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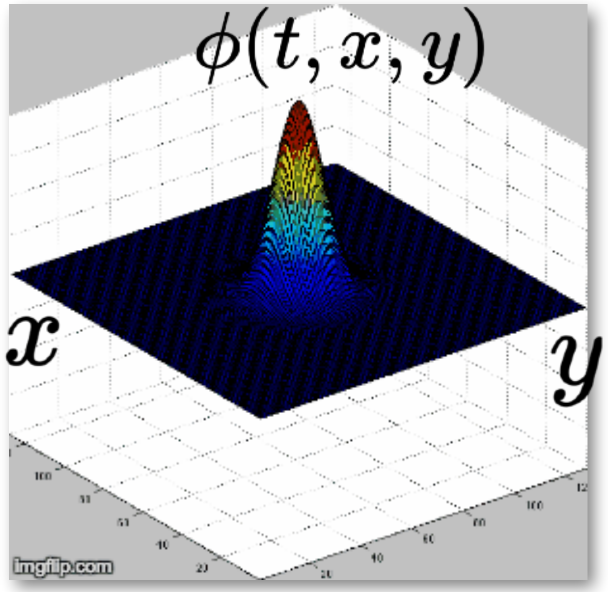



# Q-balls and oscillons

Shuang-Yong Zhou (周双勇), USTC

Shanghai, 19 Nov 2021, SPCS2021 on Emerging Frontiers of
Dark Photon, Axion, Fractional Charge Particle and Monopole New Physics

# Solitons: localized objects in PDEs

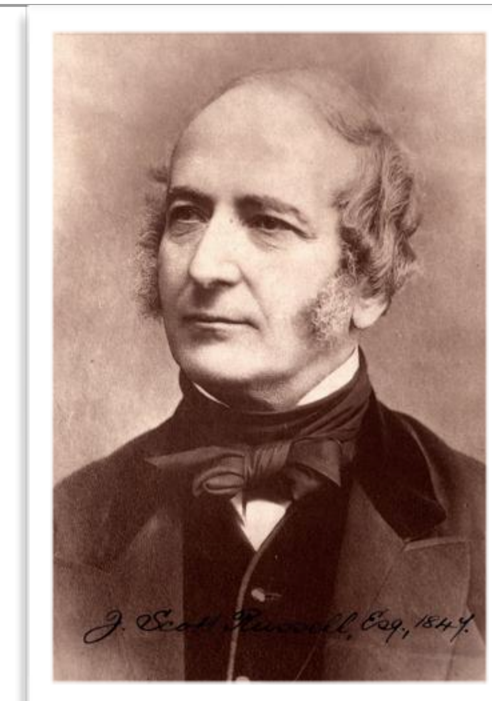
- In 1834, first described by John Scott Russell for water surface waves:

  "Wave of translation" along a canal

- Later formulated by Rayleigh, Boussinesq, Korteweg and de Vries

KdV Eq: $u_t + 6uu_x + u_{xxx} = 0$

- Nowadays, relevant in many areas: field theory, mathematics, optics, condensed matter physics, biophysics, …



John Scott Russell

Union Canal, 1995, pic from hw.ac.uk

# Static solitons

- Topological defects (topological charges, static)
  - 't Hooft-Polyakov monopole, cosmic strings, domain walls, p-branes,…

Derrick's theorem

For a set of canonical scalar fields $\mathscr{L} = \frac{1}{2}\partial_\mu\phi \cdot \partial^\mu\phi - U(\phi)$,
there is no stable, static soliton in $D > 1 + 1$ dimensions.

violated by non-canonical kinetic terms such as skirmions or galileons

Skyrme, 1961 Padilla, Saffin & SYZ, 1008.0745

# Stationary solitons

now, scalars are ok for $D > 2$

- Non-topological solitons (Noether charges, stationary)
  - Q-balls
  - boson stars (including gravity)

Nontopological solitons, T.D. Lee & Y. Pang, Phys.Rept. 221 (1992) 251-350

- Quasi-solitons (unstable but long-lived, stationary)
  - oscillons
  - oscillatons (including gravity)

# Field theory with a global symmetry

- Consider a field multiplet with a global symmetry

$$\mathcal{L} = -\frac{1}{2}\partial^{\mu}\phi_a\partial_{\mu}\phi_a - V(\phi_a)$$

- For a given total charge $Q$, one typically expects
  - particles (charges) dissipate to infinity
  - the $\phi_a = const.$ configuration is the minimum of the energy functional.

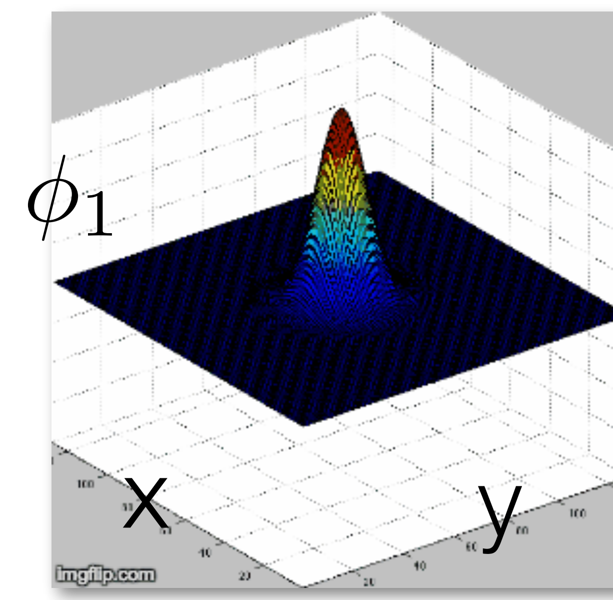
not always true…

# What is a Q-ball?

Friedberg, Lee & Sirlin 1976; Coleman 1985;
earlier works by Rosen, Kaup, Ruffini, Bonazzola, …

- A Q-ball is:

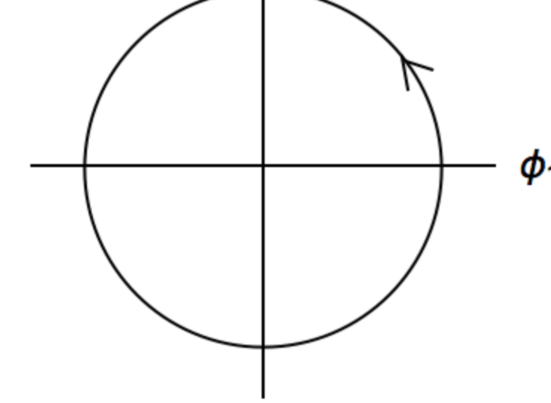
a stable, localized, time-dependent
field configuration with a Noether charge.




- Simplest case: U(1) symmetric scalar field

$$\mathcal{L} = -\partial_\mu \Phi \partial^\mu \bar{\Phi} - V(|\Phi|)$$

$$\Phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$$




Q-ball solution:

$$\Phi = \varphi(r) e^{iwt}, \quad w = const.$$

Q-ball configuration is minimal of energy functional

# When can Q-balls form?

Interacting potential: $V_{\rm nl}(|\Phi|) = V(|\Phi|) - m^2|\Phi|^2$

Attractive force:

$$V_{\rm nl} < 0 \text{ for some } |\Phi|$$

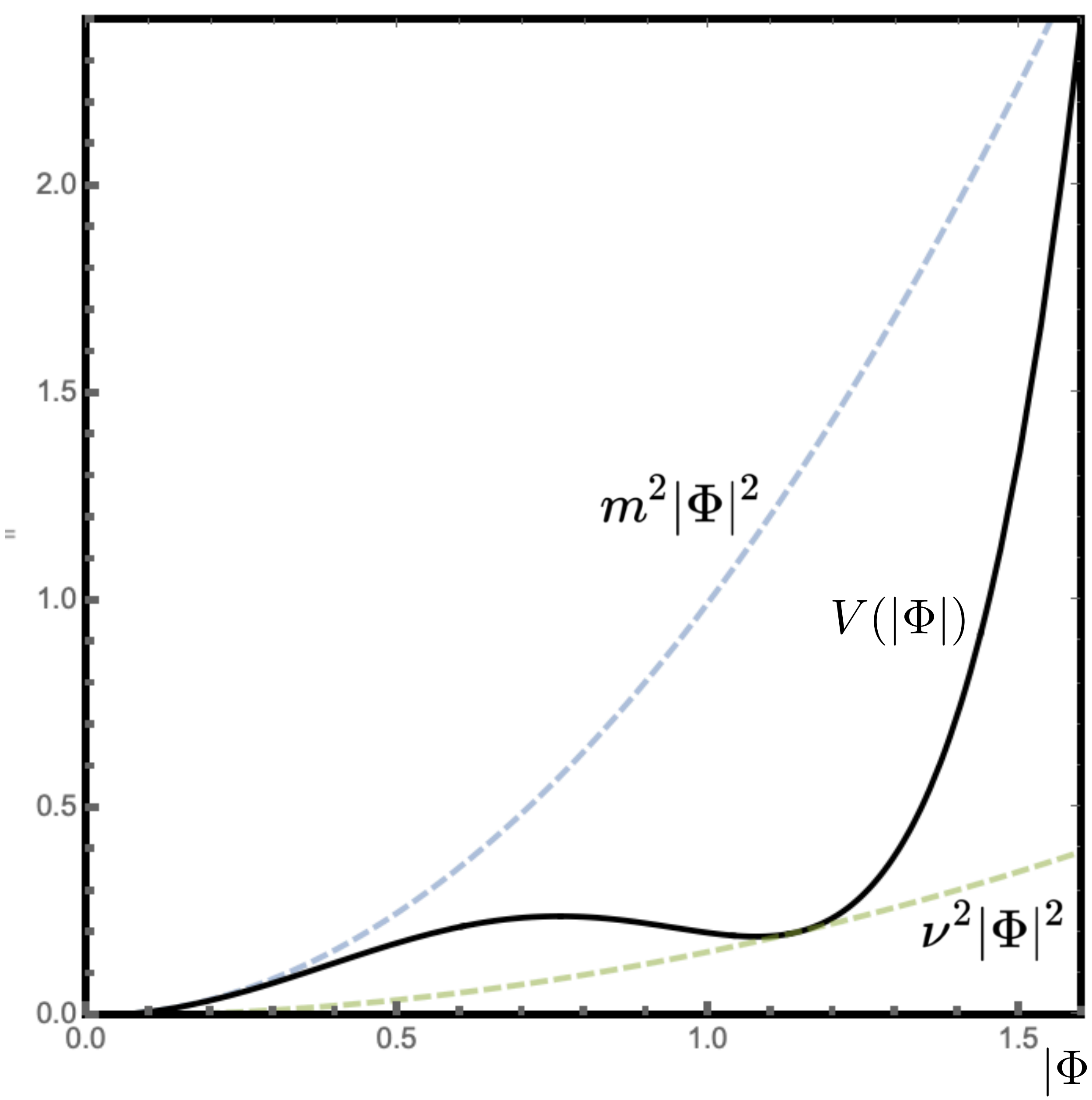
Existence condition:

$$\nu^2 \leq \omega^2 < m^2$$

Energy per particle:

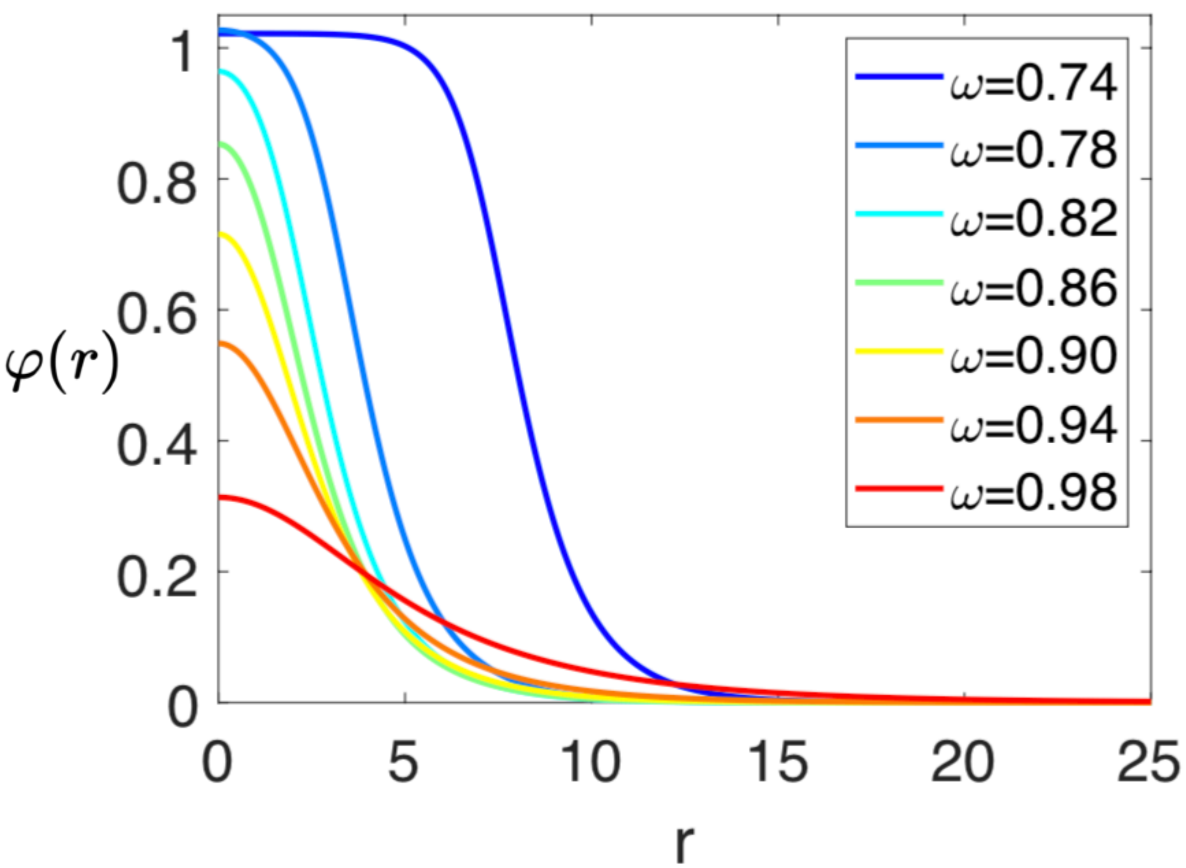
$$\left.\frac{E}{Q}\right|_{\rm Q-ball} < m$$

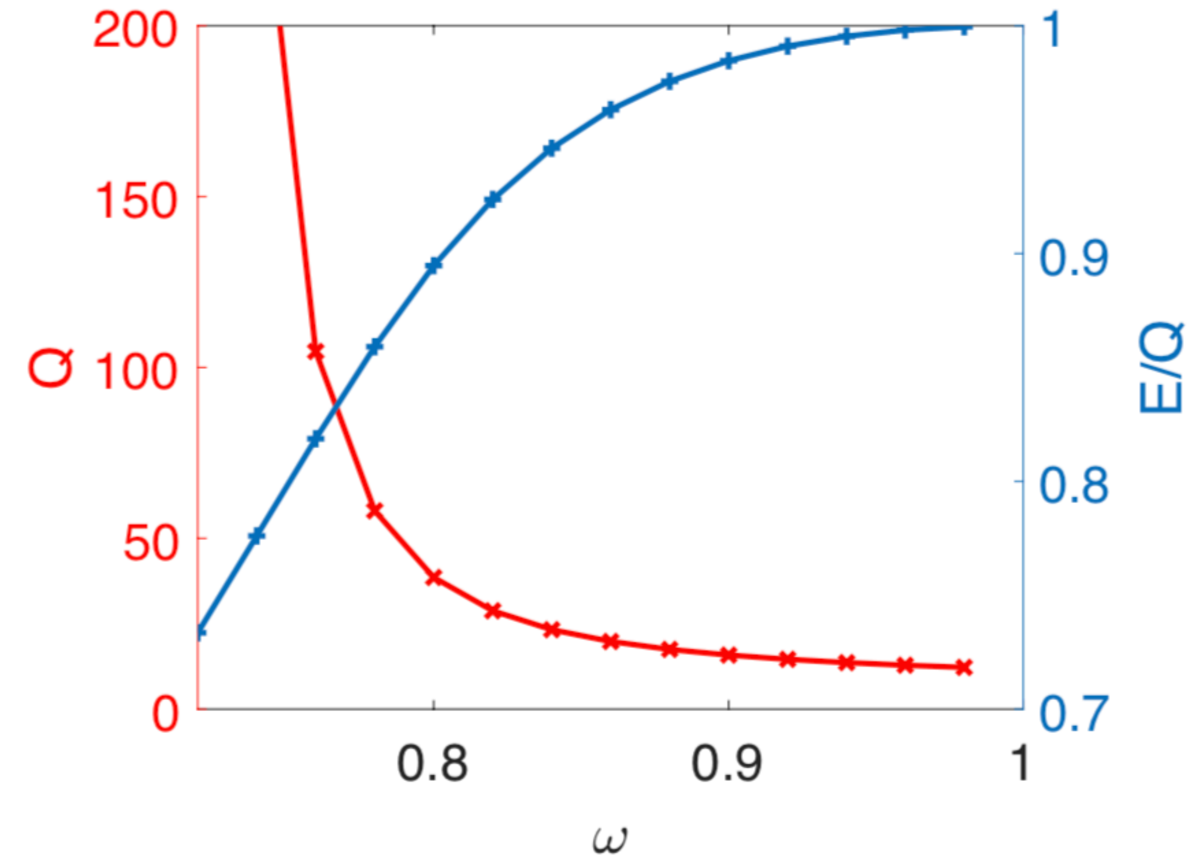
**Q-ball is the true “vacuum”!**



# Q-ball profiles

$$\Phi = \varphi(r)e^{iwt}, \quad w = const.$$




when $\omega$ is close to $m$, it is close to dissipative waves

when $\omega$ is close to $\nu$, it is close to “top hat” (thin-wall Q-ball)

# What roles may Q-balls play?

- Q-balls in cosmology
  - Q-balls from Affleck-Dine condensate decay

    Kusenko & Shaposhnikov 1997
    - Many flat directions in SUSY potentials

    Dine, Randall & Thomas 1995
  - Q-balls as dark matter candidates

    Kusenko & Shaposhnikov 1998
- (Fermionic) Q-balls to model hadrons
  - Friedberg-Lee model

    Friedberg & T.D. Lee 1998

# What is an oscillon?

An oscillon is like a Q-ball, but without Noether charges, thus quasi-stable.

Q-ball oscillon

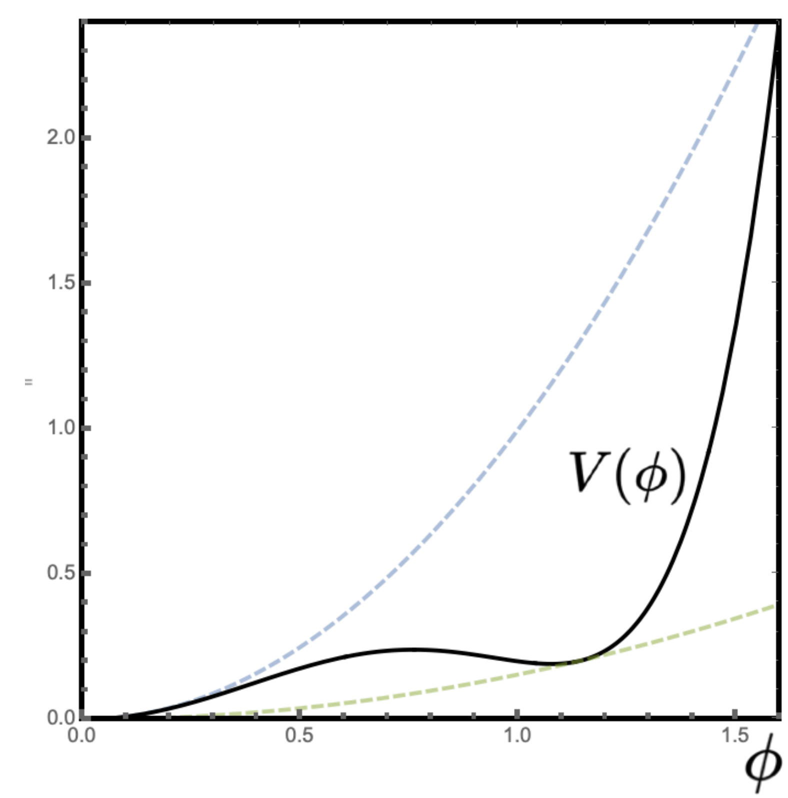
$$\mathcal{L}(\Phi) \to \mathcal{L}(\phi)$$

U(1) field real field

Also requires an attractive potential

$V_{\rm nl}(\phi) < 0$ for some $\phi$

**An oscillon is formed due to the attractive potential.**



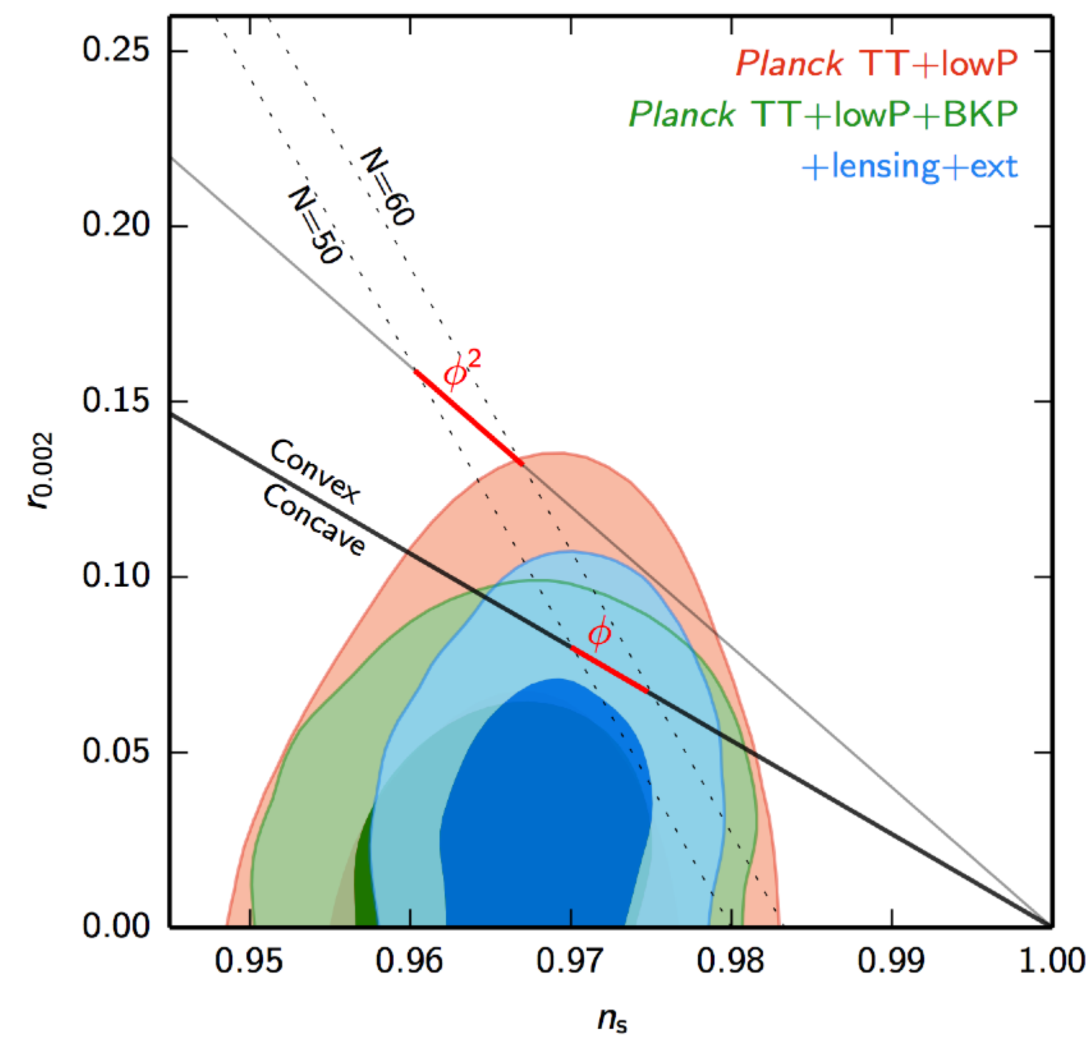
# What roles may oscillons play in cosmology?

- Inflation potential is very flat, so oscillons may arise during reheating after inflation

  Amin, Easther, Finkel, Flauger & Hertzberg, 1106.3335

  Produce gravitational waves

  SYZ, Copeland, Easther, Finkel, Mou & Saffin, 1304.6094




- Oscillons may collapse to primordial black holes

  Kou, Tian & SYZ, 1912.09658

- Oscillons may also arise in other preheating like processes in early universe

# Classical approximation of Q-balls and oscillons

- Highly non-perturbative; Can not use Feynman diagrams
- But we can use classical field approximation

because when occupation numbers are high

[redacted] $a^\dagger_{\mathbf{p}} a_{\mathbf{p}} |Q\rangle \simeq$ [redacted] $a_{\mathbf{p}} a^\dagger_{\mathbf{p}} |Q\rangle$

for large field configurations $[a^\dagger_{\mathbf{p}}, a_{\mathbf{p}}] \simeq 0$

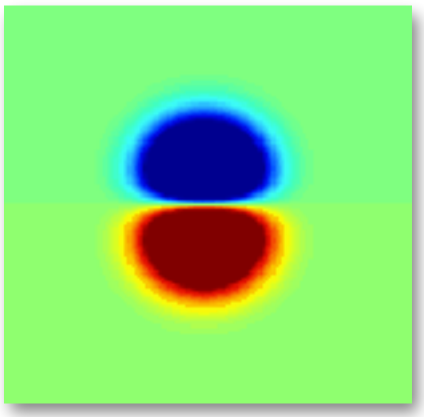
then $a_{\mathbf{p}}, a^\dagger_{\mathbf{p}}$ can be treated as stochastic c-numbers

# Charge-Swapping Q-balls

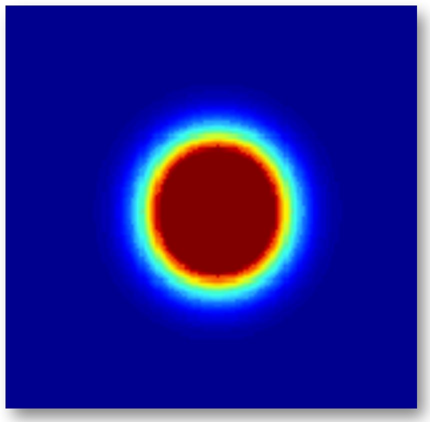
# Existence of composite Q-balls

- A Q-ball with co-existing positive and negative charges

charge density



energy density



charge swapping Q-balls (CSQs)

- Quasi-stable, but exist for a long time

- exist in **various potentials and spacetime dimensions**

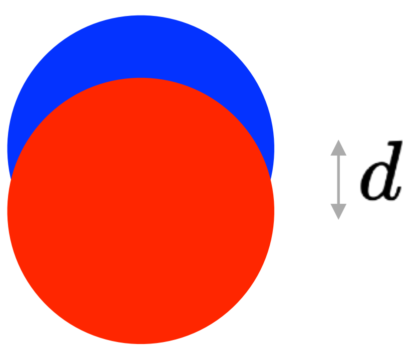
Copeland, Saffin & SYZ, PRL (Editors' Suggestion) 1409.3232

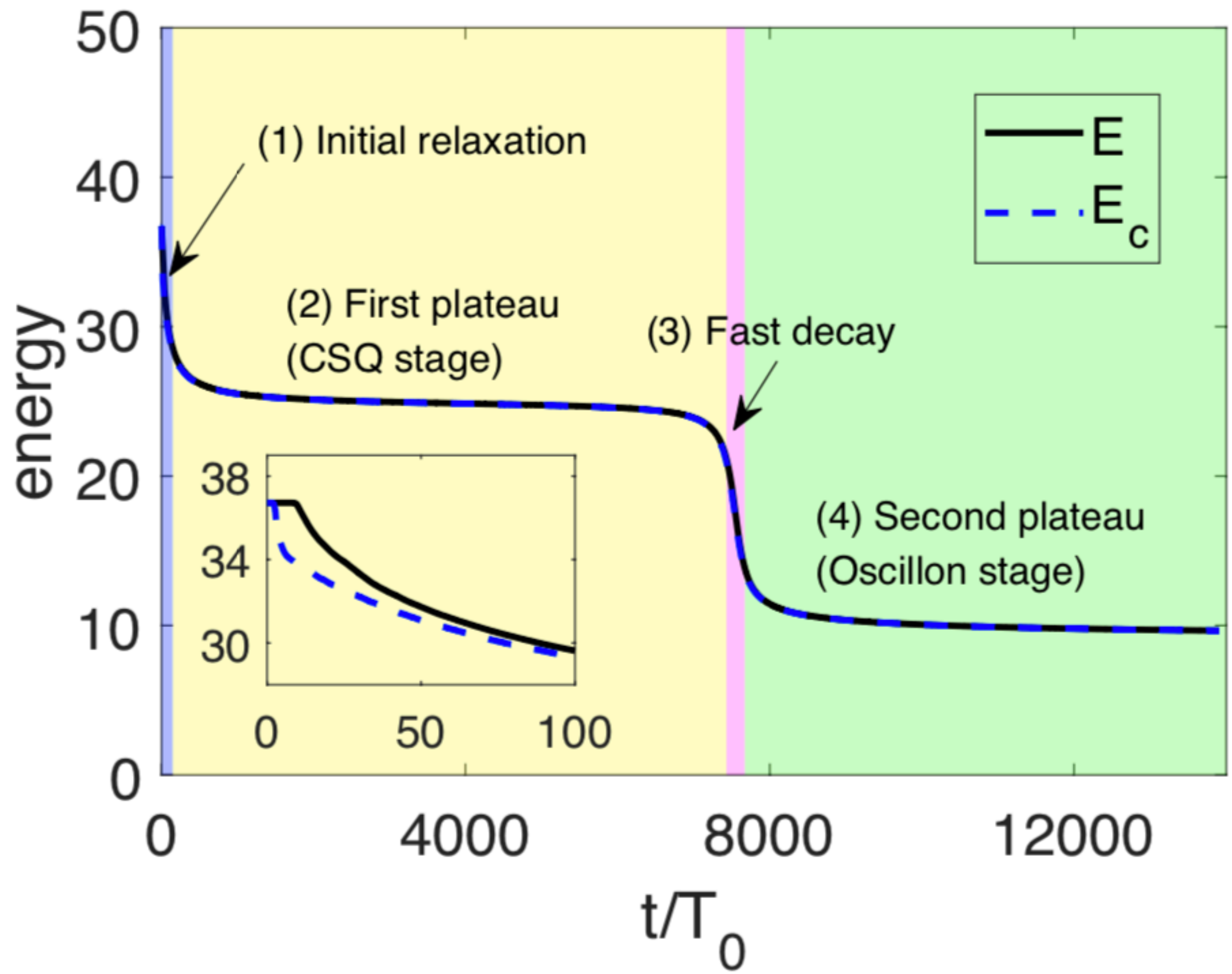
# Evolution of a CSQ

Xie, Saffin & SYZ, 2101.06988

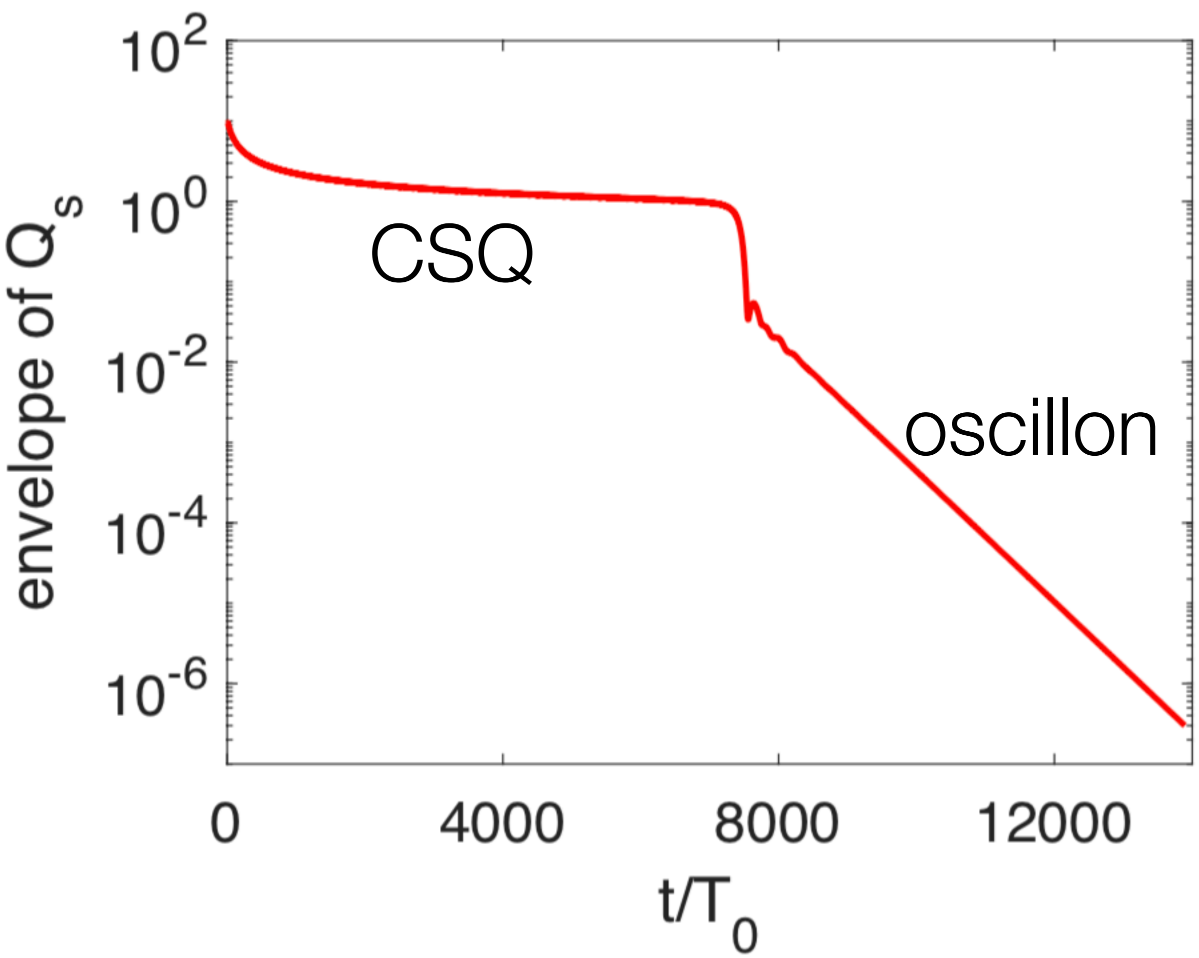
Take for example $V(|\Phi|) = |\Phi|^2 - |\Phi|^4 + g|\Phi|^6, \quad g = \frac{1}{2}$

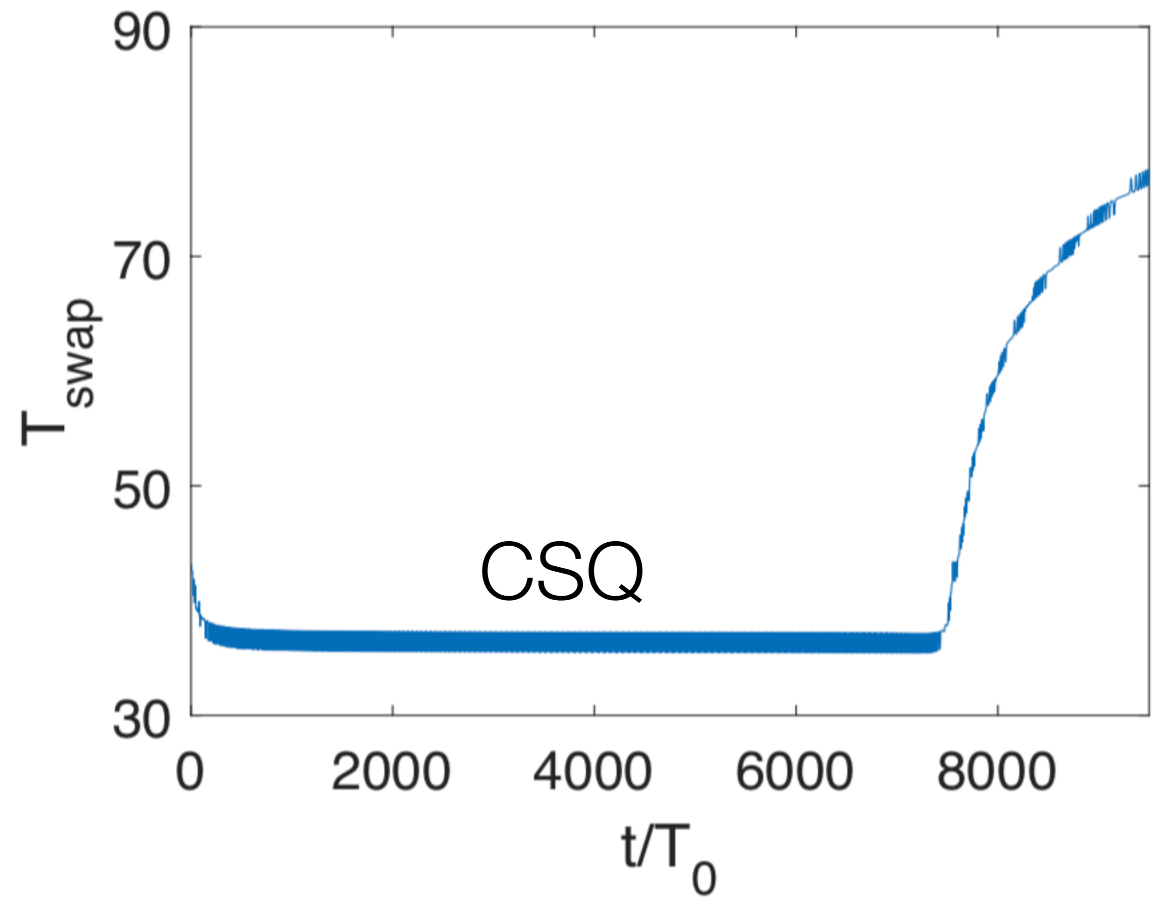
initial configuration



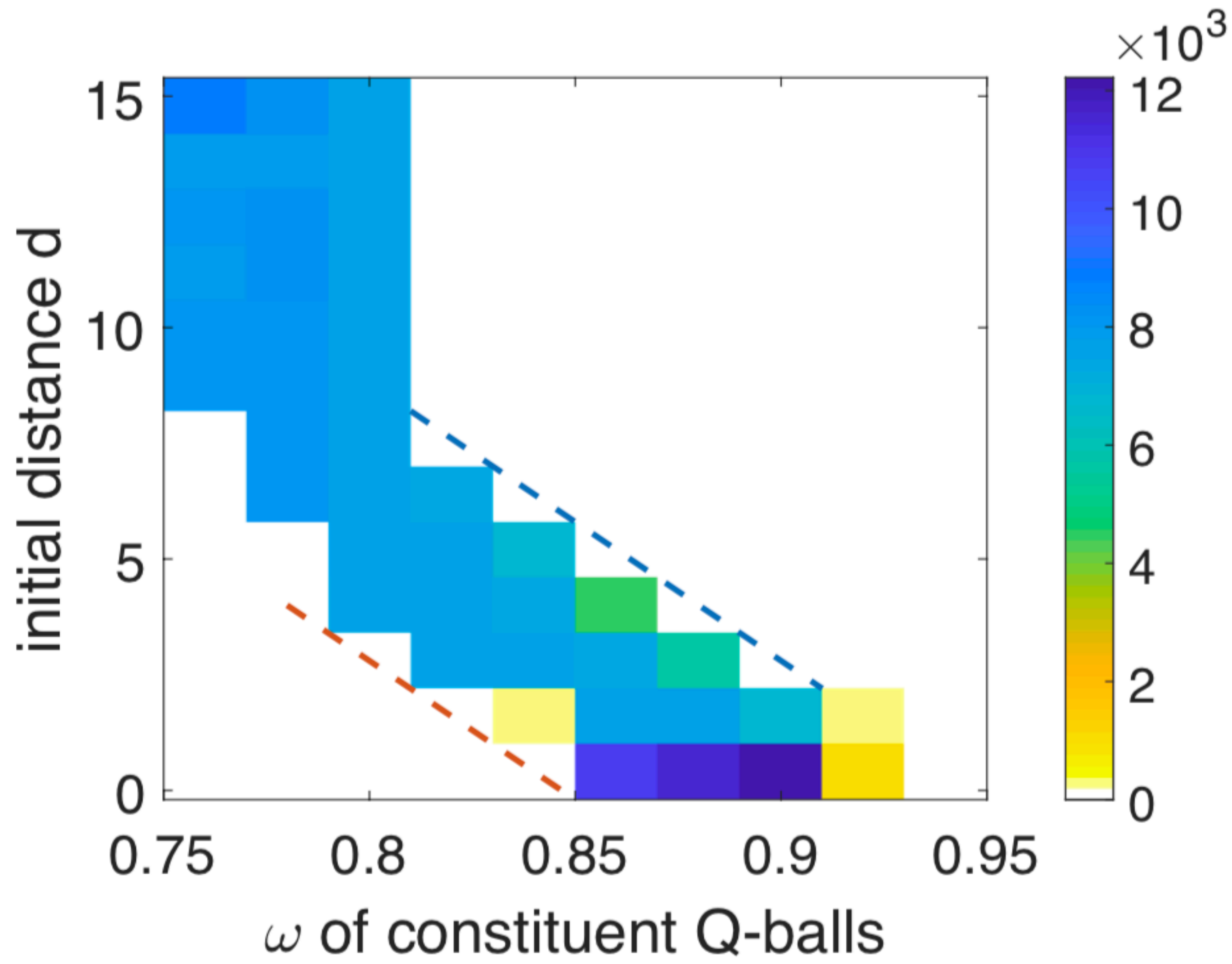
# Evolution of charges and swapping periods

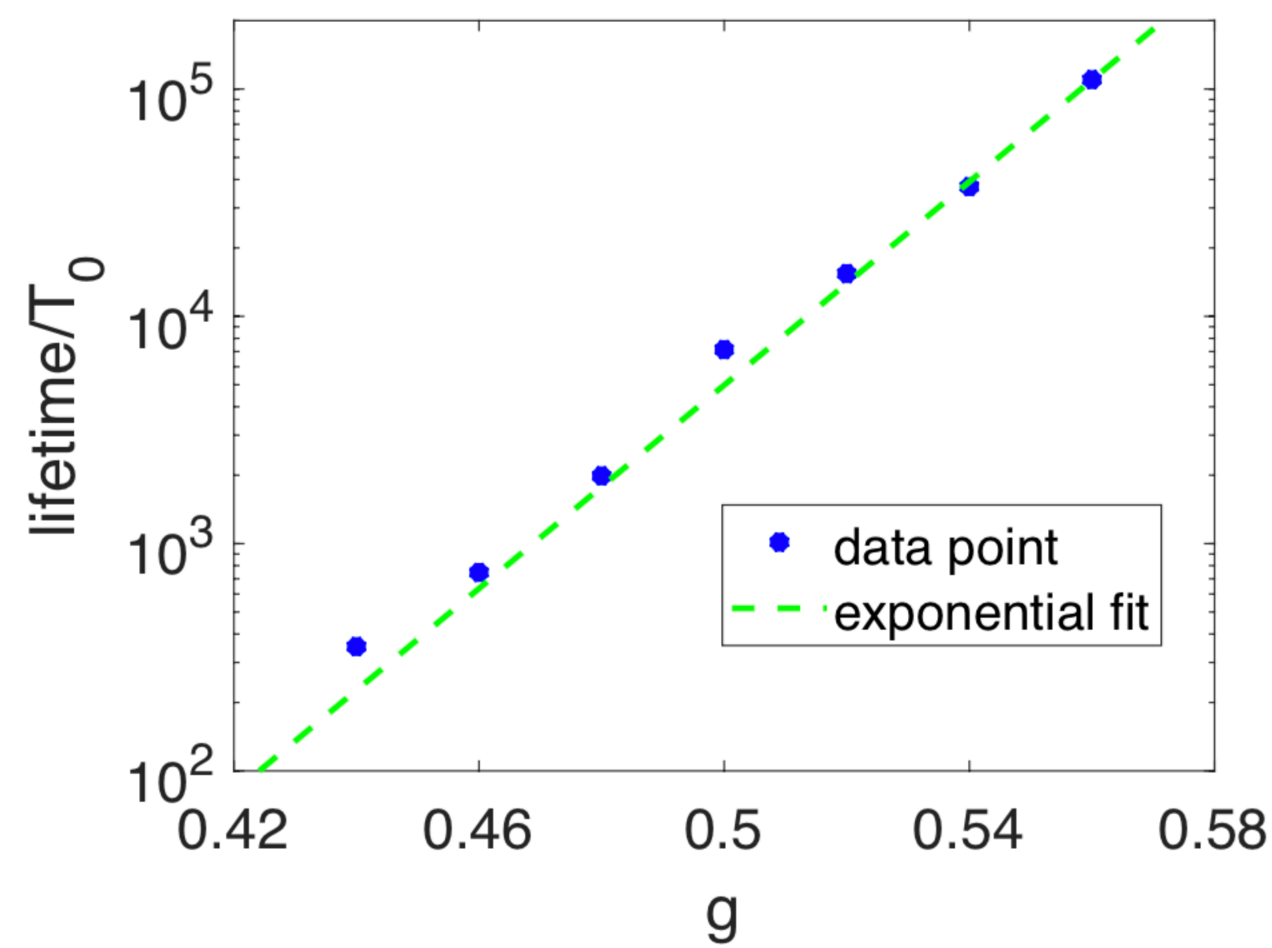


# Survey of lifetime of CSQs

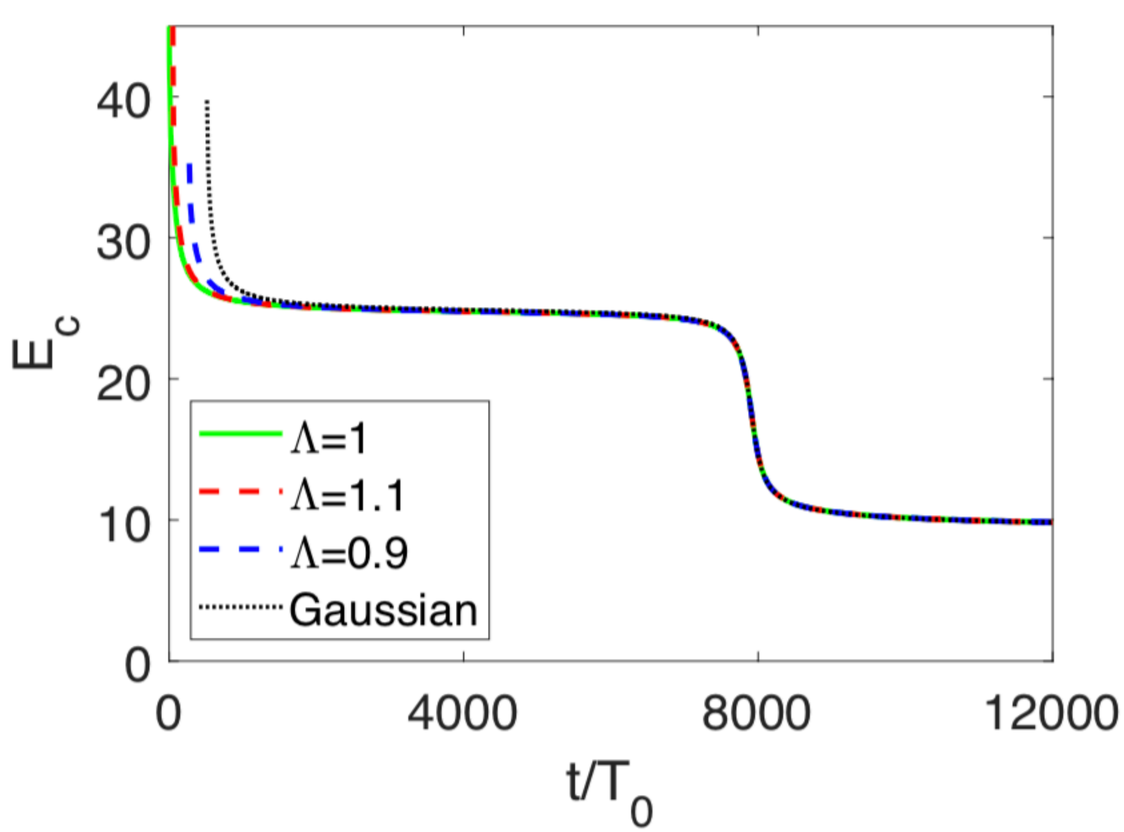
Xie, Saffin & SYZ, 2101.06988

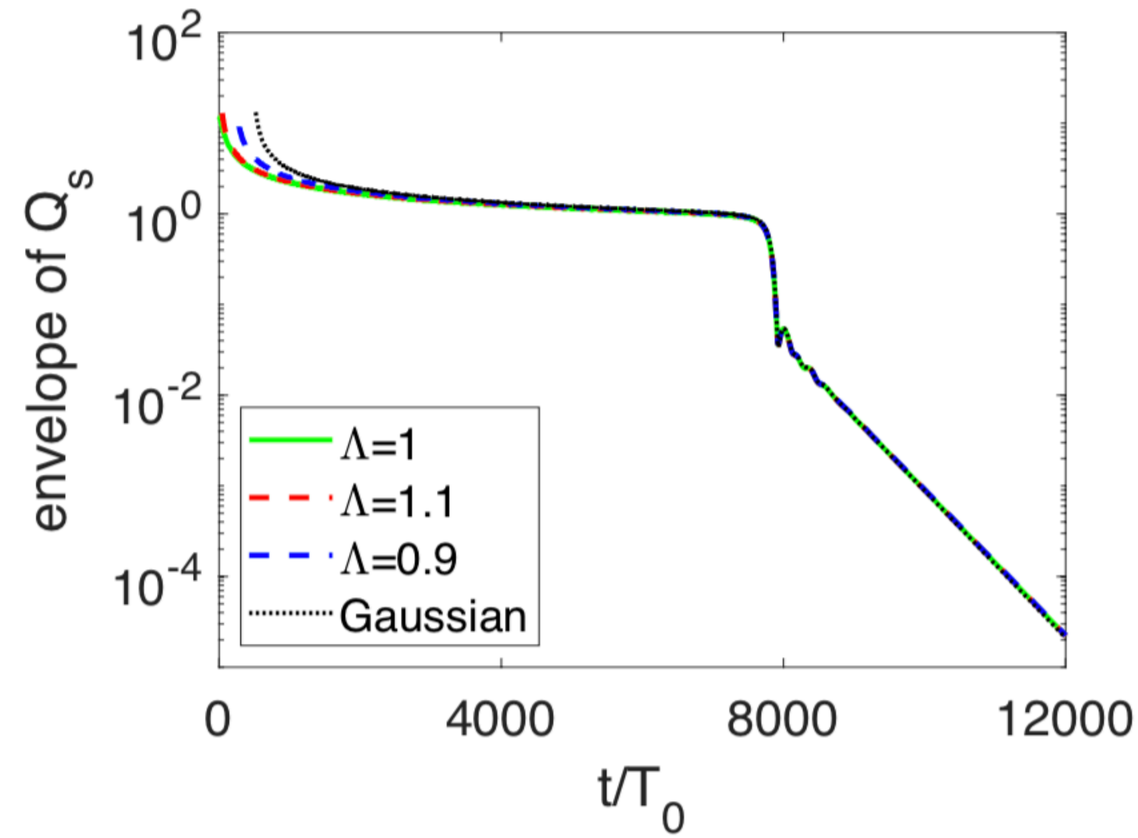



Take for example $V(|\Phi|) = |\Phi|^2 - |\Phi|^4 + g|\Phi|^6$

# Attractor behaviors from different initial conditions

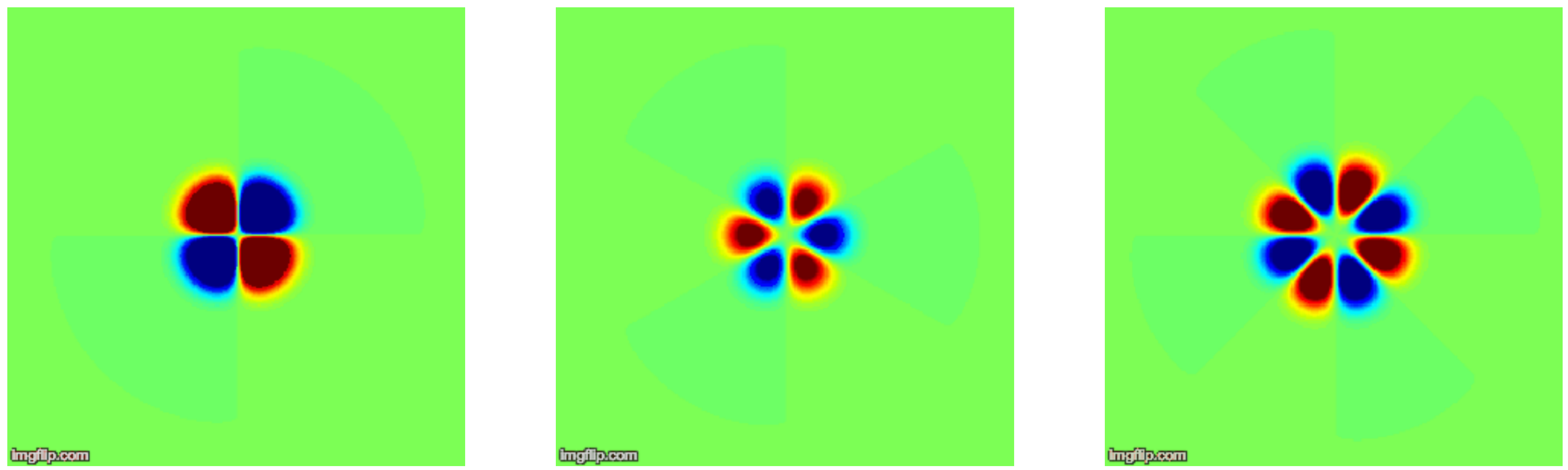


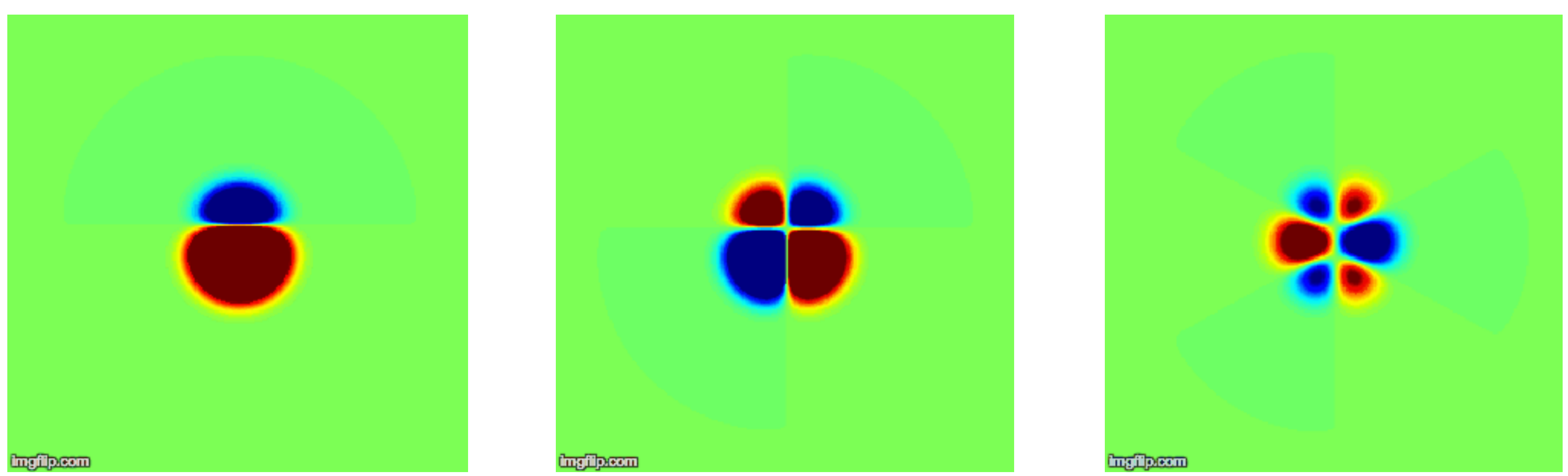

CSQs arise from fairy generic initial conditions

# CSQ zoology — more complicated cases

equal charge Q-balls



un-equal charge Q-balls



There exist a tower of new composite Q-balls!

# Oscillons and gravitational waves

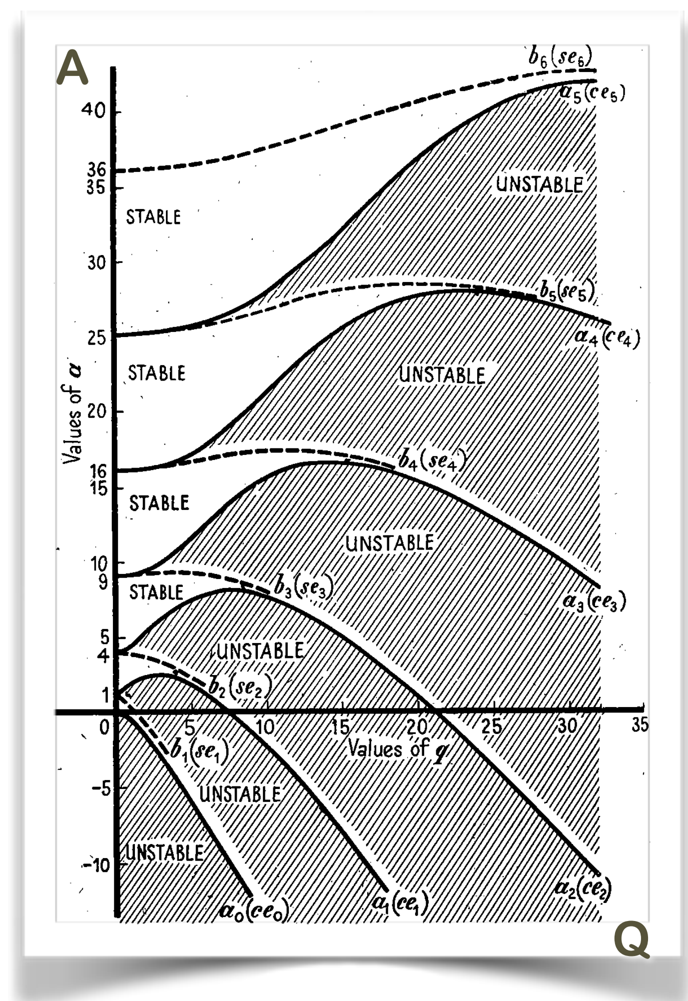
# Preheating after inflation

Inflation —reheating→ Hot Big Bang

## Preheating: parametric resonance during reheating

like Mathieu's equation

$$\ddot{x} + (A - 2Q\cos(2t))x = 0$$




## In preheating:

$\cos(2t)$ provided by homogenous inflaton $\phi_0(t)$

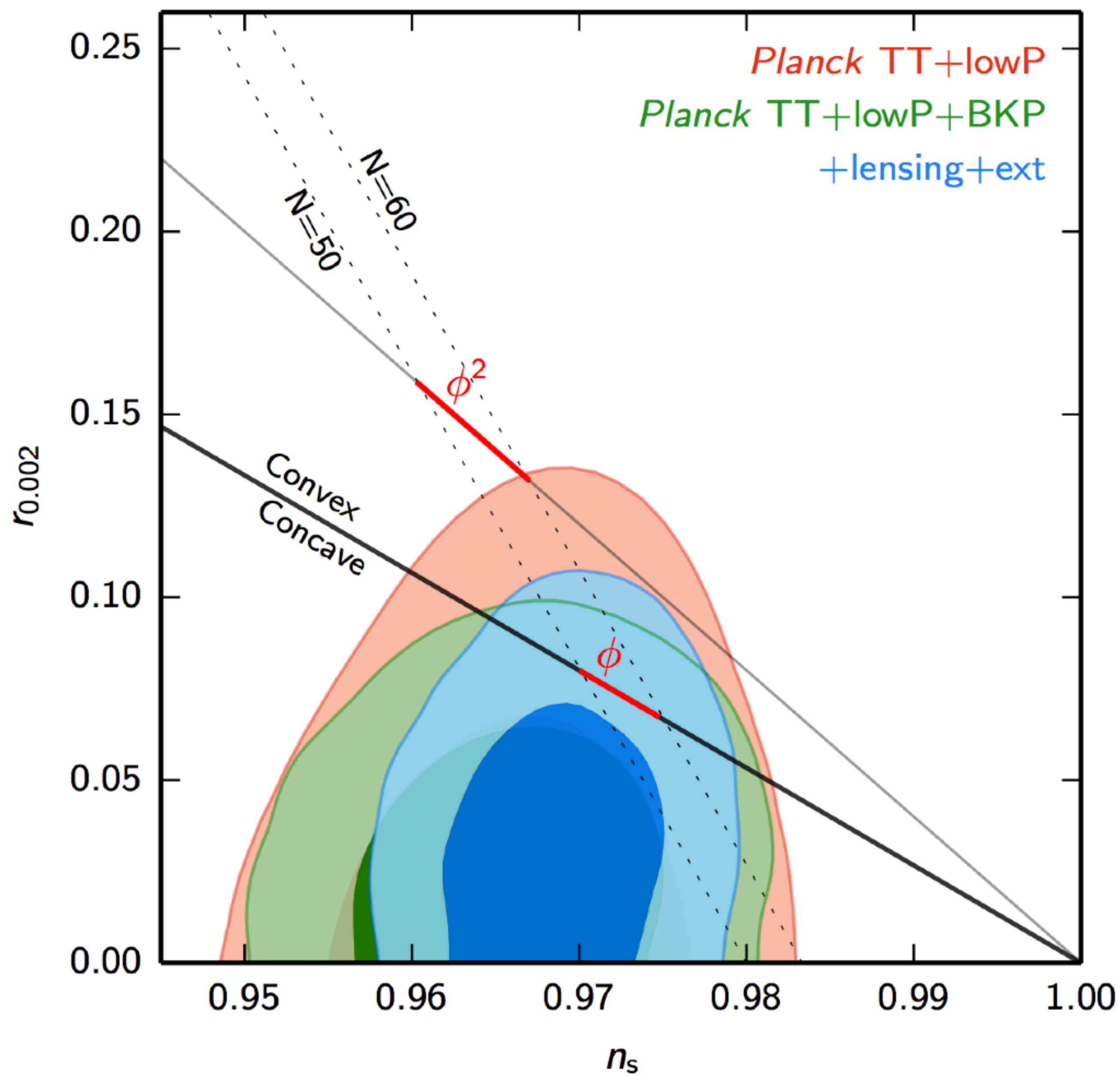
$$x(t) \rightarrow \psi_{\mathbf{k}}(t)$$

$$A \rightarrow A_{\mathbf{k}}(a(t), H(t), ...)$$

$$Q \rightarrow Q(a(t), H(t), ...)$$

# Planck's constraint on inflation potential

CMB data suggest that inflation potential is very flat



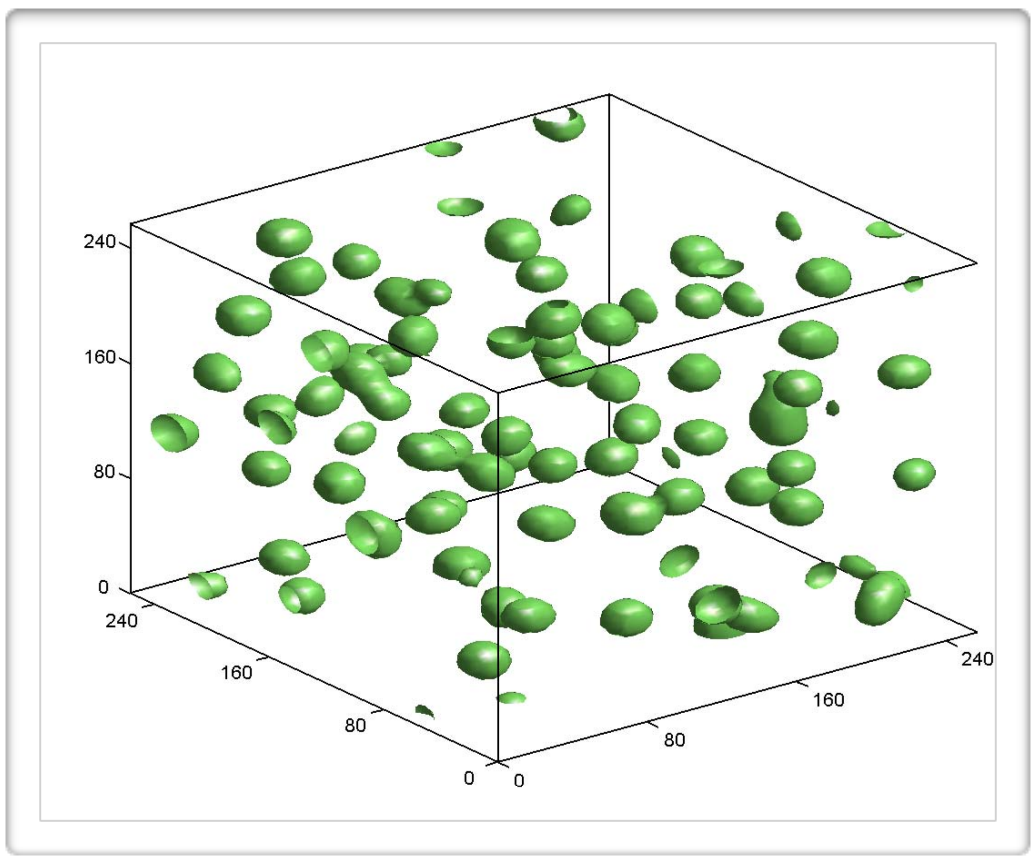

Planck 2015 XIII

# Oscillon preheating

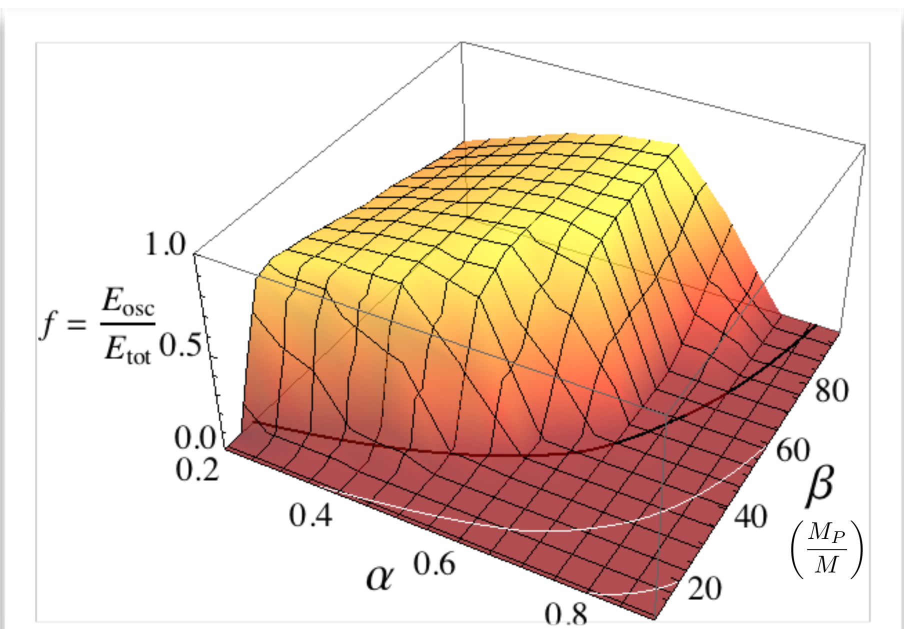
- Oscillons copiously formed in preheating
  - oscillon dominated period

Generalized axion monodromy inflation

inspired by string theory



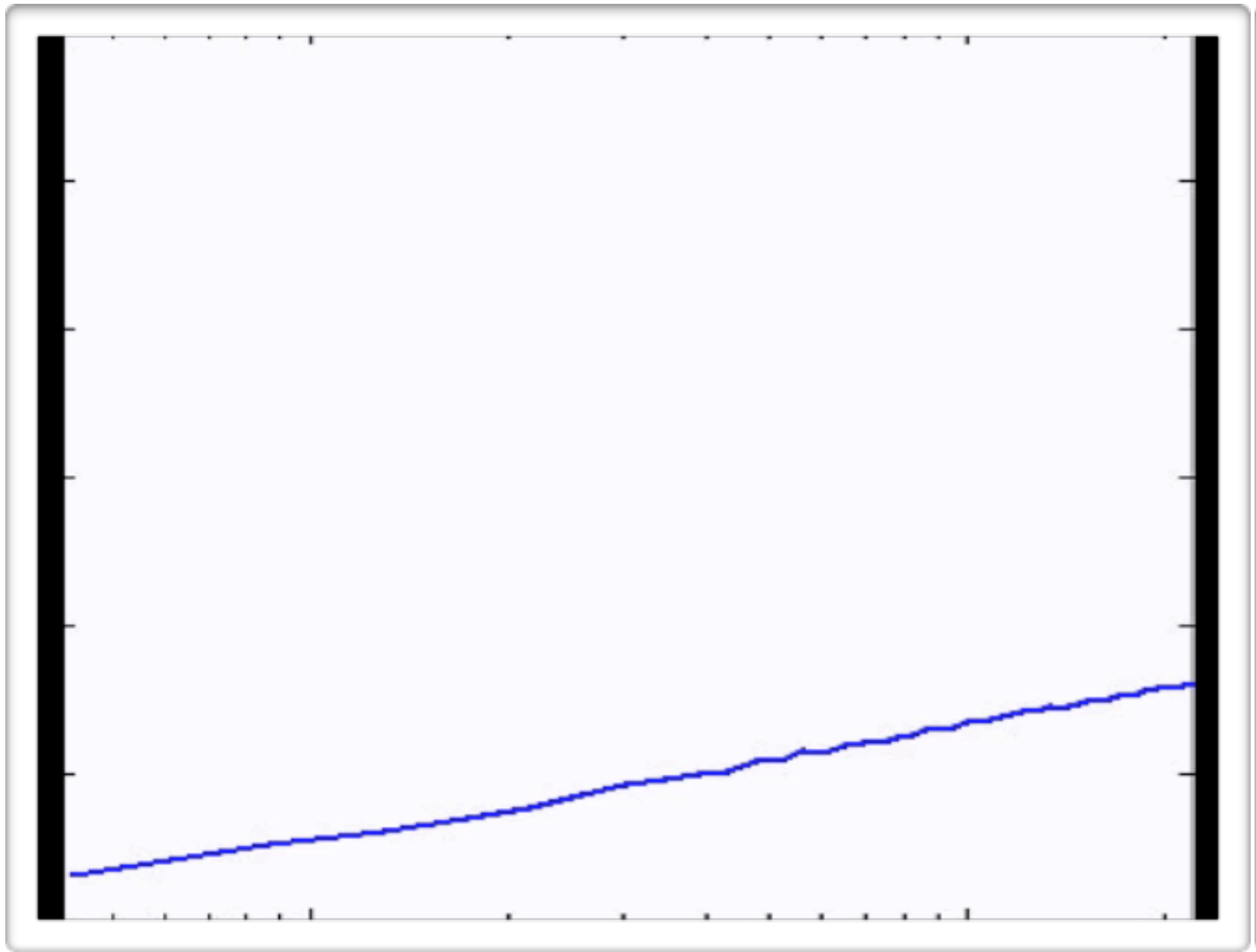

$$V(\phi) = \frac{m^2}{2\alpha\beta^2}\left[\left(1+\beta^2\phi^2\right)^{\alpha} - 1\right]$$



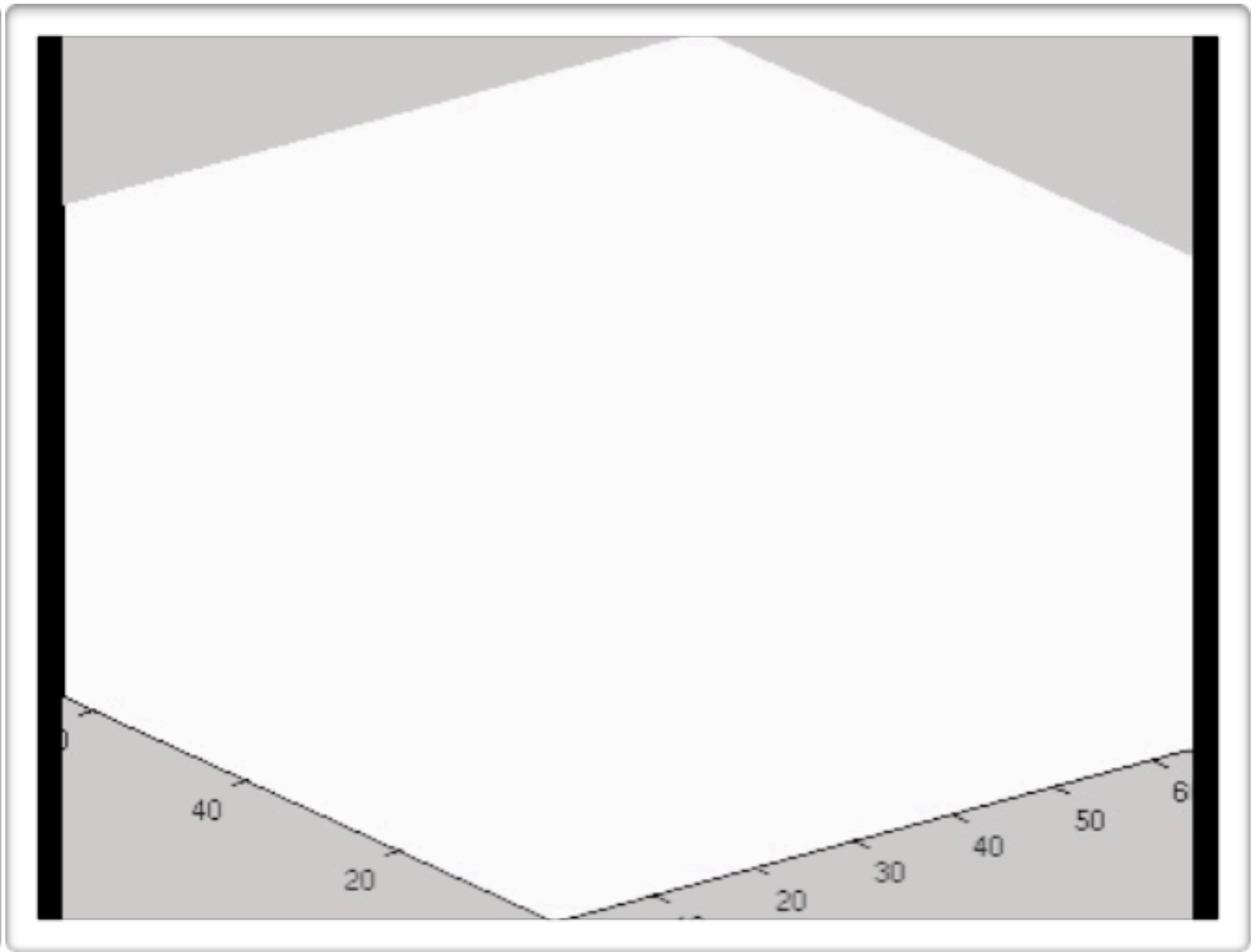

Amin, Easther, Finkel, Flauger & Hertzberg, 1106.3335

# Gravitational wave production in oscillon preheating

GW power spectrum



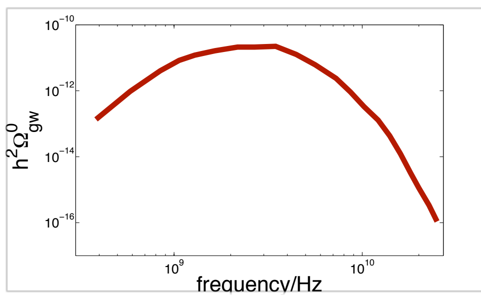
Oscillons in a 3D lattice



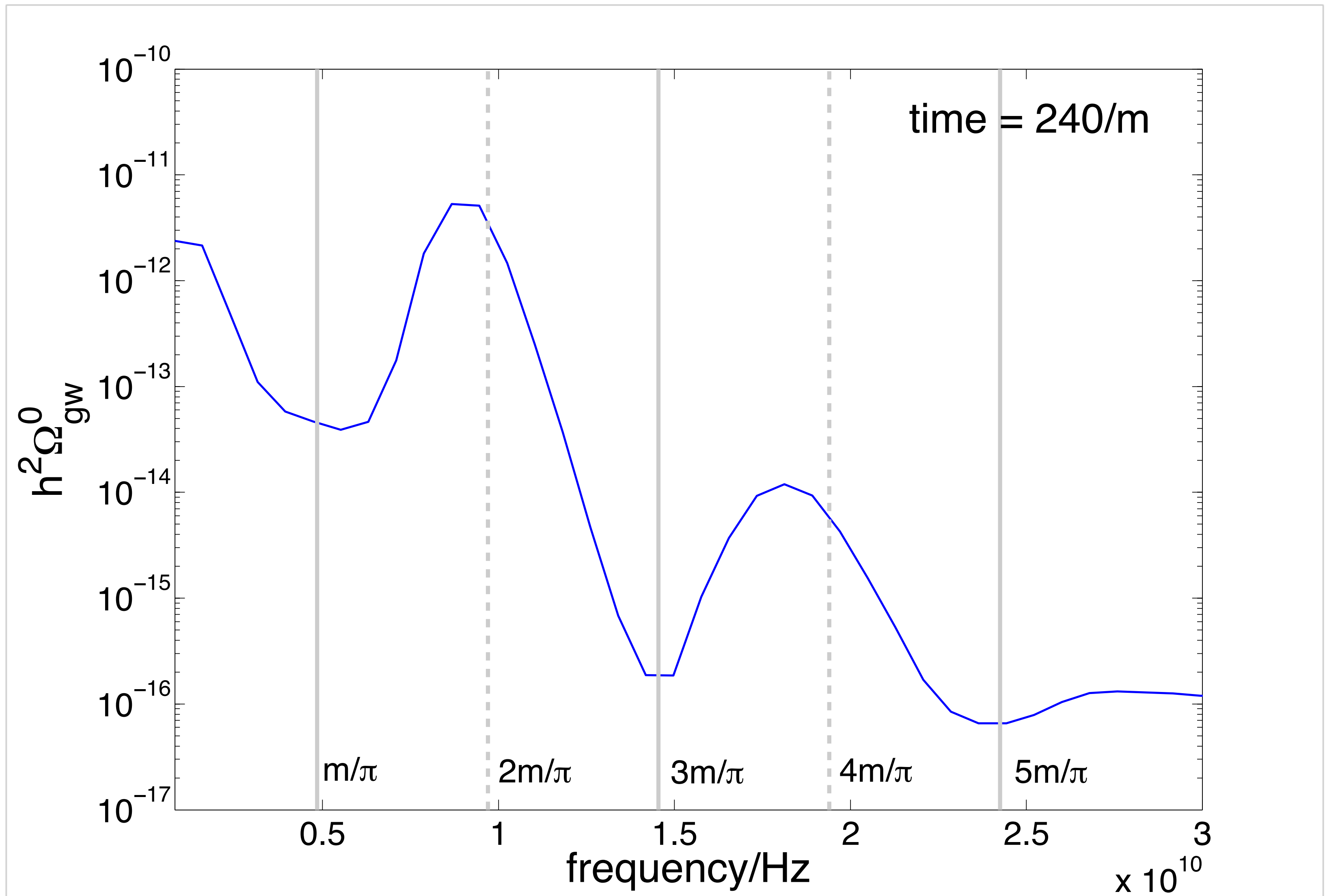

time = 80/m:240/m

SYZ, Copeland, Easther, Finkel, Mou & Saffin, 1304.6094

# Multi-peak GW power spectrum



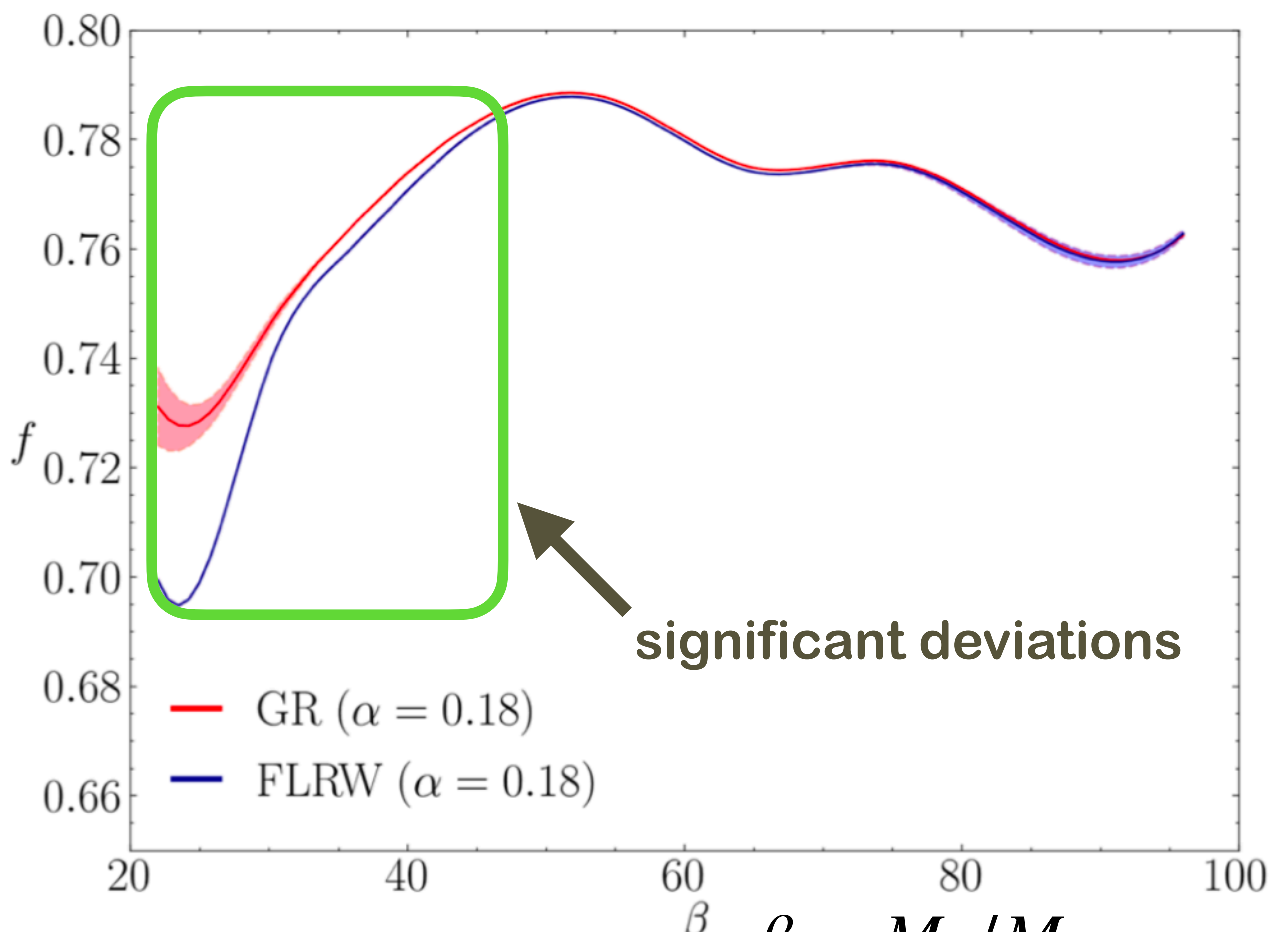

typical case




Troughs align with odd multiples of $m/\pi$ !

# Comparison with full GR simulations

Fixed FRW background **vs** Full GR (BSSN)

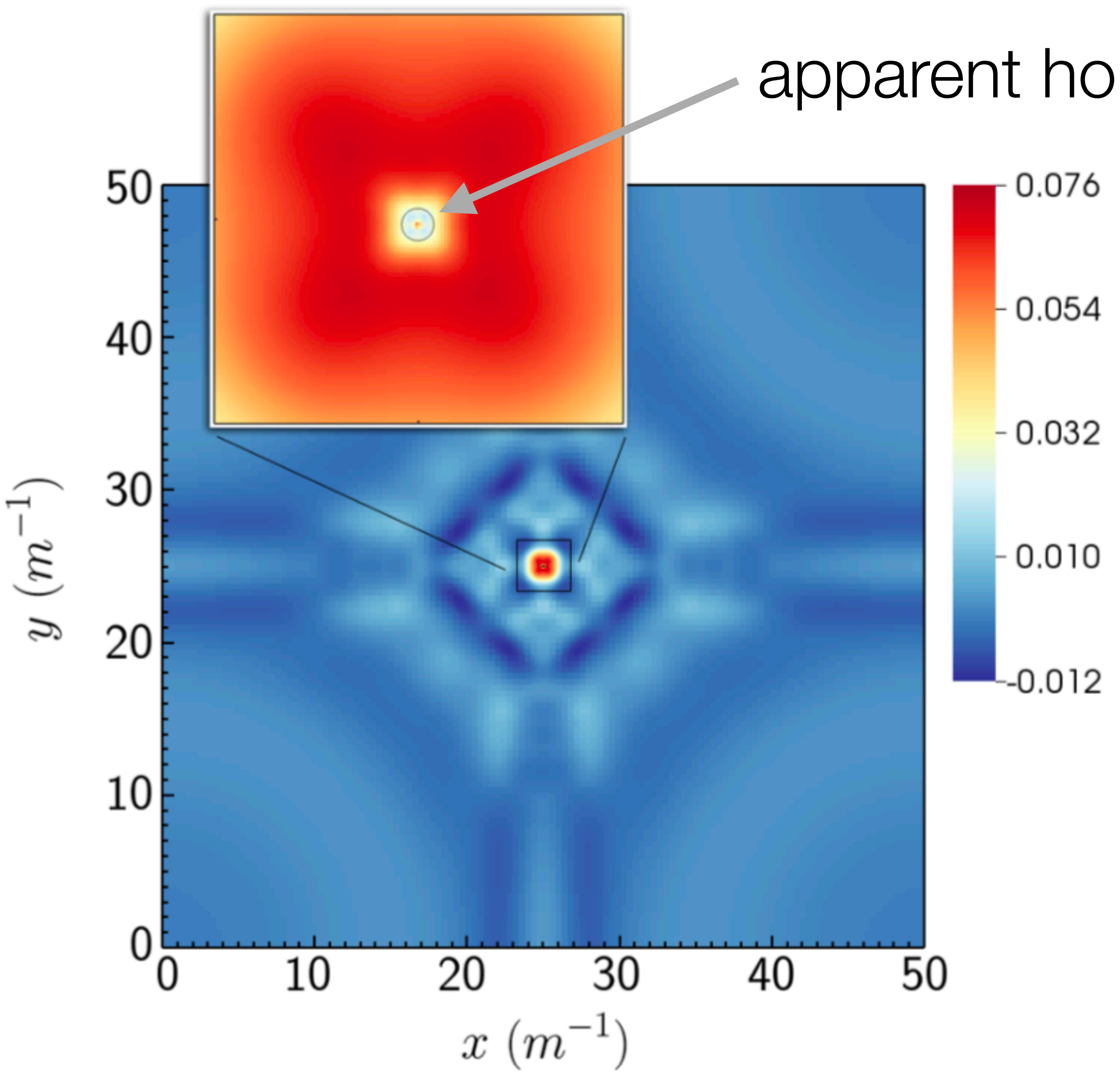


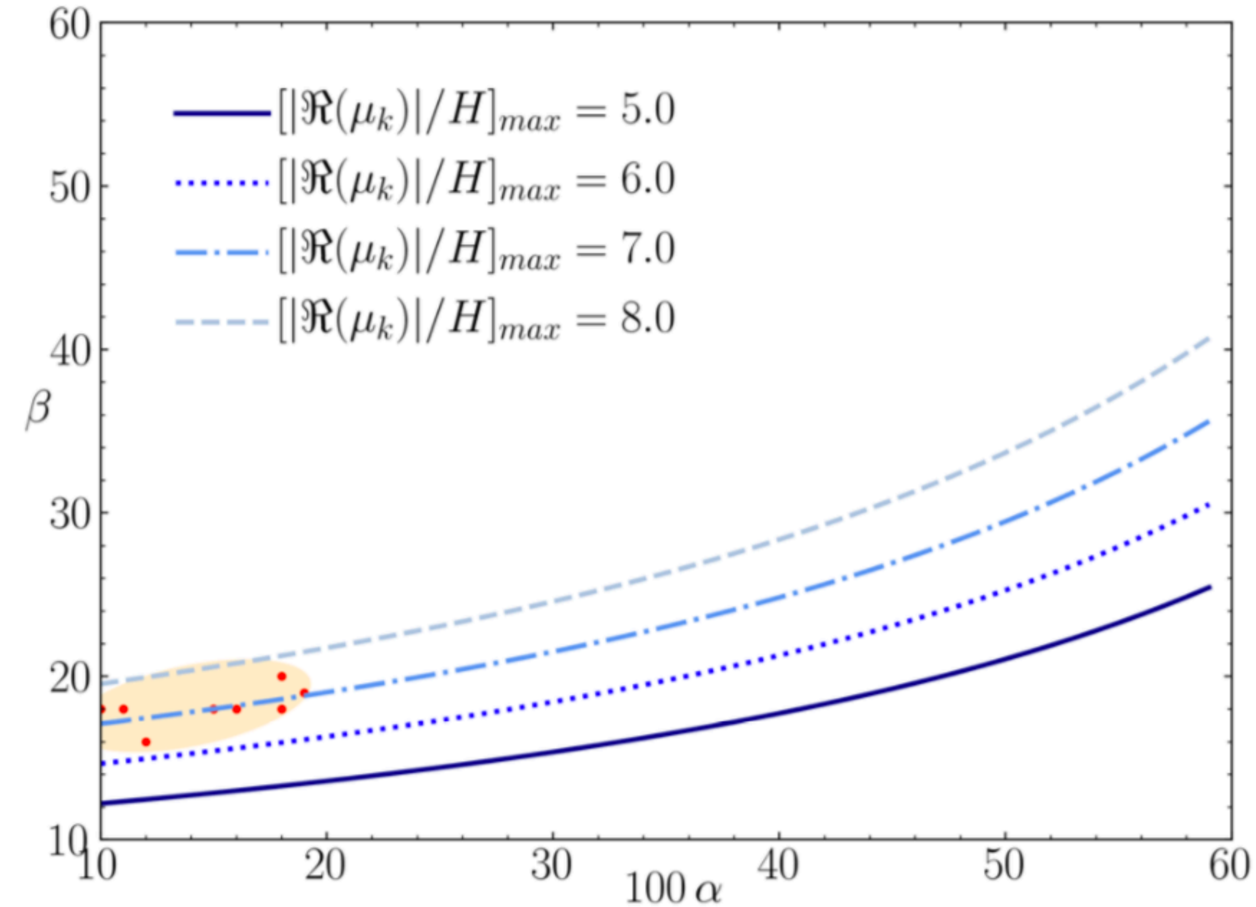

$\beta = M_P/M$

Significant deviations for higher inflation scales

Kou, Tian & SYZ, 1912.09658

# Oscillon/oscillaton collapses to black hole



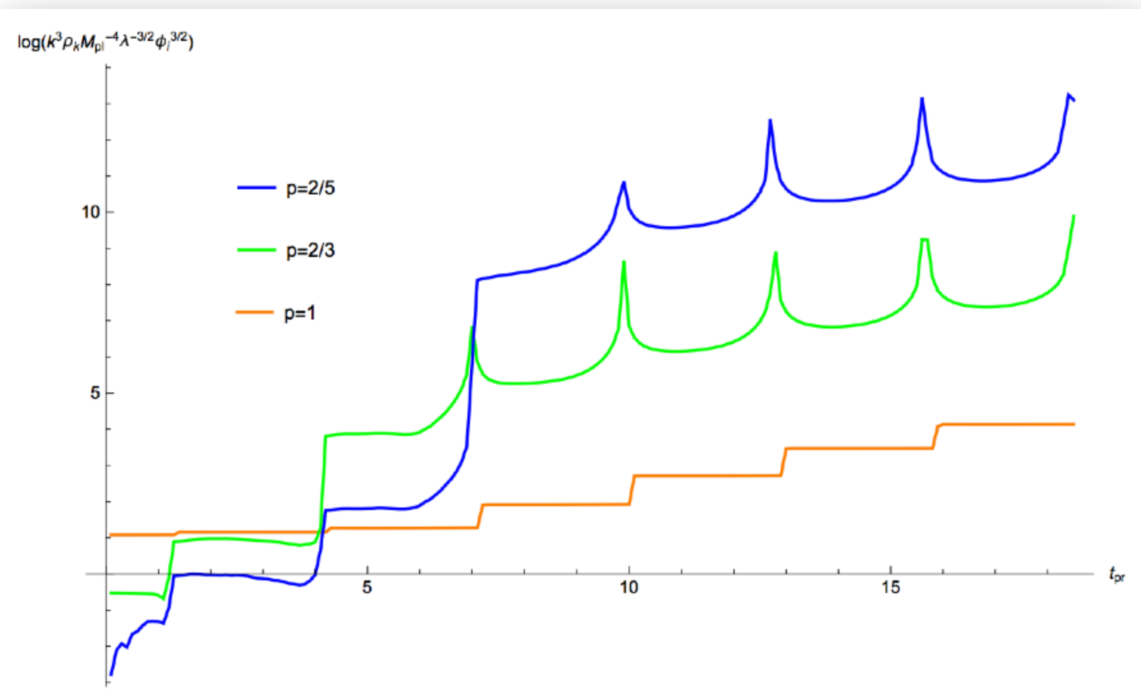

for generalized axion monodromy potential $V(\phi) = \frac{m^2}{2\alpha\beta^2}\left[\left(1+\beta^2\phi^2\right)^\alpha - 1\right]$

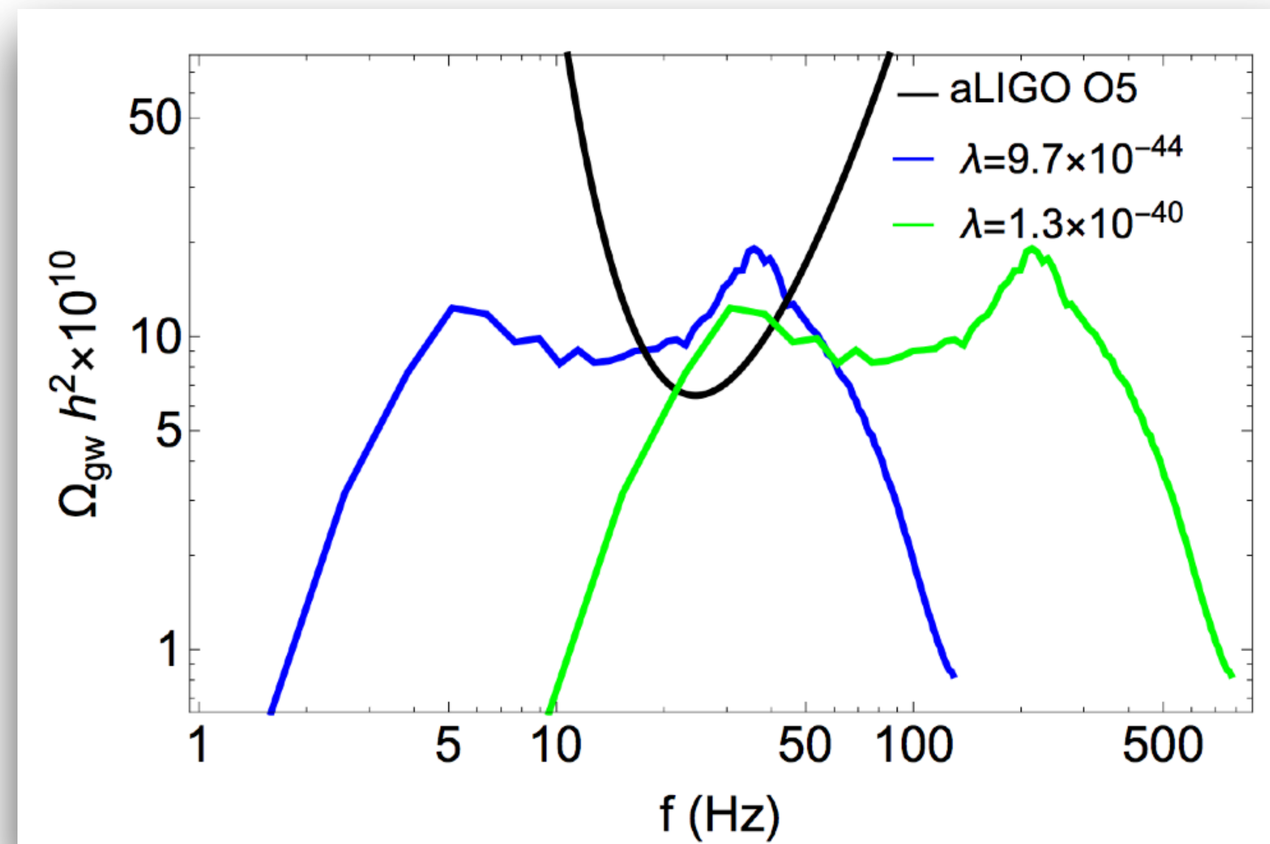
Kou, Tian & SYZ, 1912.09658

# Enhanced GWs from very shallow potentials

$$V(\phi) = \lambda M_{\rm pl}^{4-p} |\phi|^p, \quad p < 1$$

Liu, Guo, Cai & Shiu, PRL, 1707.09841



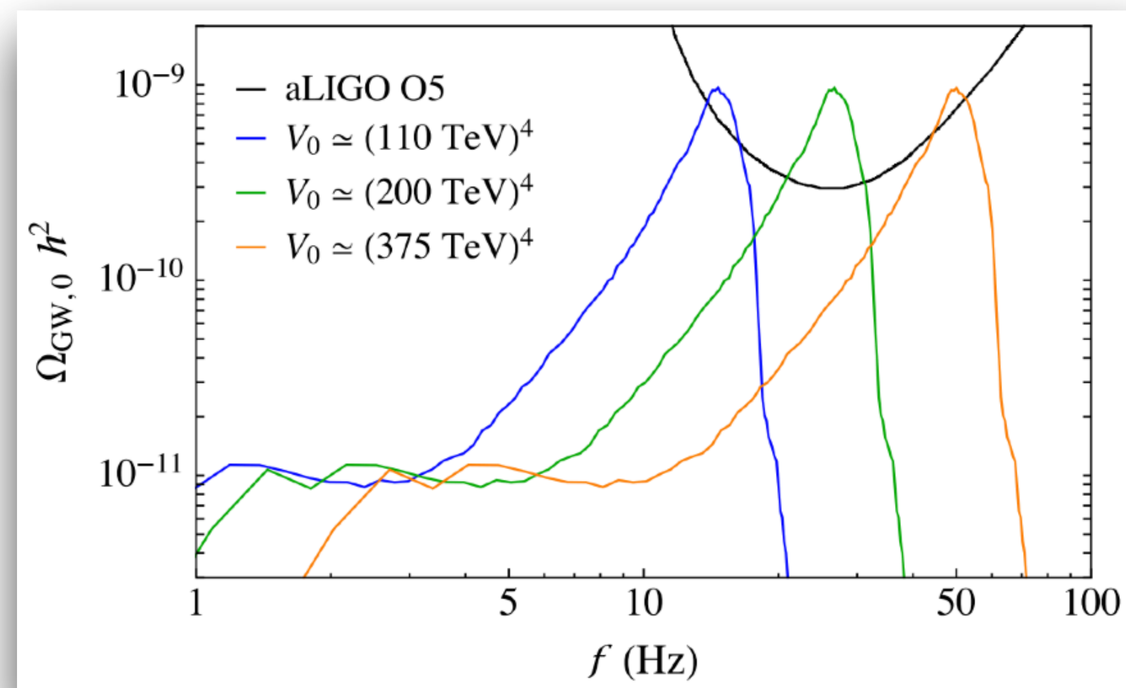



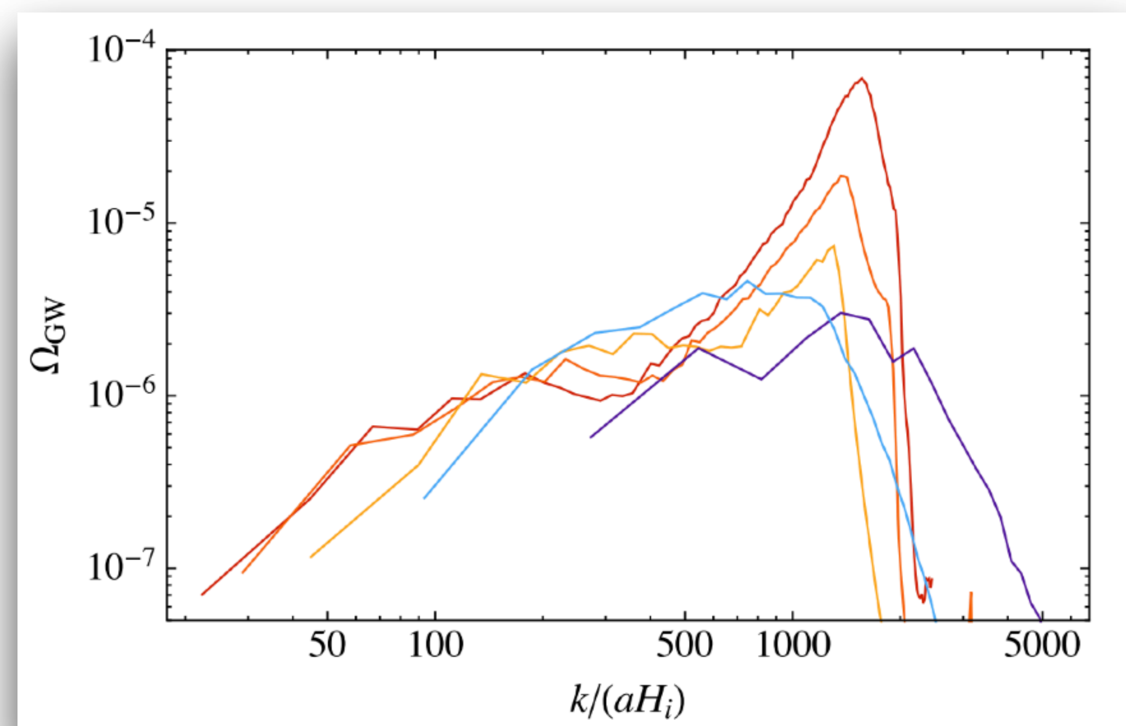
# GW enhancement from asymmetric potential?

**Gravitational Waves from Oscillons after Inflation**

Stefan Antusch, Francesco Cefalà, and Stefano Orani
Phys. Rev. Lett. **118**, 011303 – Published 6 January 2017


$$V(\phi) = V_0\left(1 - \frac{\phi^p}{v^p}\right)^2,$$




**final scale factor** $a(t) \sim 15$**!**

no rigorous convergence study

# A better simulation

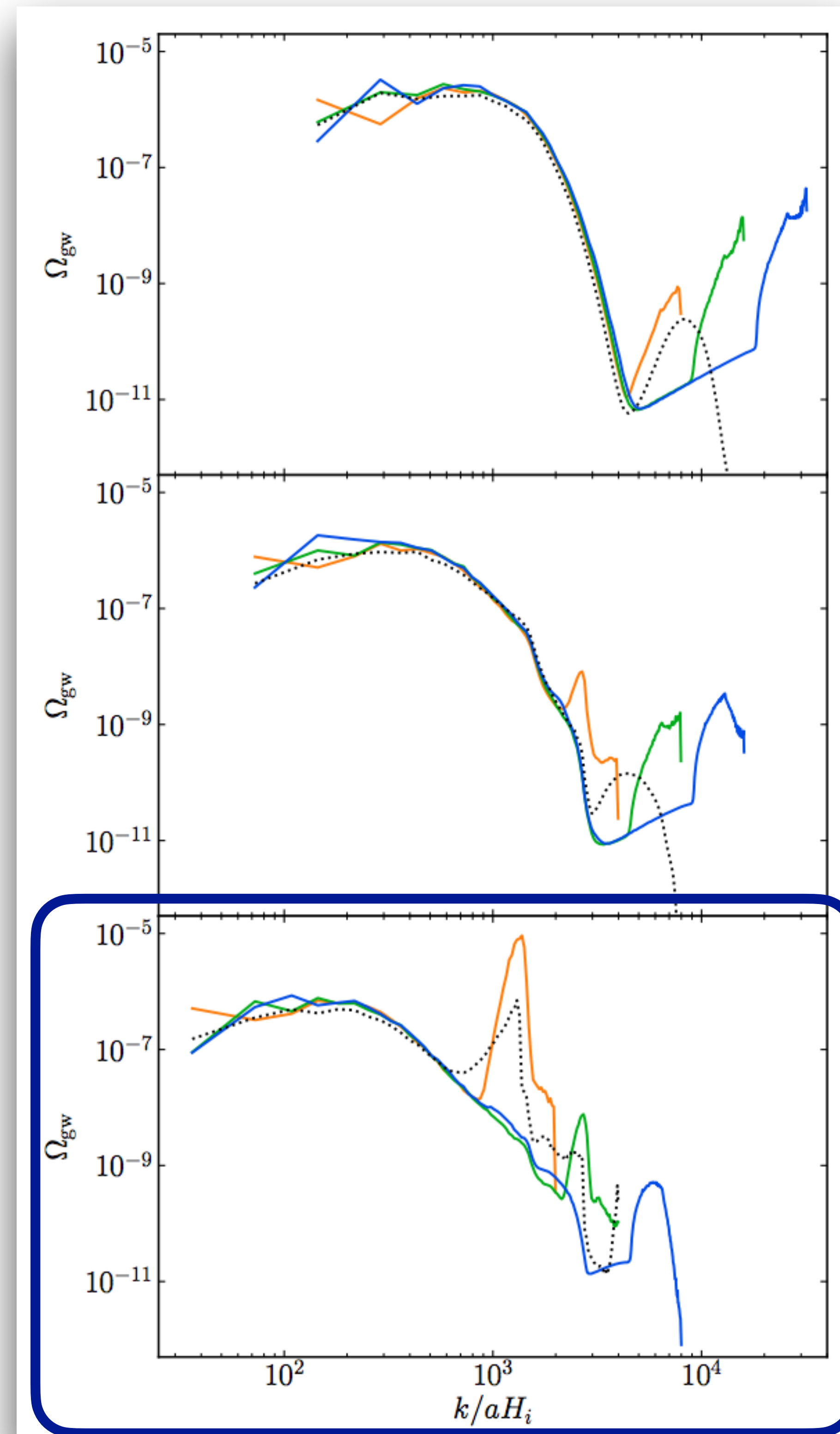
Amin, Braden, Copeland, Giblin, Solorio, Weiner & SYZ, 1803.08047

Pseudo-spectral method
GPU acceleration

**The artificial peak emerges around:**

$$a(t)dx \gtrsim m_\phi^{-1}$$

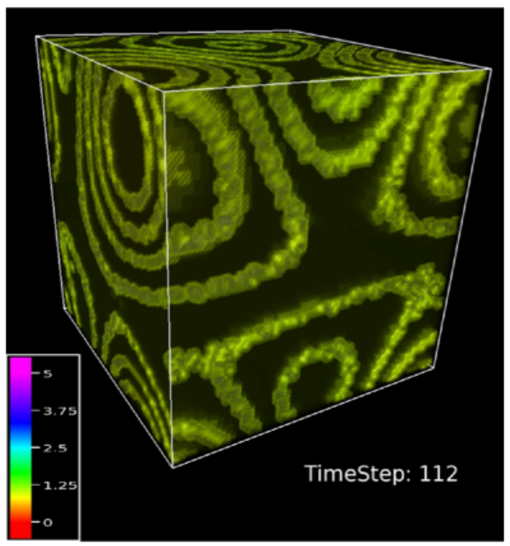
lattice spacing $\gtrsim$ Compton wavelength



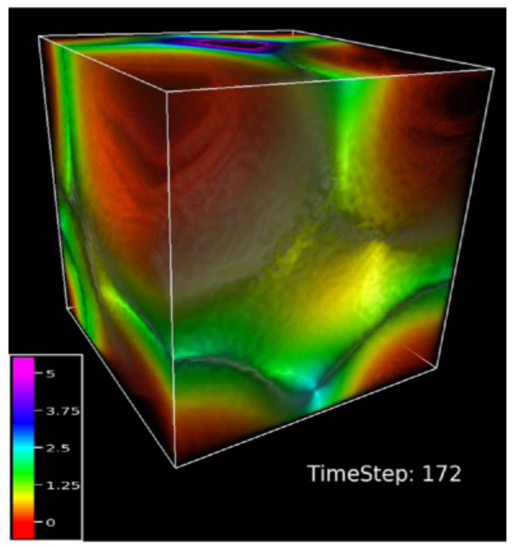
Charge-Swapping Q-balls in the early universe

# Q-balls from Affleck-Dine condensate

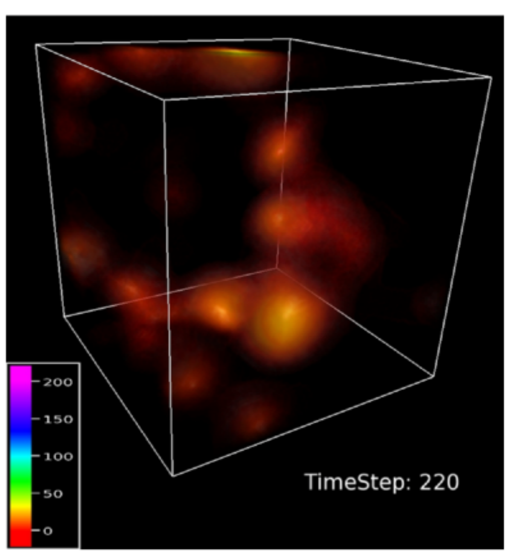
Potential $V(\Phi) = m_{\frac{3}{2}}^2 |\Phi|^2 \left(1 + K \ln \frac{|\Phi|^2}{M_P^2}\right) + A m_{\frac{3}{2}} \left(\frac{\Phi^d}{d M_P^{d-3}} + h.c.\right) + \frac{|\Phi|^{2d-2}}{M_P^{2d-6}}$



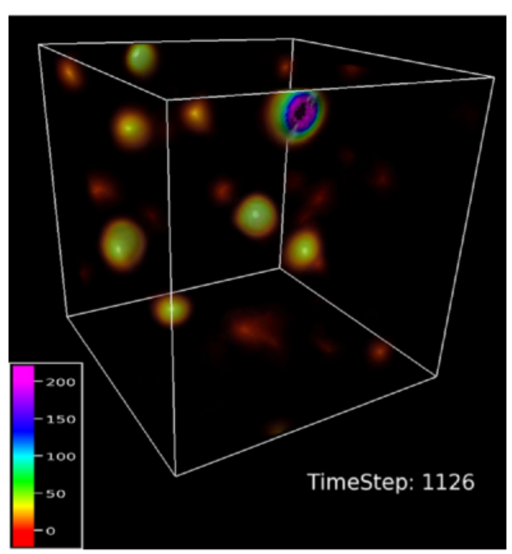
(a) Linear perturbation



(b) Fragmentation



(c) Emerging Q-balls



(d) Properly formed Q-balls

gravity mediated SUSY breaking

SUSY breaking scale: $m_{\frac{3}{2}} \gtrsim \mathcal{O}(1TeV)$

$$K \sim -\frac{\alpha_s m_{\frac{1}{2}}^2}{8\pi m_{\tilde{l}}^2} \sim (-0.1, -0.01)$$

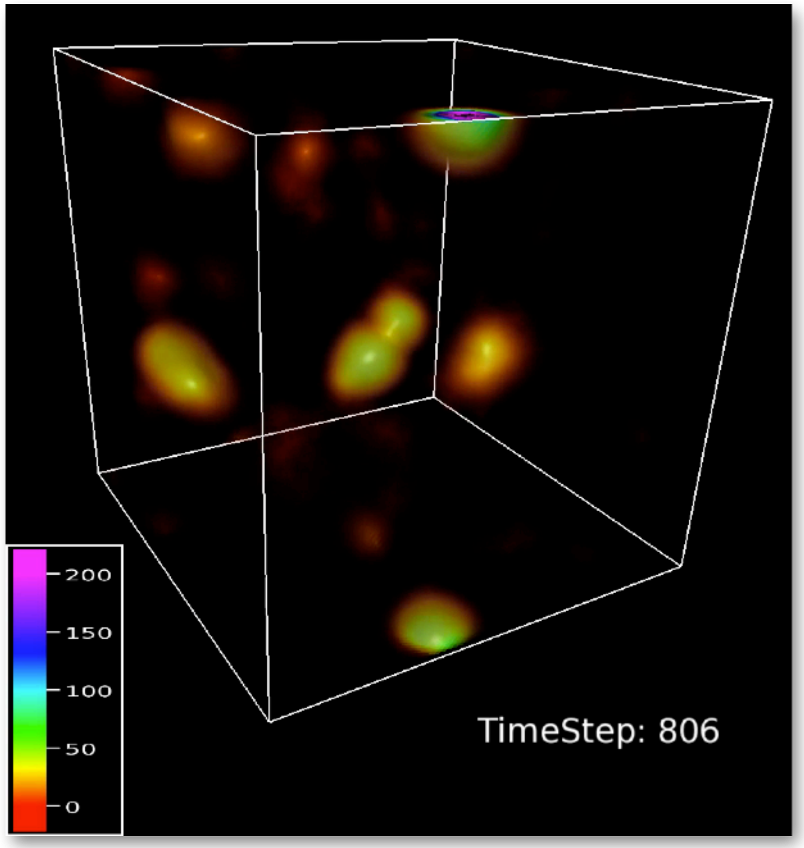
SYZ, 1501.01217

- Many flat directions in MSSM
- AD condensate formed during inflation
- After inflation, when $m \sim H$, AD condensate fragments to Q-balls
- At late time, AD field gets U(1) symmetry

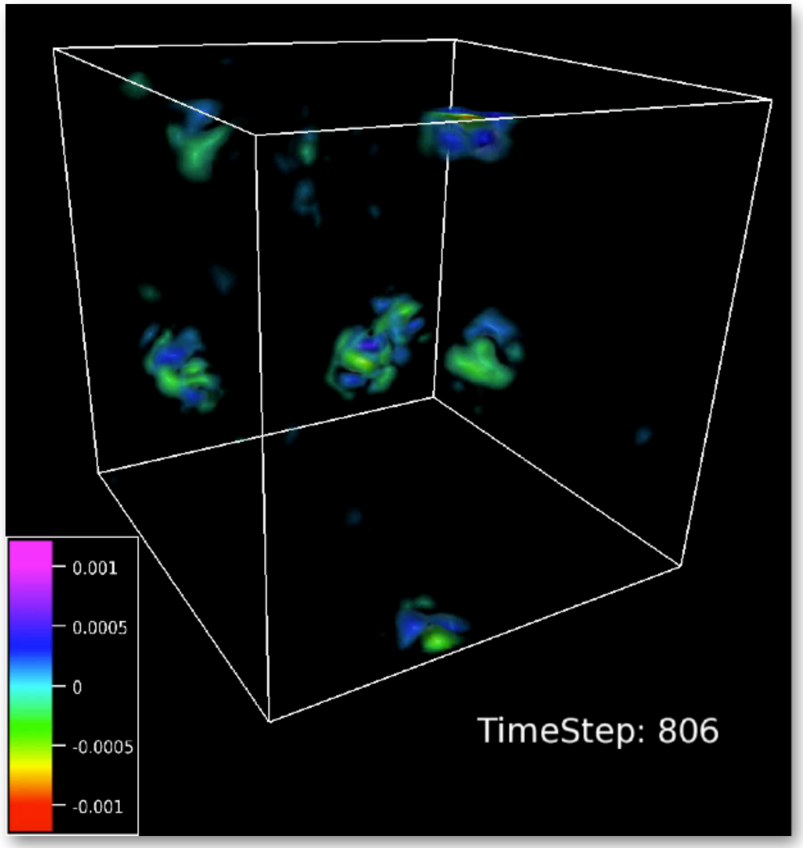
# CSQs from Affleck-Dine condensate

SYZ, 1501.01217

Energy density



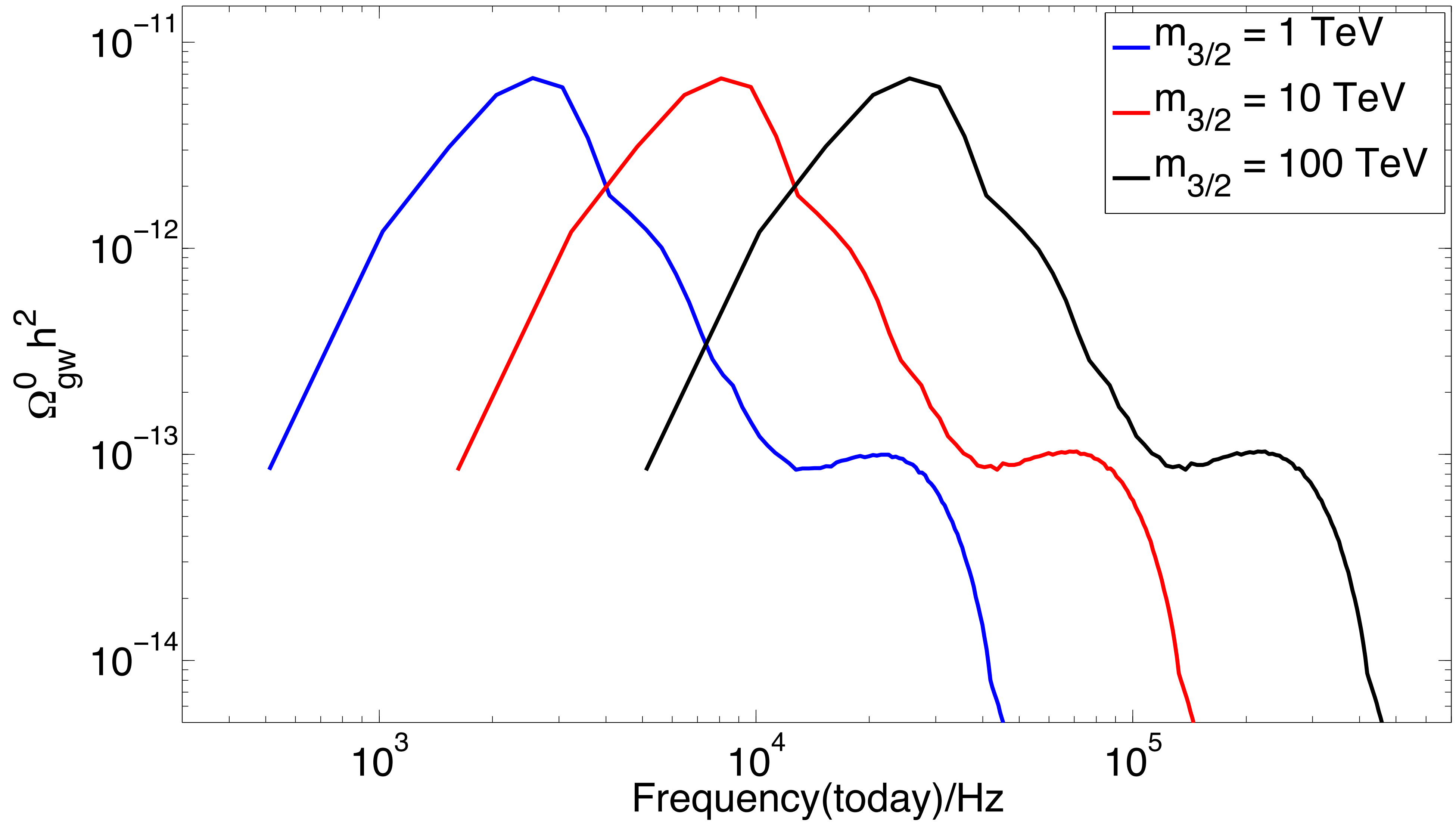

Charge density




CSQs with many multipoles

# Peaks of GW power spectra

SYZ, 1501.01217




Peaks of GWs are correlated with the SUSY breaking scale!

# Summary

- Q-balls: time-dependent non-topological solitons
- Oscillons: like Q-balls but quasi-stable

  shallow potentials lead to Q-balls and oscillons
- Charge-Swapping Q-balls:

  generic existence of composite Q-balls
- Formation of oscillons and Q-balls in the early universe
- Multi-peak stochastic GWs from oscillons and Q-balls
- Oscillons can collapse to BHs

Thank you!