$0\nu\beta\beta$-decay for LR symmetric models

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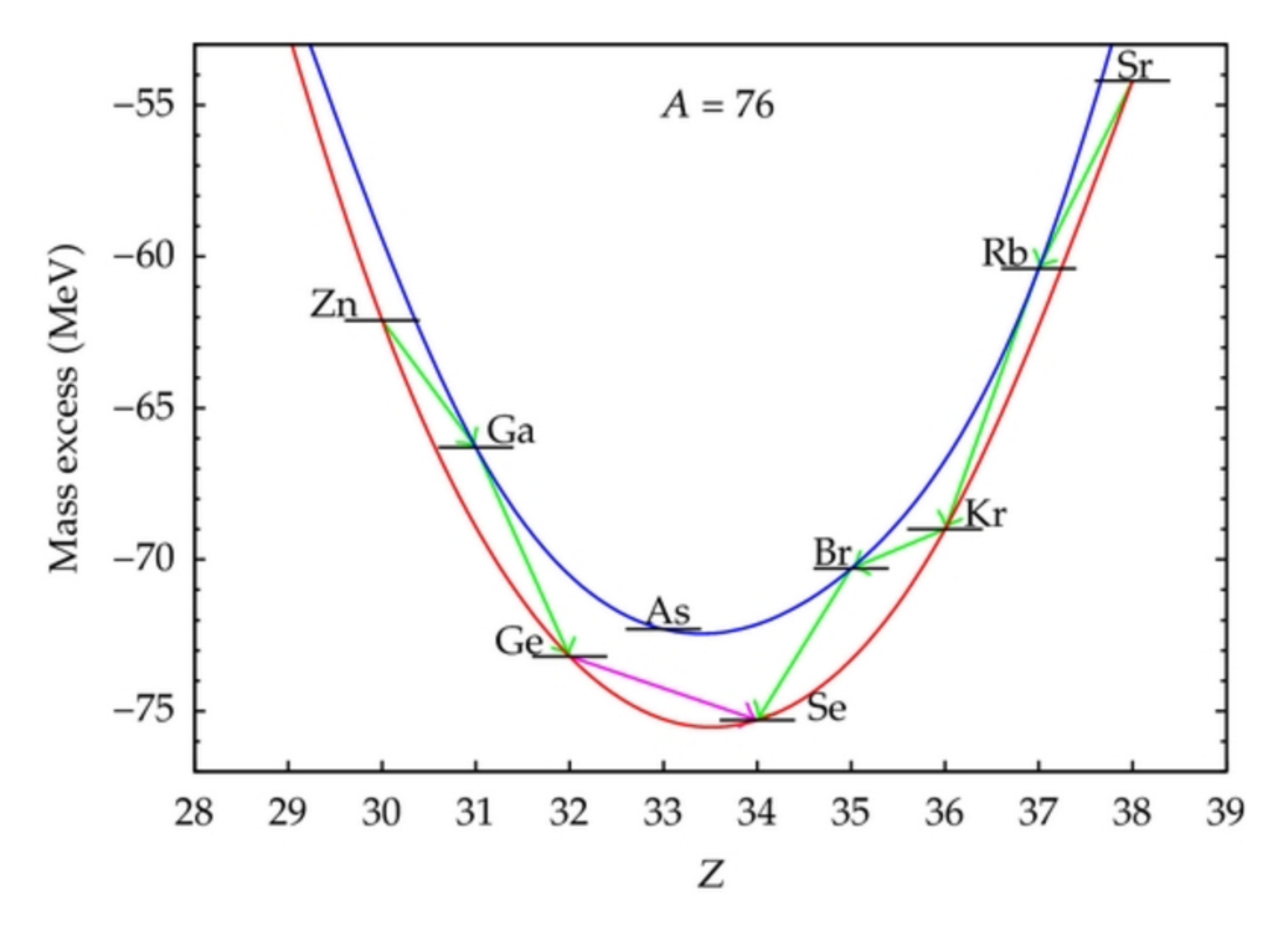
Outline

- Background
- Formalism
- Results
- Conclusion and outlook

Background



- Strong pairing force lead to nuclear double beta decay

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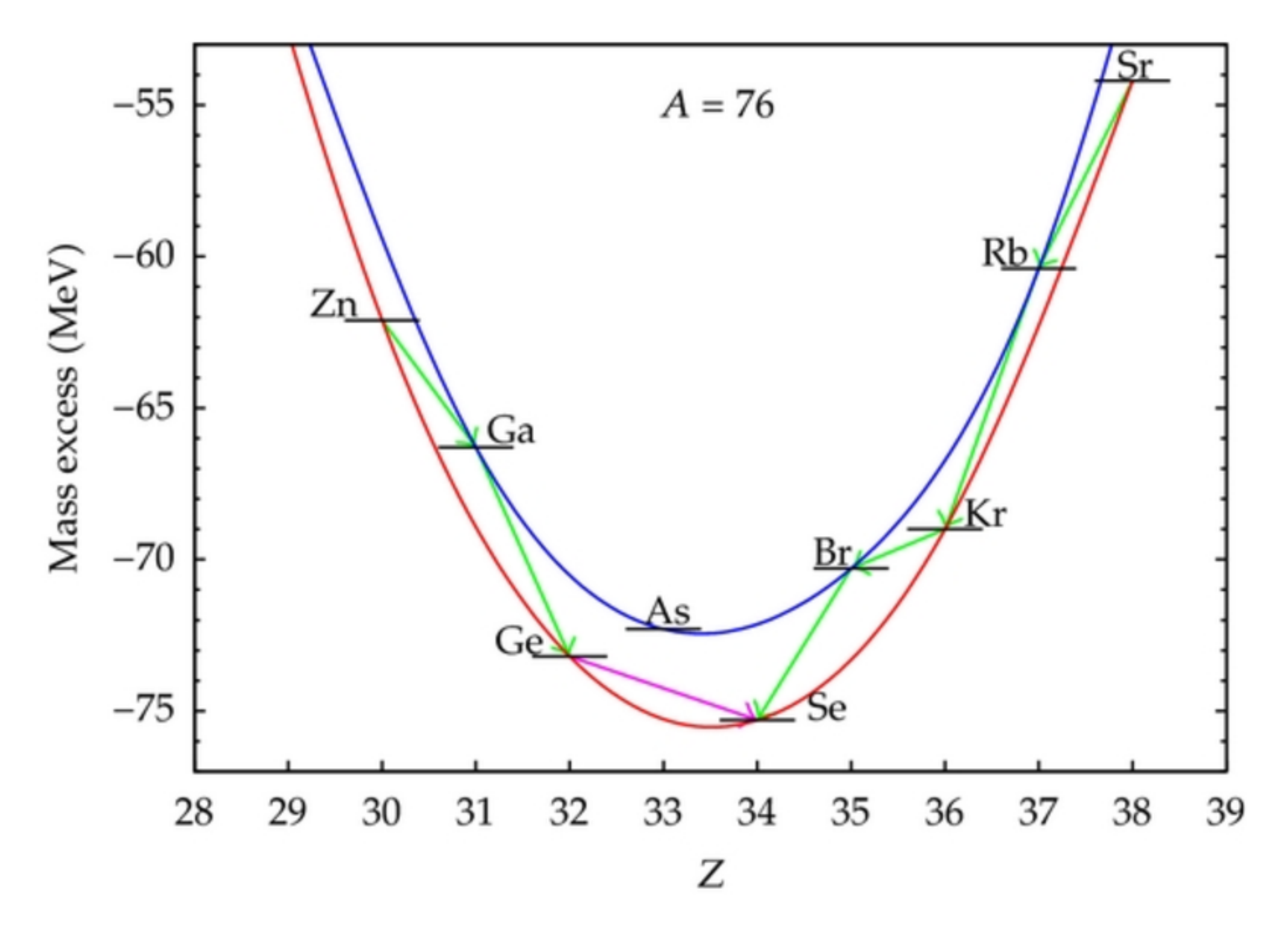

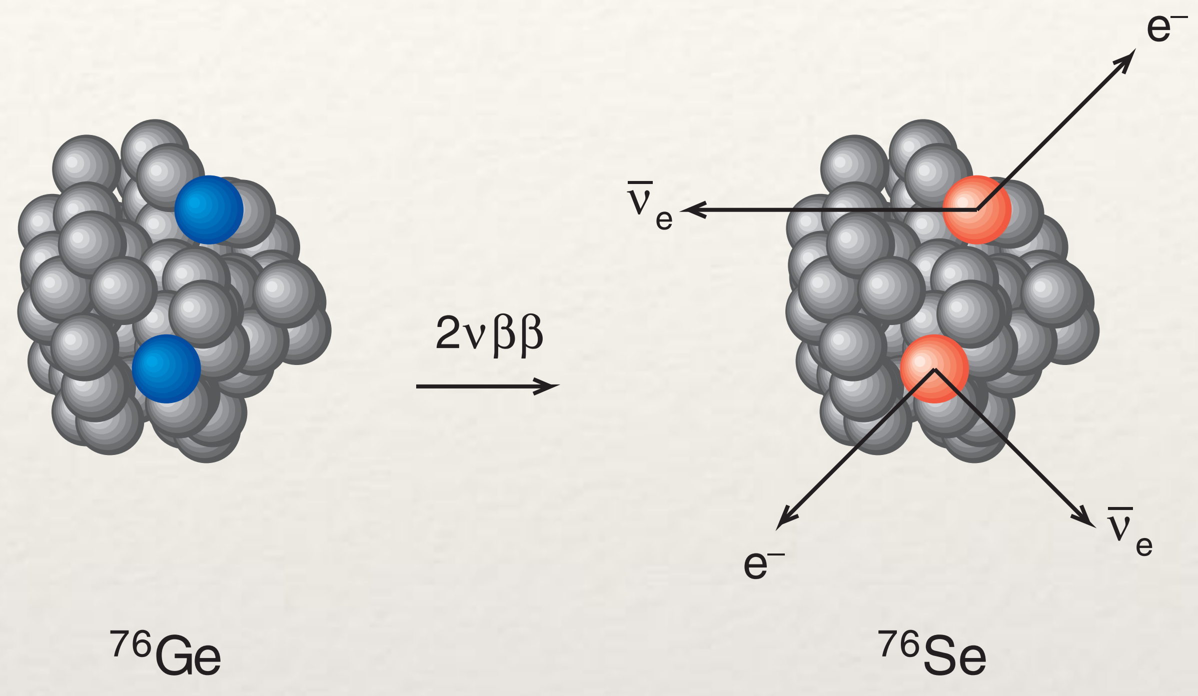

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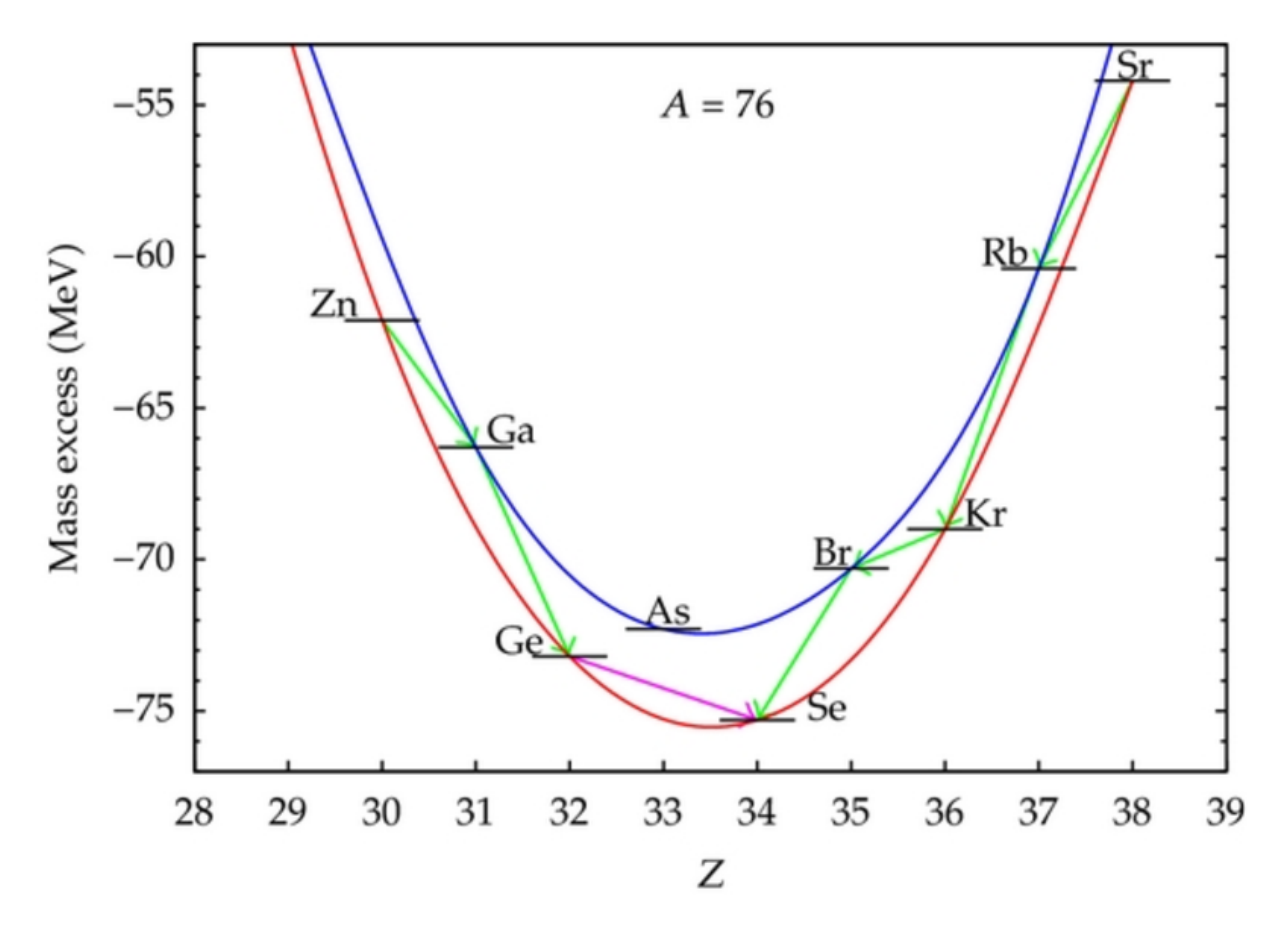

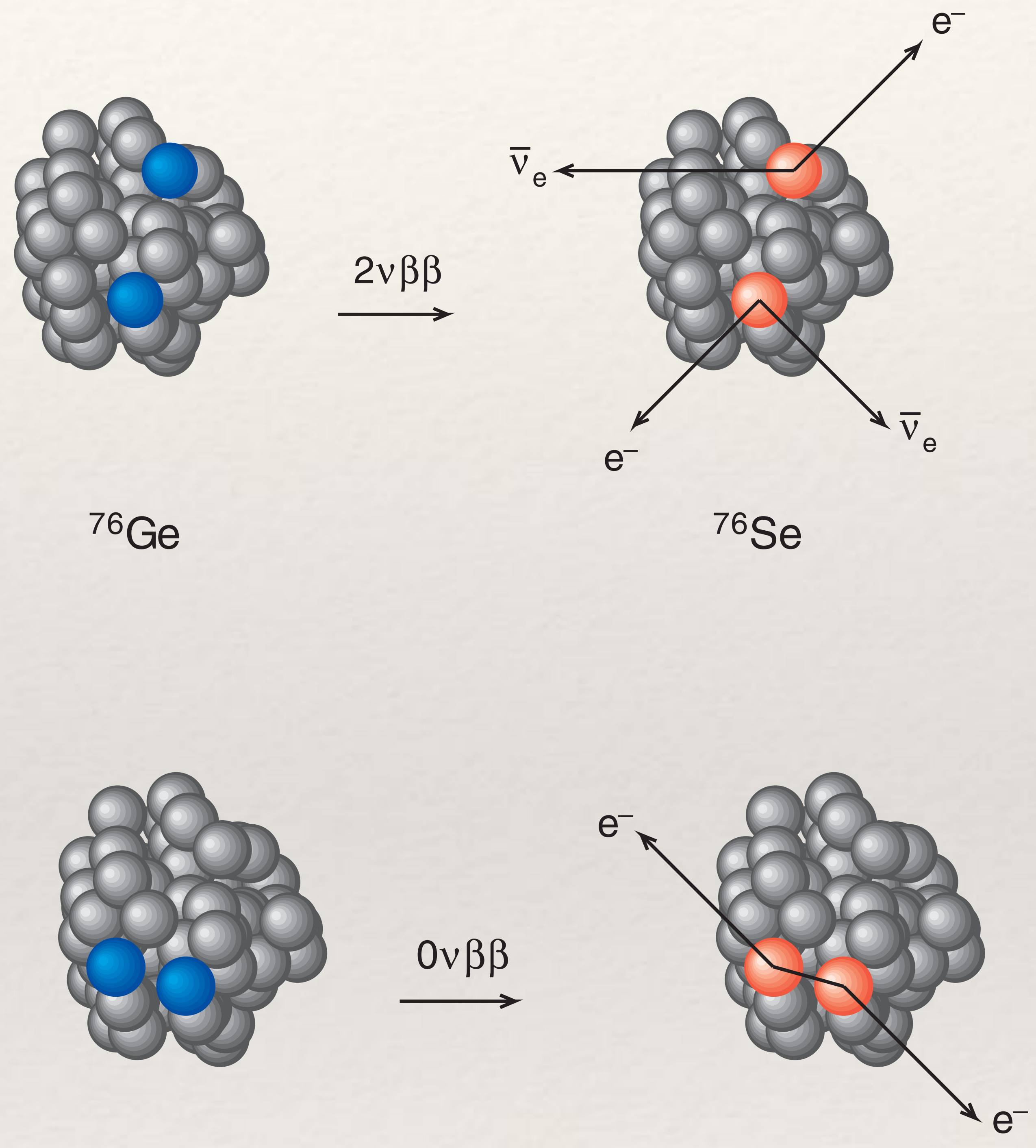

- Strong pairing force lead to nuclear double beta decay

Background

- Basics about LR symmetric model:

Mohapatra et al. PRD23,165(1981)

$$L_{iL} = \begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_L : (2,\, 1,\, -1), \qquad L_{iR} = \begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_R : (1,\, 2,\, -1), \qquad D_\mu \Psi_L = \left(\partial_\mu - i g_L \frac{\boldsymbol{\tau}}{2} \mathbf{W}_{L\mu} - i g' \frac{Y}{2} B_\mu \right) \Psi_L,$$

$$Q_{iL} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_L : (2,\, 1,\, 1/3), \qquad Q_{iR} = \begin{pmatrix} u'_i \\ d'_i \end{pmatrix}_R : (1,\, 2,\, 1/3), \qquad D_\mu \Psi_R = \left(\partial_\mu - i g_R \frac{\boldsymbol{\tau}}{2} \mathbf{W}_{R\mu} - i g' \frac{Y}{2} B_\mu \right) \Psi_R,$$

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} : (2,\, 2,\, 0) \qquad \Delta_{L,\,R} = \begin{pmatrix} \delta^+_{L,\,R}/\sqrt{2} & \delta^{++}_{L,\,R} \\ \delta^0_{L,\,R} & -\delta^+_{L,\,R}/\sqrt{2} \end{pmatrix} \qquad \begin{matrix} \Delta_L \sim (3,\, 1,\, 2) \\ \Delta_R \sim (1,\, 3,\, 2) \end{matrix}$$

- Basic gauge symmetry $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

Background

Doi et al. PTPS83,1(1985)

- Below EWSB energy scale: heavy gauge bosons acquire mass; fermions acquire mass, especially neutrino is Majorana
- Basic building blocks for double beta decay:
  - The current-current interactions

$$H_{\rm int} = \frac{G\cos\theta_C}{\sqrt{2}}(j_{L\mu}J_L^{\mu\dagger} + \kappa j_{L\mu}J_R^{\mu\dagger} + \eta j_{R\mu}J_L^{\mu\dagger} + \lambda j_{R\mu}J_R^{\mu\dagger}) + h.c.$$

$$\lambda \approx (M_1/M_2)^2 + \tan^2\zeta$$

$$\eta = \kappa \approx -\tan\zeta$$

$$\nu_{eL} = \sum_{j=1}^{3}(U_{ej}\nu_j + V_{ei}N_j^C)$$

$$\nu_{eR} = \sum_{j=1}^{3}(S_{ej}^*\nu_j^C + T_{ej}^*N_j)$$

  - The neutrino propagator

$$\langle T(N(x)\bar{N}(y))\rangle = iS_F(x-y)$$

$$\langle T(N(x)N^T(y))\rangle = iS_F(x-y)C^T$$

$$\ldots$$

$$S_F(x-y) = \int \frac{d^4q}{(2\pi)^4}\frac{(\gamma_\mu q^\mu + m_\nu)e^{-iq(x-y)}}{q^2 - m^2}$$

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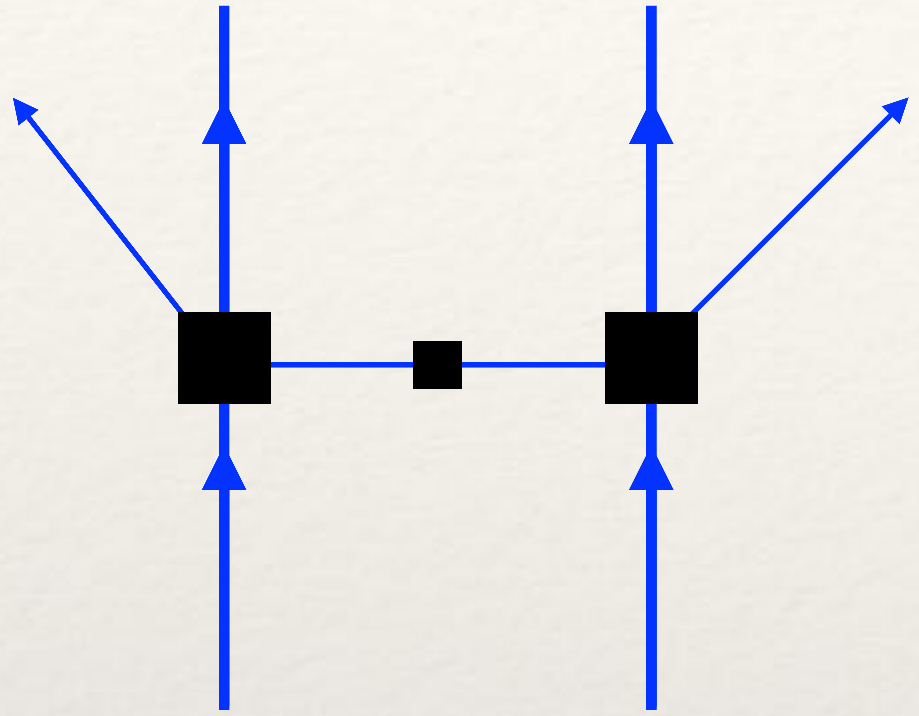
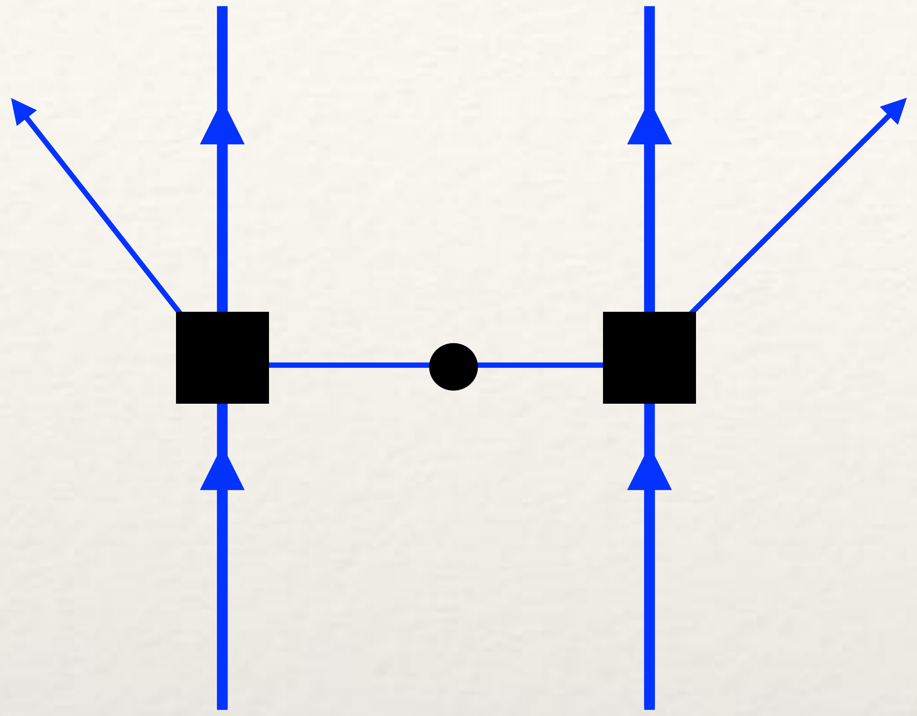
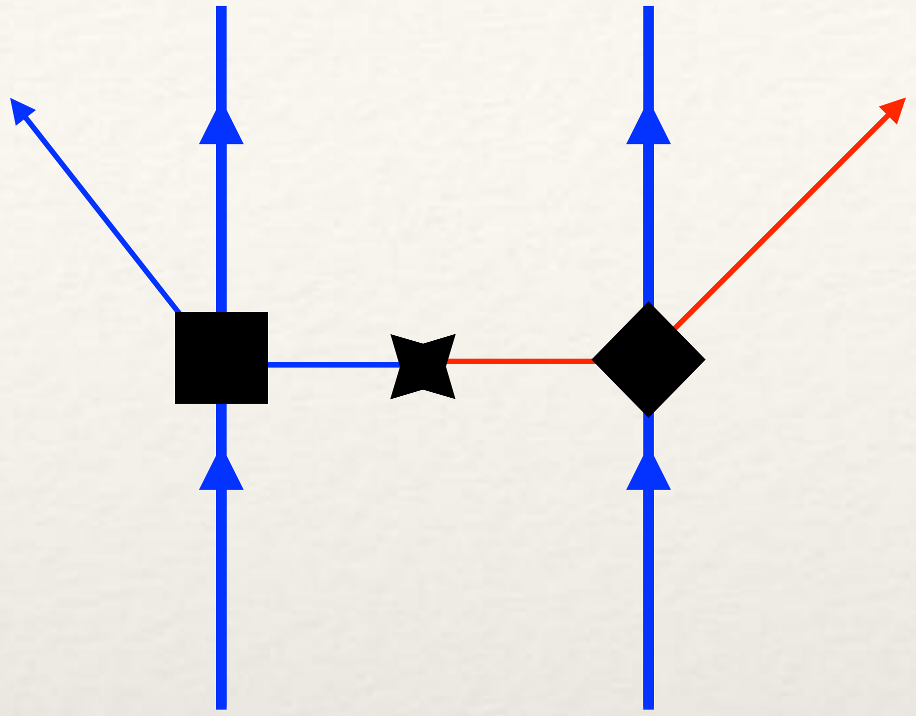


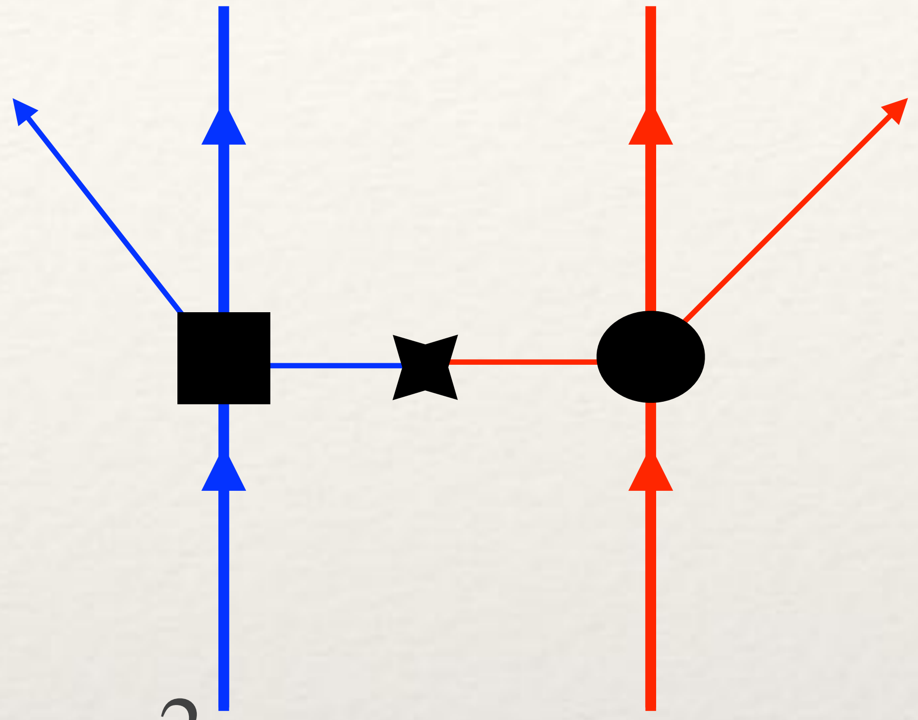
$$U_{ej}^2 \frac{m_j}{m_e} \sim \kappa_+/v_R$$



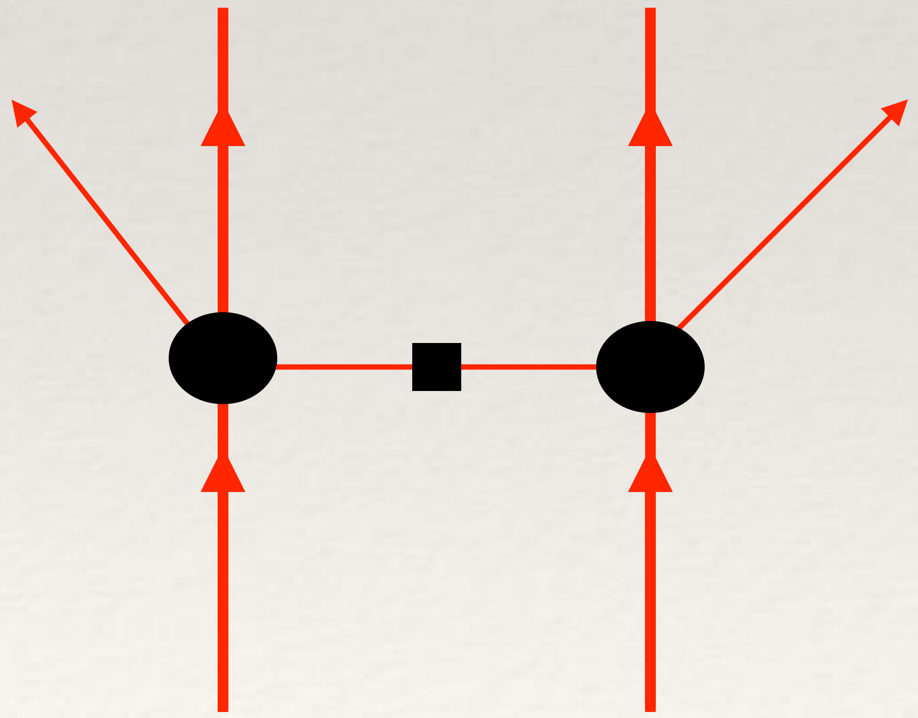
$$V_{ej}^2 \frac{m_\pi}{M_j} \sim y_L^2 \kappa_+^2 m_\pi/v_R^2$$



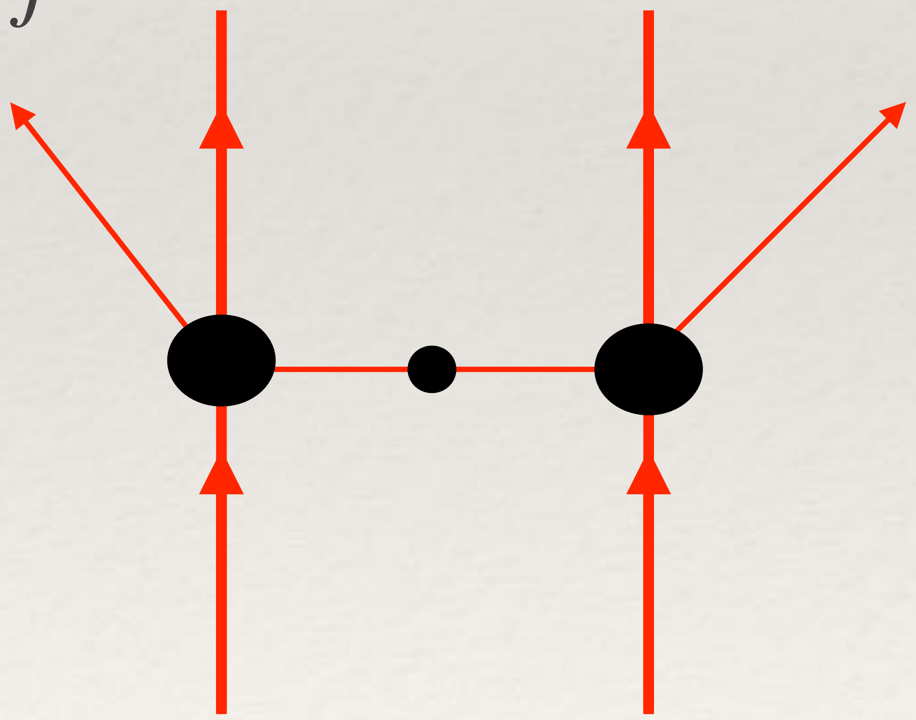
$$U_{ej} S_{ej} \tan\xi \sim y_L \kappa_1 \kappa_2 \kappa_+/v_R^3$$



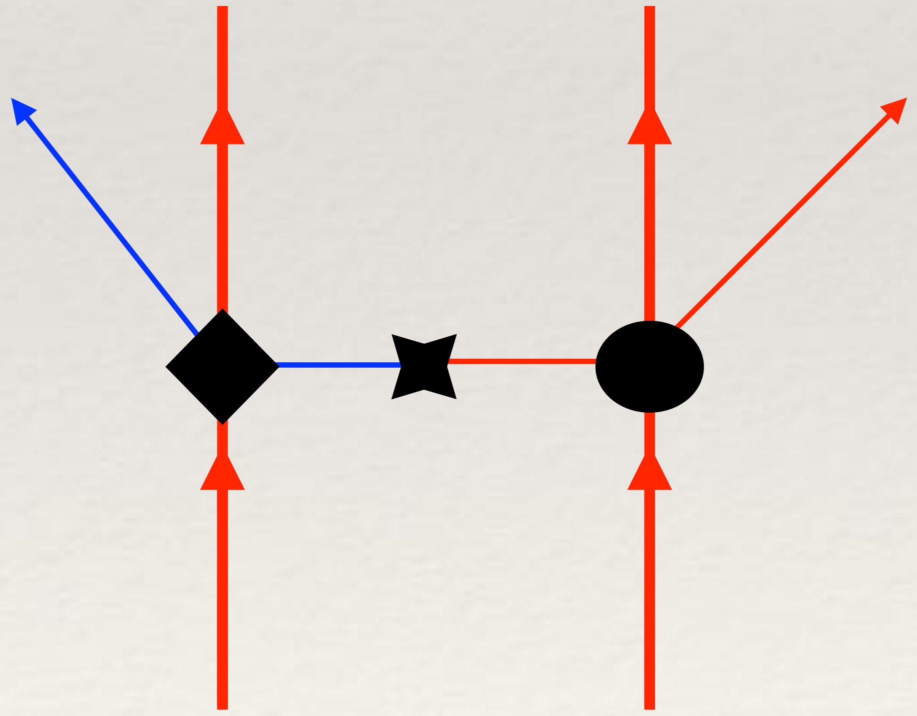
$$U_{ej} S_{ej} \frac{M_{W_1}^2}{m_{W_2}^2} \sim y_L \kappa_+^3/v_R^3$$



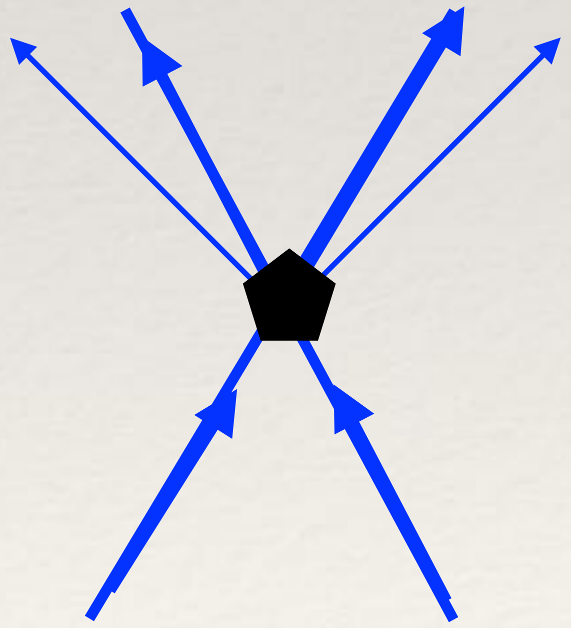
$$S_{ej}^2 \frac{m_j}{m_e} \frac{M_{W1}^4}{M_{W_2}^4} \sim y_L^2 \kappa_+^7/v_R^7$$



$$T_{ej}^2 \frac{m_\pi}{M_j} \frac{M_{W1}^4}{M_{W_2}^4} \sim \kappa_+^4 m_\pi/v_R^5$$



$$U_{ej} V_{ej} \tan\xi \frac{M_{W_1}^2}{M_{W_2}^2} \sim y_L \kappa_1 \kappa_2 \kappa_+^2 m_\pi/v_R^5$$

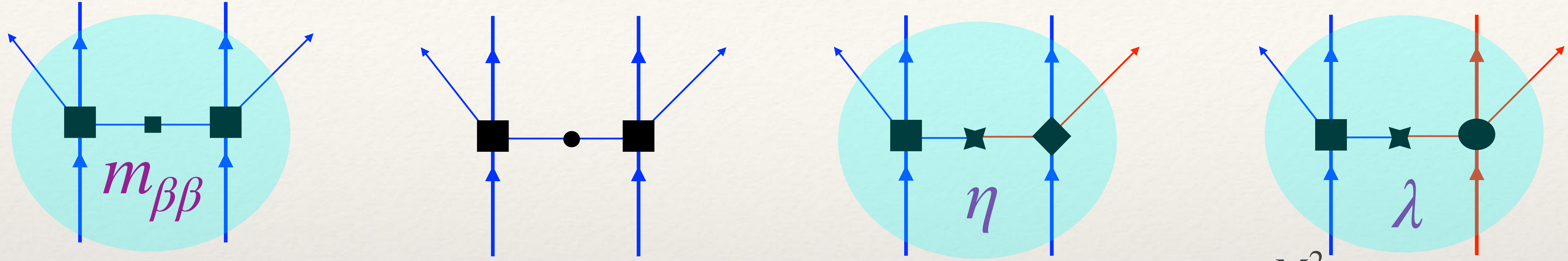


$$\sim v_L m_\pi/v_R^2$$

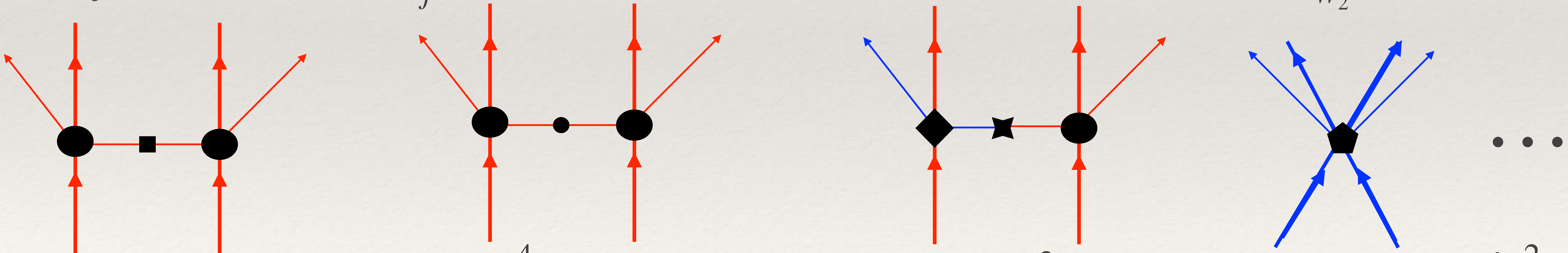
$\cdots$

Background

Doi et al. PTPS83,1(1985)



$U_{ej}^2 \frac{m_j}{m_e} \sim \kappa_+/v_R$ $V_{ej}^2 \frac{m_\pi}{M_j} \sim y_L^2 \kappa_+^2 m_\pi / v_R^2$ $U_{ej} S_{ej} \tan\xi \sim y_L \kappa_1 \kappa_2 \kappa_+ / v_R^3$ $U_{ej} S_{ej} \frac{M_{W_1}^2}{m_{W_2}^2} \sim y_L \kappa_+^3 / v_R^3$



$\sim \nu_L m_\pi / v_R^2$

$S_{ej}^2 \frac{m_j}{m_e} \frac{M_{W1}^4}{M_{W_2}^4} \sim y_L^2 \kappa_+^7 / v_R^7$ $T_{ej}^2 \frac{m_\pi}{M_j} \frac{M_{W1}^4}{M_{W_2}^4} \sim \kappa_+^4 m_\pi / v_R^5$ $U_{ej} V_{ej} \tan\xi \frac{M_{W_1}^2}{M_{W_2}^2} \sim y_L \kappa_1 \kappa_2 \kappa_+^2 m_\pi / v_R^5$

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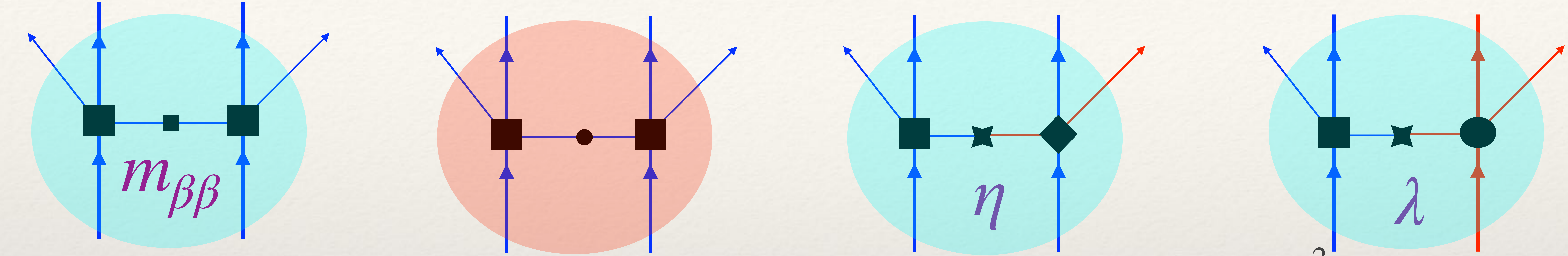
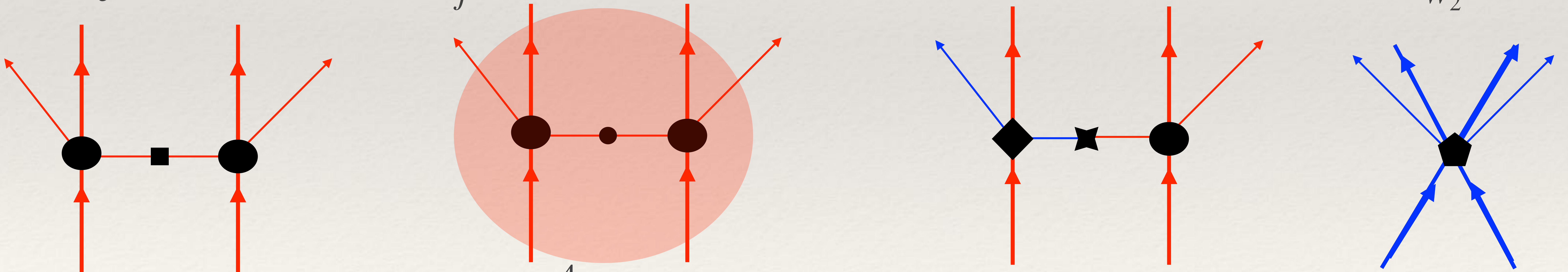


$U_{ej}^2 \frac{m_j}{m_e} \sim \kappa_+/v_R$ $\quad V_{ej}^2 \frac{m_\pi}{M_j} \sim y_L^2 \kappa_+^2 m_\pi / v_R^2$ $\quad U_{ej} S_{ej} \tan\xi \sim y_L \kappa_1 \kappa_2 \kappa_+ / v_R^3$ $\quad U_{ej} S_{ej} \frac{M_{W_1}^2}{m_{W_2}^2} \sim y_L \kappa_+^3 / v_R^3$



$\cdots$

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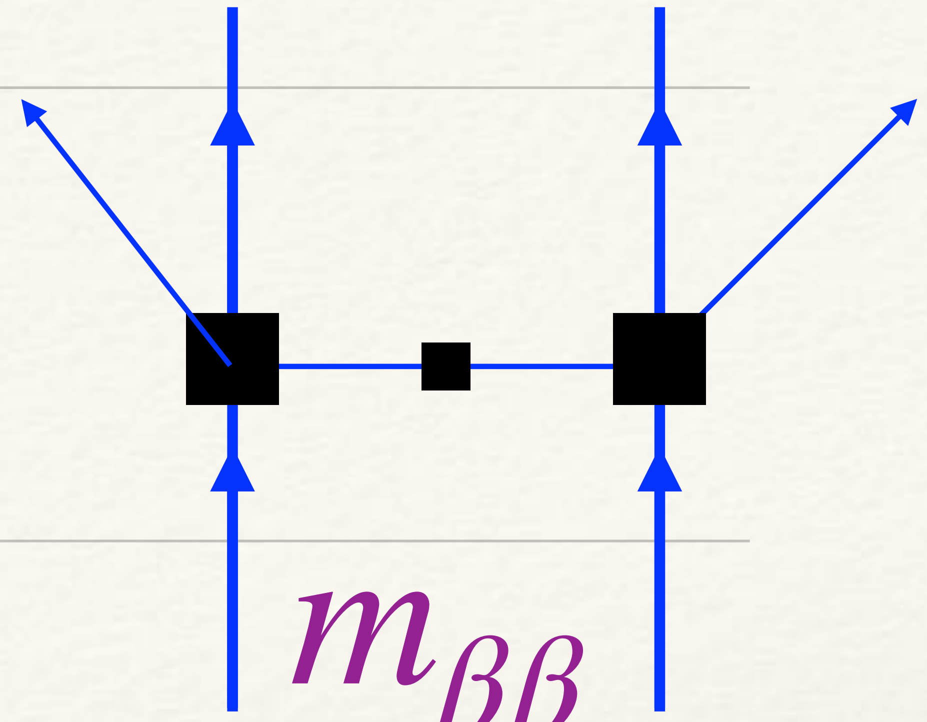


Doi et al. PTPS83,1(1985)

- Light neutrino mass mechanism
- Given that

$$J_{L(R)0}(\vec{x}) = \sum_n \delta(\vec{x} - \vec{r}_n) g_V(q^2)$$

$$J_{L(R)i}(\vec{x}) = \sum_n \delta(\vec{x} - \vec{r}_n)[\pm g_A(q^2)\sigma_{ni} \mp \frac{g_P(q^2)}{2m_N} q_i \vec{\sigma}_n \cdot \vec{q} + i\frac{g_M(q^2)}{2m_N}(\sigma_n \times \vec{q})_i]$$

- From the current-current interactions

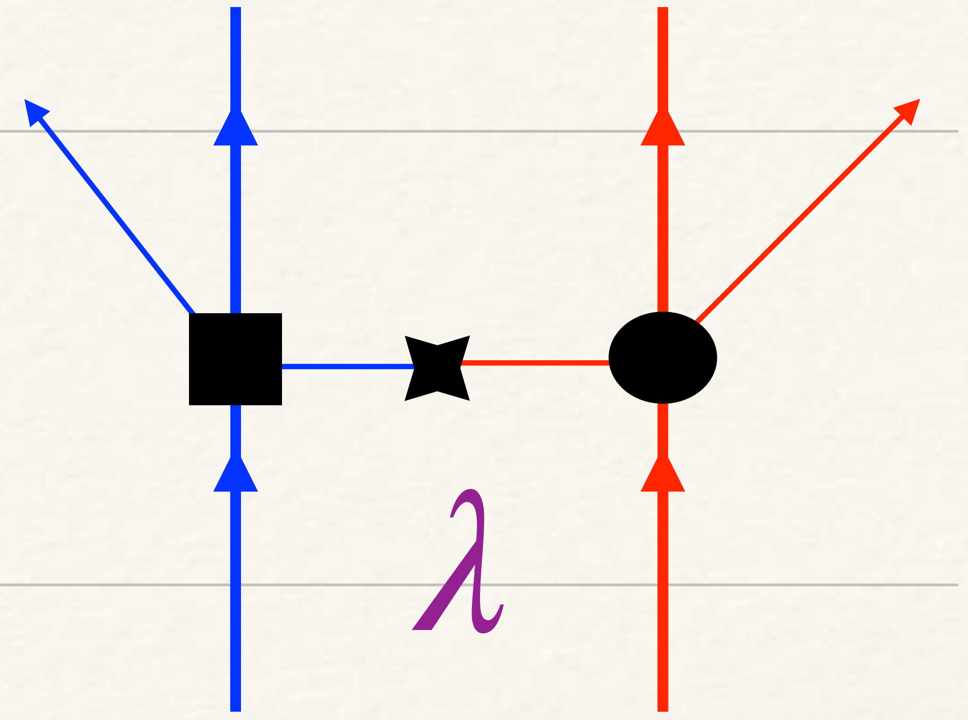
$$J_L(\vec{x})J_L(\vec{y}) \rightarrow J_{L0}(\vec{x})J_{L0}(\vec{y}) - J_{Li}(\vec{x})J_{Li}(\vec{y})$$

- We obtain the NME

$$M_m^{0\nu} = -M_F + M_{GT} + M_T$$

$\mathcal{O}(\epsilon_\chi^3)$ $\mathcal{O}(\epsilon_\chi^0)$

Background



Doi et al. PTPS83,1(1985)

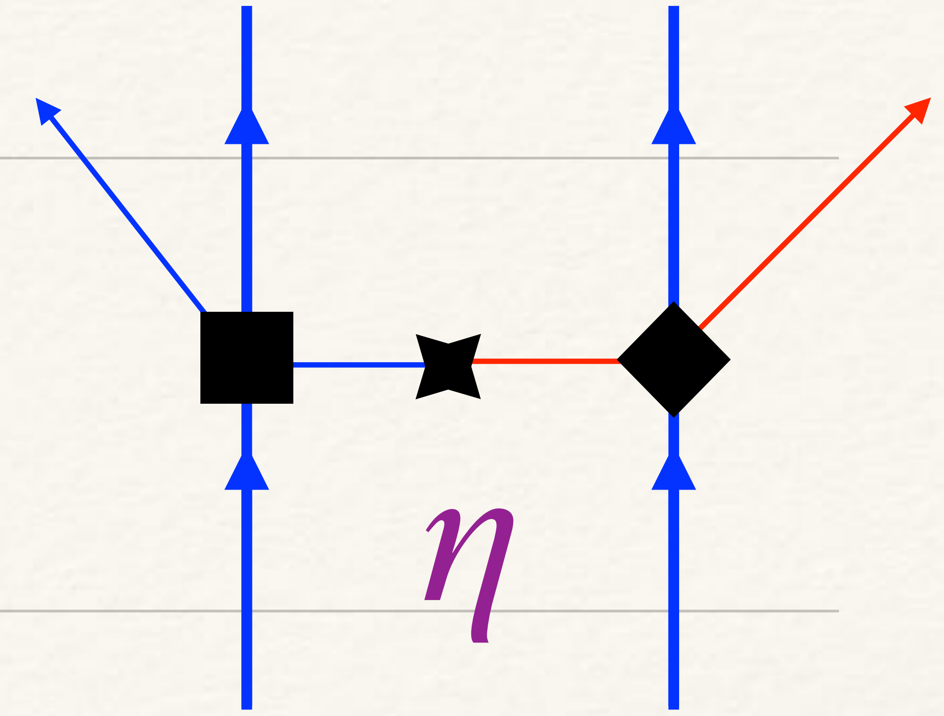
- λ mechanism:
- S-S electron wave functions

$$q_0(J_{L0}J_{R0} + J_{Li}J_{Ri}) \Rightarrow M_{\omega-} = -M_{\omega F} + M_{\omega GT-} + M_{\omega T-} \quad \mathcal{O}(\epsilon_\chi)\ \mathcal{O}(\epsilon_\chi^2)$$

- S-P electron wave functions (εR)

$$q_i r_i(J_{L0}J_{R0} + J_{Li}J_{Ri}) - q_i r_j J_{Li}(x)J_{Rj}(x) \Rightarrow M_{q+} = M_{qF} + M_{qGT+} + M_{qT+} \quad \mathcal{O}(\epsilon_\chi)\ \mathcal{O}(\epsilon_\chi^2)$$

Background



- η Mechanism:

Doi et al. PTPS83,1(1985)

  - S-S electrons
    - ω term: $q_0(J_{L0}J_{L0} + J_{Li}J_{Li})$ → $M_{\omega+} = M_{\omega F} + M_{\omega GT+} + M_{\omega T+}$ $\mathcal{O}(\epsilon_\chi)$ $\mathcal{O}(\epsilon_\chi^2)$
    - R term: $\epsilon_{ijk}q_iJ_{Lj}J_{Lk}$ → $M_R = M_{RGT} + M_{RT}$ $\mathcal{O}(\epsilon_\chi^2)$ $\mathcal{O}(\epsilon_\chi^0)$
  - S-P electrons
    - q term: $q_ir_i(J_{L0}J_{L0} + J_{Li}J_{Li}) - q_ir_jJ_{Li}J_{Lj}$ → $M_{q-} = -M_{qF} + M_{qGT-} + M_{qT-}$ $\mathcal{O}(\epsilon_\chi)$ $\mathcal{O}(\epsilon_\chi^2)$
    - P term: $\epsilon_{ijk}q_ir_{+j}J_{Lk}J_{L0}$ → M_P $\mathcal{O}(\epsilon_\chi)$ $\mathcal{O}(\epsilon_\chi^1)$

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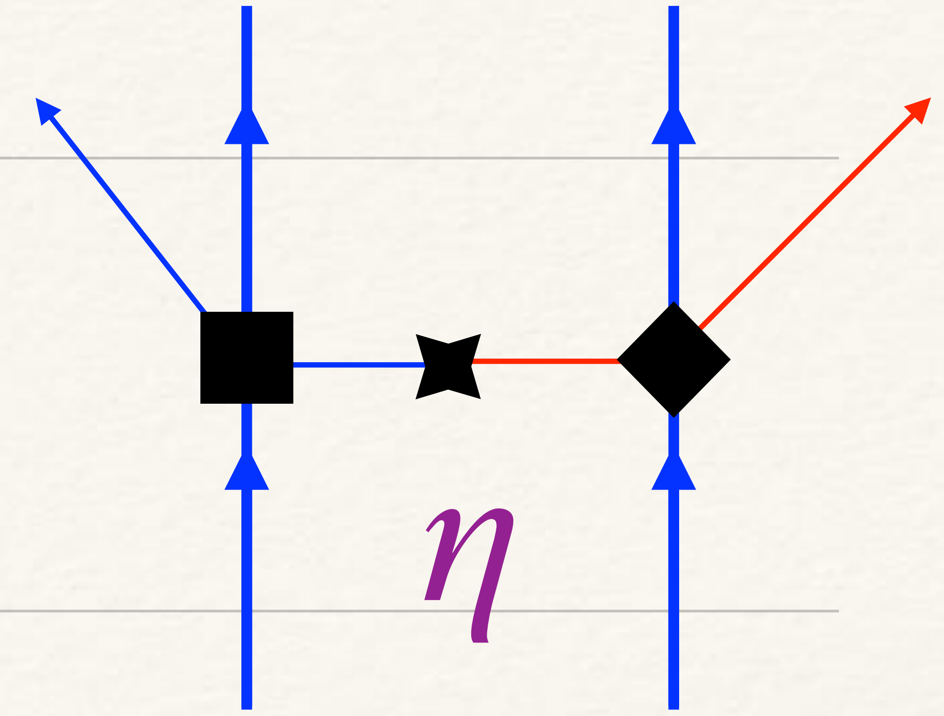

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  - S-S electrons
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    - P term: $\epsilon_{ijk}q_ir_{+j}J_{Lk}J_{L0}$ → M_P $\mathcal{O}(\epsilon_\chi)$ $\mathcal{O}(\epsilon_\chi^1)$

P-wave effects

Background

- Final expression for neutrinoless double beta decay

$$[T^{0\nu}_{1/2}]^{-1} = \mu^2_{\beta\beta}\mathcal{C}_{mm} + \mu_{\beta\beta}\langle\lambda\rangle\cos\psi_1\mathcal{C}_{m\lambda} + \mu_{\beta\beta}\langle\eta\rangle\cos\psi_2\mathcal{C}_{m\eta} + \langle\lambda\rangle^2\mathcal{C}_{\lambda\lambda} + \langle\eta\rangle^2\mathcal{C}_{\eta\eta} + \langle\lambda\rangle\langle\eta\rangle\cos(\psi_1-\psi_2)\mathcal{C}_{\lambda\eta}$$

- Here ψ's are the complex angles

Doi et al. PTPS83,1(1985), Stefanik et al. PRC92,055502(2015)

$$\begin{aligned}
C_{mm} &= G_{01}M_m^2 \\
C_{m\lambda} &= -G_{03}M_mM_{\omega-} + G_{04}M_mM_{q+} \\
C_{m\eta} &= G_{03}M_mM_{\omega+} - G_{04}M_mM_{q-} - G_{05}M_mM_P \\
&\quad + G_{06}M_mM_R \\
C_{\lambda\lambda} &= G_{02}M^2_{\omega-} + G_{011}M^2_{q+} + G_{010}M_{\omega-}M_{q+} \\
C_{\eta\eta} &= G_{02}M^2_{\omega+} + G_{011}M^2_{q-} + G_{010}M_{\omega+}M_{q-} \\
&\quad + G_{08}M_P^2 + G_{09}M_R^2 - G_{07}M_PM_R \\
C_{\lambda\eta} &= -2G_{02}M_{\omega-}M_{\omega+} - 2G_{011}M_{q+}M_{q-} \\
&\quad - G_{010}\left(M_{\omega-}M_{q-} + M_{\omega+}M_{q+}\right),
\end{aligned}$$

Formalism

- We use two kind of many-body methods in our calculations (The closure *vs.* non closure)
- Shell model with configuration interactions (NuShellX@MSU):

$$M_I = \sum_{p_1 p_2 n_1 n_2} \langle f || [[c^\dagger_{p_1} c^\dagger_{p_2}]_{J_1} \otimes [c_{\tilde{n}_2} c_{\tilde{n}_1}]_{J_2}]_J || i \rangle \langle p_1 p_2 J_1 || \mathcal{O}_J(\tilde{E}_m) || n_1 n_2 J_2 \rangle$$

- QRPA with realistic forces:

$$M_I = \sum_{p_1 p_2 n_1 n_2} \sum_m C_{J\mathcal{J}} \langle f || [c^\dagger_{p_1} c_{\tilde{n}_1}]_{J_1} || m \rangle \langle m || [c_{p_2} c_{\tilde{n}_1}]_{J_2} || i \rangle \langle p_1 p_2 \mathcal{J}_1 || \mathcal{O}_J(E_m) || n_1 n_2 \mathcal{J}_2 \rangle$$

- The latter has a large computational burden

Formalism

- The operators $\mathcal{O}_I$ can be written in the form follows:

$$\mathcal{O}_I = h_I(r, r_+)\mathcal{A}_I$$

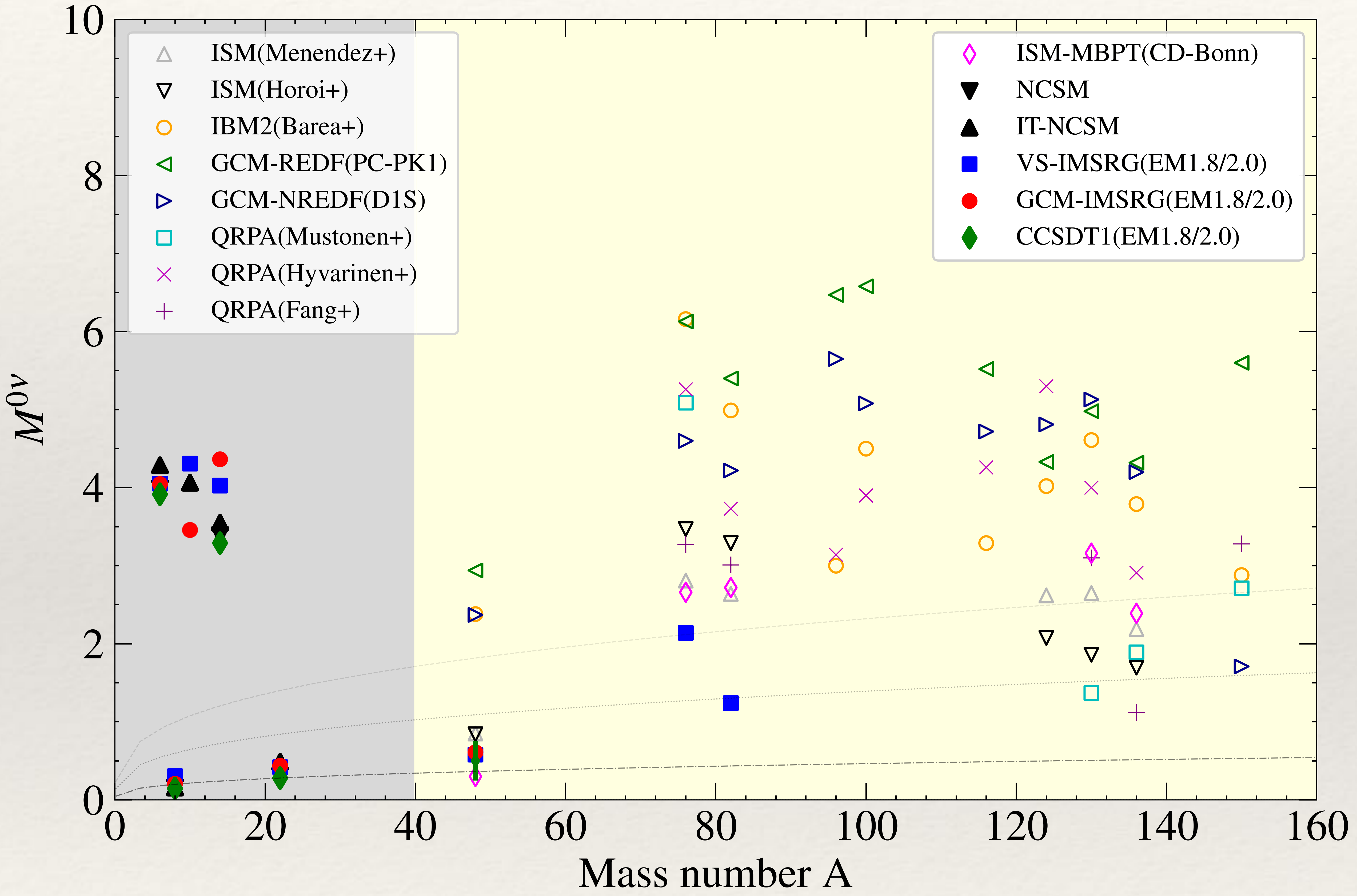
- With the radial part

$$h_I(r, r_+) = f_{src}^2(r)\frac{2R}{\pi}\int f_I(q, r, r_+)\frac{qdq}{q + \tilde{A}_m}$$

- And angular part $\mathcal{A}_I$
- A complete list of the operators see DLF *et al.* **PRC110**,045502(2024); ***ibid.*****107**,015501(2023)

Results

Yao et al. PPNP126,103965(2022)



- Results for light neutrino mass mechanism are abundant

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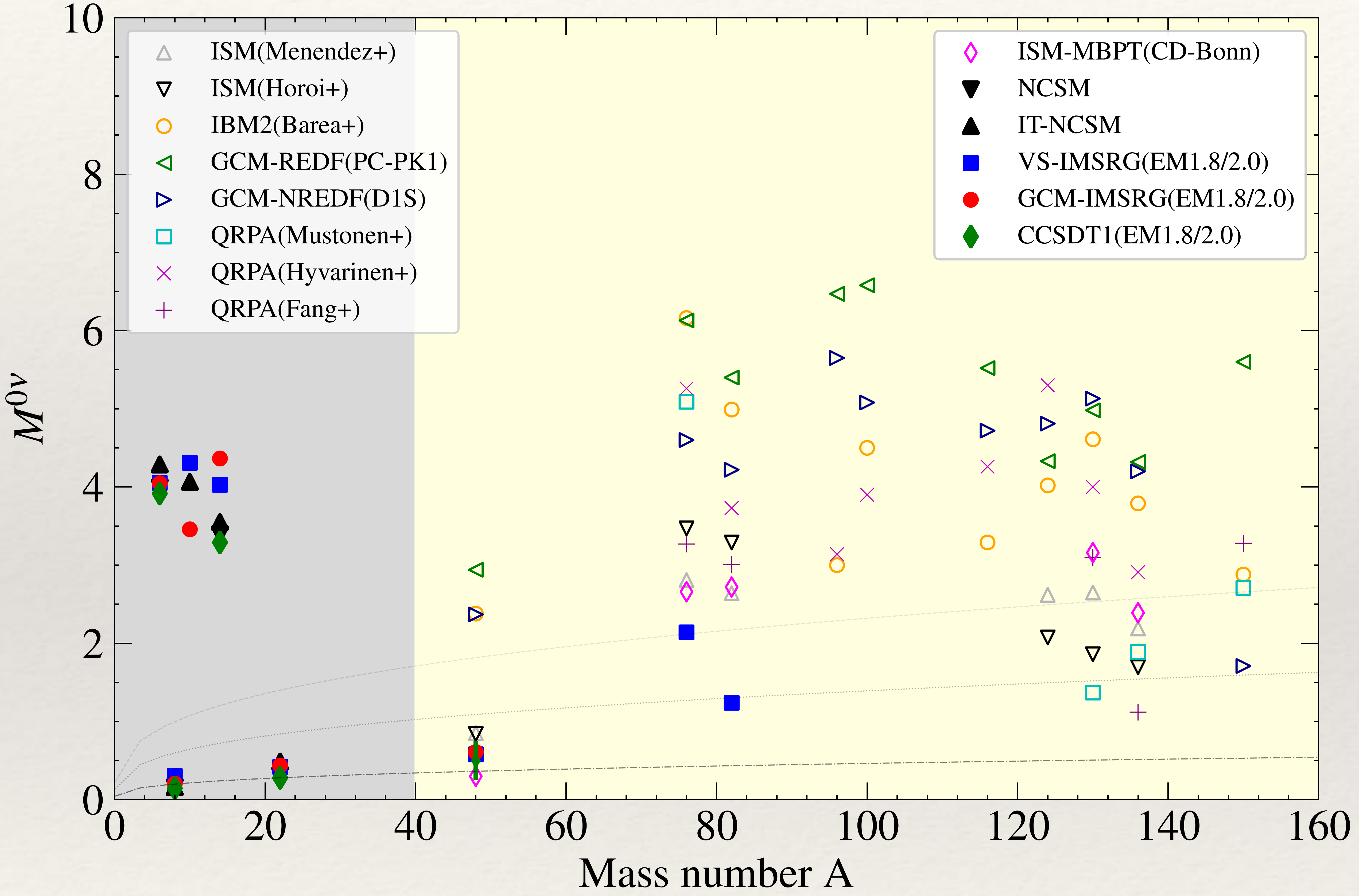
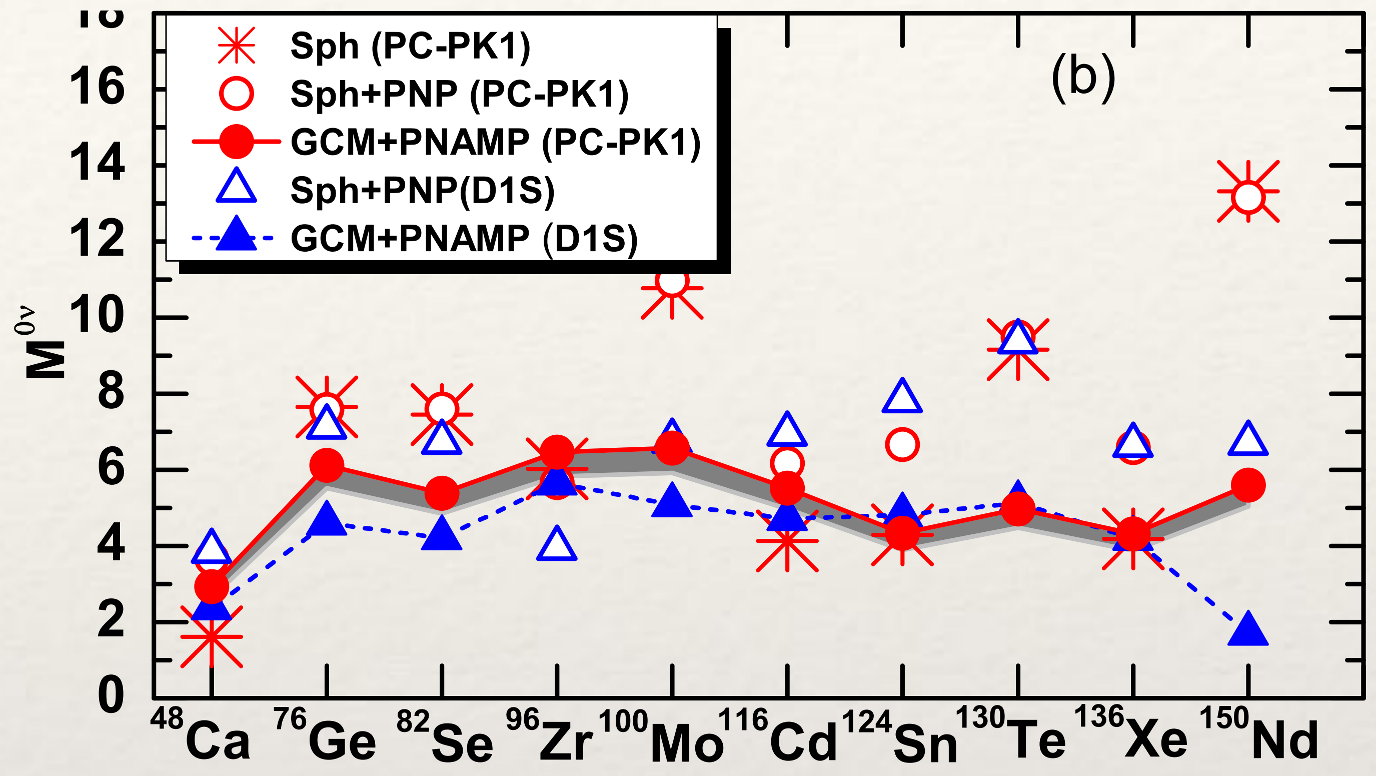
Yao et al. PPNP126,103965(2022)





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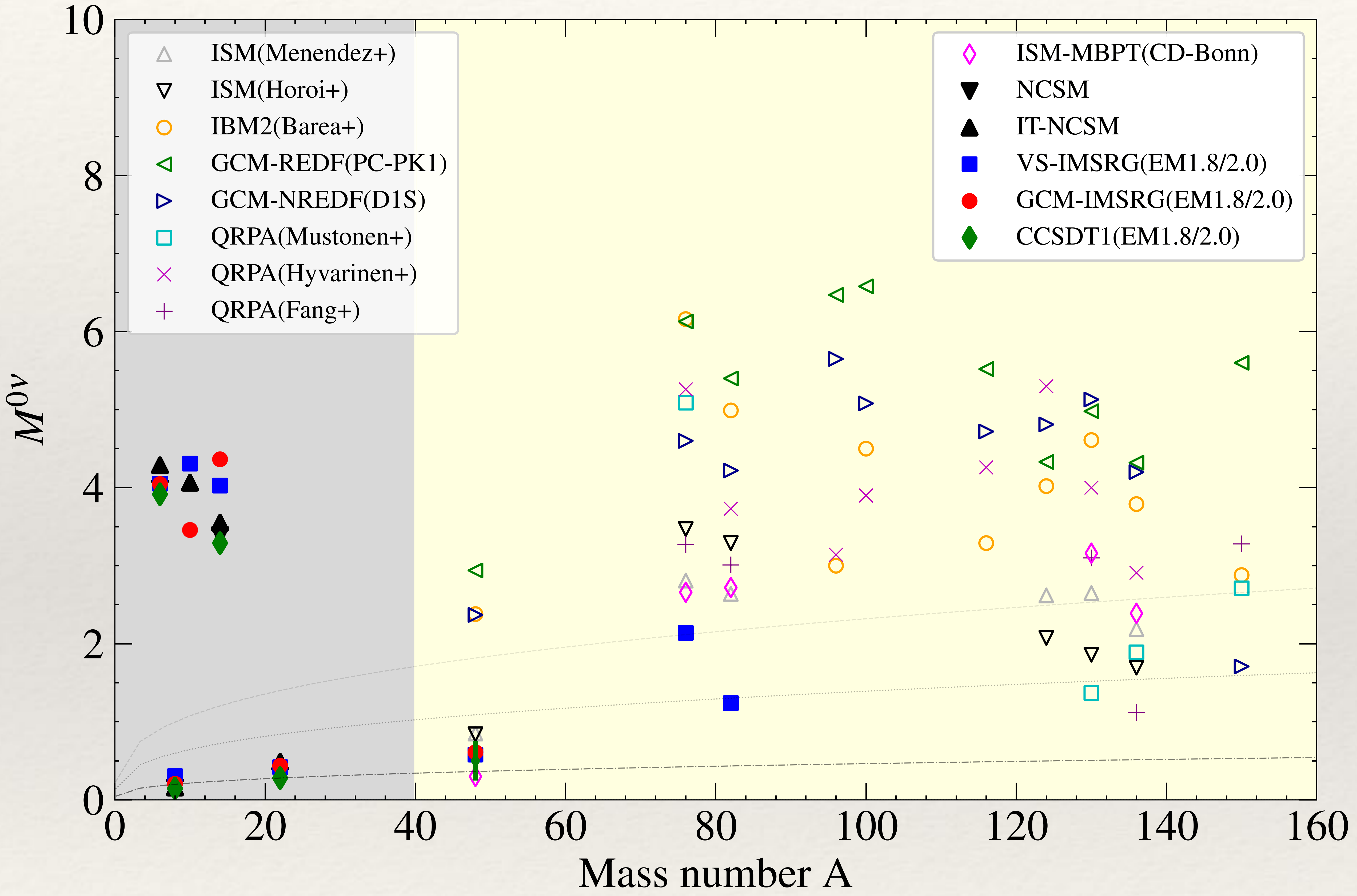

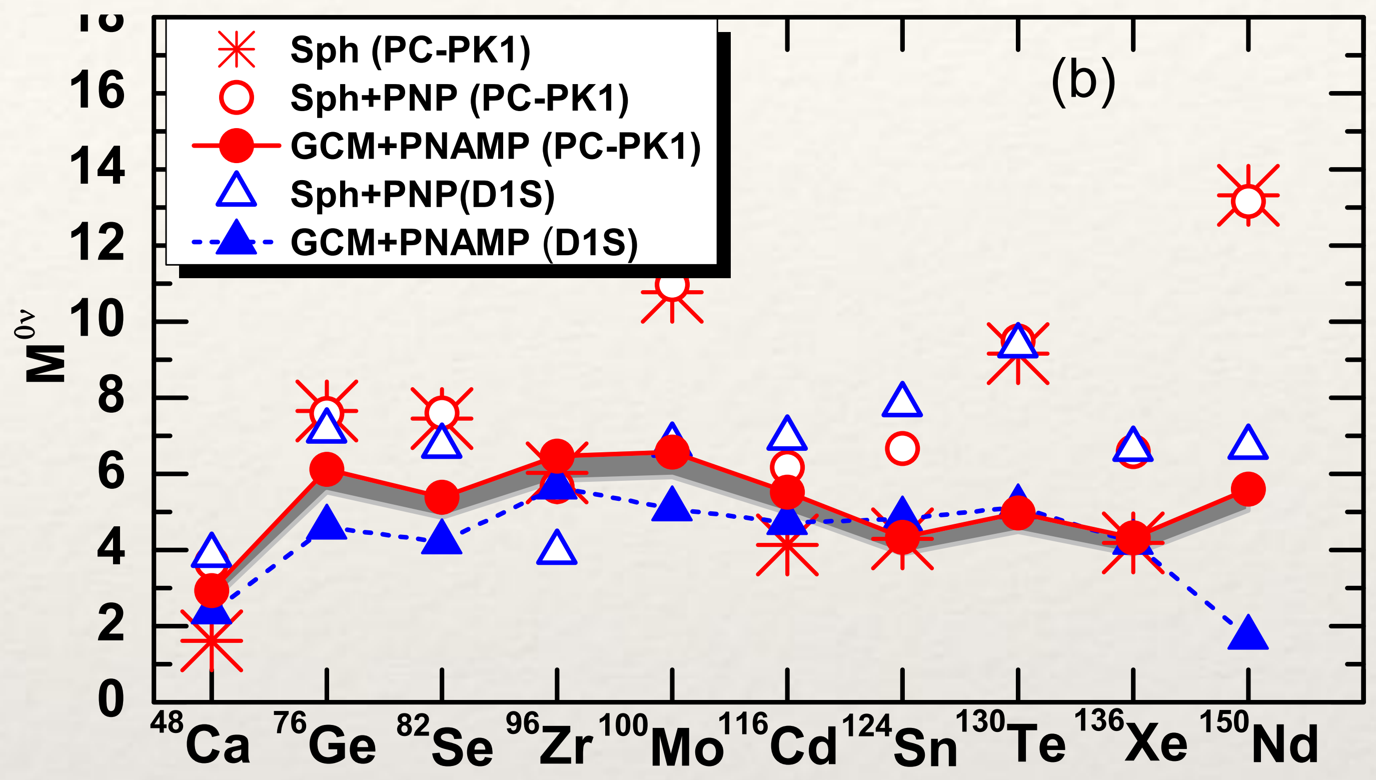

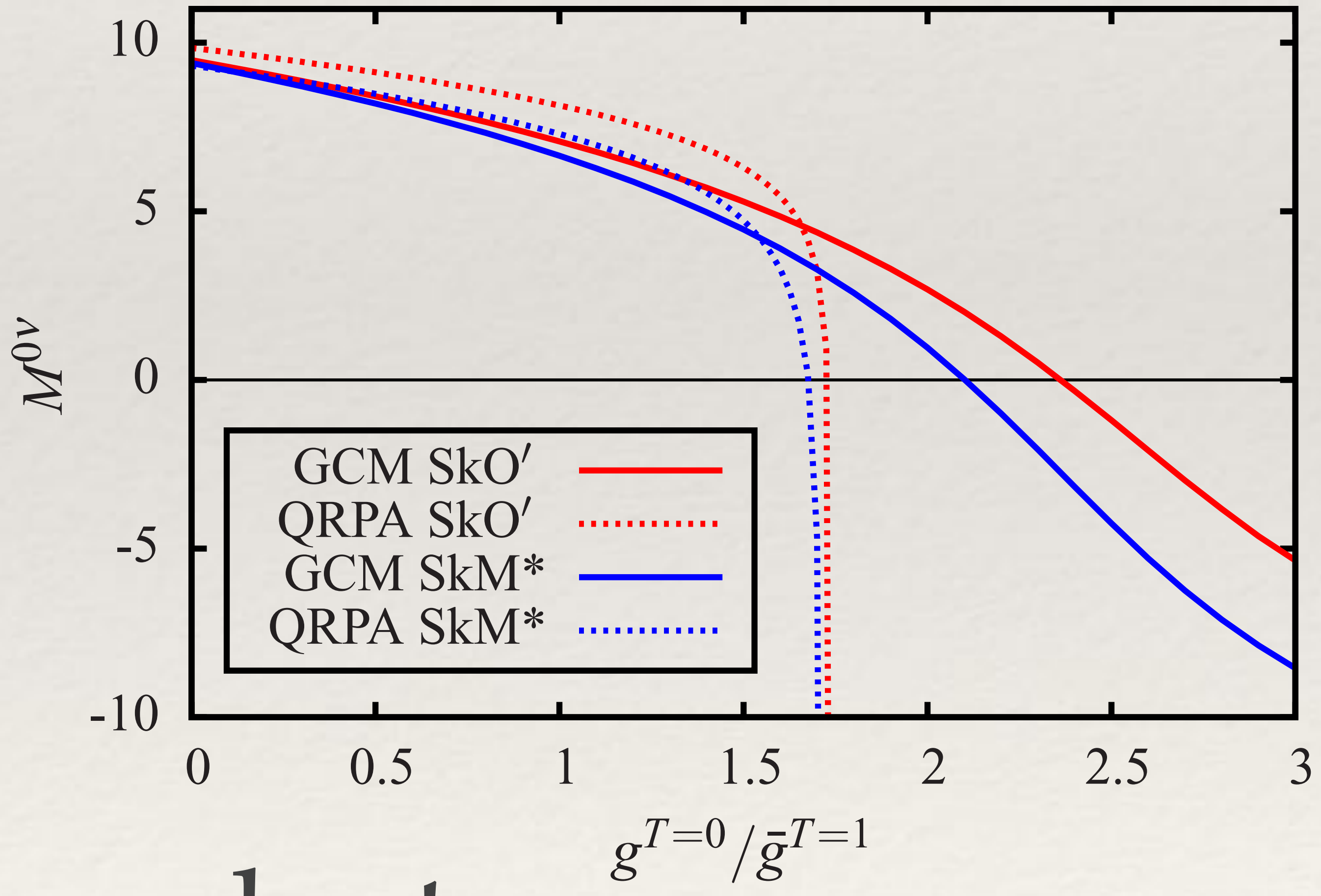

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Results

DLF et al. PRC110,045502(2024), Huang et al. arXiv:2506.13289

NMEs	^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
	QRPA	jun45	jj44b	QRPA	jun45	jj44b	QRPA	jj55a	GCN50:82	QRPA	jj55a	GCN50:82
$M_{\omega F}$	−1.290	−0.637	−0.576	−1.156	−0.597	−0.500	−1.152	−0.637	−0.669	−0.341	−0.545	−0.540
$M_{\omega GT}^{AA}$	5.036	3.276	2.980	4.339	3.073	2.596	4.025	2.883	2.931	1.450	2.427	2.351
$M_{\omega GT}^{AP}$	−1.929	−1.044	−0.919	−1.684	−0.978	−0.798	−1.665	−0.939	−0.993	−0.580	−0.786	−0.795
$M_{\omega GT}^{PP}$	0.661	0.333	0.290	0.577	0.310	0.252	0.587	0.303	0.324	0.201	0.252	0.259
$M_{\omega GT}^{MM}$	0.814	0.239	0.208	0.707	0.221	0.181	0.723	0.220	0.236	0.244	0.182	0.188
total^{+}	4.272	2.713	2.480	3.670	2.542	2.162	3.395	2.383	2.408	1.223	2.006	1.932
total^{-}	3.264	2.417	2.222	2.794	2.268	1.938	2.499	2.111	2.116	0.919	1.780	1.698
$M_{\omega T}^{AP}$	−0.785	−0.012	−0.003	−0.726	−0.013	−0.011	−0.926	0.009	0.015	−0.283	0.003	0.014
$M_{\omega T}^{PP}$	0.287	0.002	−0.001	0.265	0.003	0.003	0.330	−0.006	−0.007	0.100	−0.003	−0.006
$M_{\omega T}^{MM}$	−0.191	−0.001	0.000	−0.174	−0.001	−0.002	−0.210	0.003	0.003	−0.064	0.001	0.002
total^{+}	−0.616	−0.011	−0.004	−0.569	−0.011	−0.010	−0.726	0.006	0.010	−0.222	0.001	0.009
total^{-}	−0.380	−0.009	−0.004	−0.353	−0.009	−0.007	−0.466	0.000	0.006	−0.144	−0.001	0.007

- ω term has basically the same results as mass term, the minor revision comes from the energy denominator

Results

DLF et al. PRC110,045502(2024), Huang et al. arXiv:2506.13289

NMEs	^{76}Ge		
	QRPA	jun45	jj44b
M_F	−1.314	−0.665	−0.602
M_{GT}^{AA}	5.139	3.584	3.278
M_{GT}^{AP}	−1.961	−1.090	−0.960
M_{GT}^{PP}	0.671	0.344	0.300
M_{GT}^{MM}	0.513	0.247	0.215
total	4.362	3.085	2.833
M_T^{AP}	−0.809	−0.013	−0.004
M_T^{PP}	0.296	0.002	−0.001
M_T^{MM}	−0.075	−0.001	0.000
total	−0.588	−0.012	−0.005

NMEs	^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
	QRPA	jun45	jj44b	QRPA	jun45	jj44b	QRPA	jj55a	GCN50:82	QRPA	jj55a	GCN50:82
$M_{\omega F}$	−1.290	−0.637	−0.576	−1.156	−0.597	−0.500	−1.152	−0.637	−0.669	−0.341	−0.545	−0.540
$M_{\omega GT}^{AA}$	5.036	3.276	2.980	4.339	3.073	2.596	4.025	2.883	2.931	1.450	2.427	2.351
$M_{\omega GT}^{AP}$	−1.929	−1.044	−0.919	−1.684	−0.978	−0.798	−1.665	−0.939	−0.993	−0.580	−0.786	−0.795
$M_{\omega GT}^{PP}$	0.661	0.333	0.290	0.577	0.310	0.252	0.587	0.303	0.324	0.201	0.252	0.259
$M_{\omega GT}^{MM}$	0.814	0.239	0.208	0.707	0.221	0.181	0.723	0.220	0.236	0.244	0.182	0.188
total$^+$	4.272	2.713	2.480	3.670	2.542	2.162	3.395	2.383	2.408	1.223	2.006	1.932
total$^-$	3.264	2.417	2.222	2.794	2.268	1.938	2.499	2.111	2.116	0.919	1.780	1.698
$M_{\omega T}^{AP}$	−0.785	−0.012	−0.003	−0.726	−0.013	−0.011	−0.926	0.009	0.015	−0.283	0.003	0.014
$M_{\omega T}^{PP}$	0.287	0.002	−0.001	0.265	0.003	0.003	0.330	−0.006	−0.007	0.100	−0.003	−0.006
$M_{\omega T}^{MM}$	−0.191	−0.001	0.000	−0.174	−0.001	−0.002	−0.210	0.003	0.003	−0.064	0.001	0.002
total$^+$	−0.616	−0.011	−0.004	−0.569	−0.011	−0.010	−0.726	0.006	0.010	−0.222	0.001	0.009
total$^-$	−0.380	−0.009	−0.004	−0.353	−0.009	−0.007	−0.466	0.000	0.006	−0.144	−0.001	0.007

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Results

DLF et al. PRC110,045502(2024), Huang et al. in preparation

Bender et al. ZPA344,187(1989)

NMEs	^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
	QRPA	jun45	jj44b	QRPA	jun45	jj44b	QRPA	jj55a	GCN50:82	QRPA	jj55a	GCN50:82
M_{qF}	−0.797	−0.379	−0.352	−0.734	−0.360	−0.303	−0.683	−0.408	−0.418	−0.195	−0.358	−0.342
M_{qGT}^{AA}	1.266	1.070	0.994	1.062	1.005	0.868	0.891	0.927	0.917	0.367	0.783	0.736
M_{qGT}^{AP}	2.499	1.614	1.439	2.173	1.524	1.247	2.024	1.422	1.475	0.760	1.202	1.188
M_{qGT}^{PP}	-1.104	-0.648	-0.569	-0.967	-0.610	-0.493	-0.945	-0.577	-0.609	-0.345	-0.485	-0.489
M_{qGT}^{MM}	-1.959	-0.625	-0.545	-1.707	-0.582	-0.475	-1.734	-0.569	-0.608	-0.607	-0.473	-0.485
total^{+}	1.447	1.412	1.526	1.210	1.338	1.328	0.895	1.203	1.406	0.406	1.027	1.134
total^{-}	3.875	2.661	2.202	3.327	2.501	1.917	3.044	2.342	2.160	1.159	1.973	1.736
M_{qT}^{AA}	-2.874	-0.112	-0.066	-2.660	-0.110	-0.084	-3.456	-0.062	-0.018	-1.070	-0.062	0.004
M_{qT}^{AP}	1.534	0.008	-0.002	1.406	0.012	0.016	1.730	-0.036	-0.036	0.522	-0.014	-0.030
M_{qT}^{PP}	-0.458	0.000	0.002	-0.416	-0.002	-0.006	-0.484	0.014	0.010	-0.144	0.004	0.006
M_{qT}^{MM}	-0.178	0.000	0.000	-0.158	0.000	-0.002	-0.174	0.002	0.002	-0.052	0.000	0.002
total^{+}	-1.908	-0.104	-0.066	-1.768	-0.100	-0.076	-2.318	-0.082	-0.042	-0.724	-0.072	-0.018
total^{-}	-1.688	-0.104	-0.066	-1.572	-0.100	-0.072	-2.102	-0.086	-0.046	-0.660	-0.072	-0.022

- We redefine the q term with a overall 1/3 factor in front to make it more nature with mass and ω terms

Results

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Bender et al. ZPA344,187(1989)

NMEs	^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
	QRPA	jun45	jj44b	QRPA	jun45	jj44b	QRPA	jj55a	GCN50:82	QRPA	jj55a	GCN50:82
M_{qF}	−0.797	−0.379	−0.352	−0.734	−0.360	−0.303	−0.683	−0.408	−0.418	−0.195	−0.358	−0.342
M_{qGT}^{AA}	1.266	1.070	0.994	1.062	1.005	0.868	0.891	0.927	0.917	0.367	0.783	0.736
M_{qGT}^{AP}	2.499	1.614	1.439	2.173	1.524	1.247	2.024	1.422	1.475	0.760	1.202	1.188
M_{qGT}^{PP}	-1.104	-0.648	-0.569	-0.967	-0.610	-0.493	-0.945	-0.577	-0.609	-0.345	-0.485	-0.489
M_{qGT}^{MM}	-1.959	-0.625	-0.545	-1.707	-0.582	-0.475	-1.734	-0.569	-0.608	-0.607	-0.473	-0.485
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M_{qT}^{AP}	1.534	0.008	-0.002	1.406	0.012	0.016	1.730	-0.036	-0.036	0.522	-0.014	-0.030
M_{qT}^{PP}	-0.458	0.000	0.002	-0.416	-0.002	-0.006	-0.484	0.014	0.010	-0.144	0.004	0.006
M_{qT}^{MM}	-0.178	0.000	0.000	-0.158	0.000	-0.002	-0.174	0.002	0.002	-0.052	0.000	0.002
total^{+}	-1.908	-0.104	-0.066	-1.768	-0.100	-0.076	-2.318	-0.082	-0.042	-0.724	-0.072	-0.018
total^{-}	-1.688	-0.104	-0.066	-1.572	-0.100	-0.072	-2.102	-0.086	-0.046	-0.660	-0.072	-0.022

$$M_{1\pm} = M_{GTq} \pm 3 M_{Fq} - 6 M_T$$

- We redefine the q term with a overall 1/3 factor in front to make it more nature with mass and ω terms

Results

DLF et al. PRC110,045502(2024), Huang et al. in preparation

Bender et al. ZPA344,187(1989)

NMEs	^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
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M_{qF}	−0.797	−0.379	−0.352	−0.734	−0.360	−0.303	−0.683	−0.408	−0.418	−0.195	−0.358	−0.342
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$$M_{1\pm} = M_{GTq} \pm 3 M_{Fq} - 6 M_T$$

	^{76}Ge	^{82}Se
M_{GT}	3.014	2.847
M_F	−1.173	−1.071
$M_{GT\omega}$	2.912	2.744
$M_{F\omega}$	−1.025	−0.939
M_{GTq}	1.945	1.886
M_{Fq}	−1.058	−0.966
M_T	−0.612	−0.789
M_P	−0.530	−0.500
M_R	3.594	3.343

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NMEs	^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
	QRPA	jun45	jj44b	QRPA	jun45	jj44b	QRPA	jj55a	GCN50:82	QRPA	jj55a	GCN50:82
M_{RGT}	9.292	4.235	3.713	8.250	4.037	3.314	9.846	4.686	5.048	3.393	3.948	4.080
M_{RT}	−2.281	0.014	0.004	−2.128	0.018	0.028	−2.983	−0.056	−0.056	−0.910	−0.014	−0.042
total	7.011	4.249	3.717	6.123	4.055	3.342	6.863	4.630	4.992	2.483	3.934	4.038
M_P	−0.562	−0.431	−0.279	−0.521	−0.428	−0.152	−0.281	−0.498	−0.425	−0.203	−0.289	−0.255

- As was pointed out by various reference, R term dominates the η mechanism
- And the P term is small, this suppresses the P-wave effect

Results

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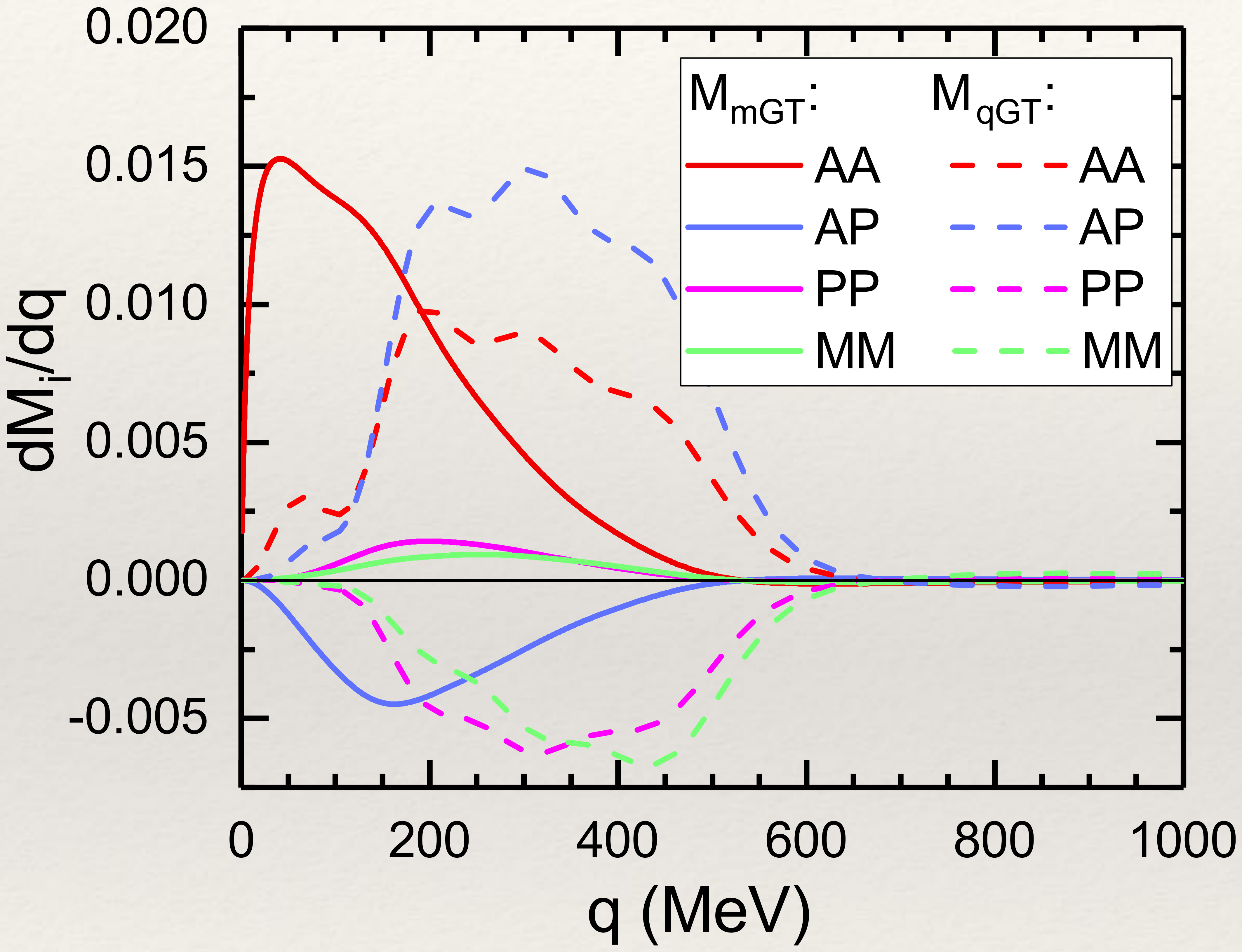
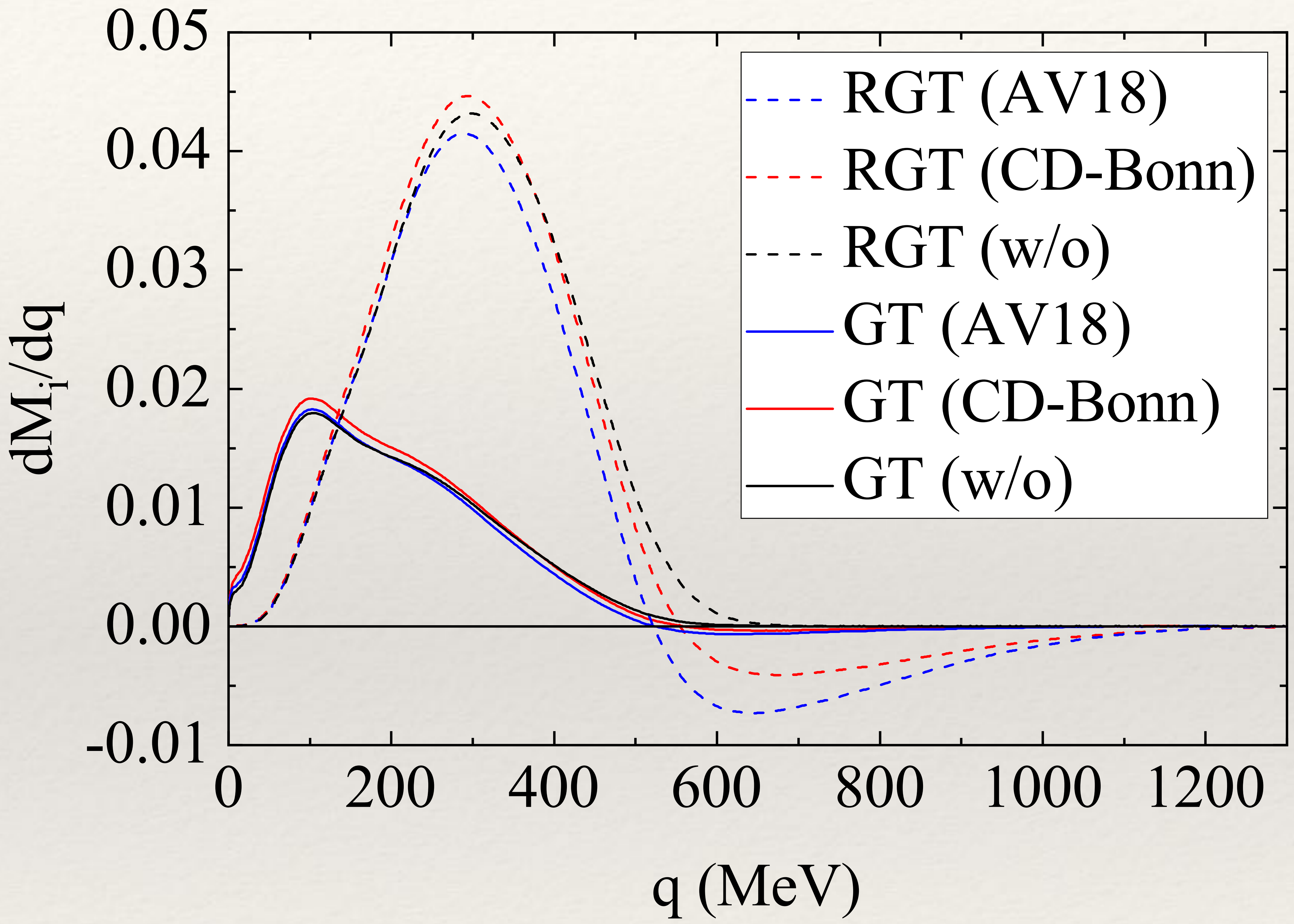




- Different typical exchange momentum changes the NME power counting

Results

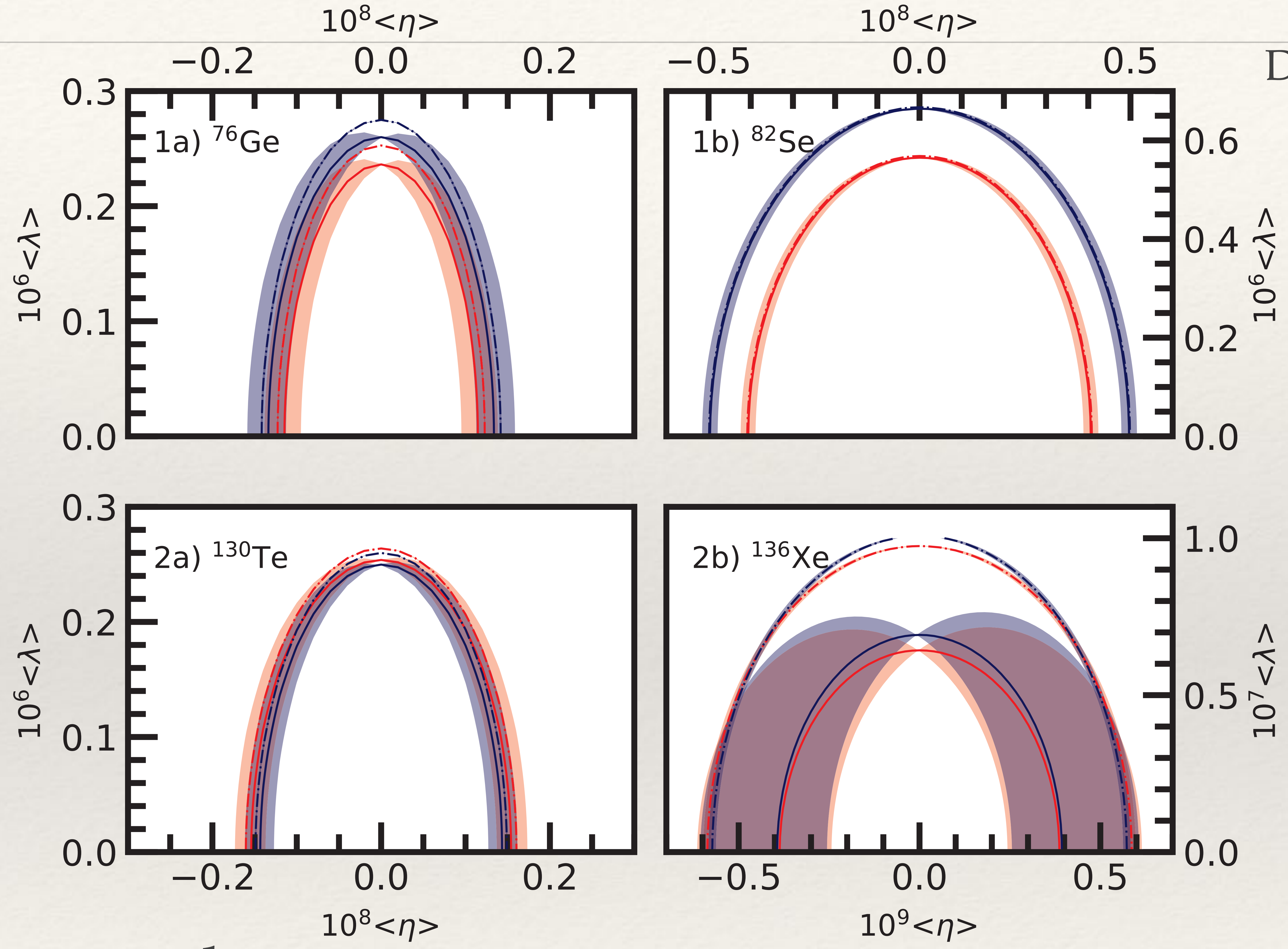
DLF et al. PRC110,045502(2024)

		rough estimation			^{76}Ge			^{82}Se			^{130}Te			^{136}Xe		
		lepton	nuclear	$\mathcal{R}$	$G_{0\nu}$	$M_{0\nu}$		$G_{0\nu}$	$M_{0\nu}$		$G_{0\nu}$	$M_{0\nu}$		$G_{0\nu}$	$M_{0\nu}$	
$\mu_{\beta\beta}$		$\mathcal{O}(1)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	0.24	5.62 5.16		1.02	5.26 4.50		1.43	5.04 5.11		1.46	4.25 4.10	
					r_e	r_N	r_R	r_e	r_N	r_R	r_e	r_N	r_R	r_e	r_N	r_R
$\langle\lambda\rangle$	M_ω	$\mathcal{O}(\epsilon_{12}/m_e)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	1.25	0.78 0.78	0.98 0.98	1.85	0.78 0.78	1.45 1.44	1.61	0.78 0.77	1.25 1.24	1.57	0.78 0.77	1.22 1.21
	M_q	$\mathcal{O}(\omega R)$	$\mathcal{O}(q/m_e)$	$\mathcal{O}(1)$	0.010	55.1 53.6	0.53 0.51	0.012	54.0 52.6	0.65 0.63	0.013	44.2 43.7	0.59 0.58	0.013	43.4 42.5	0.58 0.57
$\langle\eta\rangle$	M_ω	$\mathcal{O}(\epsilon_{12}/m_e)$	$\mathcal{O}(1)$	$\mathcal{O}(1)$	1.25	0.69 0.69	0.86 0.86	1.85	0.69 0.69	1.27 1.27	1.61	0.66 0.66	1.07 1.06	1.57	0.66 0.66	1.04 1.04
	M_q	$\mathcal{O}(\omega R)$	$\mathcal{O}(q/m_e)$	$\mathcal{O}(1)$	0.010	38.1 38.1	0.36 0.36	0.012	37.7 37.4	0.45 0.45	0.013	31.3 29.6	0.41 0.39	0.013	31.4 29.0	0.42 0.39
	M_R	$\mathcal{O}(1)$	$\mathcal{O}(q^2/(M_N m_e))$	$\mathcal{O}(\varepsilon^{-1})$	3.02	73.3 69.4	221.5 209.8	2.96	72.6 70.1	214.8 207.4	2.97	73.8 78.5	219.4 233.4	2.97	73.5 77.9	218.4 231.8
	M_P	$\mathcal{O}(\alpha Z)$	$\mathcal{O}(q/m_e)$	$\mathcal{O}(\varepsilon^{-1})$	0.34	7.40 5.21	2.49 1.75	0.33	7.65 3.18	2.50 1.04	0.27	7.97 6.71	2.19 1.84	0.25	5.41 4.94	1.37 1.25

- An overall estimate of all the terms and we find the dominance of <η> term providing that the three new physics parameters are with similar magnitude

Results

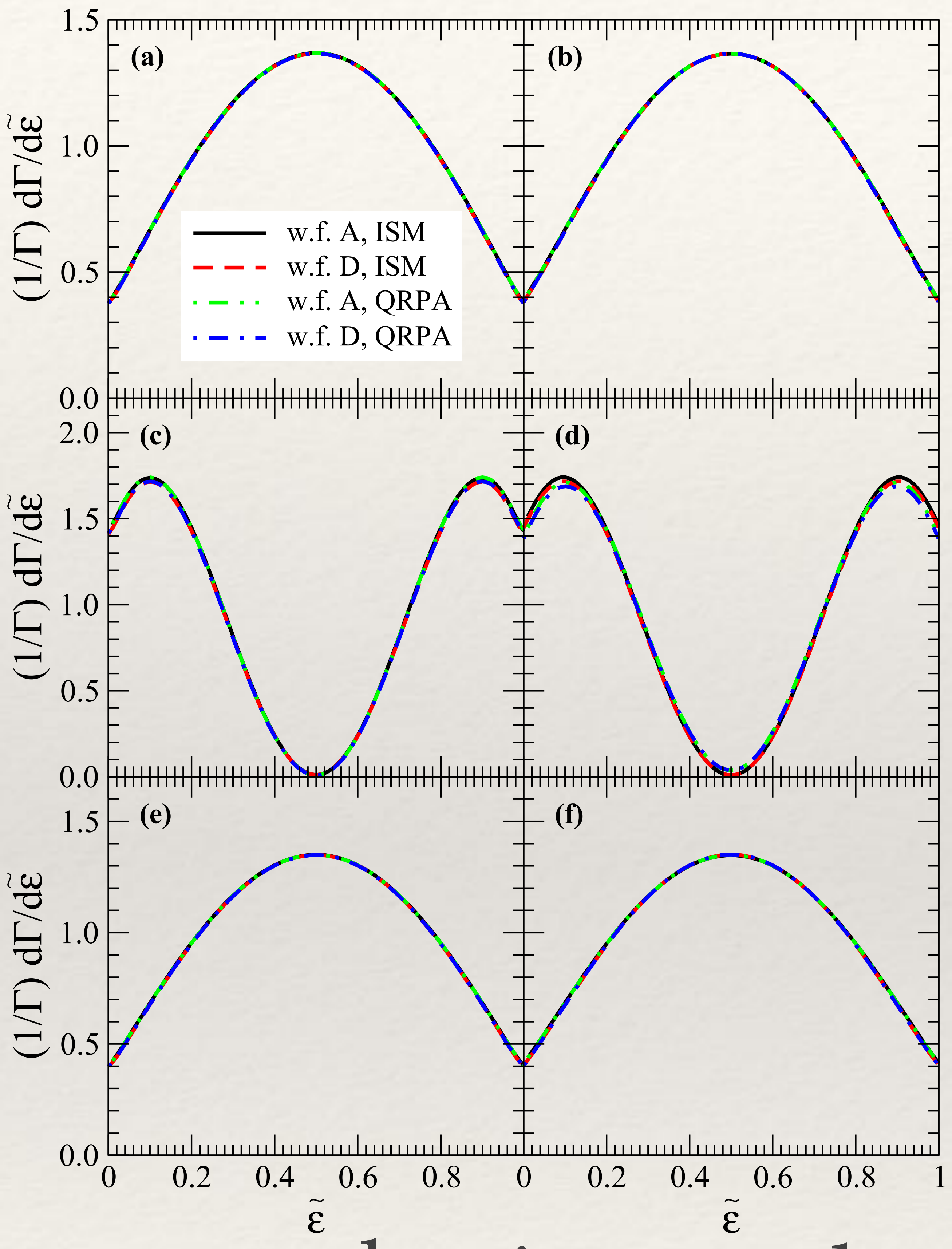
DLF et al. PRC110,045502(2024)

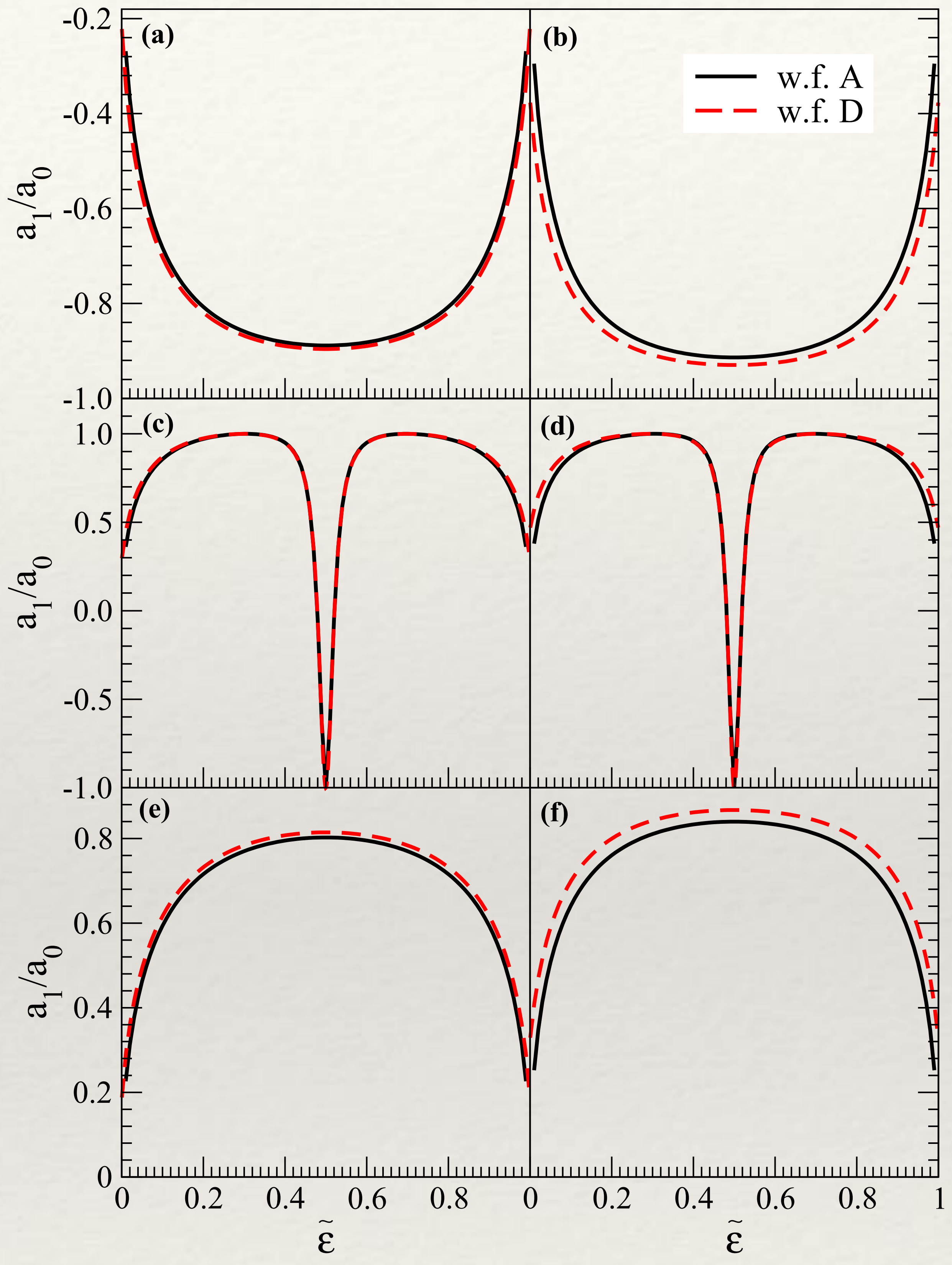


- Constraints on new physics parameters

Mechan. Identification

Stefanik et al. PRC92,055502(2015)





- The long range mechanism can be identified by the spectra and angular distribution

Conclusion

- We calculate the NME for LR symmetric models with neutrino mediated neutrino mechanisms
- We find different behaviors for these NMEs other than that for mass mechanism
- Discrepancies between QRPA and Shell Model is also observed
- We compare our formalism with master formula and decent agreement is achieved

Acknowledgement

- Collaborators for this work
  - Ri-Guang Huang, Jing-Yu Zhu, Chayan Majumdar (IMP, CAS)
  - You-Cai Chen (JLU)
  - Fedor Simkovic (Comenius)
  - Alex Brown (MSU)
  - Yu-Feng Li (IHEP, CAS)

Thanks