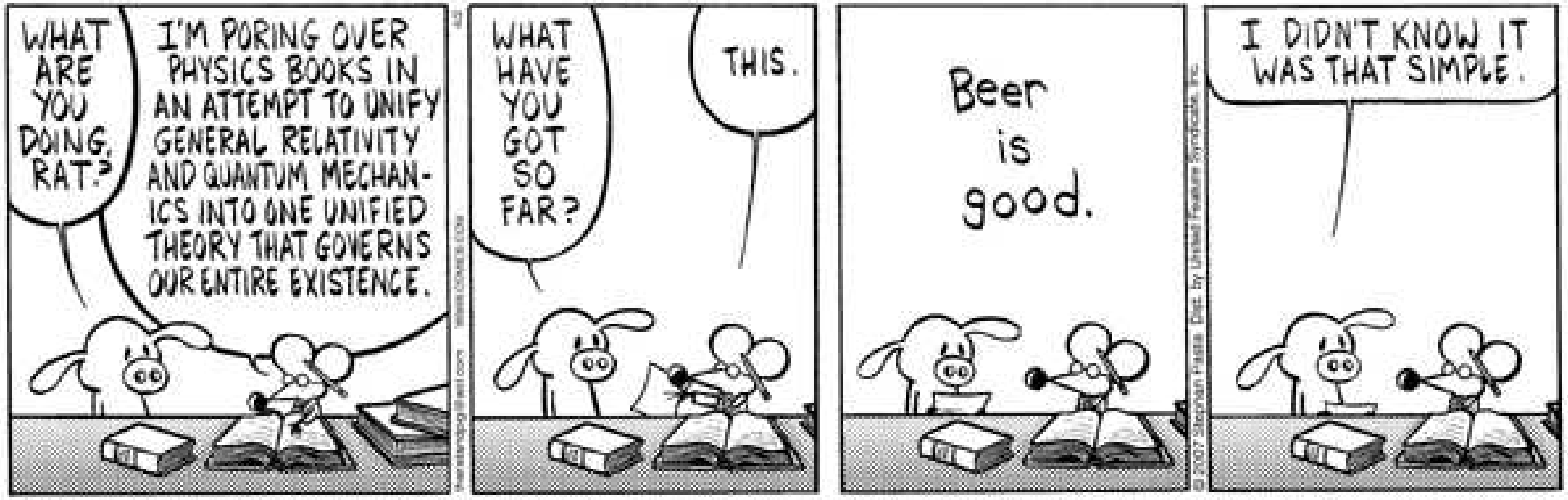
WHAT ARE YOU DOING, RAT?
I'M PORING OVER PHYSICS BOOKS IN AN ATTEMPT TO UNIFY GENERAL RELATIVITY AND QUANTUM MECHANICS INTO ONE UNIFIED THEORY THAT GOVERNS OUR ENTIRE EXISTENCE.
WHAT HAVE YOU GOT SO FAR?
THIS.
Beer is good.
I DIDN'T KNOW IT WAS THAT SIMPLE.

BSM Higgs Lecture

Sven Heinemeyer, IFT (CSIC, Madrid)

Shanghai, 10/2025

1. Motivation for BSM Higgs & BSM Higgs sectors

2. BSM Higgs sectors: collider phenomenology & more

3. Tutorial: joint calculations

BSM Higgs Lecture

Tutorial: joint calculations

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1. The sum rule & unitarity

2. Quantum corrections to the MSSM Higgs mass

1. The sum rule & unitarity

The "sum rule": $\sum_i g^2_{h_i VV} = g^2_{H_{\mathrm{SM}} VV}$

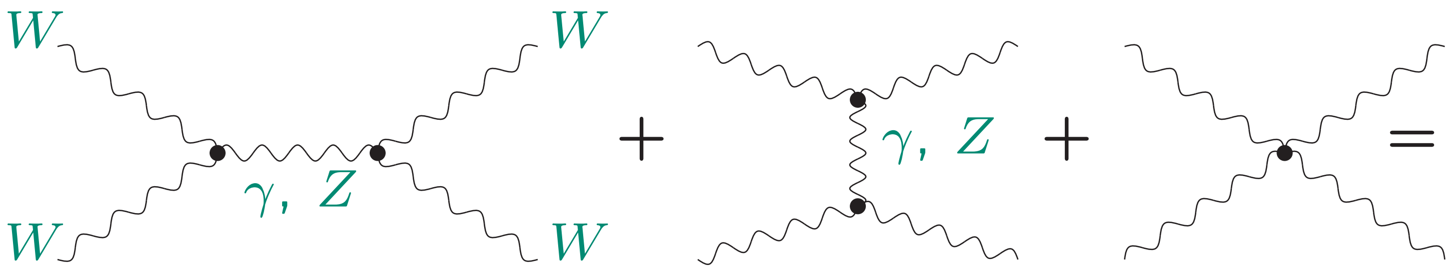
We know from LHC measurements: $g^2_{h_{125} VV} \sim g^2_{H_{\mathrm{SM}} VV}$

Connection to unitarity?

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \to W_L W_L$

$\mathcal{M}_V =$



$= -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1)$

for $E \to \infty$

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \to W_L W_L$

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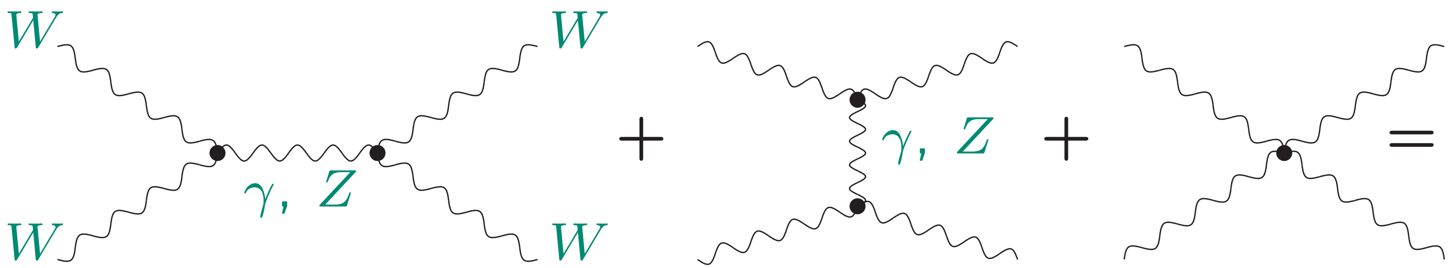


$$= -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1)$$

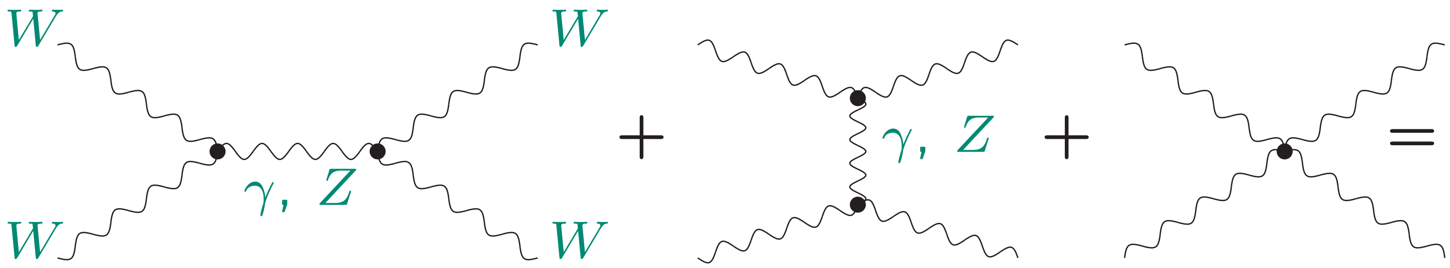
for $E \to \infty$

Q: Why is this dangerous?

Another effect of the Higgs field:

Scattering of longitudinal W bosons: $W_L W_L \to W_L W_L$

$\mathcal{M}_V =$



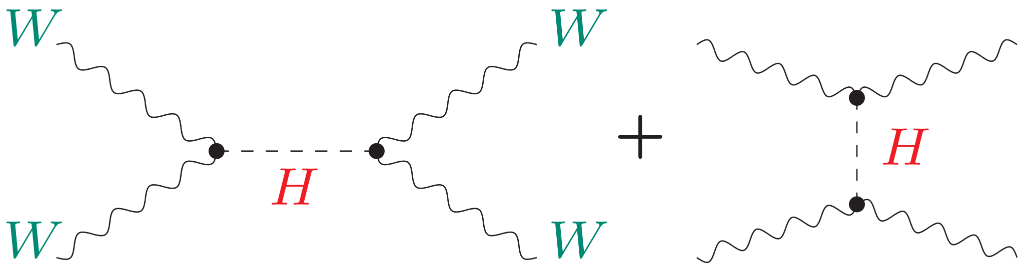
$$= -g^2 \frac{E^2}{M_W^2} + \mathcal{O}(1)$$

for $E \to \infty$

$\Rightarrow$ violation of unitarity

Contribution of a scalar particle with couplings prop. to the mass:

$\mathcal{M}_S =$



$$= g^2_{WWH} \frac{E^2}{M_W^4} + \mathcal{O}(1)$$

for $E \to \infty$

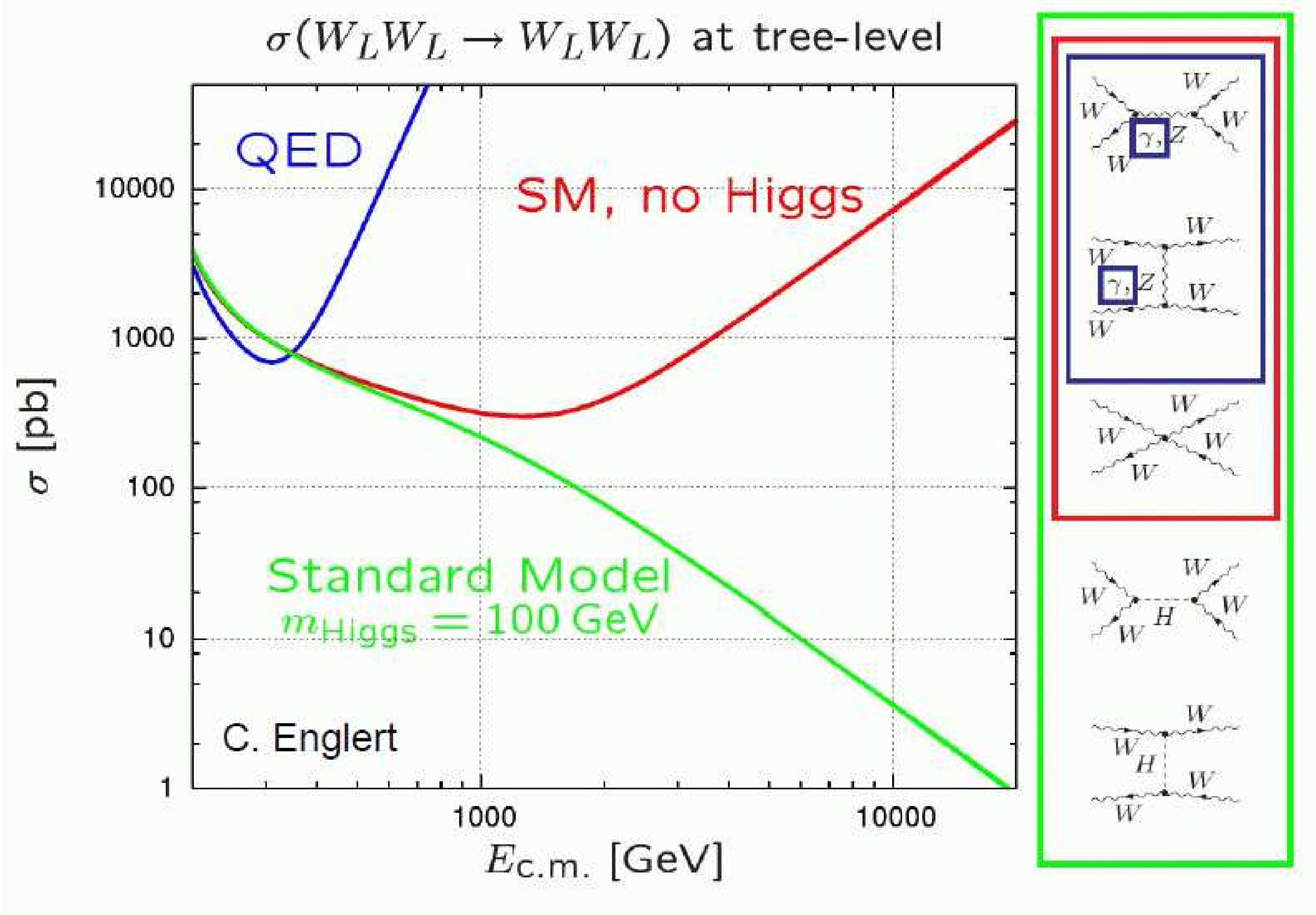
$$\mathcal{M}_{\rm tot} = \mathcal{M}_V + \mathcal{M}_S = \frac{E^2}{M_W^4}\left(g^2_{WWH} - g^2\, M_W^2\right) + \ldots$$

$\Rightarrow$ compensation of terms with bad high-energy behavior for

$$g_{WWH} = g\, M_W$$

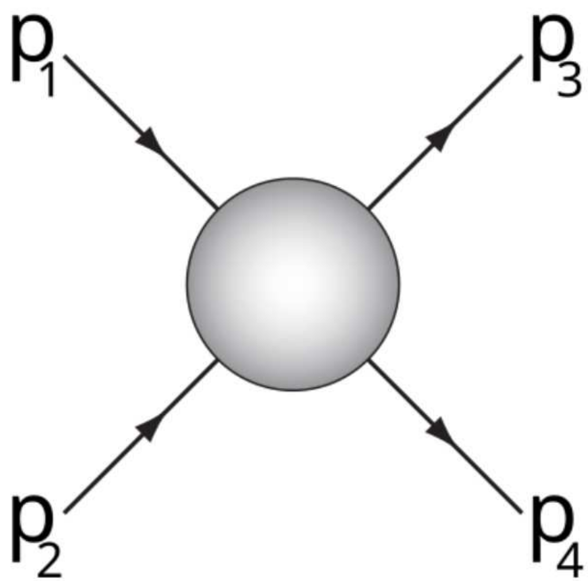
Cross section with/without the Higgs:

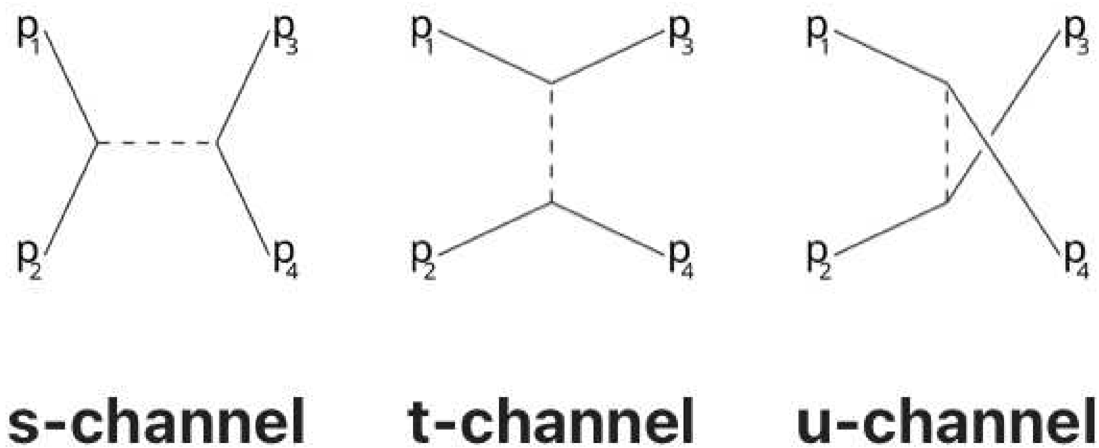
[*taken from M. Schumacher '12 / C. Englert*]



Mandelstamm variables:

[*Wikipedia*]





Mandelstamm variables:

[*Wikipedia*]

Note: for us $c = 1$

$$s = (p_1 + p_2)^2 c^2 = (p_3 + p_4)^2 c^2$$

$$t = (p_1 - p_3)^2 c^2 = (p_4 - p_2)^2 c^2$$

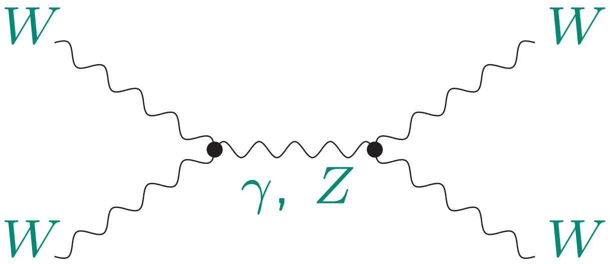
$$u = (p_1 - p_4)^2 c^2 = (p_3 - p_2)^2 c^2$$

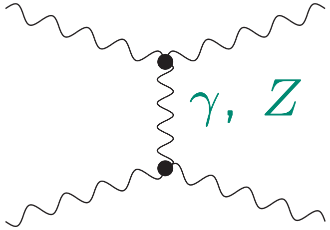
Note that

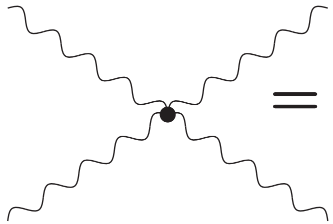
$$s + t + u = (m_1 c^2)^2 + (m_2 c^2)^2 + (m_3 c^2)^2 + (m_4 c^2)^2$$

High-energy limit: $s + t \approx -u$

Scattering of longitudinal W bosons: $W_L W_L \to W_L W_L$

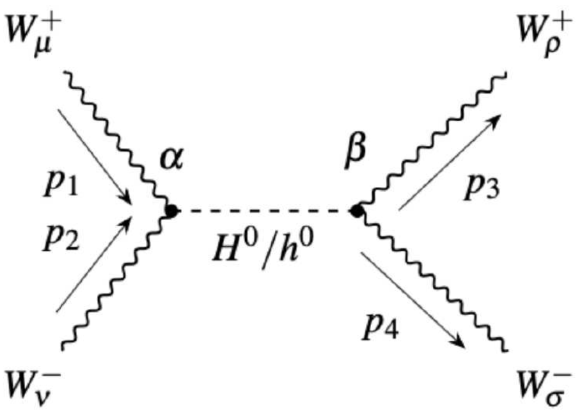
$\mathcal{M}_V =$ 

$+$ 

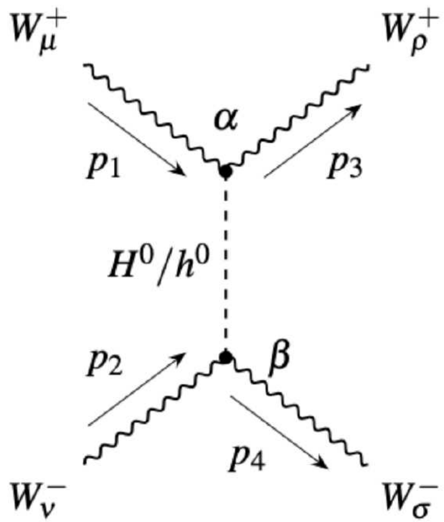
$+$ 
$= \frac{i}{v^2}(s+t) + \mathcal{O}(1)$

for $E \to \infty$

Additional Higgs contribution in the 2HDM:



s-channel



t-channel

Additional Higgs contribution in the 2HDM:

$$
\begin{aligned}
i\mathcal{M} &= -i\frac{g^2}{4m_W^2}\left[\left(\kappa_{WW}^{h^0}\right)^2\left(\frac{s^2}{s-m_{h^0}^2}+\frac{t^2}{t-m_{h^0}^2}\right)+\left(\kappa_{WW}^{H^0}\right)^2\left(\frac{s^2}{s-m_{H^0}^2}+\frac{t^2}{t-m_{H^0}^2}\right)\right], \\
&\simeq -\frac{i}{v^2}\left[\left(\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2\right)(s+t)+2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2+2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2+\cdots\right].
\end{aligned}
$$

Do the $(s+t)$ contribution cancel?

Additional Higgs contribution in the 2HDM:

$$i\mathcal{M} = -i\frac{g^2}{4m_W^2}\left[\left(\kappa_{WW}^{h^0}\right)^2\left(\frac{s^2}{s-m_{h^0}^2}+\frac{t^2}{t-m_{h^0}^2}\right)+\left(\kappa_{WW}^{H^0}\right)^2\left(\frac{s^2}{s-m_{H^0}^2}+\frac{t^2}{t-m_{H^0}^2}\right)\right],$$

$$\simeq -\frac{i}{v^2}\left[\left(\left(\kappa_{WW}^{h^0}\right)^2+\left(\kappa_{WW}^{H^0}\right)^2\right)(s+t)+2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2+2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2+\cdots\right].$$

Do the $(s+t)$ contribution cancel?

Coupling modifiers in the 2HDM:

$$\kappa_{WW}^{h^0} = \sin(\beta-\alpha), \qquad \kappa_{WW}^{H^0} = \cos(\beta-\alpha),$$

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Do the $(s+t)$ contribution cancel?

Coupling modifiers in the 2HDM:

$$\kappa_{WW}^{h^0} = \sin(\beta-\alpha), \qquad \kappa_{WW}^{H^0} = \cos(\beta-\alpha),$$

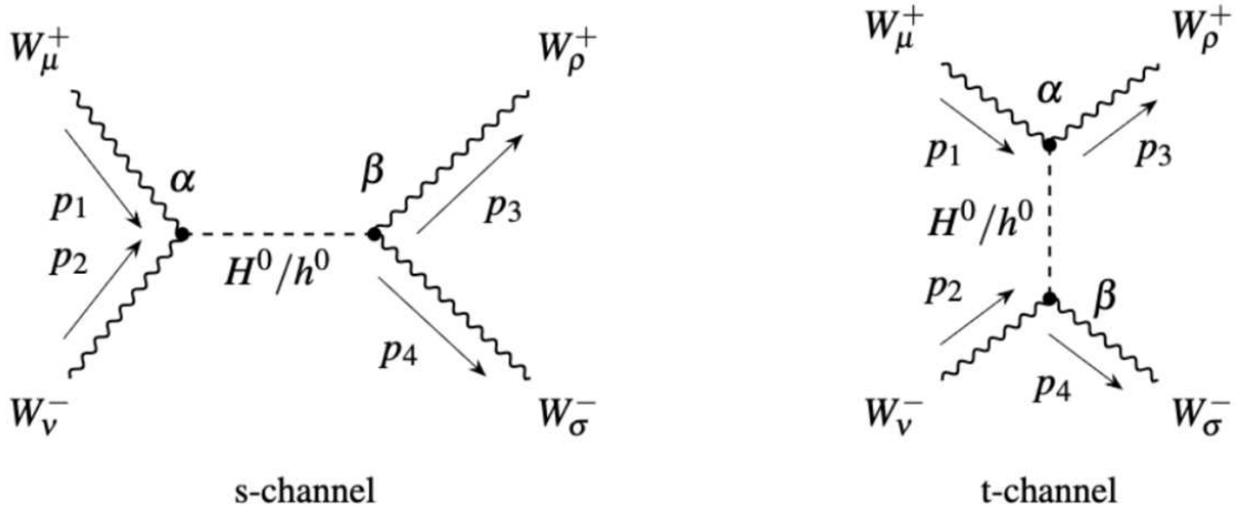
$\Rightarrow$ the $(s+t)$ contributions cancel:

$$\left(\kappa_{VV}^{h^0}\right)^2+\left(\kappa_{VV}^{H^0}\right)^2 = 1, \quad V = W, Z \text{ or}$$

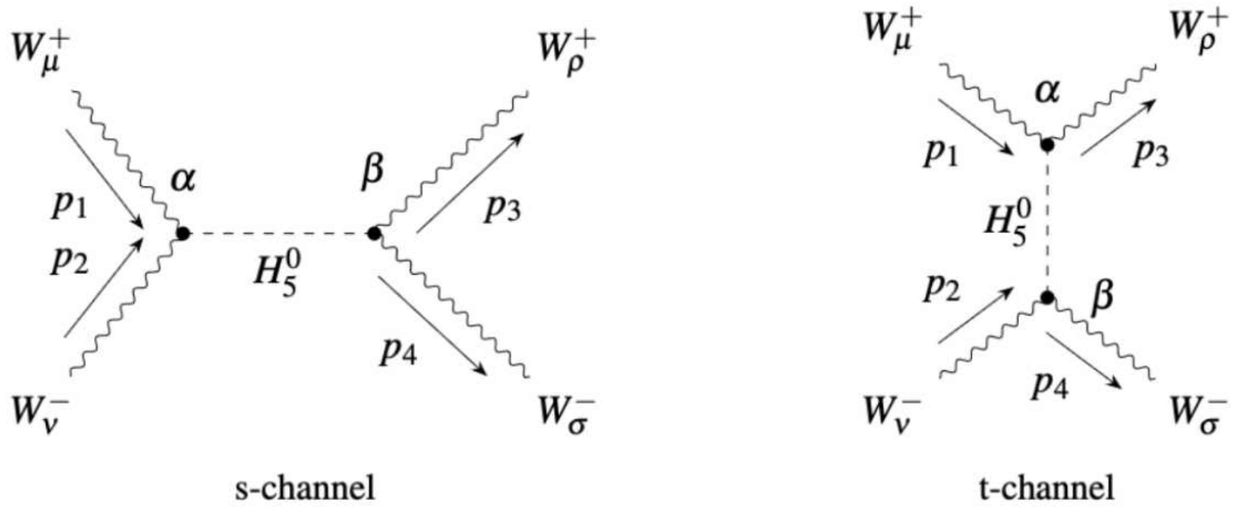
$$\sum_i g_{h_iVV}^2 = g_{H_{\rm SM}VV}^2, \quad h_i = h^0, H^0$$

Case II: GM

Additional neutral Higgs contribution in the GM:



s and t-channel h^0/H^0 diagrams **(CP even states)**



Additional neutral Higgs contribution in the GM:

$$\mathcal{M} \sim -\frac{i}{v^2}\left[\left(\kappa^{h}_{WW}\right)^2 + \left(\kappa^{H}_{WW}\right)^2 + \left(\kappa^{H_5}_{WW}\right)^2\right](s+t)$$

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Coupling modifiers of neutral Higgs bosons:

$$\kappa_V^{h^0} = \cos\theta_H \cos\alpha - \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \sin\alpha,$$

$$\kappa_V^{H^0} = \cos\theta_H \sin\alpha + \frac{\sqrt{8}}{\sqrt{3}} \sin\theta_H \cos\alpha,$$

$$\kappa_{WW}^{H_5^0} = \frac{1}{\sqrt{3}} \sin\theta_H, \qquad \kappa_{ZZ}^{H_5^0} = -\frac{2}{\sqrt{3}} \sin\theta_H.$$

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$$\kappa_{WW}^{H_5^0} = \frac{1}{\sqrt{3}} \sin\theta_H, \qquad \kappa_{ZZ}^{H_5^0} = -\frac{2}{\sqrt{3}} \sin\theta_H.$$

$$\left[\left(\kappa^h_{WW}\right)^2 + \left(\kappa^H_{WW}\right)^2 + \left(\kappa^{H_5}_{WW}\right)^2\right] = \cos^2\theta_H + 3\sin^2\theta_H$$

$\Rightarrow$ unitarity is NOT preserved??

GM is a model with triplets

Field content:

In the GM model, the SM Higgs doublet ϕ with a complex triplet χ (hypercharge $Y = 1$) and a real triplet ξ ($Y = 0$). These fields are represented as:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^{++} \\ \chi^+ \\ \chi^0 \end{pmatrix}, \quad \xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ \xi^- \end{pmatrix}. \tag{2.33}$$

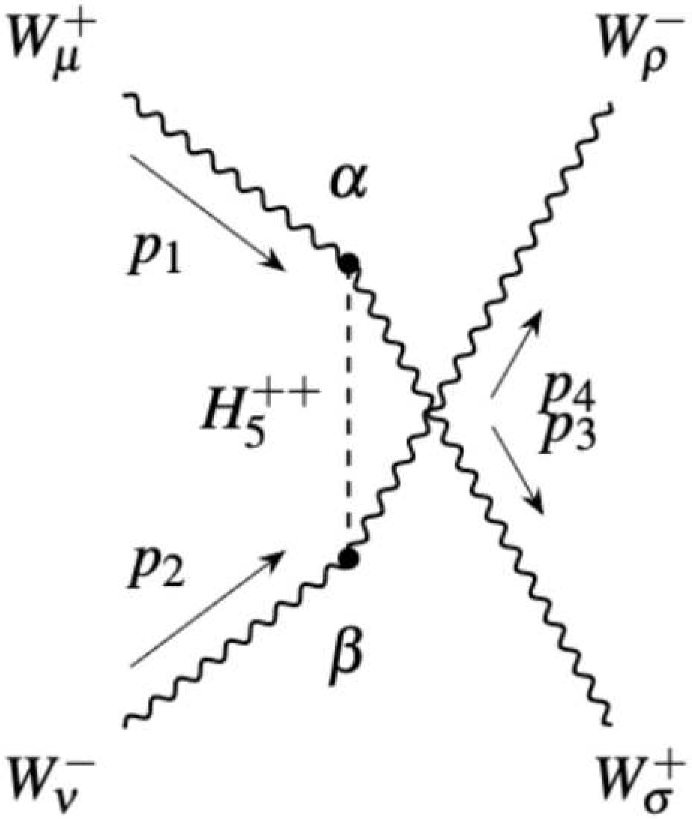
GM is a model with triplets

GM contains doubly charged Higgs bosons

GM is a model with triplets

GM contains doubly charged Higgs bosons

$\Rightarrow$ additional contribution to $W_L W_L \to W_L W_L$: $(u \sim -(s+t))$



: u-channel H_5^{++} diagram

$$\kappa_{WW}^{H^{++}} = \sqrt{2} s_H$$

Total Higgs contribution in GM:

$$\mathcal{M} \simeq -\frac{1}{v^2}\Bigg[\left(\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2\right)(s+t)$$
$$+2\left(\kappa_{WW}^{h^0}\right)^2 m_{h^0}^2 + 2\left(\kappa_{WW}^{H^0}\right)^2 m_{H^0}^2 + 2\left(\kappa_{WW}^{H_5^0}\right)^2 m_5^2 + \left(\kappa_{WW}^{H_5^{++}}\right)^2 m_5^2\Bigg]$$

Sum rule in GM:

$$\left(\kappa_{WW}^{h^0}\right)^2 + \left(\kappa_{WW}^{H^0}\right)^2 + \left(\kappa_{WW}^{H_5^0}\right)^2 - \left(\kappa_{WW}^{H_5^{++}}\right)^2 = 1,$$

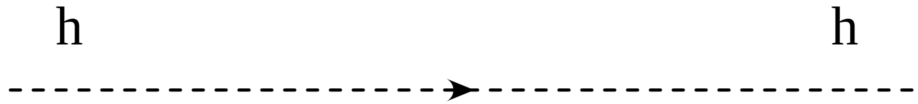
$$\sum_i g_{h_iWW}^2 - g_{h^{++}WW}^2 = g_{H_{\rm SM}WW}^2, \quad \text{where} \quad h_i = \{h^0, H^0, H_5^0\}, \quad h^{\pm\pm} = H_5^{\pm\pm}$$

$$c_H^2 + \frac{8}{3}s_H^2 + \frac{1}{3}s_H^2 - 2s_H^2 = c_H^2 + s_H^2 = 1$$

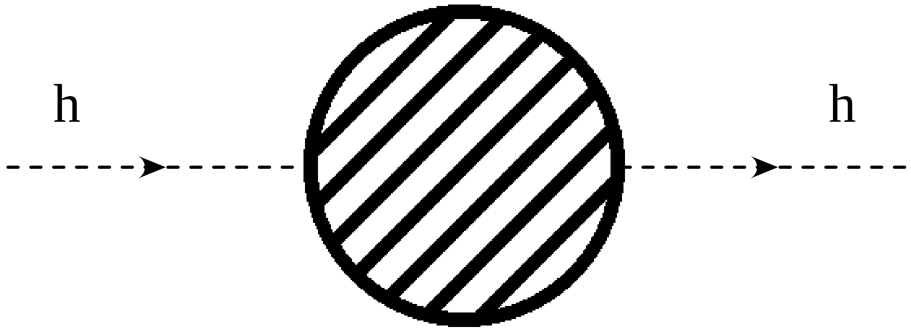
$\Rightarrow$ unitarity IS preserved in GM!

2. Quantum corrections to the MSSM Higgs mass

Higgs propagator:



quantum corrections



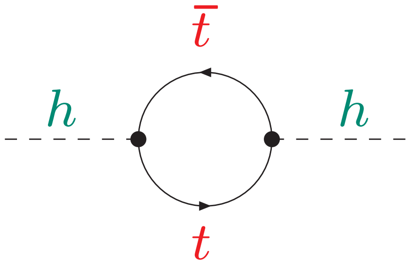
Inverse propagator:

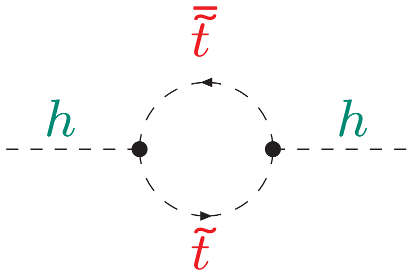
$$-i(q^2 - m^2) \quad \longrightarrow \quad -i\left(q^2 - m^2 + \hat{\Sigma}_h(q^2)\right)$$

$\hat{\Sigma}_h(q^2)$: renormalized Higgs self-energy

Loop corrected mass^2: $M^2 = m^2 - \hat{\Sigma}_h(m^2)$

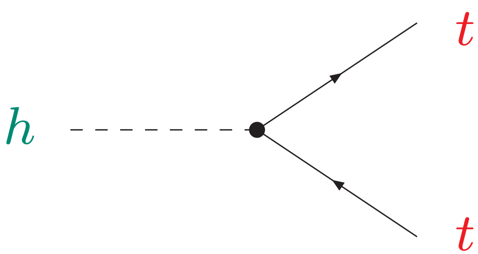
1-Loop: Feynman diagrams:

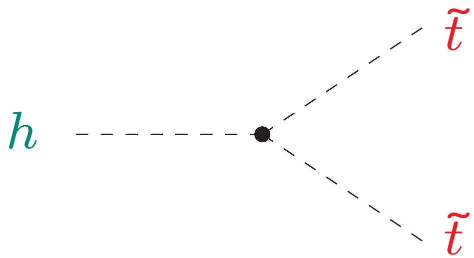




⇒ let's estimate their size!

Feynman rules:





$$\sim \frac{i\, e\, m_t}{2\, M_W s_\mathrm{W} \sin\beta} \qquad\qquad \sim \frac{i\, e\, m_t^2}{2\, M_W s_\mathrm{W} \sin\beta}$$

Consider:

- $\Delta M_h^2 = -\hat{\Sigma}_h$, dim $= [m^2]$
- SUSY is constructed in a way that corrections go with $\sim \log(m^2_{\text{SUSY}})$, (never like $\sim m_{\text{SUSY}}$ or $\sim m^2_{\text{SUSY}}$)
- logs must not have a dimensionful argument
- top diagram: m_t^2/M_W^2 from the couplings, only other mass scale available: m_t
- stop diagram: m_t^4/M_W^2 from the couplings
- color factor for tops or stops: 3
- fermion loops yield a relative factor of (-1)
- loop factor: $1/(4\,\pi^2)$

$\Rightarrow$ **estimate the formula for ΔM_h^2**

Consider:

- $\Delta M_h^2 = -\hat{\Sigma}_h$, dim = $[m^2]$
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$\Rightarrow$

$$\Delta M_h^2 \sim \frac{3\,e^2\,m_t^4}{16\,\pi^2\,M_W^2 s_{\mathrm{W}}^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Consider:

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– top diagram: m_t^2/M_W^2 from the couplings, only other mass scale available: m_t

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– color factor for tops or stops: 3

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⇒ real calculation:

$$\Delta M_h^2 \sim \frac{3\,e^2\,m_t^4}{8\,\pi^2\,M_W^2 s_{\mathrm{W}}^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

$$\text{dominant 1-loop}: \Delta M_h^2 \sim \frac{3\,e^2\,m_t^4}{8\,\pi^2\,M_W^2 s_W^2 \sin^2\beta} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

Numerical estimate:

$$\begin{aligned} e^2/(4\pi) &= \alpha = 1/137 \\ m_t &= 175 \text{ GeV} \\ M_W &= 80 \text{ GeV } (M_Z = 90 \text{ GeV}) \\ s_W &= 1/2 \\ \sin\beta &= 1 \\ m_{\tilde{t}} &= 1000 \text{ GeV} \end{aligned}$$

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$$\begin{aligned} \text{dominant 1-loop}: \Delta M_h^2 &\approx 7000 \text{ GeV}^2 \\ M_h^2 &= m_{h,\text{tree}}^2 + \Delta M_h^2 \\ m_{h,\text{tree}} &\approx M_Z \end{aligned}$$

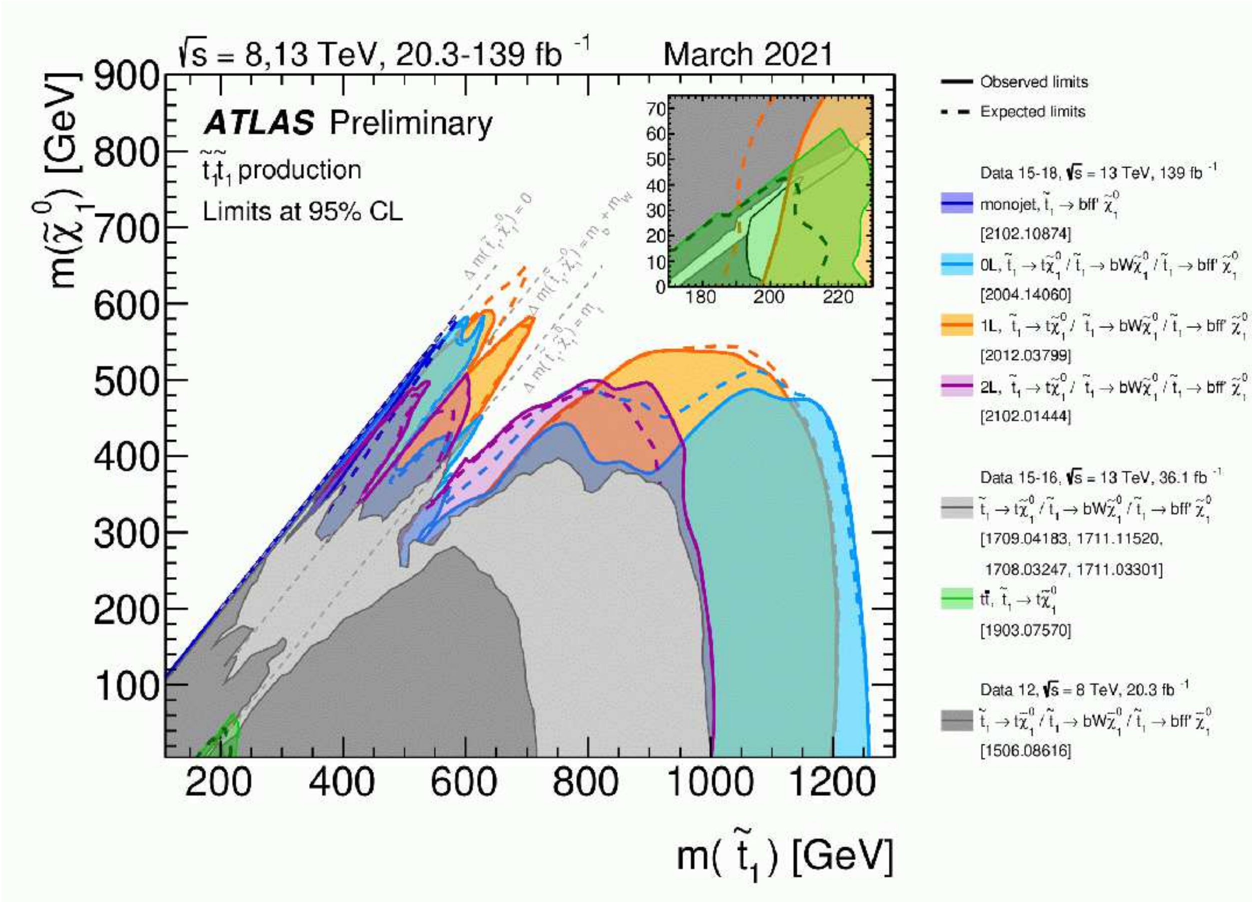
$$\text{dominant 1-loop}: M_h \approx 123 \text{ GeV}$$

dominant 1 – loop : $M_h \approx 123$ GeV

$\Rightarrow$ many, many, many, . . . more refinements have been calculated

$\Rightarrow$ stops above the TeV scale naturally yield $M_h \sim 125$ GeV

Comparison with LHC searches:



$\Rightarrow$ $M_h = 125$ GeV is in perfect agreement with the non-observation of stops