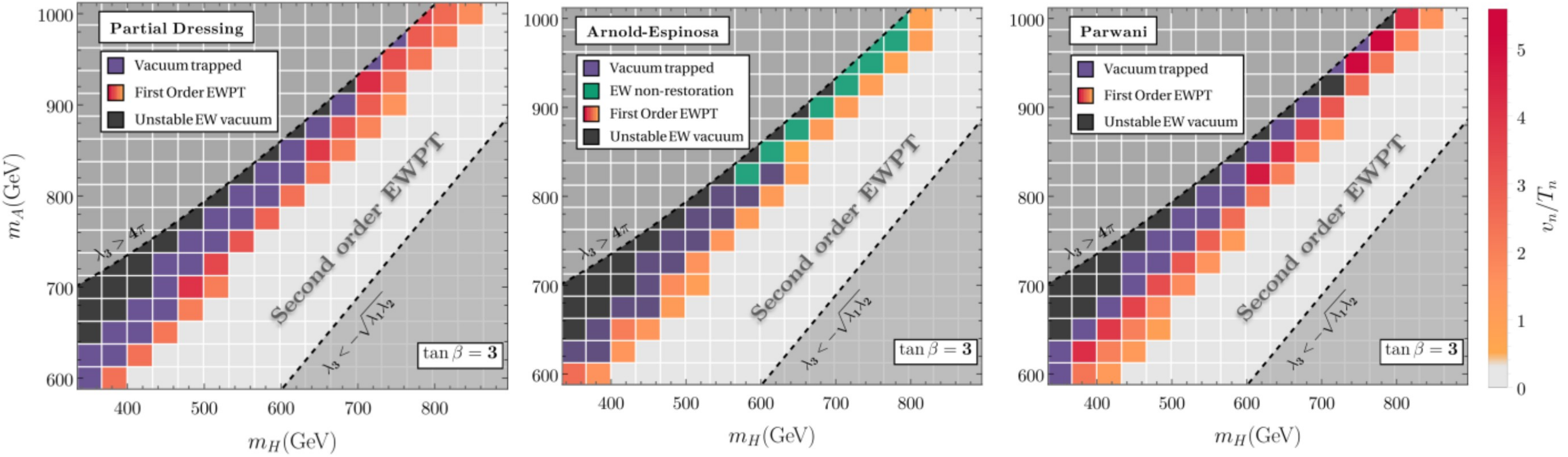
Higgs Thermal Mass



Heavy Higgs Thermal Mass




Partial Dressing
First Order EWPT
Unstable EW vacuum
$\lambda_3 > 4\pi$
Second order EWPT
$\lambda_3 < -\sqrt{\lambda_1\lambda_2}$
$\tan\beta = \mathbf{3}$
$m_A(\mathrm{GeV})$
$m_H(\mathrm{GeV})$
v_c/T_c
1000
900
800
700
600
400
500
600
700
800
0
1
2
3
4
5


Partial Dressing
Vacuum trapped
First Order EWPT
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$\lambda_3 > 4\pi$
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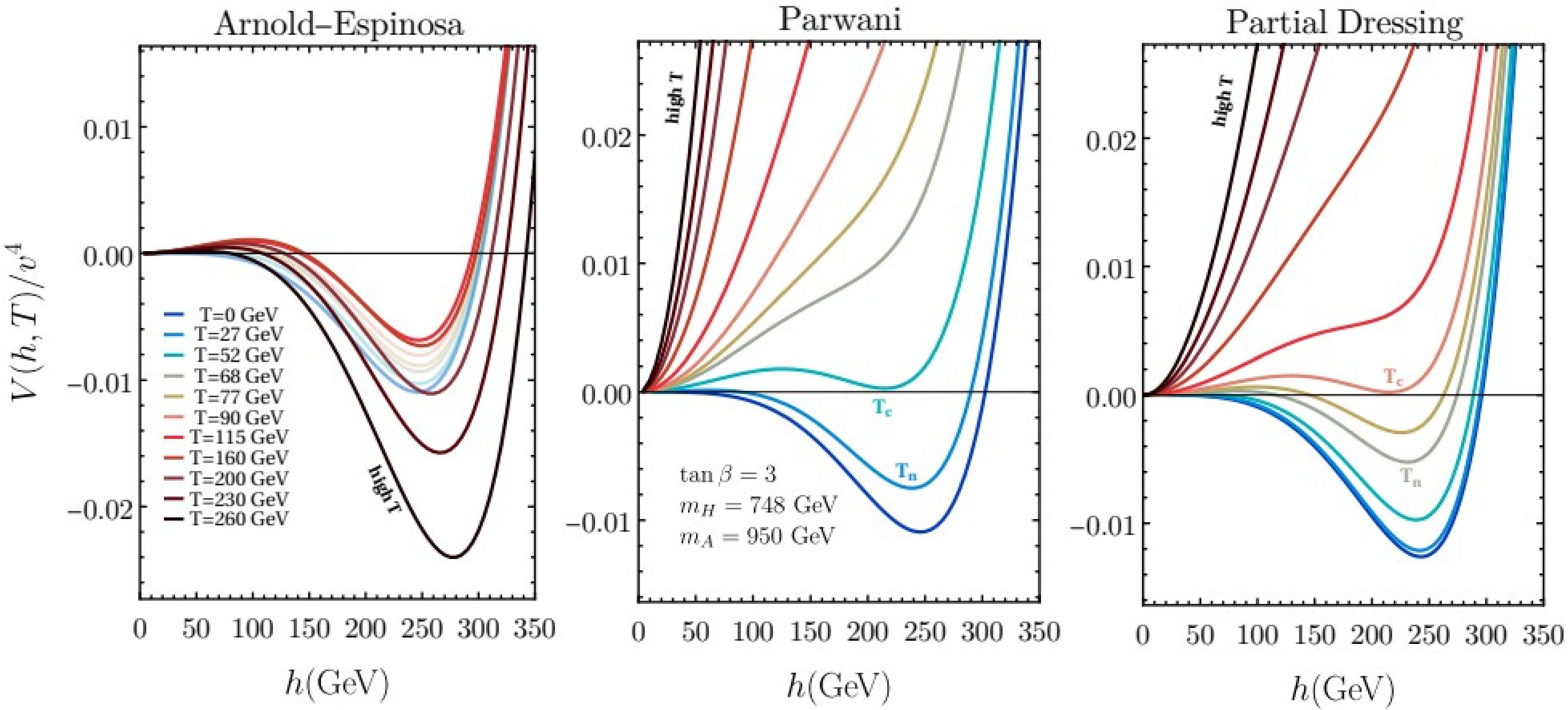
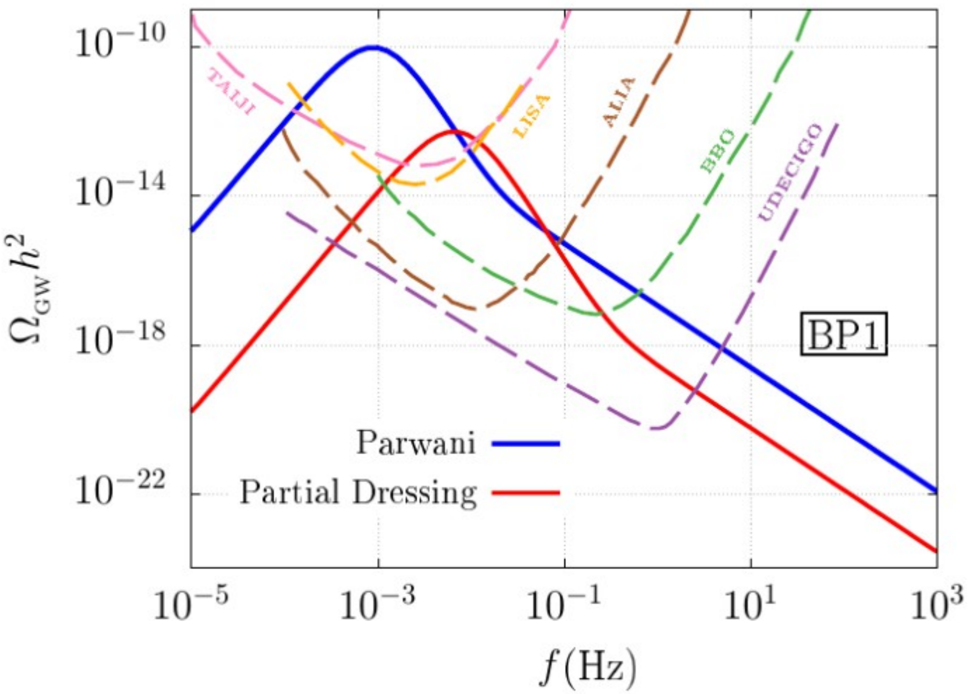
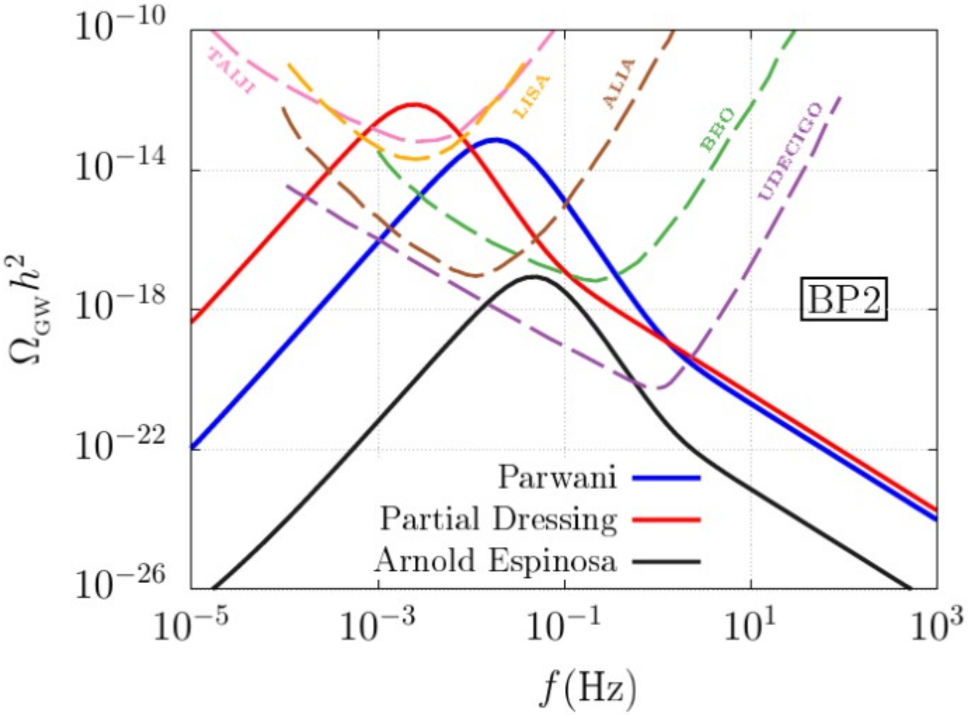
m_A(GeV)
m_H(GeV)
v_n/T_n


Arnold–Espinosa
Parwani
Partial Dressing
$V(h,T)/v^4$
h(GeV)
T=0 GeV
T=27 GeV
T=52 GeV
T=68 GeV
T=77 GeV
T=90 GeV
T=115 GeV
T=160 GeV
T=200 GeV
T=230 GeV
T=260 GeV
high T
T_c
T_n
$\tan\beta = 3$
$m_H = 748$ GeV
$m_A = 950$ GeV

Uncertainties in GW predictions





- **Partial dressing is superior to other resummation schemes**
- **But it is harder to implement...**

How much can we trust our predictions?

More refined methods are required for that

(and there has been progress)

Summary

- Studied the EW phase transition in the 2HDM using different resummation methods.

- Truncated schemes (AE, Parwani) fail near the critical temperature.

- Full dressing is self-consistent but miscounts higher-loop diagrams.

- Partial dressing is reliable and captures all relevant thermal effects.

- In 2HDM, methods disagree sharply — e.g. on symmetry non-restoration.

- **Accurate predictions (e.g. GWs) require self-consistent resummation.**

Thank you

Gravitational waves

Latent Heat:

$$\alpha = \frac{\epsilon}{\rho_{\rm rad}^*} = \frac{1}{\rho_{\rm rad}^*}\left[T\frac{{\rm d}\Delta V(T)}{{\rm d}T} - \Delta V(T)\right]\Bigg|_{T_*},$$

Time scale of the phase transition:

$$\beta = -\frac{dS_3}{dt}\Bigg|_{t_*} \simeq \frac{\dot{\Gamma}}{\Gamma} = H_* T_* \frac{d(S_3/T)}{dT}\Bigg|_{T_*},$$

$$\Omega_{\rm SW}h^2 = 2.65 \times 10^{-6}\Upsilon(\tau_{SW})\left(\frac{\beta}{H_\star}\right)^{-1} v_w \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{g_*}{100}\right)^{-\frac{1}{3}} \left(\frac{f}{f_{\rm SW}}\right)^3 \left[\frac{7}{4+3\left(\frac{f}{f_{\rm SW}}\right)^2}\right]^{\frac{7}{2}} .$$

$$f_{\rm SW} = 1.9 \times 10^{-5}\ {\rm Hz} \left(\frac{1}{v_w}\right)\left(\frac{\beta}{H_\star}\right)\left(\frac{T_n}{100\ {\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}} .$$