Probing the Supersymmetric Grand Unified Theories at the Future Proton-Proton Colliders and Hyper-Kamiokande Experiment

Tianjun Li

Institute of Theoretical Physics, Chinese Academy of Sciences

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Waqas Ahmed, TL, Shabbar Raza and Fang-Zhou Xu, arXiv:2007.15059 [hep-ph]; in preparation.

The Conference Announcement

The XXVIII International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2021), August 23-28, 2021, Beijing, P. R. China

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Motivation for New Physics beyong the Standard Model

- The convincing evidence

  Dark energy; dark matter; neutrino masses and mixing; baryon asymmetry; inflation; ...

- Fine-tuning problems

  Cosmological constant problem; gauge hierarchy problem; strong CP problem; SM fermion masses and mixings; ...

- Aesthetic problems

  Interaction and fermion unification; gauge coupling unification; charge quantization; too many parameters; ...

- The electroweak vacuum stability problem

  The stability problem can be easily solved in the new physics models.

- New Physics beyond the SM!
- Question: why do we still believe in supersymmetry?

Motivations for Supersymmetry

- Generalization of the Poincare Algebra; The most general symmetry of the S-matrix; The local supersymmetry includes gravity naturally, *i.e.*, supergravity, etc.
- Supersymmetry provides a natural solution to the gauge hiearchy problem.
- Supersymmetry partially solves the cosmological constant problem: $M_{\rm Pl} \to M_{SUSY}$.
- Supersymmetry is a bridge between the promising low energy phenomenology and the high-energy fundamental physics such as the Grand Unified Theory (GUT) and string theory.

The Supersymmetric Standard Models (SSMs)

- Solving the gauge hierarchy problem
- Gauge coupling unification
- Radiatively electroweak symmetry breaking
- Natural dark matter candidates
- Electroweak baryogenesis
- Electroweak precision: R parity
- μ problem in the MSSM: $\mu H_d H_u$.

The Grand Unified Theories: $SU(5)$ and $SO(10)$

- Unification of the gauge interactions, and unifications of the SM fermions
- Charge quantization
- Gauge coupling unification in the MSSM, and Yukawa coupling unification
- Radiative electroweak symmetry breaking due to the large top quark Yukawa coupling
- Weak mixing angle at weak scale M_Z
- Neutrino masses and mixings by seesaw mechanism
- Prediction: dimension-six proton decay via heavy gauge boson exchange.

Problems

- Gauge symmetry breaking
- Doublet-triplet splitting problem
- Proton decay problem
- Fermion mass problem: $m_e/m_\mu = m_d/m_s$

String Models

- Calabi-Yau compactification of heterotic string theory
- Orbifold compactification of heterotic string theory
- D-brane models on Type II orientifolds
- Free fermionic string model builing
- $\mathcal{F}$-Theory Model Building

Supersymmetry is a bridge between the promising low energy phenomenology and high-energy fundamental physics.

Grand Particle Physics Paradigm

$$\text{String Theory} \rightarrow \text{String Models} \rightarrow \text{SUSY GUTs} \rightarrow \text{SSMs} \rightarrow \text{SM}$$

Can we probe this grand particle physics paradiagm via the future pp colliders and other experiments? If yes, what is the center-of-mass energy needed?

We can probe the SUSY GUTs at the future experiments!

Scientific Goal: If we propose a future pp collider, what is the center-of-mass energy needed to probe the supersymmetry?

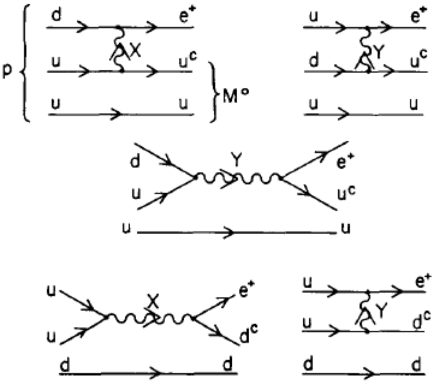
The Predictions of the SUSY GUTs: Proton Decays

- The dimension-six proton decay via superheavy $(X_\mu,\ Y_\mu)$ gauge boson exchanges

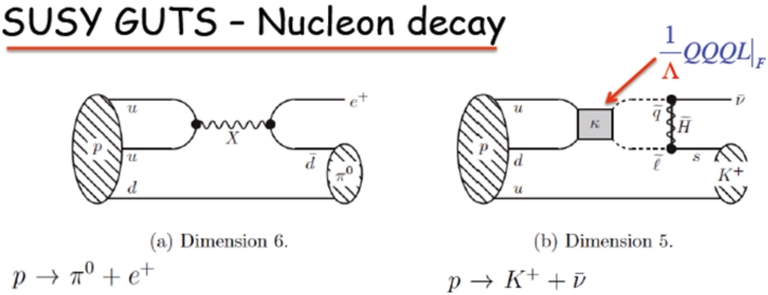
$$SU(5) \ = \ \begin{pmatrix} SU(3)_C & (\overline{X}_\mu, \overline{Y}_\mu) \\ (X_\mu, Y_\mu) & SU(2)_L \end{pmatrix} .$$

- The dimension-five proton decays via colored Higgsino exchanges in the supersymmetric GUTs.
- The operators for dimension-four proton decays from $U_i^c D_j^c D_k^c + Q_i L_j D_k^c + E_i^c L_j L_k$ are forbidden by Z_2 R parity.

The Dimension-Six Proton Decay via $(X_\mu,\ Y_\mu)$ Exchanges



The Dimension-Five Proton Decay



Proton Decays [1]

- The current bounds from Super-Kamiokande (SK)

$$\tau_{p\to e^+\pi^0} \geq 2.4 \times 10^{34}~\mathrm{yrs}~, \quad \tau_{p\to\bar{\nu}K^+} \geq 5.9 \times 10^{33}~\mathrm{yrs}~.$$

- The expected bounds from Hyper-Kamiokande (HK)

$$\tau_{p\to e^+\pi^0} \geq 1.0 \times 10^{35}~\mathrm{yrs}~, \quad \tau_{p\to\bar{\nu}K^+} \geq 2.5 \times 10^{34}~\mathrm{yrs}~.$$

[1] K. Abe *et al.* [Super-Kamiokande], Phys. Rev. D **95**, no.1, 012004 (2017) [arXiv:1610.03597 [hep-ex]]; M. Yokoyama [Hyper-Kamiokande Proto Collaboration], arXiv:1705.00306 [hep-ex]; K. Abe *et al.* [Hyper-Kamiokande], [arXiv:1805.04163 [physics.ins-det]]; A. Takenaka *et al.* [Super-Kamiokande], Phys. Rev. D **102**, 112011 (2020) [arXiv:2010.16098 [hep-ex]].

Higgs Boson Mass in the MSSM

- The SM-like Higgs boson mass is around 125 GeV.
- The tree-level Higgs boson mass is smaller than M_Z.
- The Higgs boson mass is enhanced by the top quark/squark loop corrections.
- The maximal stop mixing is needed to relax the fine-tuning.

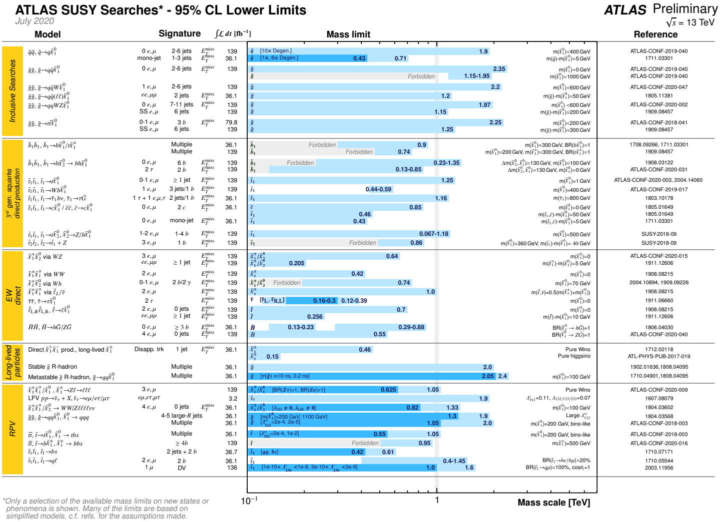
  Problem: the $SU(3)_C \times U(1)_{EM}$ symmetry breaking.
- Implication: the SM-like Higgs boson mass prefers the heavy sparticle spectrum.

The LHC Supersymmetry Search Contraints

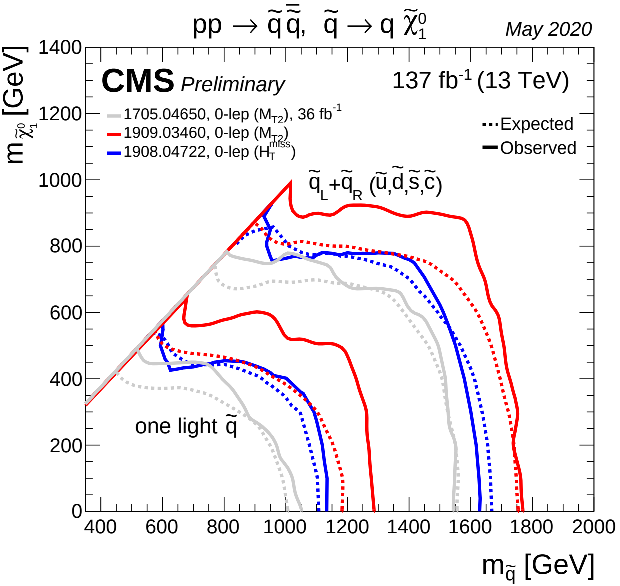
- The first two-generation squark mass low bounds are around 1.8 TeV.
- The gluino mass low bound is around 2.3 TeV.
- The stop and sbottom mass low bounds are around 1.2 and 1.3 TeV, respectively.

The SSMs are fine-tuned!!!

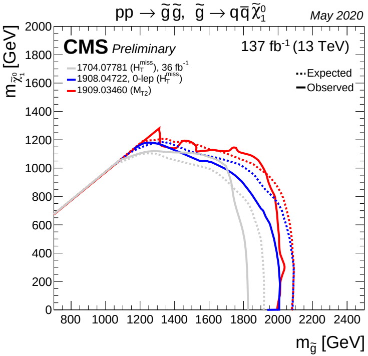
The ATLAS SUSY Searches

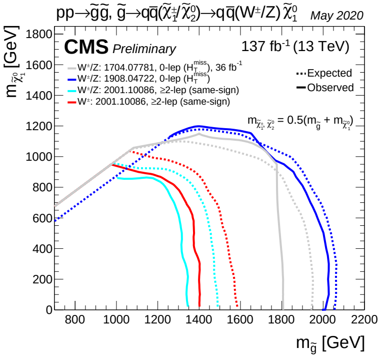


The CMS Squark Searches

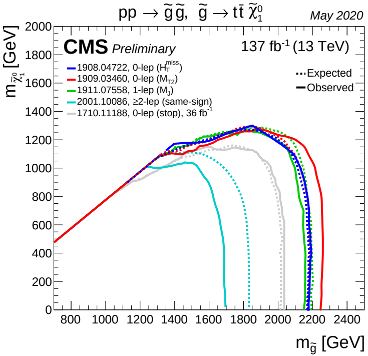


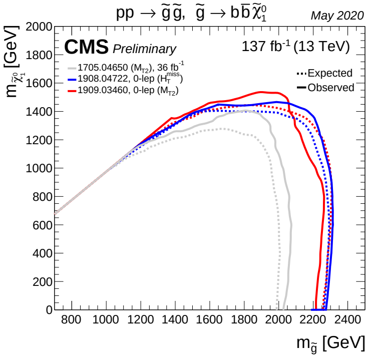
The CMS Gluino Searches



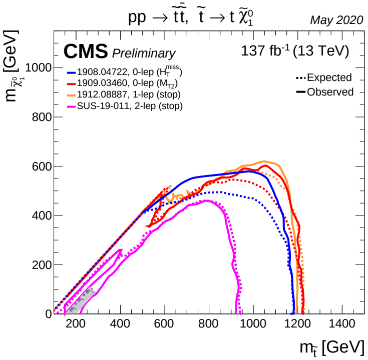


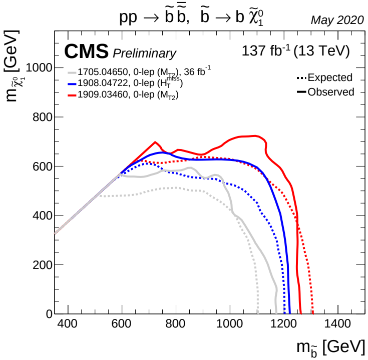
The CMS Gluino Searches





The CMS Stop and Sbottom Searches





- The wrong impression is that supersymmetry was excluded at the LHC?
- Can we rule out supersymmetry at the LHC, VLHC, FCC_{hh} and SppC?
  No! No!! No!!!
- Point: supersymmetry breaking soft mass scale can be pushed to be much higher than 1 TeV, while gauge coupling unification can still be preserved due to the logarithmic RGE running and threshold corrections around the GUT scale.
- Conclusion: supersymmetry will defintely not die in the near future!!!

Natural Supersymmetry

The interesting question: can we rule out or discover the natural supersymmetry at the $\mathrm{FCC}_{\mathrm{hh}}$ and SppC? Or can we solve the supersymmetry electroweak fine-tuning problem naturally?

The EENZ-BG Fine-Tuning Measure [2]

- Because we study the SUSY GUTs, we shall consider the EENZ-BG fine-tuning measure.
- Definition: the quantitative measure $\Delta_{\rm FT}^{\rm EENZ-BG}$ for fine-tuning is the maximum of the logarithmic derivative of M_Z with respect to all the fundamental parameters a_i at the GUT scale

$$\Delta_{\rm FT}^{\rm EENZ-BG} ~=~ {\rm Max}\{\Delta_i^{\rm GUT}\}~, \quad \Delta_i^{\rm GUT} ~=~ \left|\frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})}\right|~.$$

[2] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1**, 57 (1986); R. Barbieri and G. F. Giudice, Nucl. Phys. B **306**, 63 (1988).

Question: Super-Natural Supersymmetry

Can we propose a supersymmetry scenario whose the EENZ-BG fine-tuning measure is automatically 1 or order 1 ($\mathcal{O}(1)$)?

Fundamental physics principles: simplicity and naturalness.

Super-Natural Supersymmetry [3]

- Fine-Tuning Definition:

$$\Delta_{\rm FT} ~=~ {\rm Max}\{\Delta_i^{\rm GUT}\}~, \quad \Delta_i^{\rm GUT} ~=~ \left|\frac{\partial {\rm ln}(M_Z)}{\partial {\rm ln}(a_i^{\rm GUT})}\right| ~.$$

- Natural Solution:

$$M_Z^n ~=~ f_n \left(\frac{M_Z}{M_*}\right)~M_*^n ~.$$

$$\frac{\partial {\rm ln}(M_Z^n)}{\partial {\rm ln}(M_*^n)} ~\simeq~ \frac{M_*^n}{M_Z^n}\frac{\partial M_Z^n}{\partial M_*^n} ~\simeq~ \frac{1}{f_n} f_n \simeq \mathcal{O}(1) ~.$$

- For no-scale supergravity and M-theory on S^1/Z_2, we have $M_* = M_{1/2}$ and $M_* = M_{3/2}$, respectively.

[3] T. Leggett, T. Li, J. A. Maxin, D. V. Nanopoulos and J. W. Walker, arXiv:1403.3099 [hep-ph]; Phys. Lett. B **740**, 66 (2015) [arXiv:1408.4459 [hep-ph]]; G. Du, T. Li, D. V. Nanopoulos and S. Raza, Phys. Rev. D **92**, no. 2, 025038 (2015) [arXiv:1502.06893 [hep-ph]]; T. Li, S. Raza and X. C. Wang, Phys. Rev. D **93**, no. 11, 115014 (2016) [arXiv:1510.06851 [hep-ph]].

Supernatural Supersymmetry

- The $\mathcal{F}-SU(5)$ [4] and the MSSM [5] with no-scale supergravity [6] and Giudice-Masiero Mechanism [7].

  $\mathcal{F}-SU(5)$: the flipped $SU(5)$ model with vector-like particles and string-scale gauge coupling unification from F-theory model building.

  No-scale Supergravity: $M_{1/2} \neq 0$, and $M_0 = A_0 = B_\mu = 0$.

  Giudice-Masiero Mechanism: generating μ term from supersymmetry breaking effect, *i.e.*, $\mu \propto M_{1/2}$.

  Prolem: we cannot predict the exact mass ratio $\mu/M_{1/2}$?

- The NMSSM [8] with supersymmetry breaking soft terms from M-theory on S^1/Z_2 [9].

  The NMSSM: an singlet S, and a discrete Z_3 symmetry.

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[4] T. Leggett, T. Li, J. A. Maxin, D. V. Nanopoulos and J. W. Walker, arXiv:1403.3099 [hep-ph]; Phys. Lett. B **740**, 66 (2015) [arXiv:1408.4459 [hep-ph]].

[5] G. Du, T. Li, D. V. Nanopoulos and S. Raza, Phys. Rev. D **92**, no. 2, 025038 (2015) [arXiv:1502.06893 [hep-ph]].

[6] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, Phys. Lett. B **133**, 61 (1983); A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145**, 1 (1987).

[7] G. F. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988).

[8] T. Li, S. Raza and X. C. Wang, Phys. Rev. D **93**, no. 11, 115014 (2016) [arXiv:1510.06851 [hep-ph]].

[9] T. Li, Phys. Rev. D **59**, 107902 (1999) [hep-ph/9804243].

The Super-Natural Supersymmetry

- The Kähler potential and superpotential can be calculated in principle or at least inspired from a fundamental theory such as string theory with suitable compactifications. In other words, one cannot add arbitrary high-dimensional terms in the Kähler potential and superpotential.
- There is one and only one chiral superfield or modulus whose F-term breaks supersymmetry. And all the supersymmetry breaking soft terms are obtained from the above Kähler potential and superpotential.
- All the other mass parameters, if there exist such as the μ term in the MSSM, must arise from supersymmetry breaking.

The Effective Super-Natural Supersymmetry

There exist the correlations among the GUT-scale parameters with large fine-tuning measures [10].

[10] R. Ding, T. Li, F. Staub and B. Zhu, Phys. Rev. D **93**, no. 9, 095028 (2016) doi:10.1103/PhysRevD.93.095028 [arXiv:1510.01328 [hep-ph]].

The Interesting Questions?

- The SUSY electroweak fine-tuning problem can be solved by the super-natural supersymmetry.
- Can we probe supersymmetry at the future pp colliders? No?
- Can we probe the supersymmetric GUTs at the future pp colliders? Yes!!! [11]

[11] W. Ahmed, T. Li, S. Raza and F. Z. Xu, [arXiv:2007.15059 [hep-ph]].

The Exceptions in String Models

- ## The D-brane models: the Pati-Salam Model

In one of our models, the gauge coupling unification for $SU(4)_C \times SU(2)_L \times SU(2)_R$ can be achieved, while gaugino mass unification cannot.

- ## The F-theory models

Gauge coupling unification can be realized in the $SO(10)$ model with $U(1)_X$ flux while not necessary in the $SU(5)$ model with $U(1)_Y$ flux and the $SO(10)$ model with $U(1)_{B-L}$ flux.

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_3} = k\left(\frac{1}{\alpha_1} - \frac{1}{\alpha_3}\right) ,$$

$$\frac{M_2}{\alpha_2} - \frac{M_3}{\alpha_3} = k\left(\frac{M_1}{\alpha_1} - \frac{M_3}{\alpha_3}\right) ,$$

where $k = 5/3$.

The Proof

- Grand desert hypothesis: no new physics between the sparticle mass scale and the GUT scale.
- For the GUTs with the GUT scale $M_{GUT} \leq 1.0 \times 10^{16}$ GeV, we can probe the dimension-six proton decay via heavy gauge boson exchange at the Hyper-Kamiokande experiment.
- For the GUTs with $M_{GUT} \geq 1.0 \times 10^{16}$ GeV, we can probe the gluino and/or squarks at the future pp colliders.
- Providing the concrete "Scienctific Goal" for the future pp colliders and Hyper-Kamiokande experiment.

The GUTs with $M_{GUT} \leq 1.0 \times 10^{16}$ GeV

- The proton lifetime from the dimension-six proton decay $p \to e^+\pi^0$ via heavy gauge boson exchange is

$$\tau_p \simeq 1.0 \times 10^{34} \times \left(\frac{2.5}{A_R}\right)^2 \times \left(\frac{0.04}{\alpha_{\rm GUT}}\right)^2 \times \left(\frac{M_{\rm GUT}}{1.0 \times 10^{16}~{\rm GeV}}\right)^4 ~{\rm years}~.$$

- The current lower limit from the Super-Kamiokande experiment gives $\tau_p > 2.4 \times 10^{34}$ years [12], and thus we obtain $M_{\rm GUT} \geq 1.0 \times 10^{16}~{\rm GeV}$.
- At the future Hyper-Kamiokande experiment, we can probe the proton lifetime at least above 1.0×10^{35} years [13]. Therefore, the GUTs with $M_{\rm GUT} \leq 1.0 \times 10^{16}~{\rm GeV}$ is definitely within its reach.

[12] K. Abe *et al.* [Super-Kamiokande], Phys. Rev. D **95**, no.1, 012004 (2017) [arXiv:1610.03597 [hep-ex]]; A. Takenaka *et al.* [Super-Kamiokande], Phys. Rev. D **102**, 112011 (2020) [arXiv:2010.16098 [hep-ex]].

[13] K. Abe *et al.* [Hyper-Kamiokande], [arXiv:1805.04163 [physics.ins-det]].

Comment

- In fact, if we did not observe the dimension-six proton decay from the Super-Kamiokande experiment and future Hyper-Kamiokande experiment, we obtain $M_{X_\mu/Y_\mu} \leq 1.0 \times 10^{16}$ GeV, not $M_{\rm GUT} \leq 1.0 \times 10^{16}$ GeV.
- There indeed exists some subtleties to define the GUT scale due to threshold corrections.
- Because $M_{X_\mu/Y_\mu} \leq M_{GUT}$, our study is not affected by these subtleties.

The supersymmetry breaking mediation mechanisms

- Gravity mediated supersymmetry breaking.
- Anomaly mediated supersymmetry breaking.
- Gauge mediated supersymmetry breaking.

The Supersymmetry Searches at the Future pp Colliders

- For the 100 TeV pp Colliders such as $\mathrm{FCC_{hh}}$ and SppC, gluino $\tilde{g}$ via heavy flavor decay, gluino via light flavor decay, and the first-two generation squarks $\tilde{q}$ can be discovered for their masses up to about 11 TeV, 17 TeV, and 14 TeV, respectively. If the gluino and first-two generation squark masses are similar, they can be probed up to 20 TeV.
- To discover the gluino $\tilde{g}$ via heavy flavor decay with mass around 15 TeV, we need the 160 TeV pp collider such as the VLHC.

The Phenomenological Constraints

- The SM-like Higgs boson mass $m_h \subset [123,\ 127]\ \mathrm{GeV}$, and gluino mass $m_{\tilde{g}} \geq 2.2\ \mathrm{TeV}$.
- The constraints from rare decay processes $B_s \to \mu^+\mu^-$, $b \to s\gamma$, and $B_u \to \tau\nu_\tau$.
- To be general, we do not require the relic abundance of the LSP neutralino to satisfy the Planck bound within 5σ $0.114 \leq \Omega_{\mathrm{CDM}} h^2 \leq 0.126$.

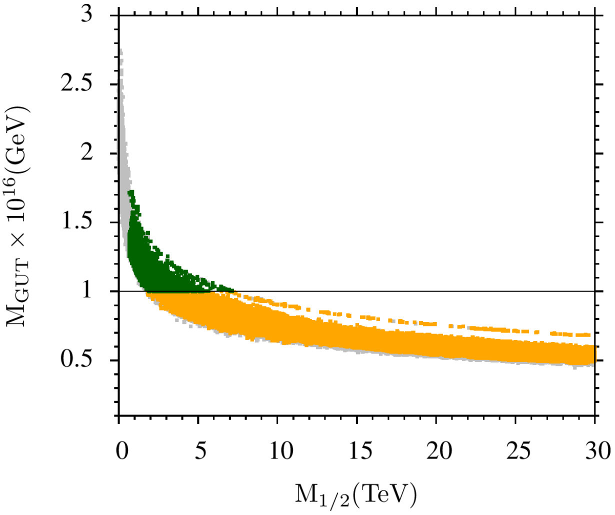
Gravity Mediated Supersymmetry Breaking

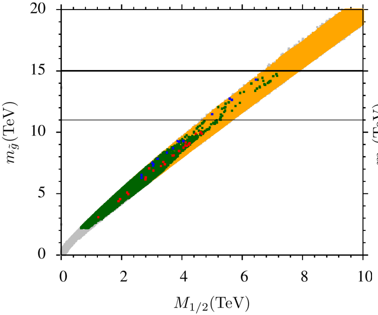
We perform the random scans for the following mSUGRA/CMSSM parameter space

$$
\begin{aligned}
0 \leq M_0 \ &\leq 90\,\mathrm{TeV}, \\
0 \leq M_{1/2} &\leq 30\,\mathrm{TeV}, \\
-3 \leq A_0/M_0 &\leq 3, \\
2 \leq \tan\beta \ &\leq 60
\end{aligned}
$$

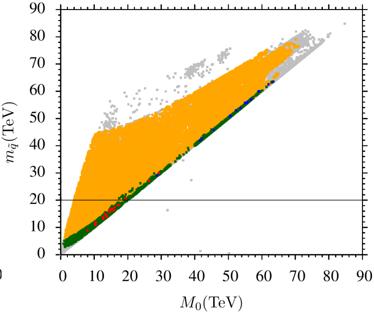
with $\mu > 0$ and $m_t = 173.2\,\mathrm{GeV}$.

Gravity Mediated Supersymmetry Breaking



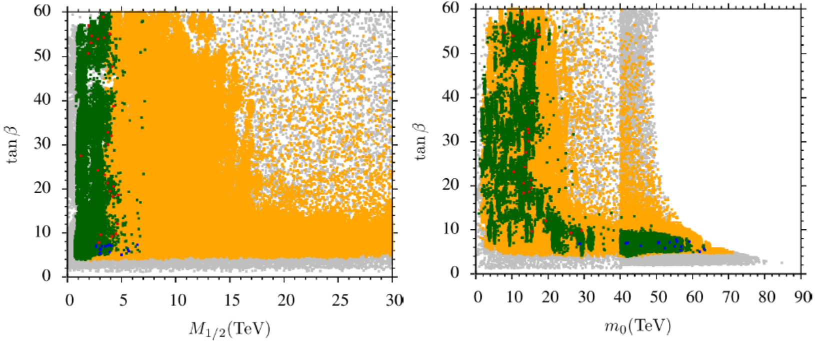


$m_{\tilde{g}}$(TeV)
$M_{1/2}$(TeV)
0
5
10
15
20
0
2
4
6
8
10



$m_{\tilde{q}}$(TeV)
M_0(TeV)
0
10
20
30
40
50
60
70
80
90

Gravity Mediated Supersymmetry Breaking



Gravity Mediated Supersymmetry Breaking

- The upper bound on the universal gaugino mass $M_{1/2}$ is about 7 TeV, and then the upper bound on gluino mass is 15 TeV.
- In our viable parameter space, the lightest squark is generically to be light stop, and thus we do have gluino via heavy flavor decay. To probe such gluino with mass up to 15 TeV, we find that the center-of-mass energy of the future pp collider needs to be about 160 TeV.
- In the viable parameter space with correct dark matter relic density and $\tan\beta > 7.5$, gluino mass is less than 10 TeV. Thus, we can probe it at the $\mathrm{FCC_{hh}}$ and SppC.

Anomaly Mediated Supersymmetry Breaking

We perform the random scans over the following parameter space of the minimal AMSB

$$
\begin{aligned}
1\,\mathrm{TeV} &\leq M_0 \leq 7.5\,\mathrm{TeV}, \\
100\,\mathrm{TeV} &\leq M_{3/2} \leq 3000\,\mathrm{TeV}, \\
2 &\leq \tan\beta \leq 60
\end{aligned}
$$

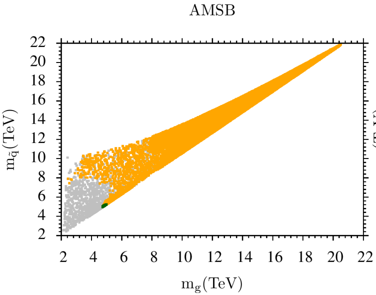
with $\mu > 0$ and $m_t = 173.2\,\mathrm{GeV}$.

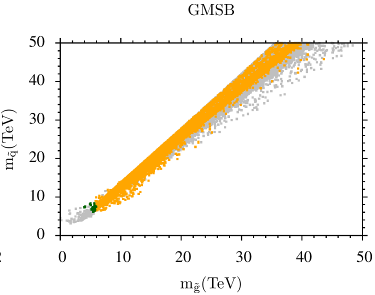
We perform random scans over the following minimal GMSB parameter space

$$
\begin{aligned}
5 \times 10^5 \,\mathrm{GeV} \ \leq \Lambda \ &\leq 10^{10} \,\mathrm{GeV}, \\
2 \times \Lambda \ \leq M_{mess} &\leq 10^{15} \,\mathrm{GeV}, \\
2 \leq \tan\beta \ &\leq 60
\end{aligned}
$$

with $\mu > 0$ and $m_t = 173.2 \,\mathrm{GeV}$.

Anomaly and Gauge Mediated Supersymmetry Breakings





Anomaly and Gauge Mediated Supersymmetry Breakings

- For anomaly mediation, the squark and gluino masses are less than 6 TeV.
- For anomaly mediation, the squark and gluino masses are less than 10 TeV and 8 TeV, respectively.
- The GUTs with anomaly and gauge mediated supersymmetry breakings are well within the reaches of the future 100 TeV pp colliders such as the $\mathrm{FCC_{hh}}$ and SppC.

Summary

- Supersymmetry is a bridge between the low energy phenomenology and high-energy fundamental physics, and thus is the promising new physics beyond the SM.
- Gauge coupling unification in the supersymmetric SM strongly implies the GUTs.
- With the grand desert hypothesis, we show that the supersymmetric GUTs can be probed at the future pp colliders and Hyper-Kamiokande experiment.
- The super-natural supersymmetry can definitely be probed at the $\mathrm{FCC_{hh}}$ and SppC.

Thank You Very Much for Your Attention!