
HAVE YOU HEARD? THEY DISCOVERED THE GOD PARTI...
IT'S CALLED THE HIGGS BOSON!

BSM Higgs Lecture

Sven Heinemeyer, IFT (CSIC, Madrid)

Shanghai, 10/2025

1. Motivation for BSM Higgs & BSM Higgs sectors

2. BSM Higgs sectors: collider phenomenology & more

3. Tutorial: joint calculations

BSM Higgs Lecture

Motivation for BSM Higgs & BSM Higgs sectors

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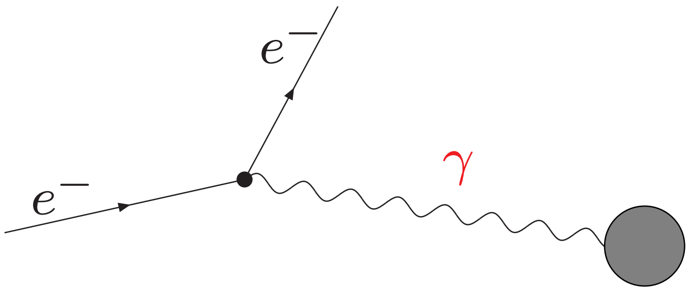
1. Why Higgs?
2. The SM-like Higgs at the LHC
3. Motivation for BSM Higgs
4. BSM Higgs Introduction
5. Higgs Singlet and Doublet Extensions
6. The Higgs Sector of the (N)MSSM

1. Why Higgs?

Construction principle of the SM: gauge invariance

Example: Quantum electro-dynamics (QED)

field quanta: photon A_μ



Lagrangian:

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi \ .$$

ψ is the electron spinor, A_μ is the photon (vector) field
with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

D_μ denotes the covariant derivative

$$D_\mu = \partial_\mu + i\, e\, A_\mu \ .$$

Construction principle of the SM: gauge invariance

The QED Lagrangian is invariant under the local $U(1)$ gauge symmetry,

$$\psi \to e^{-i\alpha(x)}\psi \;, \tag{1}$$

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu\alpha(x) \;. \tag{2}$$

Introducing a mass term for the photon,

$$\mathcal{L}_{\text{photon mass}} = \frac{1}{2} m_A^2 A_\mu A^\mu \;,$$

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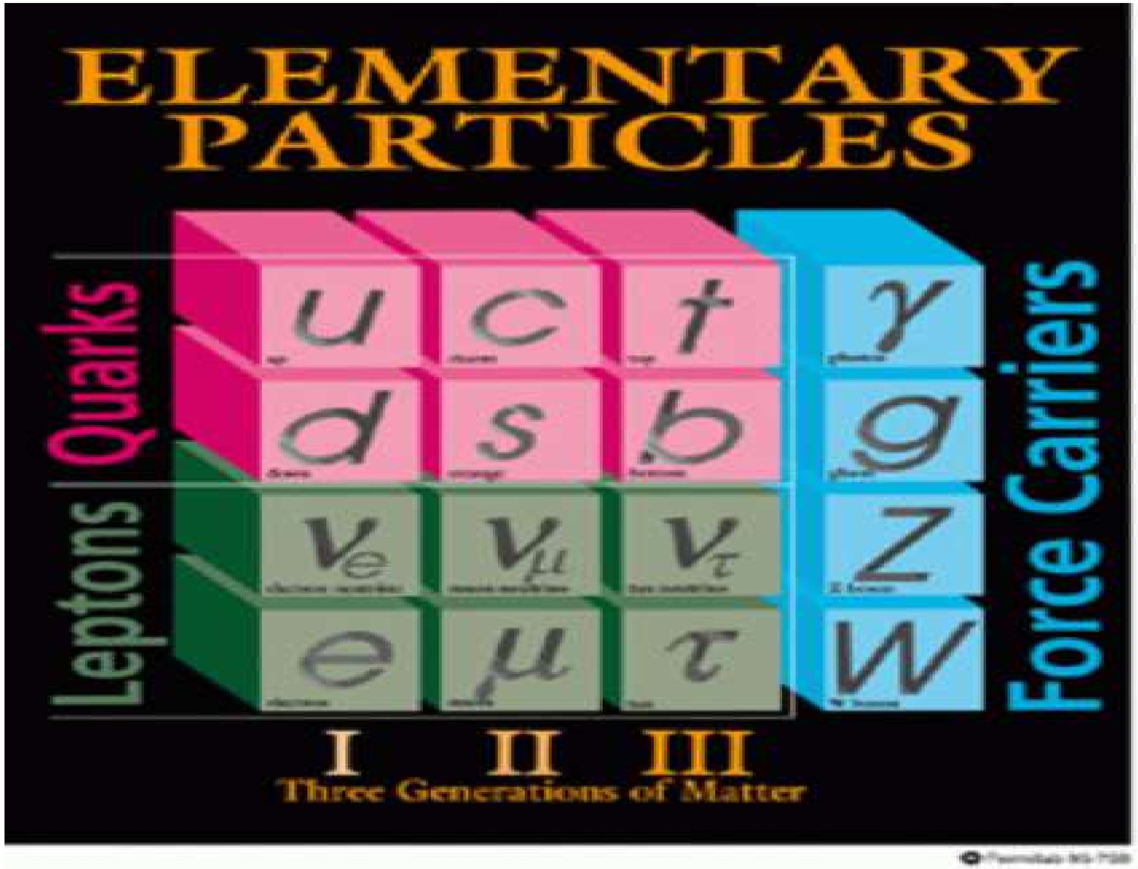
however, is not gauge-invariant. Applying eq. (2) yields

$$\frac{1}{2} m_A^2 A_\mu A^\mu \to \frac{1}{2} m_A^2 \left[A_\mu A^\mu + \frac{2}{e} A^\mu \partial_\mu \alpha + \frac{1}{e^2} \partial_\mu \alpha \, \partial^\mu \alpha \right] \ .$$

⇒ the gauge bosons must be massless!

Current status of knowledge: the Standard Model (SM)



$\Rightarrow$ all particles experimentally seen (as of 2011)

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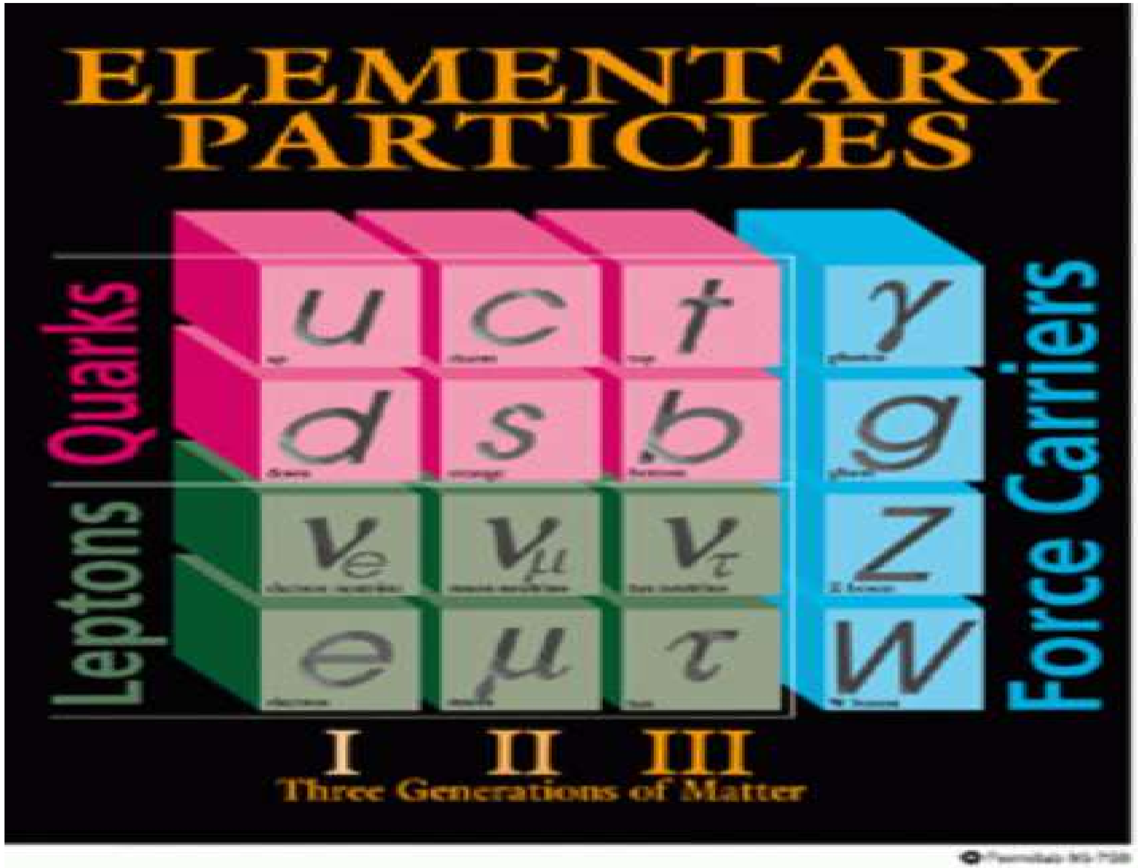


$\Rightarrow$ all particles experimentally seen (as of 2011)

$\Rightarrow$ but it predicts massless gauge bosons ...

Problem:

Gauge fields Z, W^+, W^- are massive

explicite mass terms in the Lagrangian $\Leftrightarrow$ breaking of gauge invariance

Solution: Higgs mechanism

scalar field postulated, mass terms from coupling to Higgs field

Higgs sector in the Standard Model:

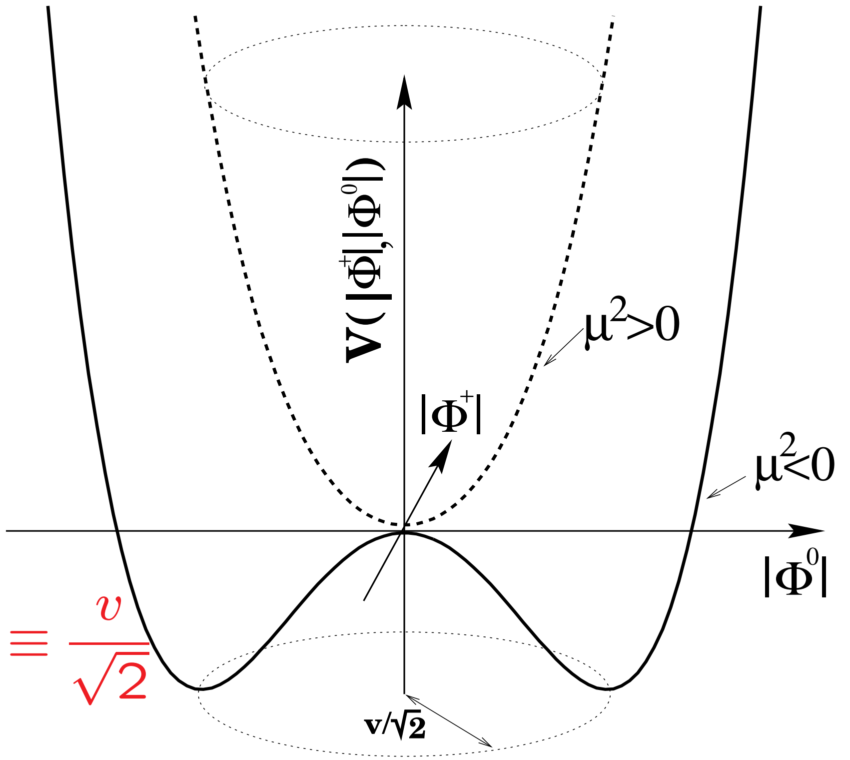
Scalar SU(2) doublet: $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$

Higgs potential:

$$V(\phi) = \mu^2 \left|\Phi^\dagger \Phi\right| + \lambda \left|\Phi^\dagger \Phi\right|^2, \quad \lambda > 0$$

$\mu^2 < 0$: Spontaneous symmetry breaking

minimum of potential at $|\langle \Phi_0 \rangle| = \sqrt{\frac{-\mu^2}{2\,\lambda}} \equiv \frac{v}{\sqrt{2}}$



$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} \quad \text{(unitary gauge)}$$

H: elementary scalar field, Higgs boson

Lagrange density:

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = \quad & (D_\mu \Phi)^\dagger (D^\mu \Phi) \\ & - g_d \bar{Q}_L \Phi d_R - g_u \bar{Q}_L \Phi_c u_R \\ & - V(\Phi) \end{aligned}$$

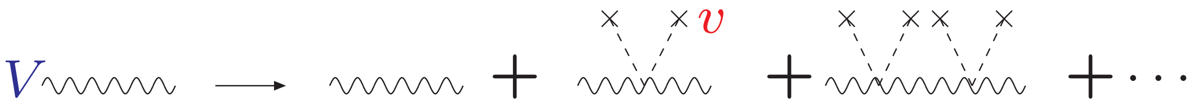
with

$$\begin{aligned} iD_\mu &= i\partial_\mu - g_2 \vec{I} \vec{W}_\mu - g_1 Y B_\mu \\ \Phi_c &= i\sigma_2 \Phi^* \qquad\qquad Q_L \sim \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \; \Phi \sim \begin{pmatrix} 0 \\ v \end{pmatrix}, \; \Phi_c \sim \begin{pmatrix} v \\ 0 \end{pmatrix} \end{aligned}$$

Gauge invariant coupling to gauge fields

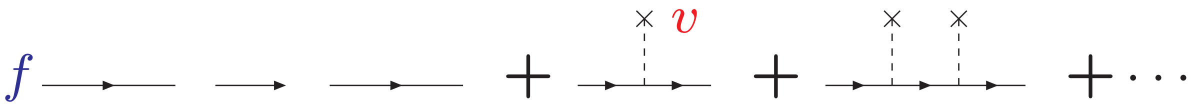
$\Rightarrow$ mass terms for gauge bosons and fermions

1.) $VV\Phi\Phi$ coupling:



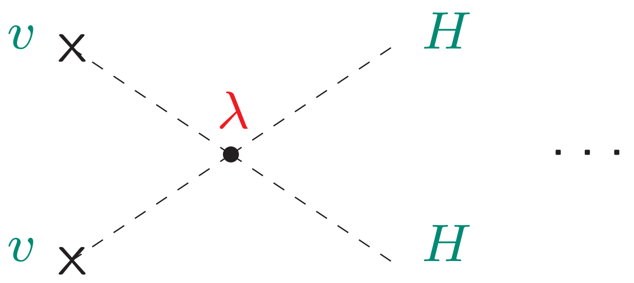
$$\frac{1}{q^2} \to \frac{1}{q^2} + \sum_j \frac{1}{q^2}\left[\left(\frac{gv}{\sqrt{2}}\right)^2 \frac{1}{q^2}\right]^j = \frac{1}{q^2 - M^2} \ : \ M^2 = g^2\frac{v^2}{2} \qquad \Rightarrow M \propto g$$

2.) fermion mass terms: Yukawa couplings:



$$\frac{1}{\not{q}} \to \frac{1}{\not{q}} + \sum_j \frac{1}{\not{q}}\left[\frac{g_f v}{\sqrt{2}} \frac{1}{\not{q}}\right]^j = \frac{1}{\not{q} - m_f} \ : \ m_f = g_f\frac{v}{\sqrt{2}} \qquad \Rightarrow m_f \propto g_f$$

3.) mass of the Higgs boson: self coupling



$$\lambda = M_H^2/v^2$$

$M_H = v\sqrt{\lambda}$ free parameter

$\rightarrow$ last unknown (now measured) parameter of the SM

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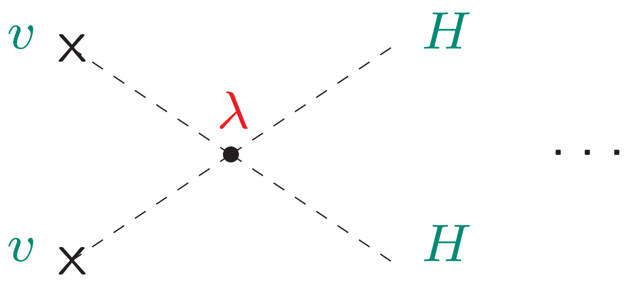

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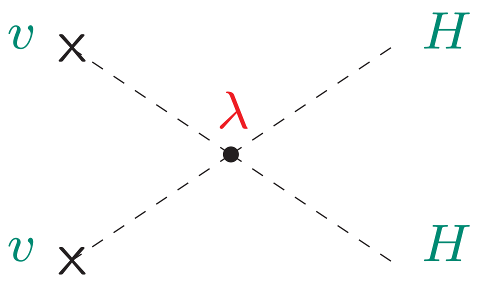

$\dots$

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Q1: How can we measure Higgs couplings?

Q2: What else should be measured?

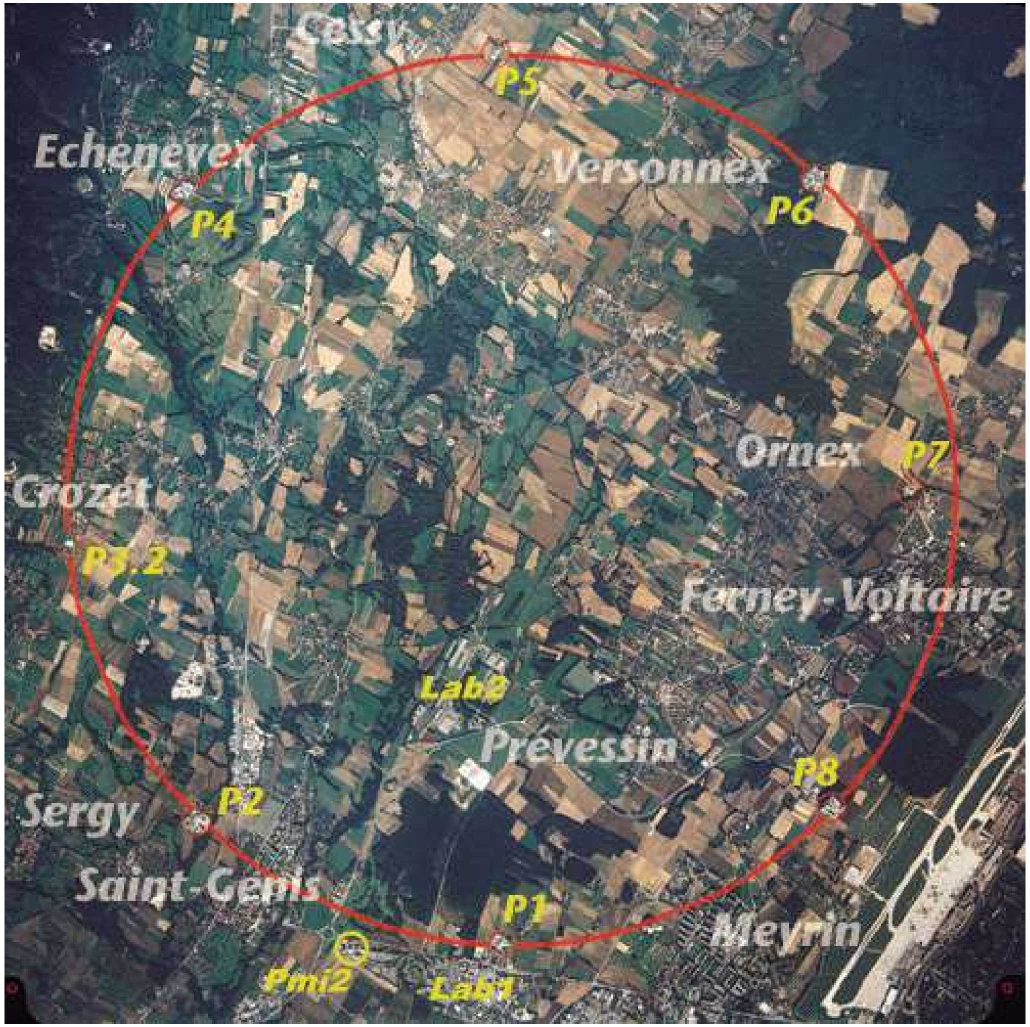
2. The SM-like Higgs at the LHC

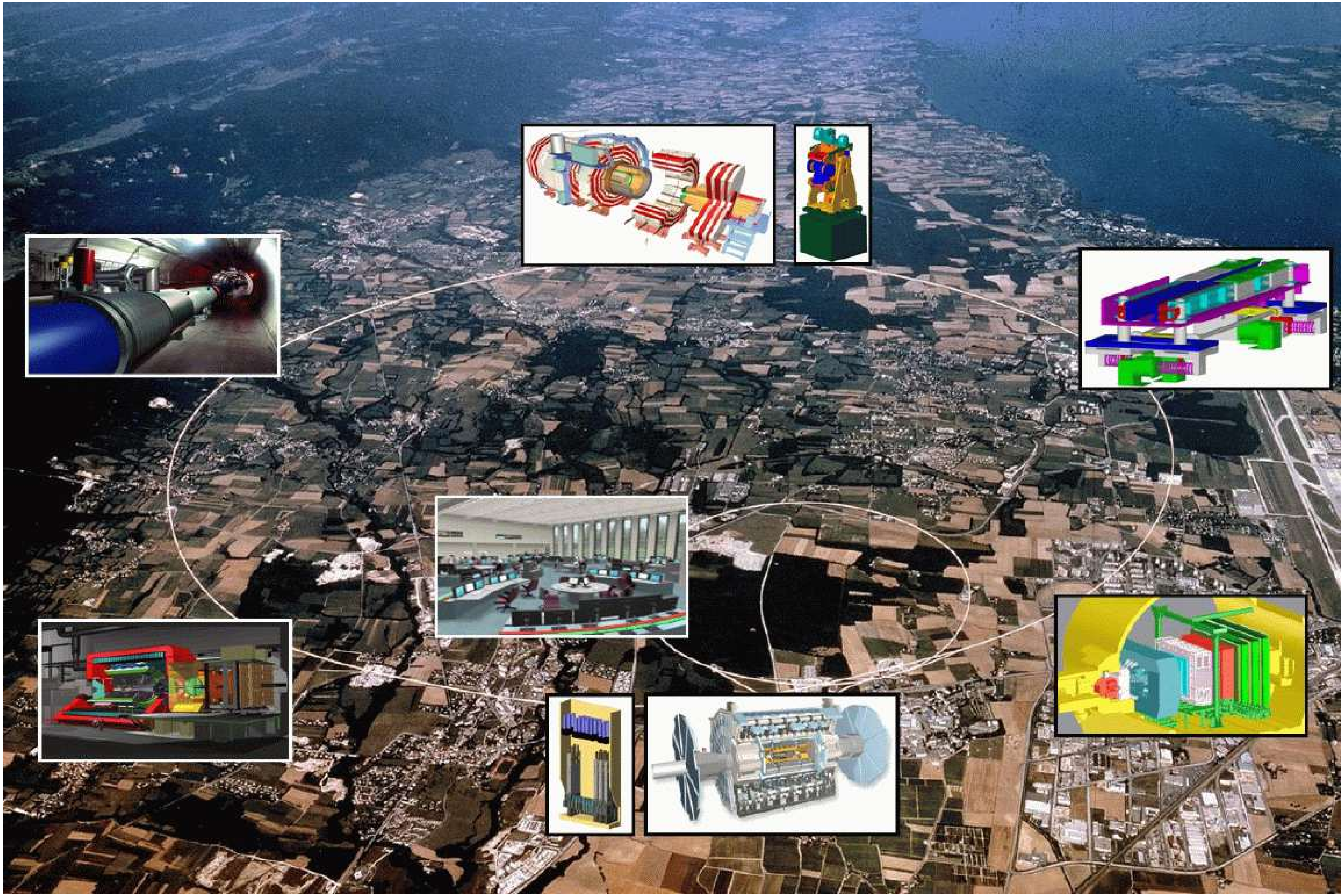
The LHC

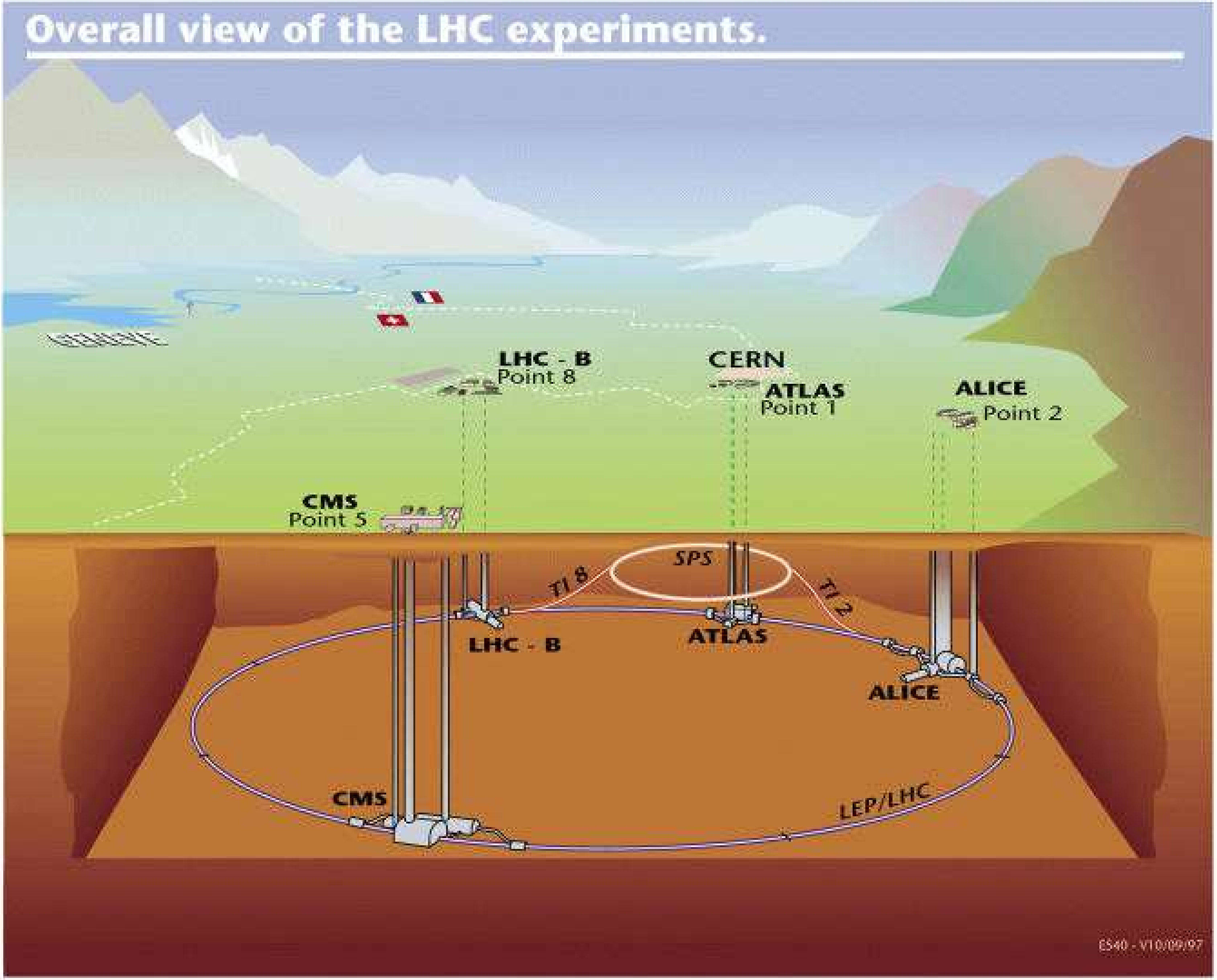
LHC:

$p\,p$ collisions at $\sqrt{s} \lesssim 14$ TeV

- 27 km circumference
- two general purpose detectors: ATLAS and CMS
- one B physics detector: LHCb
- one heavy ion detector: Alice






Overall view of the LHC experiments.
LHC - B
Point 8
CERN
ATLAS
Point 1
ALICE
Point 2
CMS
Point 5
SPS
TI 8
TI 2
LHC - B
ATLAS
ALICE
CMS
LEP/LHC
ES40 - V10/09/97

The (un)official (optimistic?) LHC time line:

03/2010: first collisions at record breaking energy

2010: $\lesssim 0.05\,\text{fb}^{-1}$ (at $\sqrt{s} = 7$ TeV)

2011: $\sim 5\,\text{fb}^{-1}$ (at $\sqrt{s} = 7$ TeV) $\Rightarrow$ first physics results!

2012: $\sim 20\,\text{fb}^{-1}$ (at $\sqrt{s} = 8$ TeV) $\Rightarrow$ **Higgs discovery!**

2013 – 2014 shutdown, further splice checks, repairs, . . .

2015 – 2018: $\sim 40\,\text{fb}^{-1}$ per year $\Rightarrow$ physics results at $\sqrt{s} = 13$ TeV

2019 – 2021: shutdown, preparation for "higher luminosity"

2022 – 2026: $\gtrsim 50\,\text{fb}^{-1}$ per year $\Rightarrow$ physics results with "high" luminosity

2026 – 2028: upgrade to HL-LHC

2029 + X (X > 0): HL-LHC

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YOU live in an exciting time!!!

LHC Results: Executive Summary (take home message)

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Groundbreaking discovery in the Higgs searches!

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Groundbreaking discovery in the Higgs searches!

No clear evidence for BSM physics - yet! But . . . :-)

Properties of the SM Higgs boson

1.) Decay to fermions:

coupling:

$$g_{f\bar{f}H} = \left[\sqrt{2}\,G_\mu\right]^{1/2} m_f$$

decay width:

$$\Gamma(H \to f\bar{f}) = N_c \frac{G_\mu\,M_H}{4\sqrt{2}\,\pi} m_f^2(M_H^2) \left(1 - 4\frac{m_f^2}{M_H^2}\right)^{3/2}$$

with N_c = number of colors

Bulk of QCD corrections for decays to quarks are mapped into

$$m_q^2(\text{pole}) \to m_q^2(M_H^2)$$

Q: What is the strongest/most important decay?

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Dominant decay process: $H \to b\bar{b}$

2.) Decay to heavy gauge bosons ($V = W, Z$):

coupling:

$$g_{VVH} = 2\left[\sqrt{2}\,G_\mu\right]^{1/2} M_V^2$$

on-shell decay width ($M_H > 2M_V$):

$$\Gamma(H \to VV) = \delta_V \frac{G_\mu M_H^3}{16\sqrt{2}\,\pi}\left(1 - 4\frac{M_V^2}{M_H^2} + 12\frac{M_V^4}{M_H^4}\right)\left(1 - 4\frac{M_V^2}{M_H^2}\right)^{1/2}$$

with $\delta_{W,Z} = 2, 1$

off-shell decay width ($M_H < 2M_V$):

$$\Gamma(H \to VV^*) = \delta_V' \frac{3G_\mu^2 M_H}{16\,\pi^3} M_V^4 \times \text{Integral}$$

3.) Decay to massless gauge bosons (gg, $\gamma\gamma$):

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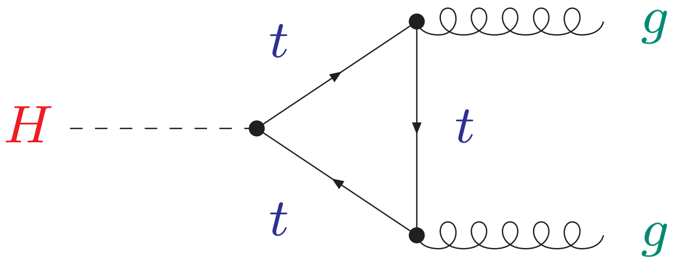
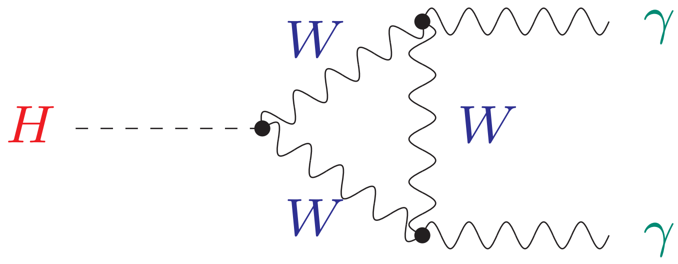
3.) Decay to massless gauge bosons (gg, $\gamma\gamma$):





$$\Gamma(H \to gg) \;=\; \frac{G_\mu\, \alpha_s^2(M_H^2)\, M_H^3}{36\,\sqrt{2}\,\pi^3}\left[1 + C\frac{\alpha_s(\mu)}{\pi}\right]$$

via the top quark loop with

$$C = \frac{215}{12} - \frac{23}{6}\log\left(\frac{\mu^2}{M_H^2}\right) + \mathcal{O}(\alpha_s)$$

$\Rightarrow$ huge QCD corrections

$$\Gamma(H \to \gamma\gamma) \;=\; \frac{G_\mu\, \alpha^2\, M_H^3}{128\,\sqrt{2}\,\pi^3}\left|\frac{4}{3}e_t^2 - 7\right|^2$$

via the top quark and W boson loop

Total width:

sum over all decay widths

$$\begin{aligned}\Gamma_{H,\text{tot}} := \sum_{\text{dd}'} \Gamma(H \to \text{dd}') \\ = \quad & \Gamma(H \to t\bar{t}) + \Gamma(H \to b\bar{b}) + \Gamma(H \to c\bar{c}) + \ldots \\ & + \Gamma(H \to \tau^+\tau^-) + \Gamma(H \to \mu^+\mu^-) + \ldots \\ & + \Gamma(H \to WW^{(*)}) + \Gamma(H \to ZZ^{(*)}) + \Gamma(H \to \gamma\gamma) + \ldots \\ & + \ldots\end{aligned}$$

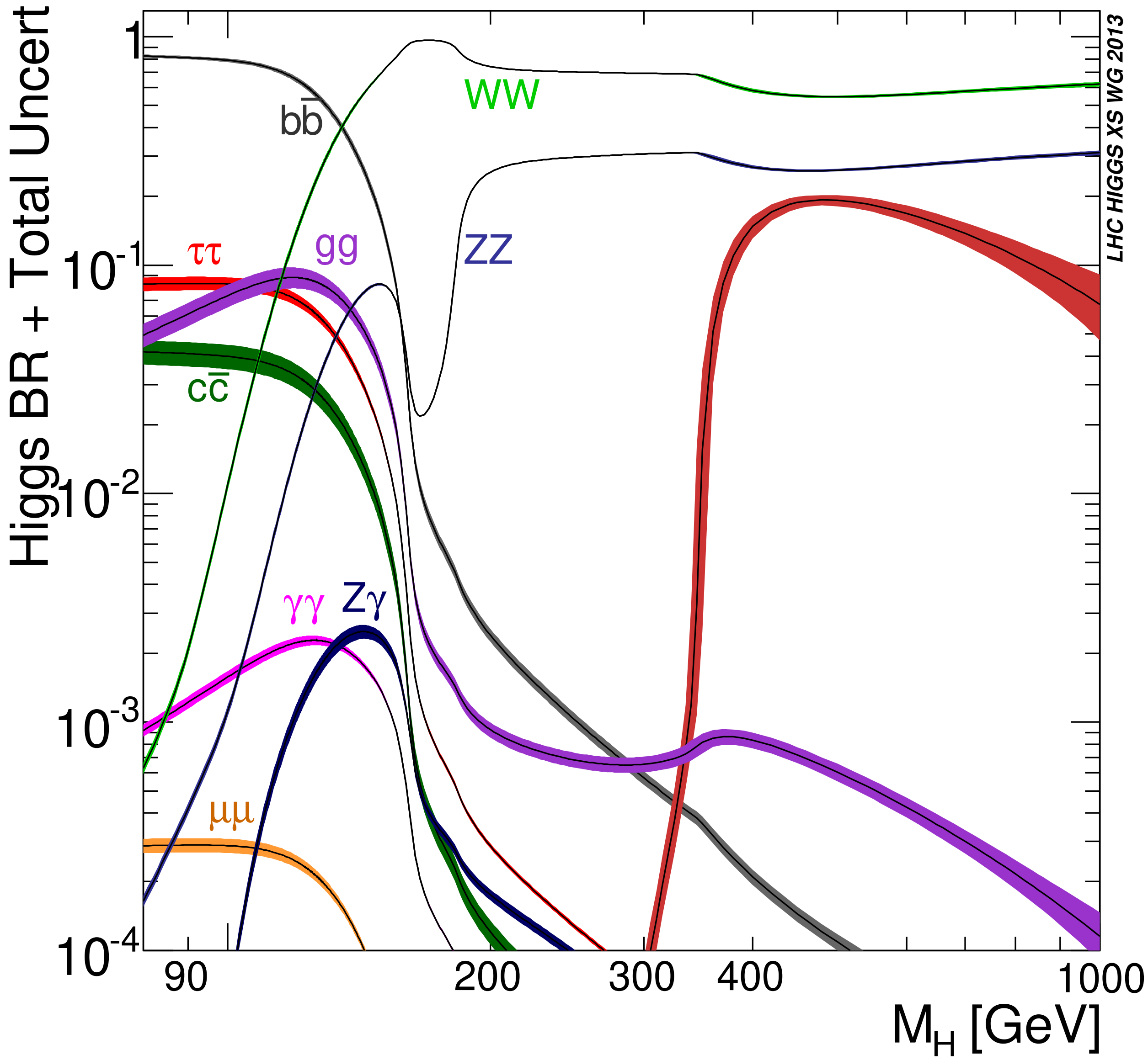
Branching ratio:

probability that a particle decays to a certain final state

$$\text{BR}(H \to \text{dd}') := \frac{\Gamma(H \to \text{dd}')}{\Gamma_{H,\text{tot}}}$$

Theory predictions for the SM Higgs: branching ratios

[*LHC Higgs XS WG '13*]



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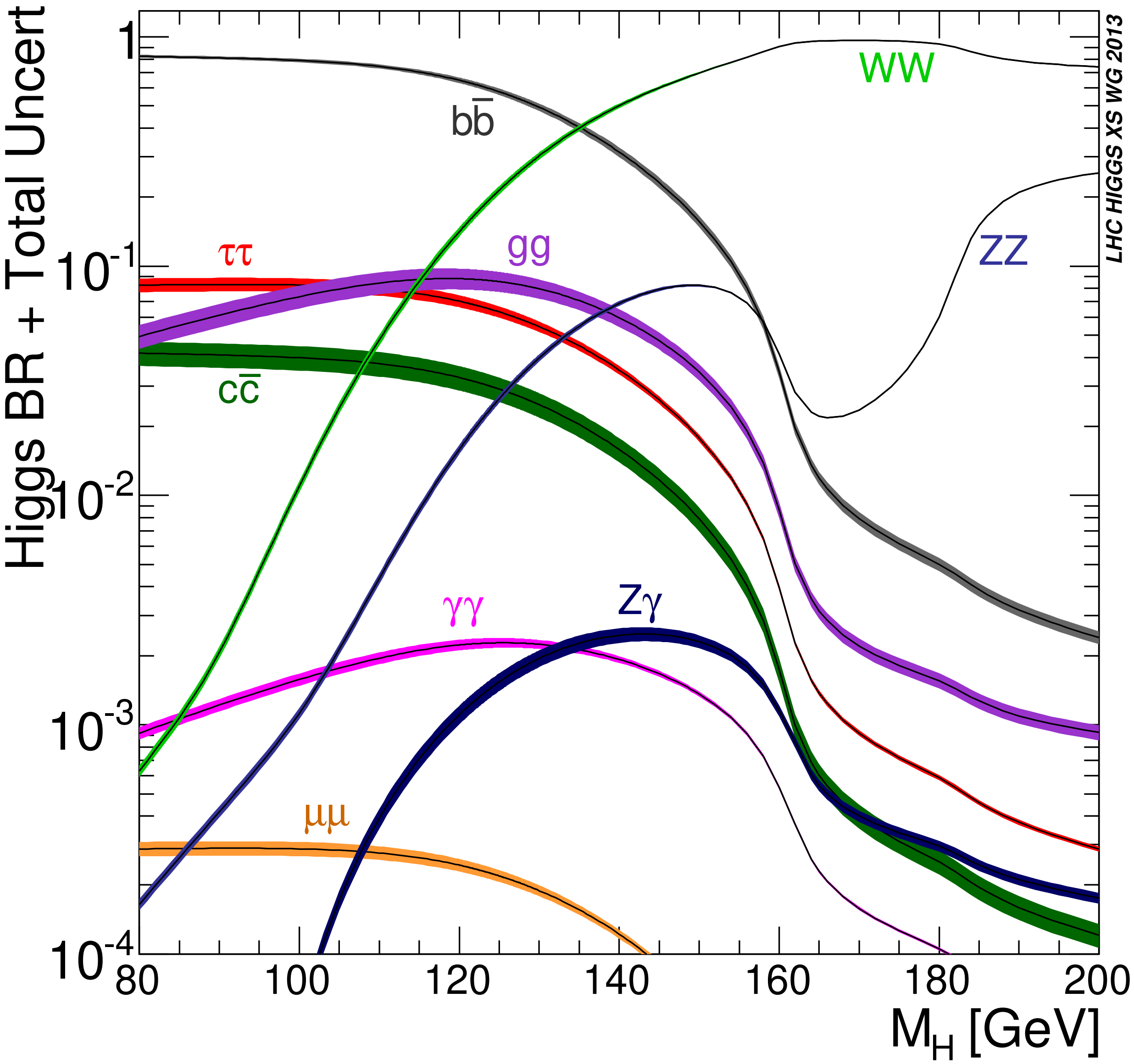
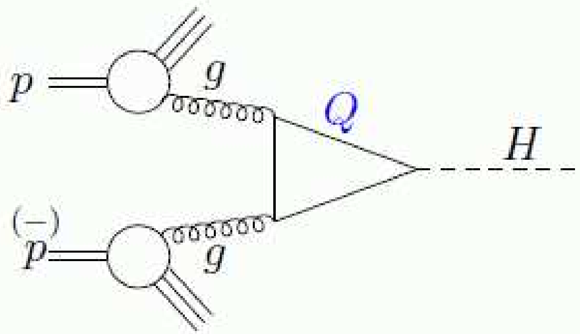


Higgs production modes at the LHC:

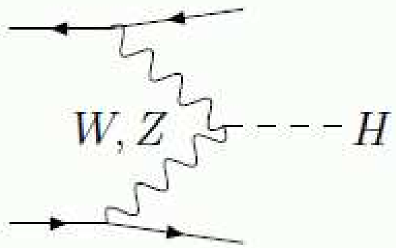
- **Gluon Gluon Fusion**

  $pp \to gg \to H$



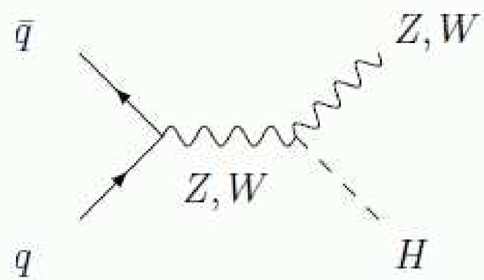
- **W/Z Fusion**

  $pp \to qq \to qq + WW/ZZ \to qq + H$



- **Higgs-strahlung**

  $pp \to W^*/Z^* \to W/Z + H$



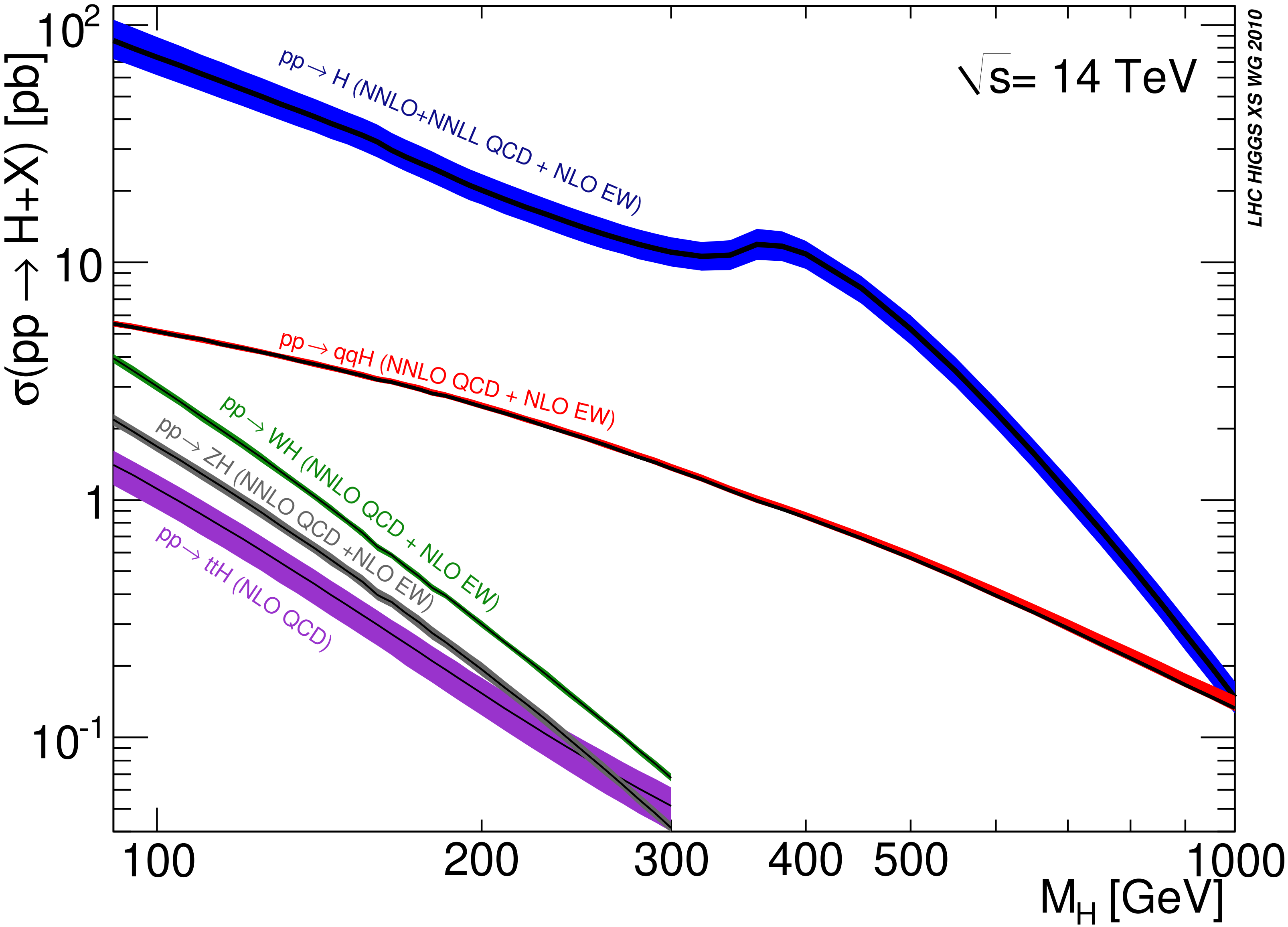
- **Associated production with $t\bar{t}$**

  $pp \to t\bar{t} + H$

[*taken from M. Mühlleitner*]

Theory predictions for the SM Higgs: LHC production XS

[*LHC Higgs XS WG '10*]



Various possible (LHC) “observables” for the Higgs discovery:

$\to$ normally given as a function of M_H

- Local p_0 value:

  probability that the observed signal/number of events is caused by “background only” (i.e. the SM without a Higgs)

- $\sigma_{\text{excl.}}/\sigma_{\text{SM}}$:

  excluded cross section divided by SM cross section:
  if for a certain M_H a cross section smaller than the SM cross section excluded, this M_H value is excluded, as it would have led to more events than observed.

- signal strength μ:

$$\mu := \frac{\sigma(pp \to H) \times \text{BR}(H \to dd')_{\text{observed}}}{\sigma(pp \to H) \times \text{BR}(H \to dd')_{\text{SM}}}$$

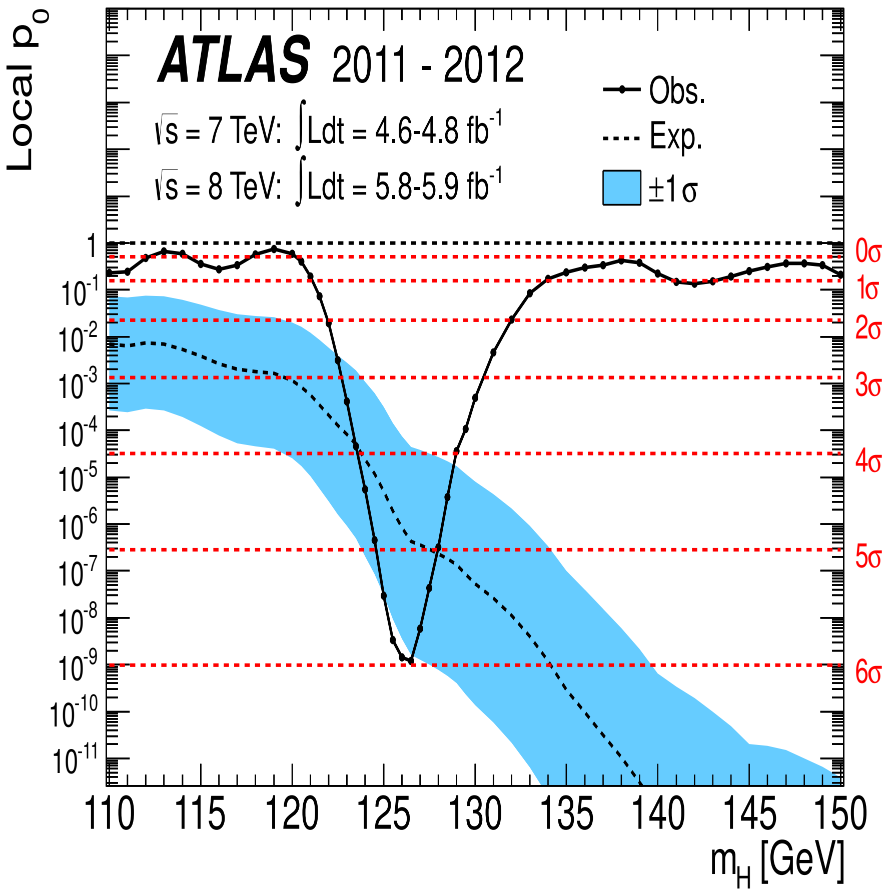
  should be “around 1” to find agreement with SM

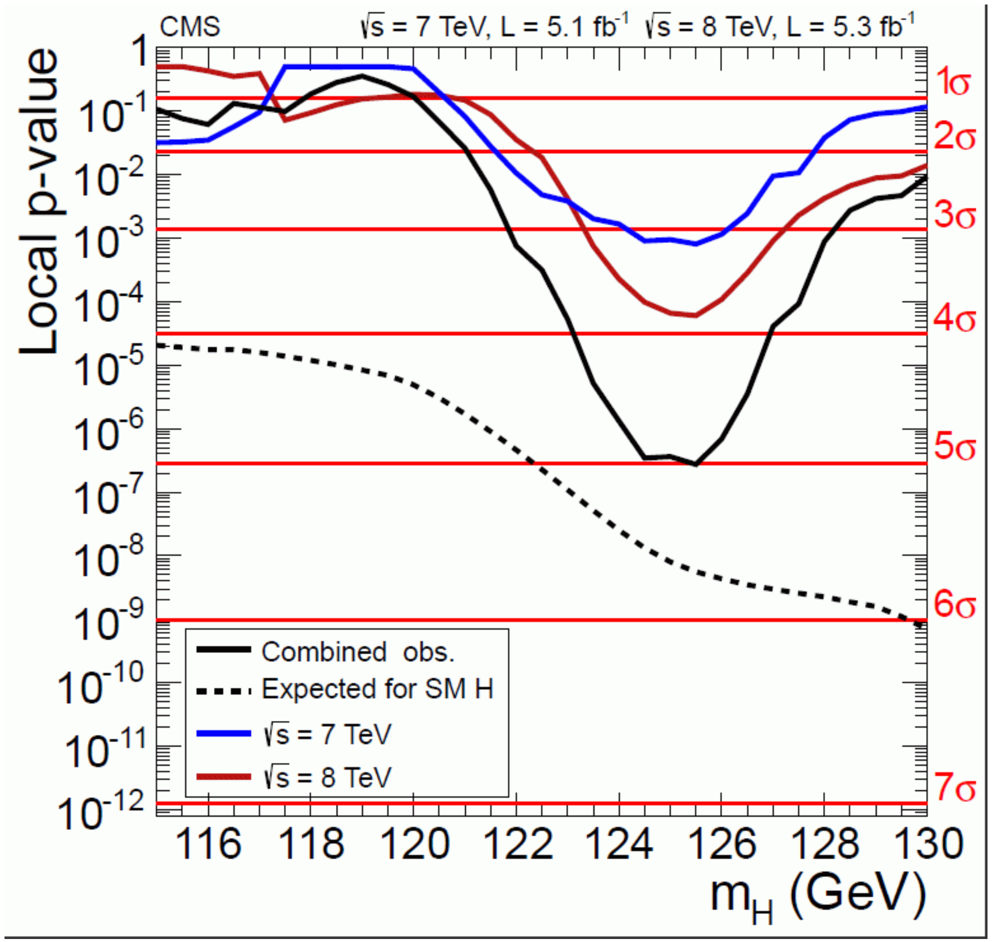
Why July 4th is celebrated (not only in the US): Q: Why?

3. Motivation for BSM Higgs

Fact I: The physics world changed on 04.07.2012:

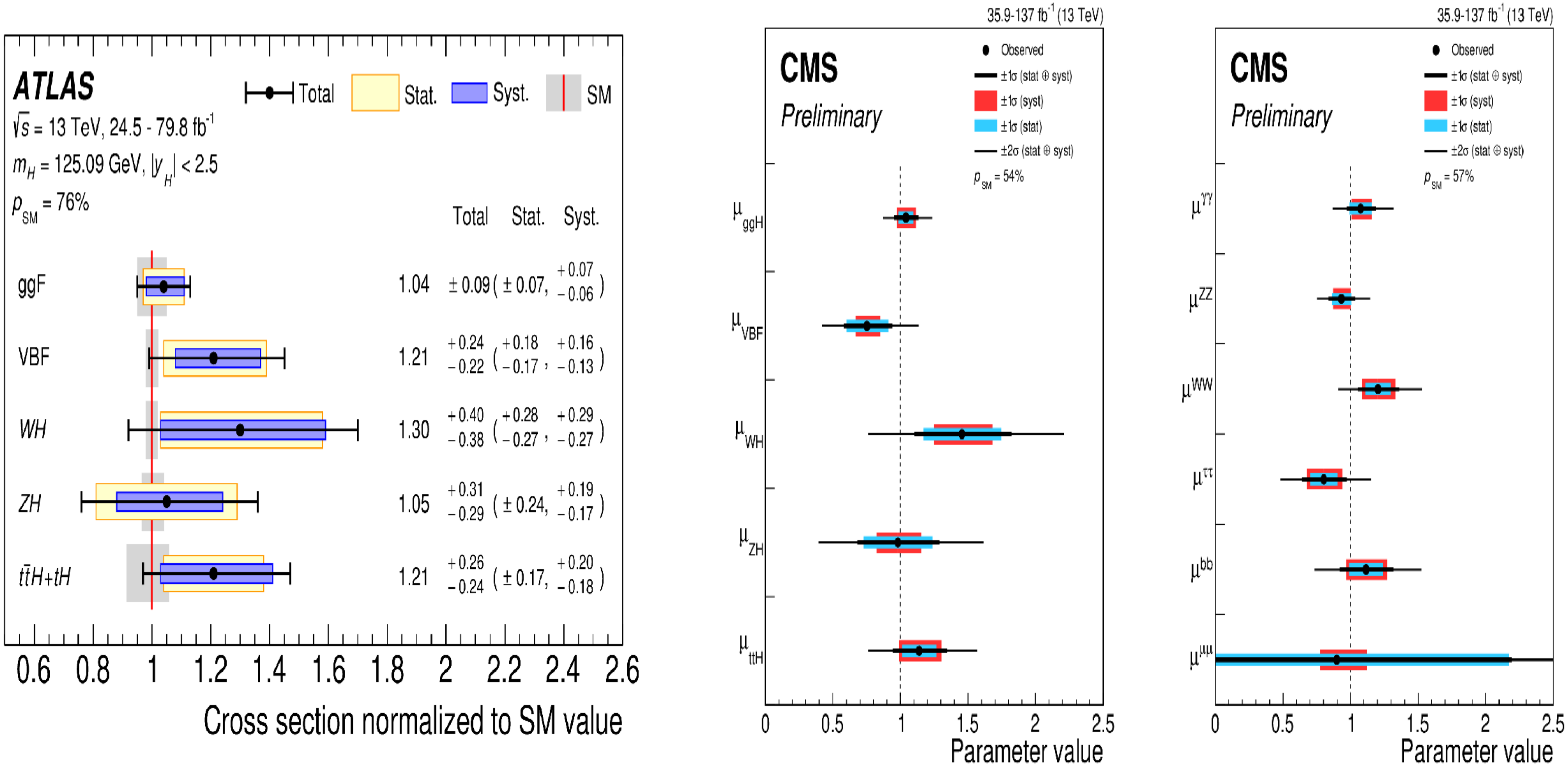
We have a discovery!





Fact I: The physics world changed on 04.07.2012:

We have an SM-like discovery!



Fact II:

$\boxed{\text{The SM cannot be the ultimate theory!}}$

Some sub-facts:

1. gravity is not included
2. the hierarchy problem
3. no unification of the three forces
4. Dark Matter is not included
5. Baryon Asymmetry of the Universe cannot be explained
6. neutrino masses are not included
7. Some interesting experimental anomalies . . .

Fact I & II:

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Conclusion: The discovered Higgs cannot be “the SM Higgs”!

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Q: Does the BSM physics have any (relevant) impact on the Higgs?
$\Rightarrow$ any hints from LHC results (as guideline/toy example)?

Q': Which model?

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Conclusion: The discovered Higgs cannot be "the SM Higgs"!

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$\Rightarrow$ any hints from LHC results (as guideline/toy example)?
Q': Which model?

A1: check changed properties of the h_{125}

A2: check for additional Higgs bosons
check for additional Higgs bosons above and below 125 GeV
$\Rightarrow$ any evidence yet? :-)

Back to fact II:

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Some sub-facts:

1. gravity is not included $\Leftarrow$ and here?
2. the hierarchy problem $\Leftarrow$ and here?
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Back to fact II:

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Some sub-facts:

1. gravity is not included $\Rightarrow$ SUSY
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6. neutrino masses are not included $\Rightarrow$ SUSY
7. Some interesting experimental anomalies . . . $\Rightarrow$ SUSY

My favorite motivation

My favorite motivation

My favorite motivation

$\Rightarrow$ Why is there more matter than antimatter? $\Rightarrow$ (EW) baryogenesis

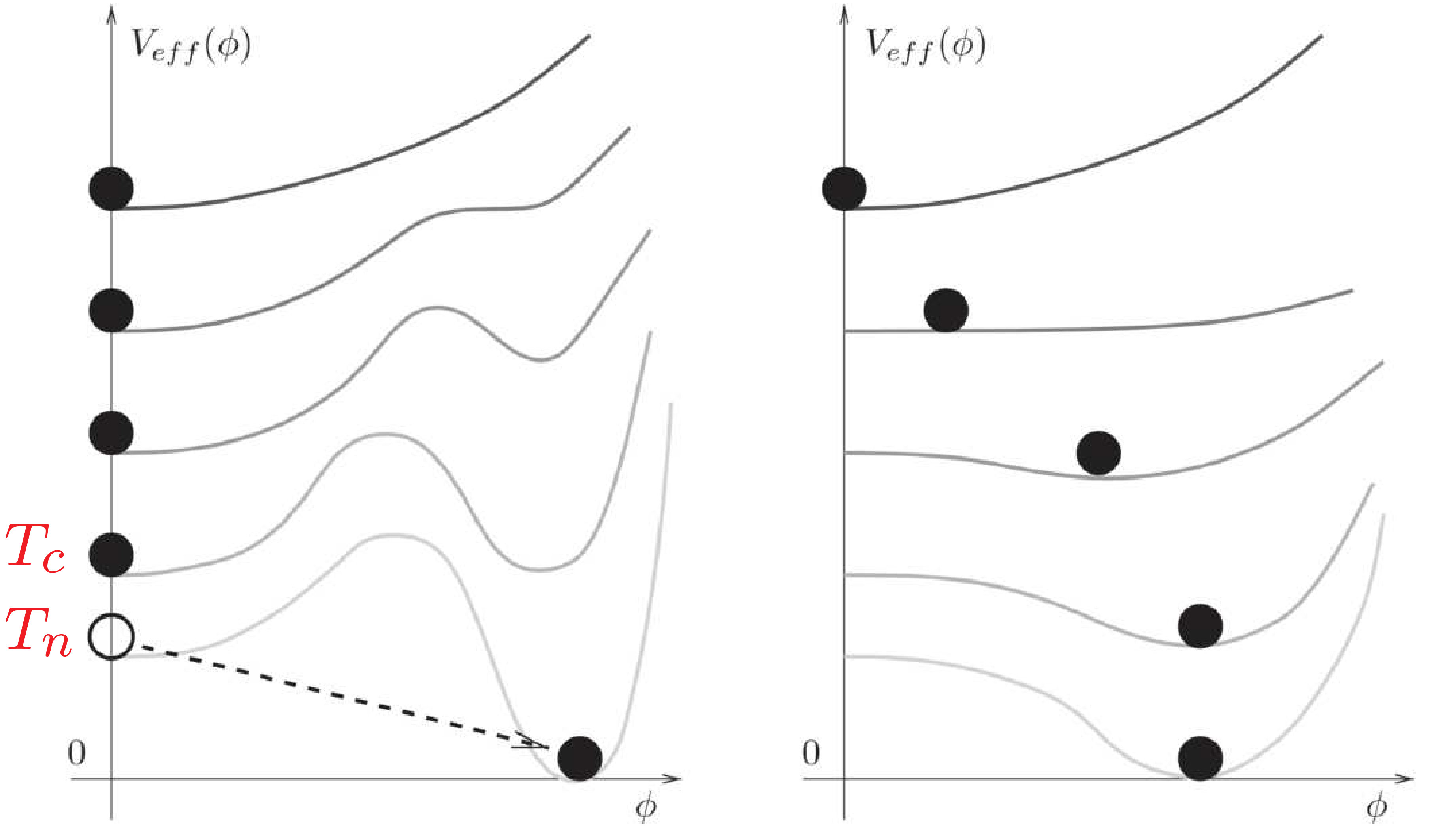
$\Rightarrow$ requires First Order EW Phase Transition (FOEWPT)

FOEWPT not possible in the SM $\Rightarrow$ BSM Higgs sector required

FOEWPT can cause Gravitational Waves (GW), detectable with LISA, . . .

Phase transition: BSM vs. SM

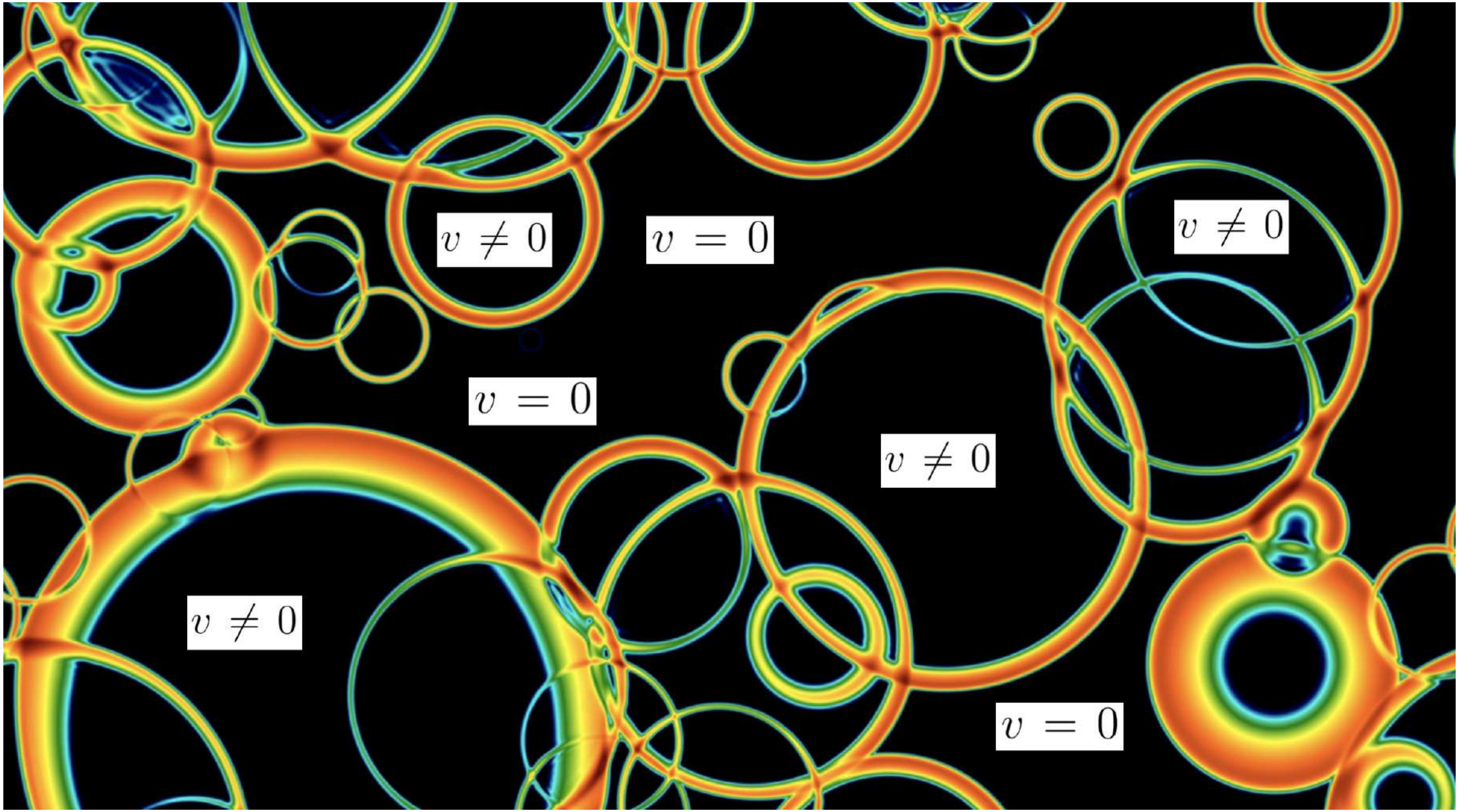
[*taken from V. A. Rubakov and D. S. Gorbunov*]



$\Rightarrow$ BSM Higgs sector required to realized FOEWPT

Bubble formation can lead to Gravitational Waves

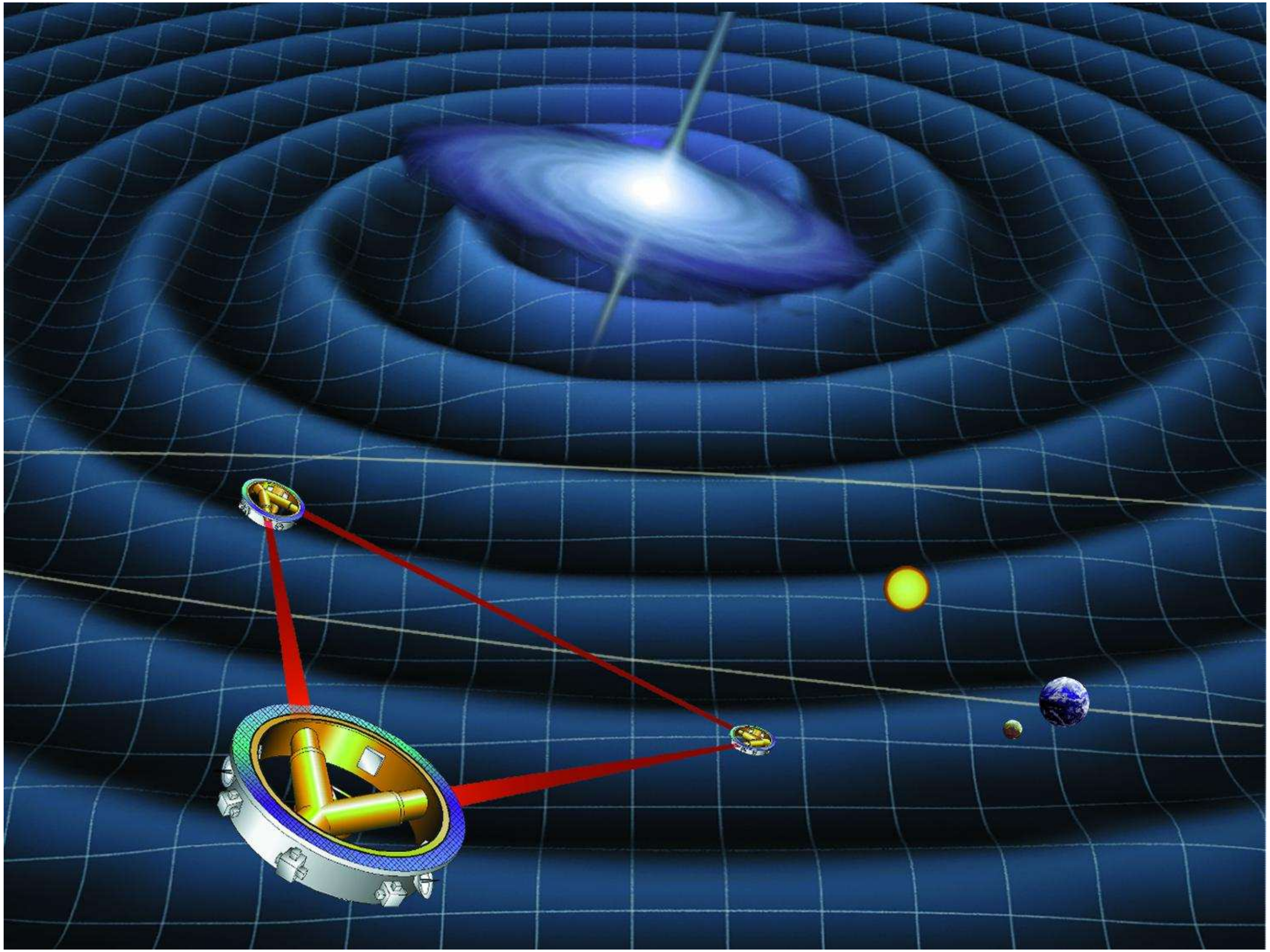
[*taken from D. Weir*]



$\Rightarrow$ Interesting interplay of Higgs physics and Gravitational Waves!

GW observatory: LISA

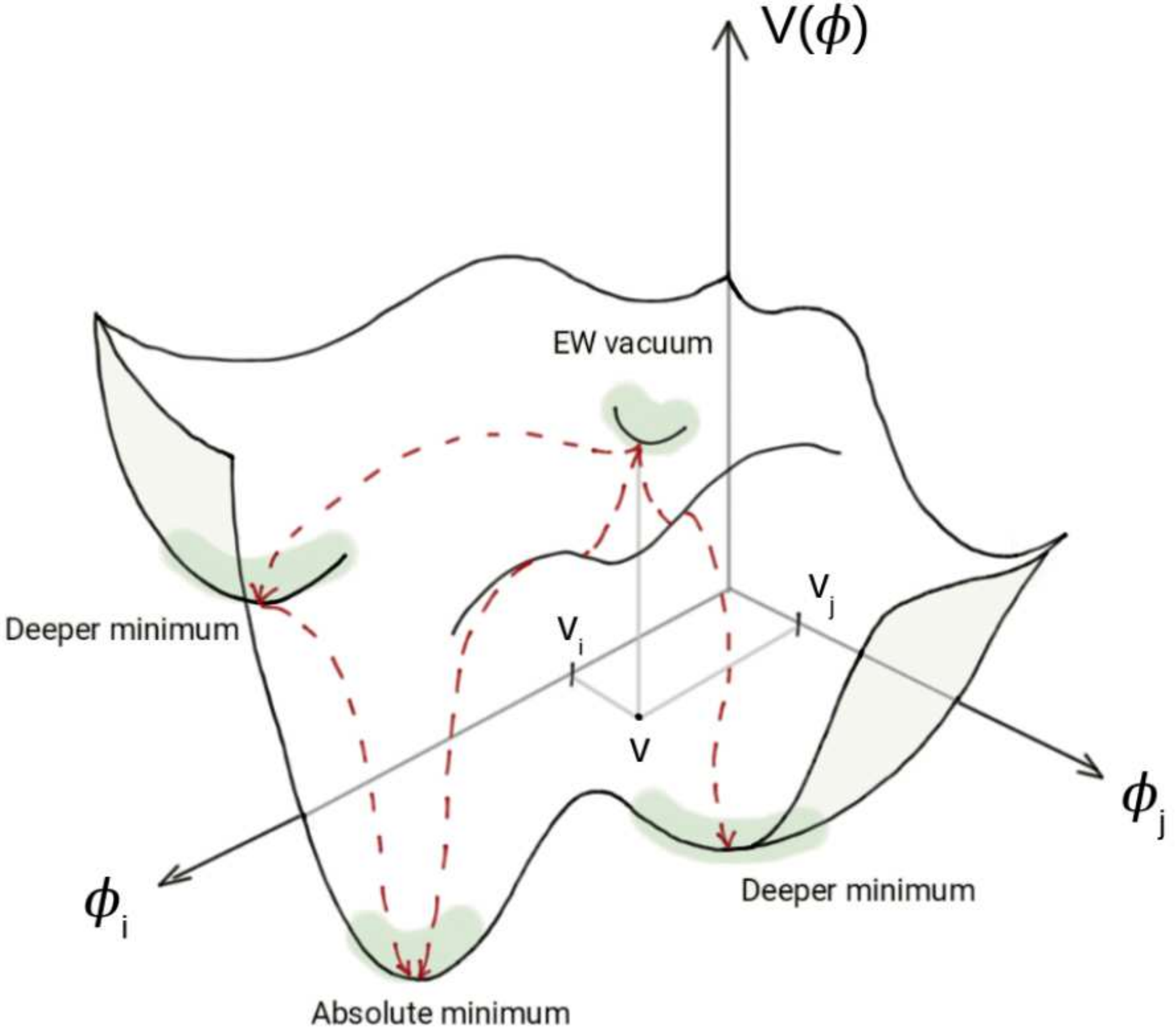
[NASA]



Approved launch date: $\sim$ 2035

BSM Higgs sectors can (should) behave very different from SM Higgs

[K. Radchenko '24]



$\Rightarrow$ much more complicated structure, very^2 little known . . .

4. BSM Higgs Introduction

The main questions:

- What are the couplings of this particle to other known elementary particles? Is its coupling to each particle proportional to that particle's mass, as required by the BEH mechanism?

- What are the mass, total width, spin and $\mathcal{CP}$ properties of this particle? Are there additional sources of $\mathcal{CP}$ violation in the Higgs sector?

- What is the value of the particle's self-coupling? Is this consistent with the expectation from the symmetry-breaking potential?

- Is this particle a single, fundamental scalar as in the SM, or is it part of a larger structure? Is it part of a model with additional scalar singlets/doublets/...?
  Or, could it be a composite state, bound by new interactions?

- Does this particle couple to new particles with no other couplings to the SM ("Higgs portal")? Is the particle mixed with new scalars of exotic origin, for example, the radion of extra-dimensional models?

Models with extended Higgs sectors:

Models with extended Higgs sectors:

Q: Do you know any BSM Higgs models? :-)

Models with extended Higgs sectors:

1. SM with addional Higgs singlet (RxSM, . . .)
2. Two Higgs Doublet Model (2HDM): type I, II, III, IV
3. 2HDM with singlet extensions: N2HDM, S2HDM, 2HDMS, . . .
4. Minimal Supersymmetric Standard Model (MSSM)
5. MSSM with one extra singlet (NMSSM)
6. MSSM with more extra singlets ($\mu\nu$SSM)
7. SM/MSSM with Higgs triplets (GM, . . .)
8. . . .

Extended Higgs sectors

Compatibility with the experimental results requires:

- A SM-like Higgs at ~ 125 GeV
- Properties of the other Higgs bosons (masses, couplings,...) have to be such that they are in agreement with the present bounds

The "sum rule": $\sum_i g^2_{h_i VV} = g^2_{H_{\rm SM} VV}$ – and we know $g^2_{h_{125} VV} \sim g^2_{H_{\rm SM} VV}$

$\Rightarrow$ not much room left for BSM Higgs couplings to gauge bosons

Sum rule "violated" only by triplets or higher representations ...

Model Motivation?

Model Motivation? $\Rightarrow$ experimental data as guidance!

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Some "recent" measurements:

– top quark mass
– Higgs boson mass
– Higgs boson "couplings"
– Dark Matter (properties)

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- Higgs boson "couplings"
- Dark Matter (properties)

⇒ **good motivation to look at SUSY! :-)**

But we cover everything (RxSM, 2HDM, N2HDM, . . .)

5. Higgs Singlet and Doublet Extensions

Models with extended Higgs sectors:

1. SM with addional Higgs singlet (RxSM, . . .)
2. Two Higgs Doublet Model (2HDM): type I, II, III, IV
3. 2HDM with singlet extensions: N2HDM, S2HDM, 2HDMS, . . .
4. Minimal Supersymmetric Standard Model (MSSM)
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6. MSSM with more extra singlets ($\mu\nu$SSM)
7. SM/MSSM with Higgs triplets (GM, . . .)
8. . . .

The simplest showcase: has all the interesting features

SM plus one real singlet (RxSM):

Fields:

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \rho + i\eta) \end{pmatrix}, \; S = \frac{1}{\sqrt{2}}(v_S + \rho_S)$$

Lagrangian:

$$\mathcal{L}_{\text{RxSM}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) + (\partial^\mu S)(\partial_\mu S) - V(\Phi, S)$$

Potential: (with an additional Z_2 symmetry)

$$\begin{aligned} V(\Phi, S) &= -m^2 \Phi^\dagger \Phi - \mu^2 S^2 + \left(\Phi^\dagger \Phi, S^2\right) \begin{pmatrix} \lambda_1 & \frac{1}{2}\lambda_3 \\ \frac{1}{2}\lambda_3 & \lambda_2 \end{pmatrix} \begin{pmatrix} \Phi^\dagger \Phi \\ S^2 \end{pmatrix} \\ &= -m^2 \Phi^\dagger \Phi - \mu^2 S^2 + \lambda_1 (\Phi^\dagger \Phi)^2 + \lambda_2 S^4 + \lambda_3 (\Phi^\dagger \Phi) S^2 \end{aligned}$$

Original basis: λ_1, λ_2, λ_3, v, v_S

Potential is bounded from below for

$$4\lambda_1\lambda_2 - \lambda_3^2 > 0$$
$$\lambda_1, \lambda_2 > 0$$

Unitary gauge:

$$\Phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+\rho) \end{pmatrix}$$
$$S = \frac{1}{\sqrt{2}}(v_S + \rho_S)$$

Expansion around the minimum:

$$\mathcal{L}_{\text{mass}} = (\rho, \rho_S) \quad \mathcal{M}^2 \quad \begin{pmatrix} \rho \\ \rho_S \end{pmatrix}$$

$$= (\rho, \rho_S) \begin{pmatrix} 2\lambda_1 v^2 & \lambda_3 v v_S \\ \lambda_3 v v_S & 2\lambda_2 v_S^2 \end{pmatrix} \begin{pmatrix} \rho \\ \rho_S \end{pmatrix}$$

The matrix is diagonalized with the angle α:

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho \\ \rho_S \end{pmatrix}$$

Mass eigenvalues:

$$m^2_{h,H} = \lambda_1 v^2 + \lambda_2 v_S^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 v_S^2)^2 + (\lambda_3 v v_S)^2}$$

One eigenvalue ~ 125 GeV, the other one above or below

α is given by

$$\sin 2\alpha = \frac{\lambda_3 v v_S}{\sqrt{(\lambda_1 v^2 - \lambda_2 v_S^2)^2 + (\lambda_3 v v_S)^2}}$$

$$\cos 2\alpha = \frac{\lambda_2 v_S^2 - \lambda_1 v^2}{\sqrt{(\lambda_1 v^2 - \lambda_2 v_S^2)^2 + (\lambda_3 v v_S)^2}}$$

Physical basis: m_h, m_H, α, v, $\tan\beta := v/v_S$ with $v \equiv v_{\mathrm{SM}} \sim 246$ GeV

Express original basis in terms of physical basis:

$$\lambda_1 = \frac{1}{2v^2}\left(m_h^2 \cos^2\alpha + m_H^2 \sin^2\alpha\right)$$
$$\lambda_2 = \frac{1}{2v_S^2}\left(m_h^2 \sin^2\alpha + m_H^2 \cos^2\alpha\right)$$
$$\lambda_3 = \frac{1}{2vv_S}\left(m_H^2 - m_h^2\right)\sin 2\alpha$$

Only the SM doublet couples to SM particles (with doublet component ρ)

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho \\ \rho_S \end{pmatrix}$$

$\Rightarrow$ All Higgs couplings are suppressed w.r.t. the SM

$$g_{h\text{–SM–SM}} = \cos\alpha\, g_{H_{\mathrm{SM}}\text{–SM–SM}}$$
$$g_{H\text{–SM–SM}} = \sin\alpha\, g_{H_{\mathrm{SM}}\text{–SM–SM}}$$

SM limit (alignment limit):

– $m_h \sim 125$ GeV: $\alpha \to 0$, $\cos\alpha \to 1$

– $m_H \sim 125$ GeV: $\alpha \to \pi/2$, $\sin\alpha \to 1$

Sum rule:

$$\sum_i (g_{h_i VV})^2 = g^2_{hVV} + g^2_{HVV} = (\cos^2\alpha + \sin^2\alpha) g^2_{H_{\mathrm{SM}}VV} \equiv g^2_{H_{\mathrm{SM}}VV}$$

Decay widths:

$$\Gamma(h \to \mathrm{SM}) = \cos^2\alpha\, \Gamma(H_{\mathrm{SM}} \to \mathrm{SM}) \quad \text{with} \quad m_{H_{\mathrm{SM}}} \to m_h$$
$$\Gamma(H \to \mathrm{SM}) = \sin^2\alpha\, \Gamma(H_{\mathrm{SM}} \to \mathrm{SM}) \quad \text{with} \quad m_{H_{\mathrm{SM}}} \to m_H$$

[These mass setting are implicitely assumed in the following.]

Trilinear Higgs Couplings:

$$\lambda_{hhh} = \frac{1}{v}\Big[\lambda_3 v_S c_\alpha^2 \sin\alpha + 2\lambda_3 v \cos\alpha \sin^2\alpha$$
$$+ 4\lambda_2 v_S \sin^3\alpha + \lambda_1 v \cos^3\alpha\Big]$$

$\rightarrow \lambda_1 \equiv \lambda_{\mathrm{SM}}$ in SM limit

$$\lambda_{hhH} = \frac{1}{v}\Big[\lambda_3 v_S \cos^3\alpha + v(2\lambda_3 - 3\lambda_1)\cos^2\alpha \sin\alpha$$
$$- v_S(\lambda_3 - 12\lambda_2)\cos\alpha \sin^2\alpha - \lambda_3 v \sin^3\alpha\Big]$$

$\rightarrow \frac{v_S}{v}\lambda_3 \equiv 0$ in SM limit

$$\lambda_{hHH} = \ldots$$

$$\lambda_{HHH} = \ldots$$

New possible decay mode (for $m_H > 2m_h$)

$$\Gamma(H \to hh) = \frac{|\lambda_{hhH}|^2}{8\,\pi\,m_H}\sqrt{1 - \frac{4m_h^2}{m_H^2}}$$

Total decay widths:

$$\Gamma_{h,\text{tot}} = \cos^2\alpha\,\Gamma_{H_{\text{SM}},\text{tot}}$$
$$\Gamma_{H,\text{tot}} = \sin^2\alpha\,\Gamma_{H_{\text{SM}},\text{tot}} + \Gamma(H \to hh)$$

Branching ratios:

$$\text{BR}(h \to \text{SM}) = \frac{\Gamma(h \to \text{SM})}{\Gamma_{h,\text{tot}}} = \text{BR}(H_{\text{SM}} \to \text{SM})$$
$$\text{BR}(H \to \text{SM}) = \frac{\Gamma(H \to \text{SM})}{\Gamma_{H,\text{tot}}} = \sin^2\alpha\,\frac{\Gamma(H_{\text{SM}} \to \text{SM})}{\Gamma_{H,\text{tot}}} \left(= \text{BR}(H_{\text{SM}} \to \text{SM}) \text{ for } m_H < 2m_h\right)$$
$$\text{BR}(H \to hh) = \frac{\Gamma(H \to hh)}{\Gamma_{H,\text{tot}}}$$

Two Higgs Doublet Model (2HDM):

Fields:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}$$

Potential:

$$\begin{aligned} V \; = \; & m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.) + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + h.c.] \end{aligned}$$

Physical states: h, H, ($\mathcal{CP}$-even), A ($\mathcal{CP}$-odd), $H^\pm$ (charged)

Mixing angles: α diag. $\mathcal{CP}$-even sector, β diag. $\mathcal{CP}$-odd/charged sector

“Physical” input parameters:

$$c_{\beta-\alpha}\,, \quad \tan\beta\,, \quad v\,, \quad M_h, \quad M_H\,, \quad M_A\,, \quad M_{H^\pm}\,, \quad m_{12}^2$$

Assumption (often): $h \sim h_{125}$

Z_2 symmetry to avoid FCNC:

$$\Phi_1 \to \Phi_1 \,,\; \Phi_2 \to -\Phi_2$$

Extension of the Z_2 symmetry to fermions determines four types:

	u-type	d-type	leptons	
type I	Φ_2	Φ_2	Φ_2	
type II	Φ_2	Φ_1	Φ_1	$\to$ MSSM type
type III (lepton-specific)	Φ_2	Φ_2	Φ_1	
type IV (flipped)	Φ_2	Φ_1	Φ_2	

Unitarity/perturbativity and EWPO: $\Rightarrow M_A \sim M_H \sim M_{H^\pm}$

SM/alignment limit: $c_{\beta-\alpha} \to 0$ for $h \sim h_{125}$
$s_{\beta-\alpha} \to 0$ for $H \sim h_{125}$

Higgs-boson couplings in the 2HDM

$$\mathcal{L} = -\sum_{f=u,d,l} \frac{m_f}{v}\left[\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H + i\xi_A^f \bar{f}\gamma_5 f A\right]$$
$$+ \sum_{h_i=h,H,A}\left[g M_W \xi_{h_i}^W W_\mu W^\mu h_i + \frac{1}{2} g M_Z \xi_{h_i}^Z Z_\mu Z^\mu h_i\right],$$

$$\xi_h^V = s_{\beta-\alpha}\,, \quad \xi_H^V = c_{\beta-\alpha}\,, \quad \xi_A^V = 0$$

Sum rule:

$$g_{hVV}^2 + g_{HVV}^2 + g_{AVV}^2 = (\sin^2(\beta-\alpha) + \cos^2(\beta-\alpha))g_{H_{\mathrm{SM}}VV}^2 \equiv g_{H_{\mathrm{SM}}VV}^2$$

	Type I	Type II	Type III/Flipped/Y	Type IV/Lepton-specific/X
ξ_h^u	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$
ξ_h^d	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$
ξ_h^l	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$
ξ_h^V	$s_{\beta-\alpha}$	$s_{\beta-\alpha}$	$s_{\beta-\alpha}$	$s_{\beta-\alpha}$
ξ_H^u	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$
ξ_H^d	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$
ξ_H^l	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$
ξ_H^V	$c_{\beta-\alpha}$	$c_{\beta-\alpha}$	$c_{\beta-\alpha}$	$c_{\beta-\alpha}$
ξ_A^u	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$
ξ_A^d	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
ξ_A^l	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
ξ_A^V	0	0	0	0

Higgs-boson couplings in the 2HDM

$\Rightarrow$ check alignment limit!

$$\mathcal{L} = -\sum_{f=u,d,l} \frac{m_f}{v}\left[\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H + i \xi_A^f \bar{f} \gamma_5 f A\right]$$
$$+ \sum_{h_i=h,H,A}\left[g M_W \xi_{h_i}^W W_\mu W^\mu h_i + \frac{1}{2} g M_Z \xi_{h_i}^Z Z_\mu Z^\mu h_i\right],$$

$$\xi_h^V = s_{\beta-\alpha}\,, \quad \xi_H^V = c_{\beta-\alpha}\,, \quad \xi_A^V = 0$$

Sum rule:

$$g_{hVV}^2 + g_{HVV}^2 + g_{AVV}^2 = (\sin^2(\beta-\alpha) + \cos^2(\beta-\alpha))\, g_{H_{\rm SM}VV}^2 \equiv g_{H_{\rm SM}VV}^2$$

	Type I	Type II	Type III/Flipped/Y	Type IV/Lepton-specific/X
ξ_h^u	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$
ξ_h^d	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$
ξ_h^l	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$	$s_{\beta-\alpha} + c_{\beta-\alpha}\cot\beta$	$s_{\beta-\alpha} - c_{\beta-\alpha}\tan\beta$
ξ_h^V	$s_{\beta-\alpha}$	$s_{\beta-\alpha}$	$s_{\beta-\alpha}$	$s_{\beta-\alpha}$
ξ_H^u	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$
ξ_H^d	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$
ξ_H^l	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$	$c_{\beta-\alpha} - s_{\beta-\alpha}\cot\beta$	$c_{\beta-\alpha} + s_{\beta-\alpha}\tan\beta$
ξ_H^V	$c_{\beta-\alpha}$	$c_{\beta-\alpha}$	$c_{\beta-\alpha}$	$c_{\beta-\alpha}$
ξ_A^u	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$
ξ_A^d	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
ξ_A^l	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
ξ_A^V	0	0	0	0

Next-Two Higgs Doublet Model (N2HDM): $\rightarrow$ (nearly) NMSSM type

Fields:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \rho_1 + i\eta_1) \end{pmatrix}, \; \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \rho_2 + i\eta_2) \end{pmatrix}, \; \Phi_S = v_S + \rho_S$$

Potential:

$$\begin{aligned} V \; = \; & m_{11}^2 |\Phi_1|^2 + m_{22}^2 |\Phi_2|^2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + h.c.) + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + h.c.] \\ & + \frac{1}{2} m_S^2 \Phi_S^2 + \frac{\lambda_6}{8} \Phi_S^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger \Phi_1)\Phi_S^2 + \frac{\lambda_8}{2}(\Phi_2^\dagger \Phi_2)\Phi_S^2 \end{aligned}$$

Z_2 symmetry: $\Phi_1 \to \Phi_1 \, , \; \Phi_2 \to -\Phi_2 \, , \; \Phi_S \to \Phi_S$

Z_2' symmetry: $\Phi_1 \to \Phi_1 \, , \; \Phi_2 \to \Phi_2 \, , \; \Phi_S \to -\Phi_S$ (broken by $v_S \Rightarrow$ no DM)

Physical states: h_1, h_2, h_3 ($\mathcal{CP}$-even), A ($\mathcal{CP}$-odd), $H^\pm$ (charged)

Extension of the Z_2 symmetry to fermions determines four types:

	u-type	d-type	leptons
type I	Φ_2	Φ_2	Φ_2
type II	Φ_2	Φ_1	Φ_1
type III (lepton-specific)	Φ_2	Φ_2	Φ_1
type IV (flipped)	Φ_2	Φ_1	Φ_2

$\Rightarrow$ exactly as in 2HDM

Three neutral $\mathcal{CP}$-even Higgses:

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_S \end{pmatrix}, \quad R = \begin{pmatrix} c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\ -(c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3}) & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\ -c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & -(c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3} \end{pmatrix}$$

Coupling to massive gauge bosons: (identical for all four types)

	$c_{h_iVV} = c_\beta R_{i1} + s_\beta R_{i2}$
h_1	$c_{\alpha_2} c_{\beta-\alpha_1}$
h_2	$-c_{\beta-\alpha_1} s_{\alpha_2} s_{\alpha_3} + c_{\alpha_3} s_{\beta-\alpha_1}$
h_3	$-c_{\alpha_3} c_{\beta-\alpha_1} s_{\alpha_2} - s_{\alpha_3} s_{\beta-\alpha_1}$

Coupling to fermions: (same pattern as in 2HDM)

	u-type (c_{h_itt})	d-type (c_{h_ibb})	leptons ($c_{h_i\tau\tau}$)
type I	$\frac{R_{i2}}{s_\beta}$	$\frac{R_{i2}}{s_\beta}$	$\frac{R_{i2}}{s_\beta}$
type II	$\frac{R_{i2}}{s_\beta}$	$\frac{R_{i1}}{c_\beta}$	$\frac{R_{i1}}{c_\beta}$
type III (lepton-specific)	$\frac{R_{i2}}{s_\beta}$	$\frac{R_{i2}}{s_\beta}$	$\frac{R_{i1}}{c_\beta}$
type IV (flipped)	$\frac{R_{i2}}{s_\beta}$	$\frac{R_{i1}}{c_\beta}$	$\frac{R_{i2}}{s_\beta}$

"Physical" input parameters:

$$\alpha_{1,2,3}\,, \quad \tan\beta\,, \quad v\,, \quad v_S\,, \quad m_{h_{1,2,3}}\,, \quad m_A\,, \quad M_{H^\pm}\,, \quad m_{12}^2$$

A model with triplets: Georgi-Machacek Model (GM)

[*Material taken from S. Chopra*]

Field content:

In the GM model, the SM Higgs doublet ϕ with a complex triplet χ (hypercharge $Y = 1$) and a real triplet ξ ($Y = 0$). These fields are represented as:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}. \tag{2.33}$$

Neutral fields:

$$\phi^0 = \frac{1}{\sqrt{2}}(v_\phi + \phi_r^0 + i\phi_i^0), \quad \chi^0 = v_\chi + \frac{1}{\sqrt{2}}(\chi_r^0 + i\chi_i^0), \quad \xi^0 = v_\chi + \xi_r^0.$$

Two vevs:

$$v^2 = v_\phi^2 + 8v_\chi^2 = \frac{4M_W^2}{g^2} \approx (246\,\mathrm{GeV})^2.$$

New mixing angle:

$$\cos\theta_H = \frac{v_\phi}{v}, \qquad \sin\theta_H = \frac{\sqrt{8}\,v_\chi}{v}.$$

Three neutral $\mathcal{CP}$-even Higgs bosons:

$$h^0 = \cos\alpha\,\phi^{0,r} - \sin\alpha\left(\frac{1}{\sqrt{3}}\chi^{0,r} + \sqrt{\frac{2}{3}}\,\xi^{0,r}\right),$$

$$H^0 = \sin\alpha\,\phi^{0,r} + \cos\alpha\left(\frac{1}{\sqrt{3}}\chi^{0,r} + \sqrt{\frac{2}{3}}\,\xi^{0,r}\right),$$

$$H_5^0 = \sqrt{\frac{2}{3}}\,\xi^{0,r} - \frac{1}{\sqrt{3}}\,\chi^{0,r},$$

Coupling modifiers of neutral Higgs bosons:

$$\kappa_V^{h^0} = \cos\theta_H \cos\alpha - \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \sin\alpha,$$

$$\kappa_V^{H^0} = \cos\theta_H \sin\alpha + \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \cos\alpha,$$

$$\kappa_{WW}^{H_5^0} = \frac{1}{\sqrt{3}} \sin\theta_H, \qquad \kappa_{ZZ}^{H_5^0} = -\frac{2}{\sqrt{3}} \sin\theta_H.$$

Difference w.r.t. 2HDM:

$$\left(\kappa_V^{h^0}\right)^2 + \left(\kappa_V^{H^0}\right)^2 = \cos^2\theta_H + \frac{8}{3}\sin^2\theta_H \geq 1$$

$\Rightarrow$ couplings larger than in SM possible!

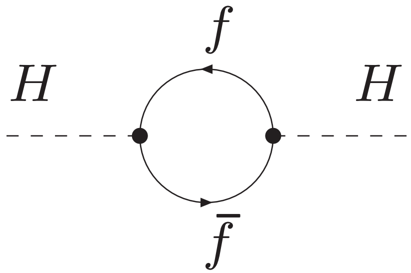
6. The Higgs sector of the (N)MSSM

Motivation for BSM physics: the Hierarchy problem

Mass is what determines the properties of the free propagation of a particle

Free propagation: 

inverse propagator: $i(p^2 - M_H^2)$

Loop corrections: 

inverse propagator: $i(p^2 - M_H^2 + \Sigma_H^f)$

QM: integration over all possible loop momenta k

dimensional analysis:

$$\Sigma_H^f \;\sim\; N_f\,\lambda_f^2 \int d^4k \left(\frac{1}{k^2 - m_f^2} + \frac{2m_f^2}{(k^2 - m_f^2)^2} \right)$$

$$\text{for } \Lambda \to \infty : \qquad \Sigma_H^f \;\sim\; N_f\,\lambda_f^2 \left(\underbrace{\int \frac{d^4k}{k^2}}_{\sim\,\Lambda^2} \;+\; 2m_f^2 \underbrace{\int \frac{dk}{k}}_{\sim\,\ln\Lambda} \right)$$

$\Rightarrow$ quadratically divergent!

For $\Lambda = M_{\text{Pl}}$:

$$\Sigma_H^f \approx \delta M_H^2 \sim M_{\text{Pl}}^2 \quad \Rightarrow \quad \delta M_H^2 \approx 10^{30}\, M_H^2$$

(for $M_H \lesssim 1$ TeV)

– no additional symmetry for $M_H = 0$

– no protection against large corrections

$\Rightarrow$ Hierarchy problem is instability of small Higgs mass to large corrections in a theory with a large mass scale in addition to the weak scale

E.g.: Grand Unified Theory (GUT): $\delta M_H^2 \approx M_{\text{GUT}}^2$

Note however: there is another fine-tuning problem in nature, for which we have no clue so far – cosmological constant

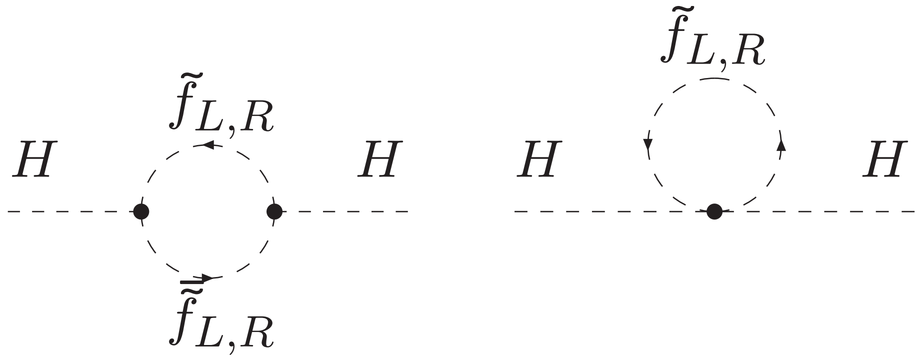
Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m^2_{\tilde{f}_L}} + \frac{1}{k^2 - m^2_{\tilde{f}_R}} \right) + \text{ terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2\, \Lambda^2$

$\Rightarrow$ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} \; = \; N_f$$
$$\lambda^2_{\tilde{f}} \; = \; \lambda^2_f$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \lambda^2_{\tilde{f}} = \lambda^2_f$

$$\Rightarrow \; \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda^2_f \, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1 \, \text{TeV}$$

$$\boxed{\Rightarrow \text{SUSY at TeV scale provides attractive solution of hierarchy problem}}$$

Supersymmetry (SUSY) : Symmetry between

$$\text{Bosons} \leftrightarrow \text{Fermions}$$

$$Q \;|\text{Fermion}\rangle \rightarrow |\text{Boson}\rangle$$
$$Q \;|\text{Boson}\rangle \rightarrow |\text{Fermion}\rangle$$

Simplified examples:

$$Q \;|\text{top}, t\rangle \rightarrow |\text{scalar top}, \tilde{t}\rangle$$
$$Q \;|\text{gluon}, g\rangle \rightarrow |\text{gluino}, \tilde{g}\rangle$$

$\Rightarrow$ each SM multiplet is enlarged to its double size

Unbroken SUSY: All particles in a multiplet have the same mass

Reality: $m_e \neq m_{\tilde{e}}$ $\Rightarrow$ SUSY is broken . . .

. . . via soft SUSY-breaking terms in the Lagrangian (added by hand)

SUSY particles are made heavy: $\boxed{M_{\text{SUSY}} = \mathcal{O}(1\text{ TeV})}$

The Higgs sector of the MSSM

Comparison with SM case:

$$\mathcal{L}_{\rm SM} = \underbrace{m_d \bar{Q}_L \Phi d_R}_{\text{d-quark mass}} + \underbrace{m_u \bar{Q}_L \Phi_c u_R}_{\text{u-quark mass}}$$

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad \Phi_c = i\sigma_2 \Phi^*, \quad \Phi \to \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \Phi_c \to \begin{pmatrix} v \\ 0 \end{pmatrix}$$

In SUSY: term $\bar{Q}_L \Phi^*$ not allowed

Superpotential is holomorphic function of chiral superfields, i.e. depends only on φ_i, not on φ_i^*

No soft SUSY-breaking terms allowed for chiral fermions

$\Rightarrow$ $H_d (\equiv H_1)$ and $H_u (\equiv H_2)$ needed to give masses to down- and up-type fermions

Furthermore: two doublets also needed for cancellation of anomalies, quadratic divergences

The MSSM Higgs sector:

Enlarged Higgs sector: Two Higgs doublets

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
V &= m_1^2 H_1 \bar{H}_1 + m_2^2 H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
&\quad + \underbrace{\frac{g'^2 + g^2}{8}}_{} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \underbrace{\frac{g^2}{2}}_{} |H_1 \bar{H}_2|^2
\end{aligned}
$$

gauge couplings, in contrast to SM

physical states: $h^0, H^0, A^0, H^\pm$ Goldstone bosons: $G^0, G^\pm$

Input parameters: (to be determined experimentally)

$$
t_\beta = \frac{v_2}{v_1}, \qquad M_A^2 = -m_{12}^2 (t_\beta + \cot\beta)
$$

Tree-level result for m_h, m_H:

$$m^2_{H,h} = \frac{1}{2}\left[M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2 M_A^2 \cos^2 2\beta}\right]$$

$\Rightarrow m_h \leq M_Z$ at tree level

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Q: What are the implications?

Tree-level result for m_h, m_H:

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$\Rightarrow$ Light Higgs boson h required in SUSY

Measurement of m_h, Higgs couplings

$\Rightarrow$ test of the theory (more directly than in SM)

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Measurement of m_h, Higgs couplings

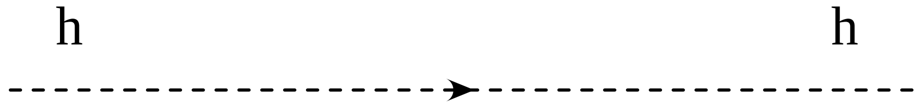
$\Rightarrow$ test of the theory (more directly than in SM)

LHC measurements: $M_h \sim 125$ GeV

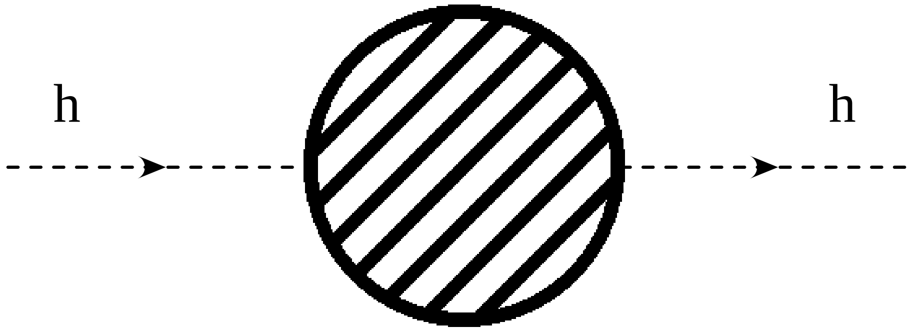
Is this a problem?

Quantum corrections to the MSSM Higgs mass

Higgs propagator:



$\Downarrow$ quantum corrections



Inverse propagator:

$$-i(q^2 - m^2) \quad \longrightarrow \quad -i\left(q^2 - m^2 + \hat{\Sigma}_h(q^2)\right)$$

$\hat{\Sigma}_h(q^2)$: renormalized Higgs self-energy

Loop corrected mass^2: $M^2 = m^2 - \hat{\Sigma}_h(m^2)$

dominant 1-loop: $\Delta M_h^2 \sim XXX$ $\Rightarrow$ Tutorial on Wednesday

Numerical estimate:

$$
\begin{aligned}
e^2/(4\pi) &= \alpha = 1/137 \\
m_t &= 175 \text{ GeV} \\
M_W &= 80 \text{ GeV } (M_Z = 90 \text{ GeV}) \\
s_\mathrm{W} &= 1/2 \\
\sin\beta &= 1 \\
m_{\tilde{t}} &= 1000 \text{ GeV}
\end{aligned}
$$

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Numerical estimate:

$$e^2/(4\pi) = \alpha = 1/137$$
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$$s_\mathrm{W} = 1/2$$
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$$\text{dominant 1-loop}: \Delta M_h^2 \approx 7000 \text{ GeV}^2$$
$$M_h^2 = m_{h,\text{tree}}^2 + \Delta M_h^2$$
$$m_{h,\text{tree}} \approx M_Z$$

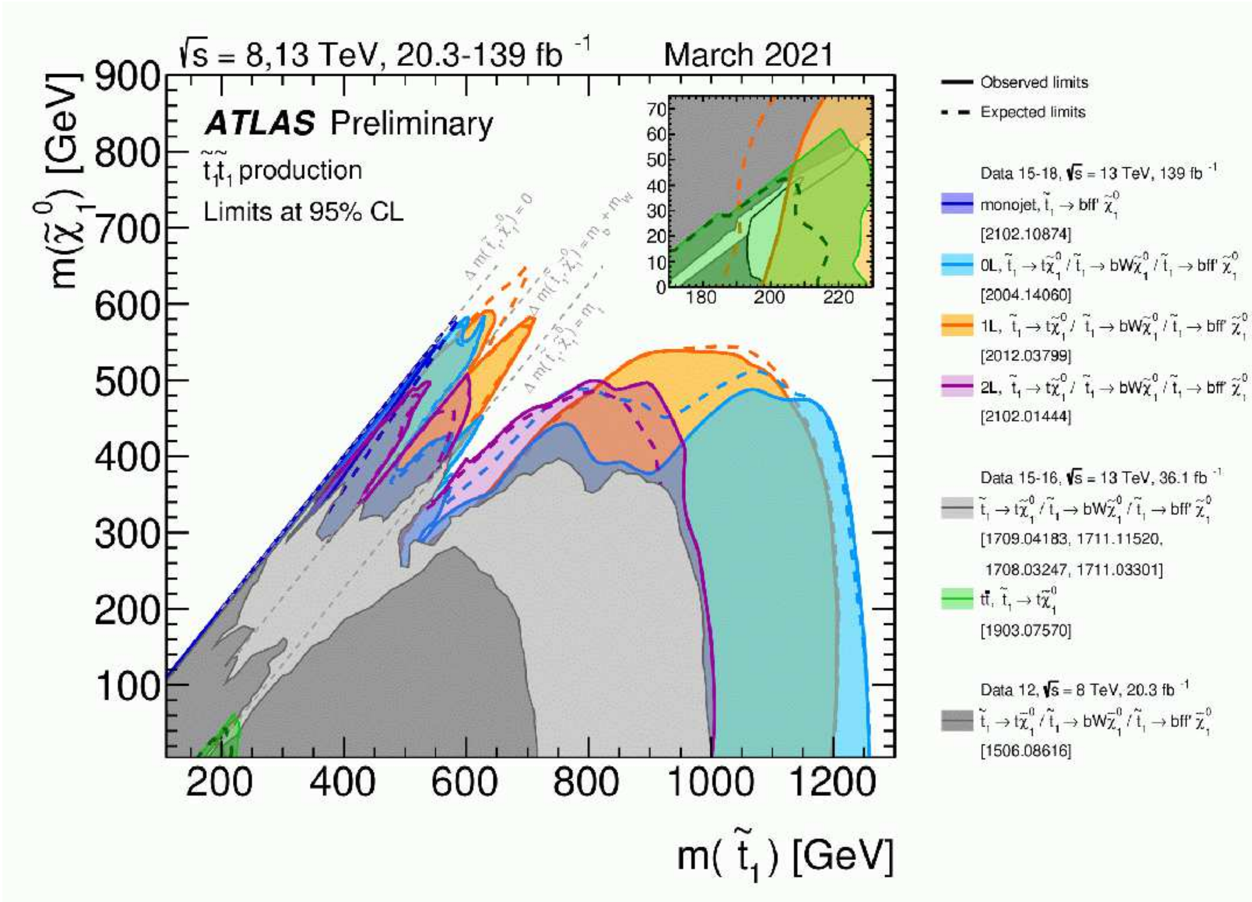
$$\text{dominant 1-loop}: M_h \approx 123 \text{ GeV}$$

dominant 1 – loop : $M_h \approx 123$ GeV

⇒ many, many, many, . . . more refinements have been calculated

⇒ stops above the TeV scale naturally yield $M_h \sim 125$ GeV

Comparison with LHC searches:



⇒ $M_h = 125$ GeV is in perfect agreement with the non-observation of stops

Upper bound on M_h in the MSSM:

"Unconstrained MSSM":

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$\boxed{M_h \lesssim 135 \text{ GeV}}$$

for $m_t = 173.2 \pm 0.9\,\text{GeV}$ and $m_{\tilde{t}} \lesssim 2$ TeV

(including theoretical uncertainties from unknown higher orders)

$\Rightarrow$ in agreement with all LHC Higgs measurements and stop searches

Obtained with:

FeynHiggs

`www.feynhiggs.de`

[*H. Bahl, T. Hahn, S.H., W. Hollik, S. Passehr, H. Rzehak, G. Weiglein '98 – '25*]

$\rightarrow$ all Higgs masses, couplings, BRs, XSs (easy to link, easy to use :-)

Upper bound on M_h in the MSSM:

"Unconstrained MSSM":

M_A, $\tan\beta$, 5 parameters in $\tilde{t}$–$\tilde{b}$ sector, μ, $m_{\tilde{g}}$, M_2

$$\boxed{M_h \lesssim 135 \text{ GeV}} \qquad \text{Note}: 125 < 135!$$

for $m_t = 173.2 \pm 0.9\,\text{GeV}$ and $m_{\tilde{t}} \lesssim 2$ TeV
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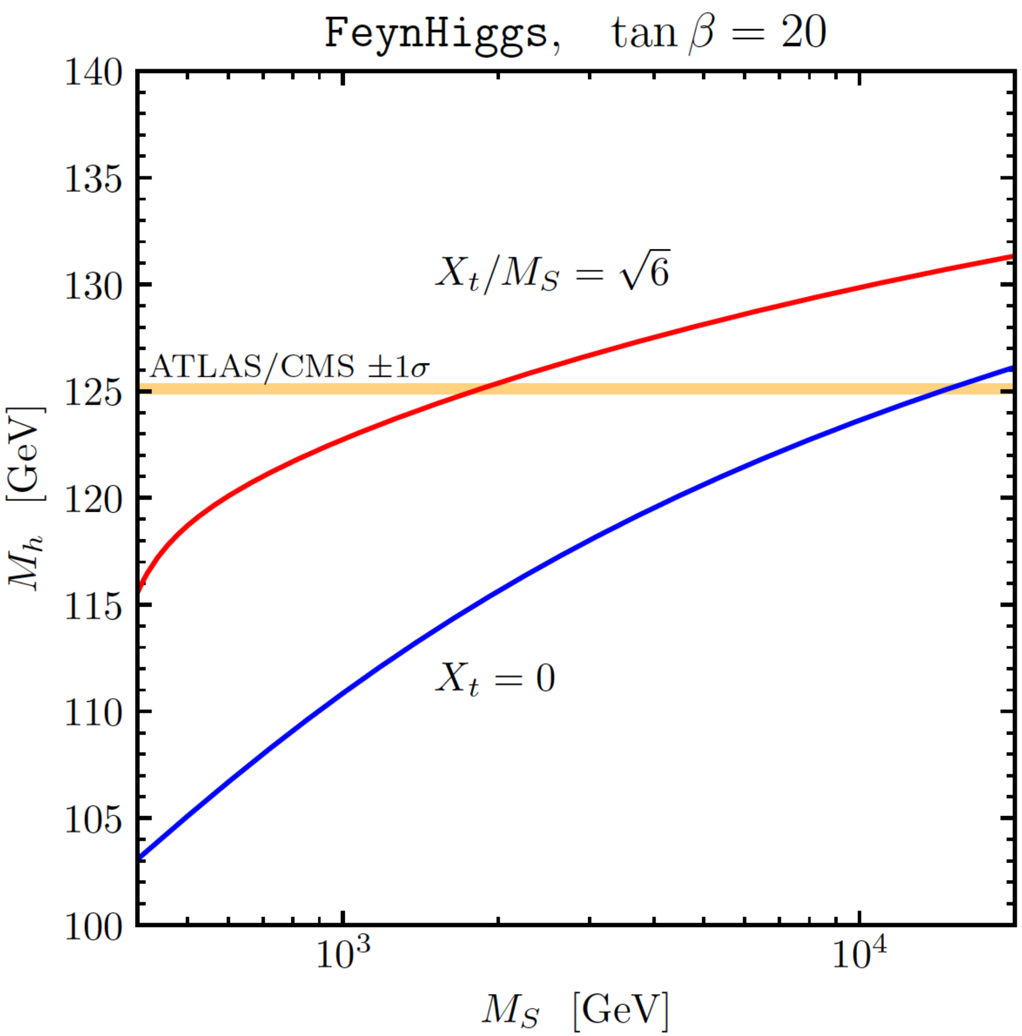
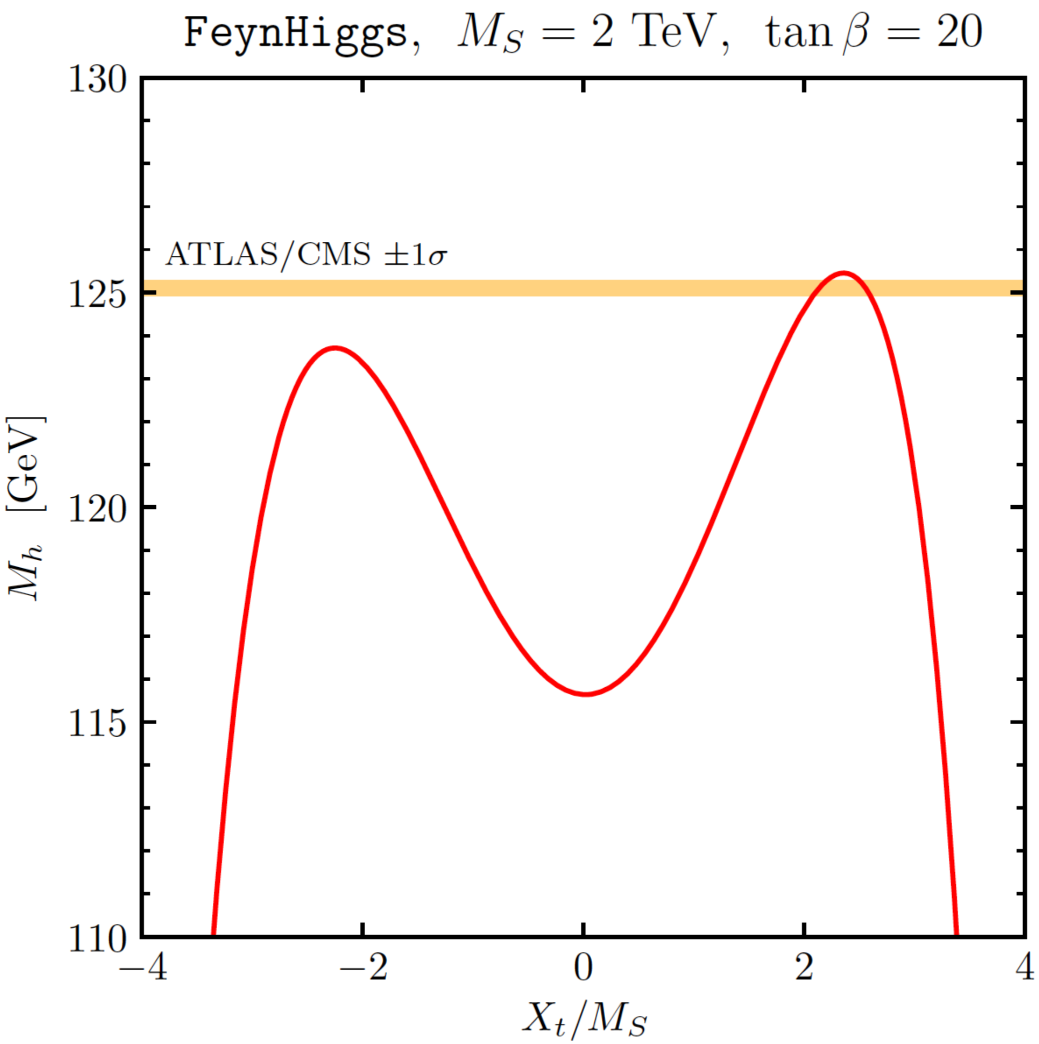
Example for M_h prediction in a simple MSSM scenario (I): [KUTS report '20]



$\Rightarrow M_h \sim 125$ GeV can easily be reached

Example for M_h prediction in a simple MSSM scenario (II): [*KUTS report '20*]



⇒ $M_h \sim 125$ GeV "easier" with mixing in the scalar top sector

The SM/alignment limit:

[*J. Gunion, H. Haber, hep-ph/0207010*]

$\to \mathcal{CP}$ conserving 2HDM in the Higgs basis ($\langle H_1 \rangle = v/\sqrt{2}$, $\langle H_2 \rangle = 0$)

$$\mathcal{V} = \ldots + \tfrac{1}{2} Z_1 (H_1^\dagger H_1)^2 + \ldots + \left[\tfrac{1}{2} Z_5 (H_1^\dagger H_2)^2 + Z_6 (H_1^\dagger H_1)(H_1^\dagger H_2) + \text{h.c.}\right] + \ldots$$

$\Rightarrow \mathcal{CP}$-even mass matrix:

$$\mathcal{M}^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & M_A^2 + Z_5 v^2 \end{pmatrix}$$

with mixing angle $\cos(\beta - \alpha) \equiv c_{\beta-\alpha}$

Decoupling limit: $M_A^2 \gg Z_i v^2$

$\Rightarrow m_h^2 \sim Z_1 v^2$, $|c_{\beta-\alpha} \ll 1|$, h is SM-like

Alignment limit: $Z_6 = 0$ and $Z_1 < Z_5 + M_A^2/v^2$

$\Rightarrow h$ is identical to the SM Higgs, $c_{\beta-\alpha} = 0$

$Z_6 = 0$ and $Z_1 > Z_5 + M_A^2/v^2$

$\Rightarrow H$ is identical to the SM Higgs, $c_{\beta-\alpha} = 1$

Some NMSSM Higgs theory (Z_3 invariant NMSSM)

MSSM Higgs sector: Two Higgs doublets

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
V = {} & (\tilde{m}_1^2 + |\mu \quad |^2) H_1 \bar{H}_1 + (\tilde{m}_2^2 + |\mu \quad |^2) H_2 \bar{H}_2 - m_{12}^2 (\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
& + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2
\end{aligned}
$$

Some NMSSM Higgs theory (Z_3 invariant NMSSM)

NMSSM Higgs sector: Two Higgs doublets + one Higgs singlet

$$
\begin{aligned}
H_1 &= \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1 + i\chi_1)/\sqrt{2} \\ \phi_1^- \end{pmatrix} \\
H_2 &= \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2 + i\chi_2)/\sqrt{2} \end{pmatrix} \\
S &= v_s + S_R + I S_I
\end{aligned}
$$

$$
\begin{aligned}
V = \; & (\tilde{m}_1^2 + |\cancel{\mu}\lambda S|^2) H_1 \bar{H}_1 + (\tilde{m}_2^2 + |\cancel{\mu}\lambda S|^2) H_2 \bar{H}_2 - m_{12}^2(\epsilon_{ab} H_1^a H_2^b + \text{h.c.}) \\
& + \frac{g'^2 + g^2}{8} (H_1 \bar{H}_1 - H_2 \bar{H}_2)^2 + \frac{g^2}{2} |H_1 \bar{H}_2|^2 \\
& + |\lambda(\epsilon_{ab} H_1^a H_2^b) + \kappa S^2|^2 + m_S^2 |S|^2 + (\lambda A_\lambda (\epsilon_{ab} H_1^a H_2^b) S + \frac{\kappa}{3} A_\kappa S^3 + \text{h.c.})
\end{aligned}
$$

Free parameters:

$$\lambda,\ \kappa,\ A_\kappa,\ M_{H^\pm},\ \tan\beta,\ \mu_{\text{eff}} = \lambda v_s$$

Higgs spectrum:

$$\mathcal{CP}\text{−even} : h_1, h_2, h_3$$
$$\mathcal{CP}\text{−odd} : a_1, a_2$$
$$\text{charged} : H^+, H^-$$
$$\text{Goldstones} : G^0, G^+, G^-$$

Neutralinos:

$$\mu \to \mu_{\text{eff}}$$

compared to the MSSM: one singlino more

$$\to \tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0$$

Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m^2_{h,\text{tree},\text{NMSSM}} = m^2_{h,\text{tree},\text{MSSM}} + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta$$

Mass of the $\mathcal{CP}$-odd Higgs:

$$\text{MSSM}: M_A^2 = -m_{12}^2(t_\beta + \cot\beta) = \mu B(t_\beta + \cot\beta)$$

$$\text{NMSSM}: ``M_A^2" = \mu_{\text{eff}} B_{\text{eff}} (t_\beta + \cot\beta)$$

with $B_{\text{eff}} = A_\lambda + \kappa\, s,\ \mu_{\text{eff}} = \lambda\, s$ $\Rightarrow$ one very light a_1

Mass of the charged Higgs:

$$\text{MSSM}: M_{H^\pm}^2 = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2$$

$$\text{NMSSM}: M_{H^\pm}^2 = M_A^2 + v^2 \left(\frac{g^2}{2} - \lambda^2 \right)$$

Mass of the lightest $\mathcal{CP}$-even Higgs:

$$m^2_{h,\text{tree,NMSSM}} = m^2_{h,\text{tree,MSSM}} + M_Z^2 \frac{\lambda^2}{g^2} \sin^2 2\beta$$

Mass of the $\mathcal{CP}$-odd Higgs:

$$\text{MSSM}: M_A^2 = -m_{12}^2(t_\beta + \cot\beta) = \mu B(t_\beta + \cot\beta)$$

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with $B_{\text{eff}} = A_\lambda + \kappa\, s,\ \mu_{\text{eff}} = \lambda\, s$ $\Rightarrow$ one very light a_1

Mass of the charged Higgs:

$$\text{MSSM}: M^2_{H^\pm} = M_A^2 + M_W^2 = M_A^2 + \frac{1}{2} v^2 g^2$$

$$\text{NMSSM}: M^2_{H^\pm} = M_A^2 + v^2 \left(\frac{g^2}{2} - \lambda^2 \right)$$

$\Rightarrow M_{h_1}^{\text{MSSM,tree}} \leq M_{h_1}^{\text{NMSSM,tree}}$, one light a_1, $M_{H^\pm}^{\text{MSSM,tree}} \geq M_{H^\pm}^{\text{NMSSM,tree}}$