Sterile Neutrino Oscillations with a Crossing-width Term

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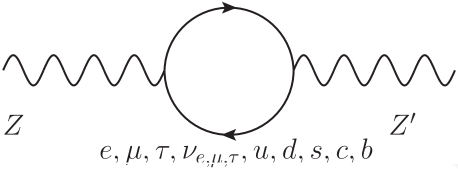
Contents

Problem 0, What is a “Particle”?

- For stable particles, “trajectories” in the detectors correspond to “asymptotic states”.
- For unstable particles, sometimes the decay products as well as the reconstructed “secondary vertiex” indicate the particles, which cannot be regarded as “asymptotic”.
- Sometimes, Breit-Wigner propagators are utilized to describe their contribution in the Feynmann Diagram.
- What if the Breit-Wigner resonances overlap? Can these two scenarios be compatible?

Problem 1, Crossing-width Term in the Fermionic Case

- In our JHEP05(2024)167, we discussed a nearly degenerate Z-Z' system. The two resonances overlap and interfere with each other.
- In this case "crossing-width term" arise and might be non-negligible. Such a term should also be resummed as well as the usual width terms.



Problem 1, Crossing-width Term in the Fermionic Case

$$\begin{pmatrix} m_{\hat{Z}'}^2 + ic_{\hat{Z}'\hat{Z}'} & \hat{g}'(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) & -\hat{g}(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) \\ \hat{g}'(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) & \frac{\hat{g}'^2}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) & -\frac{\hat{g}'\hat{g}}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) \\ -\hat{g}(\delta m^2 - \frac{ic_{\hat{Z}'\hat{Z}}}{\sqrt{\hat{g}'^2+\hat{g}^2}}) & -\frac{\hat{g}'\hat{g}}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) & \frac{\hat{g}^2}{4}(\hat{v}^2 + \delta v^2 + \frac{4ic_{\hat{Z}\hat{Z}}}{\hat{g}'^2+\hat{g}^2}) \end{pmatrix}.$$

Note that it is the resummed propagators that are "diagonalized", rather than "fields".

- Traditional algorithm: Diagonalization → Compute the width.
- Our proposal to reverse the algorithm: Compute the (crossing-)width → Diagonalization.

Problem 1, Crossing-width Term in the Fermionic Case

Fermionic case?

- Pseudo-Dirac sterile neutrino. (Discussed in this work)
- neutron oscillation in the framework of some GUTs.

Complexity: Imaginary elements can arise in the original mass matrix. How can we distinct the phase of the mass matrix elements with the (crossing-)width terms?

Problem 1, Crossing-width Term in the Fermionic Case

Pseudo-Dirac sterile neutrino can arise from the "inverse", "linear", or "radiation-inverse" seesaw models ($\mu_1 \approx \mu_2$)

$$\mathcal{M}_{N\nu} = \begin{pmatrix} 0_{3\times3} & m_{D3\times1} & m'_{D3\times1} \\ m^T_{D3\times1} & \mu_1 & m_R \\ m'^T_{D3\times1} & m_R & \mu_2 \end{pmatrix}.$$

Diagonalizing the $\mathcal{M}_{N\nu2} = \mathcal{M}_{N\nu}\mathcal{M}^\dagger_{N\nu} = \mathcal{M}^T_{N\nu}\mathcal{M}^*_{N\nu}$ gives

$$U^T \mathcal{M}^T_{N\nu2} U^* = U^\dagger \mathcal{M}_{N\nu2} U = \begin{pmatrix} \hat{m}_1^2 & 0 & 0 \\ 0 & \hat{m}_2^2 & 0 \\ 0 & 0 & \hat{m}_3^2 \end{pmatrix},$$

and the corresponding fields are reorganized into three "mass eigen-states". Traditionally, the (diagonal-)width terms are then computed and added in the particle propagators.

Problem 1, Crossing-width Term in the Fermionic Case

In order to consider the crossing-width term, we revise the algorithm steps:

- Diagonalize the $\mathcal{M}_{N\nu}$ as before to rotate the Weyl spinors χ by a "U" matrix.
- Combine the "mass-eigenstate" 2-spinors χ into the 4-spinors $\mathcal{N}_i = \begin{pmatrix} \chi_i \\ i\sigma^2 \chi_i^* \end{pmatrix}$. Here the $\mathcal{N}_i^C = \mathcal{N}_i$. Calculate all the diagonal- and crossing-width terms in these basis.
- Diagonalize the mass matrix with the widths considered by a matrix "V".

$$\begin{pmatrix} \hat{m}_1 & 0 & 0 \\ 0 & \hat{m}_2 - i\frac{\Gamma_{22}}{2} & -i\frac{\Gamma_{23}}{2} \\ 0 & -i\frac{\Gamma_{23}}{2} & \hat{m}_3 - i\frac{\Gamma_{33}}{2} \end{pmatrix} \rightarrow \begin{pmatrix} \hat{m}_1 & 0 & 0 \\ 0 & m_{\mathcal{N}_2'} - \frac{i}{2}\Gamma_{\mathcal{N}_2'} & 0 \\ 0 & 0 & m_{\mathcal{N}_3'} - \frac{i}{2}\Gamma_{\mathcal{N}_3'} \end{pmatrix}.$$

Problem 1, Crossing-width Term in the Fermionic Case

Let us compare the traditional algorithm and the one we revised:

- Traditional algorithm: Diagonalization in the 2-spinor basis → Compute the width.
- Our proposal: Diagonalization in the 2-spinor basis (rotate out the mass phase) → Compute the (corssing-)width → Re-diagonalization in the 4-spinor bases (resum the imaginary part of the self-energy diagram).

Separate the two imaginary contributions. Dubbed Re-diagonalization algorithm?

Problem 1, Crossing-width Term in the Fermionic Case

Note that the second matrix V is not unitary,! the "diagonalized" fields $\mathcal{N}'_{1,2,3}$ no longer satisfy the "self-conjugate" conditions $\mathcal{N}'^{C}_{1,2,3} = \mathcal{N}'_{1,2,3}$. Again, this is still only a mathematical technique to resum the crossing-width term contributions.

- However, these $\mathcal{N}'_{1,2,3}$ can be straightforwardly input into the simulators like MadGraph, **without modifying** the code.
- No asymptotic external $N'_{2,3}$ should arise, and they can only be regarded as propagators

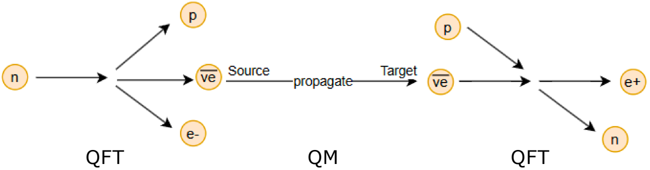
the Model

In the standard inverse/linear seesaw models, the crossing-width terms Γ_{23} disappear due to the approximate chiral symmetry rotating each 2-spinor element of the $\mathcal{N}_D$. We introduce a dark sector coupling with the sterile neutrino to generate such a term.

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\overline{\chi}(i\gamma^\mu\partial_\mu - m_\chi)\chi + \overline{N_D}(i\gamma^\mu\partial_\mu - m_{N_D})N_D + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi - m_\phi^2\phi^2) \\
&+ (\mu_1\overline{N_D^C}P_L N_D + \mu_2\overline{N_D^C}P_R N_D + \text{h.c.}) + \frac{\lambda}{4}\phi^4 + \lambda_{h\phi}\phi^2 H^\dagger H \\
&+ (y_{\chi D}\overline{\chi}N_D\phi + iy_{\chi D5}\overline{\chi}\gamma^5 N_D\phi + y_{Ni}\overline{N}P_L l_i\cdot H + y_{NCi}\overline{N^C}P_L l_i\cdot H \\
&+ \text{h.c.}) + \mathcal{L}_{\text{SM}}.
\end{aligned}
$$

Problem 2, Oscillation in the QFT Framework

How can we compute the oscillation of the sterile neutrino introduced in this model?
Traditionally, oscillation is understood from the aspect of the quantum mechanics.



Simulator's code should be modified to compute the generation and decay processes by QFT, while computing the propagation of the neutrino by QM.

Problem 2, Oscillation in the QFT Framework

It is not new to understand the oscillation processes within the framework of the QFT, but we would like a pragmatic algorithm in a full QFT framework?

Notice in the usually S-matrix, the space information is concealed

$$S = \cdots \int \frac{d^4 p}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(p_1 + p_2 + \cdots - p)$$

$$\sum_i A_i \Delta_{(F)i}(p) (2\pi)^4 \delta^{(4)}(p - p_1' - p_2' - \ldots) \ldots.$$

Actually they are hidden in the δ-functions, which originate from the integration over the full space-time,

$$\int d^4 x_1 \, e^{i(p_1 + p_2 + \cdots - p)\cdot x_1} \to (2\pi)^4 \delta^{(4)}(p_1 + p_2 + \cdots - p),$$

$$\int d^4 x_2 \, e^{i(p - p_1' - p_2' - \ldots)\cdot x_2} \to (2\pi)^4 \delta^{(4)}(p_1 + p_2 + \cdots - p).$$

Problem 2, Oscillation in the QFT Framework

Restore the space information by replacing the δ-function. Keep only the distance of the "flying" propagator while integrating out all other space-time parameters gives

$$S \approx \cdots \int d|\Delta\vec{x}| \frac{(2\pi)^4}{2} \delta^4(p_1 + p_2 + \cdots - p_1' - p_2' - \dots) e^{-i|\vec{p}_1+\vec{p}_2+\dots||\Delta\vec{x}|}$$
$$\frac{-2\pi}{i|\vec{p}|} \times \sum_i \frac{i}{4\pi} A_i (\not{p}+m_i) \left. \frac{e^{i\sqrt{(p^0)^2 - m_i^2 + i m_i \Gamma_i}|\Delta\vec{x}|}}{2} \right|_{p^0 = p_1^0 + p_2^0 + \dots} \dots .$$

It is then easy to acquire the oscillation probability

$$P_{|\Delta\vec{x}|} \propto \left| \cdots \sum_i A_i i(p' \cdot \gamma + m_i) e^{i\sqrt{(p^0)^2 - m_i^2 + i m_i \Gamma_i}|\Delta\vec{x}|} \dots \right|^2 .$$

Problem 2, Oscillation in the QFT Framework

We develop a simple way to recover the oscillation information from the S-matrix:

- Follow the normal way to calculate the S-matrix for the total probability when the intermediate $\mathcal{N}'_{2,3}$ are nearly on-shell.
- Replace all the $\frac{i}{p^2-m_i^2+im_i\Gamma_i}$ with the corresponding $e^{i\sqrt{(p^0)^2-m_i^2+im_i\Gamma_i}|\Delta\vec{x}|}$ factors to formulate the probability function $P_{|\Delta\vec{x}|}$.
- Generate the $|\Delta\vec{x}|$ randomly with the probability distribution function $P_{|\Delta\vec{x}|}$ for the displaced vertex distance value.

Such a algorithm requires the detailed S-matrix formulas. We are still too lazy to hack into the generator's codes to get them...

Problem 2, Oscillation in the QFT Framework

A more simple way: to calculate the "full propagators" of the "flavor eigenstates" $\mathcal{N}_D$ and its anti-particle $\overline{\mathcal{N}_D}$, it is easier to compute the four full propagators,

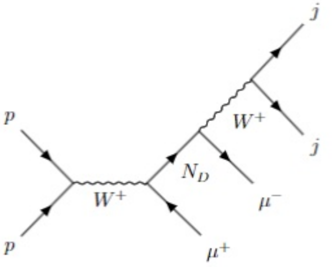
$$\langle 0|\, T[N_D \overline{N_D}]\, |0\rangle, \langle 0|\, T[N_D \overline{N^c_D}]\, |0\rangle,$$
$$\langle 0|\, T[N^c_D \overline{N_D}]\, |0\rangle, \langle 0|\, T[N^c_D \overline{N^c_D}]\, |0\rangle.$$

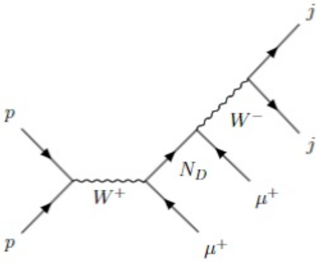
Rotating from the "diagonalized propagators" of the "mass eigenstates", one can acquire

$$\begin{aligned}
P_{N_D,\rho\to N_D,\lambda} &\propto \delta_{\lambda\rho}[\, c_1 c_1^* e^{-\frac{m\Gamma_{N_2'}}{E}t} + c_2 c_2^* e^{-\frac{m\Gamma_{N_3'}}{E}t} + 2\mathrm{Re}(\, c_1 c_2^* e^{i(E_2-E_1)t})\, e^{-\frac{m(\Gamma_{N_2'}+\Gamma_{N_3'})}{2E}t}],\\
P_{N_D,\lambda_1\to\overline{N}_D,\lambda_2}(t) &\propto \delta_{\lambda\rho}[\, c_3 c_3^* e^{-\frac{m\Gamma_{N_2'}}{E}t} + c_4 c_4^* e^{-\frac{m\Gamma_{N_3'}}{E}t} + 2\mathrm{Re}(\, c_3 c_4^* e^{i(E_2-E_1)t})\, e^{-\frac{m(\Gamma_{N_2'}+\Gamma_{N_3'})}{2E}t}],\\
P_{\overline{N}_D,\lambda_1\to\overline{N}_D\lambda_2}(t) &\propto \delta_{\lambda\rho}[\, c_5 c_5^* e^{-\frac{m\Gamma_{N_2'}}{E}t} + c_6 c_6^* e^{-\frac{m\Gamma_{N_3'}}{E}t} + 2\mathrm{Re}(\, c_5 c_6^* e^{i(E_2-E_1)t})\, e^{-\frac{m(\Gamma_{N_2'}+\Gamma_{N_3'})}{2E}t}],\\
P_{\overline{N}_D,\lambda_1\to N_D,\lambda_2}(t) &\propto \delta_{\lambda\rho}[\, c_7 c_7^* e^{-\frac{m\Gamma_{N_2'}}{E}t} + c_8 c_8^* e^{-\frac{m\Gamma_{N_3'}}{E}t} + 2\mathrm{Re}(\, c_7 c_8^* e^{i(E_2-E_1)t})\, e^{-\frac{m(\Gamma_{N_2'}+\Gamma_{N_3'})}{2E}t}],
\end{aligned}$$

Parts of the Numerical Results

One can discriminate the N_D and $\overline{N_D}$ by the by-products. For an example, if an N_D is produces, then an anti-lepton is generated to balance the lepton number in the vertex. The decay of an $\overline{N_D}$ also accompanies an anti-lepton. Therefore, the observable: $R_{ll} = \frac{\sigma_{\text{LNV}}}{\sigma_{\text{LNC}}}$ indicates the oscillation probabilities. When more and more same-sign leptons are created, more “majorana-like” the sterile neutrino becomes.

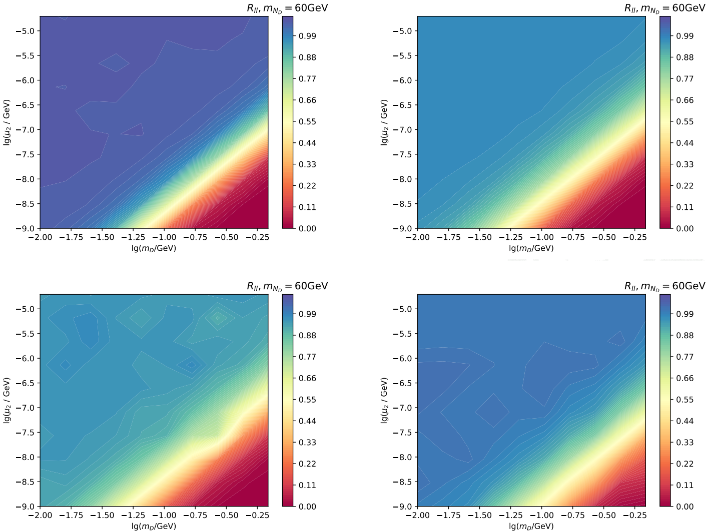




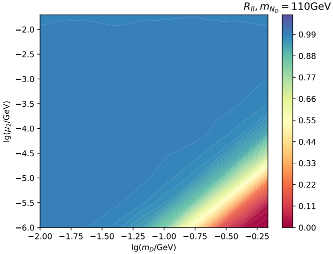
Parts of the Numerical Results

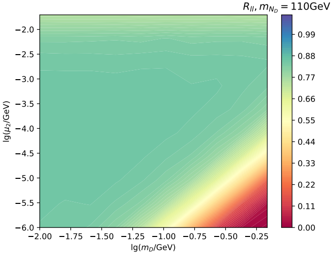
$\frac{\Gamma_{23}}{\Delta M} = 0.1, 0.5, 1, 5$ respectively, where $\Delta M = |m_{\mathcal{N}_2} - m_{\mathcal{N}_3}|$.
$m_{N_D} = 60, 110$ GeV.

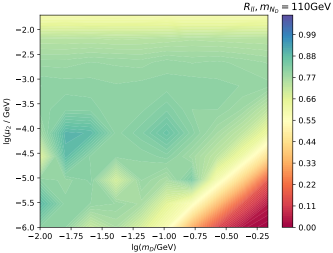
Parts of the Numerical Results

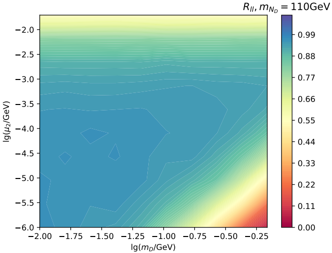


Parts of the Numerical Results



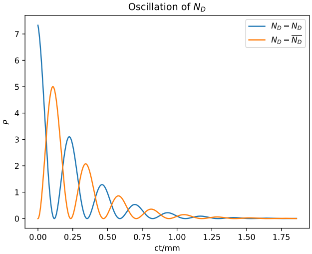






Parts of the Numerical Results

Here are the simulations of the sterile neutrino's oscillation between the $\mathcal{N}_D$-$\overline{\mathcal{N}_D}$ states. Here $m_{N_D} < m_W$ so that the lifetime arises, leaving us the possibility to observe the macroscopic displaced vertices.



Summary

- By our two-step diagonaliztion processes, the crossing-width term can be considered in the Fermionic cases.
- Oscillation can be understood and calculated in the QFT framework in which the space-time information should be retrieved to compute the distance of the displaced vertex.

Thanks!