Quantum Error Correction for Noise Mitigation in Wave-like Dark Matter Search

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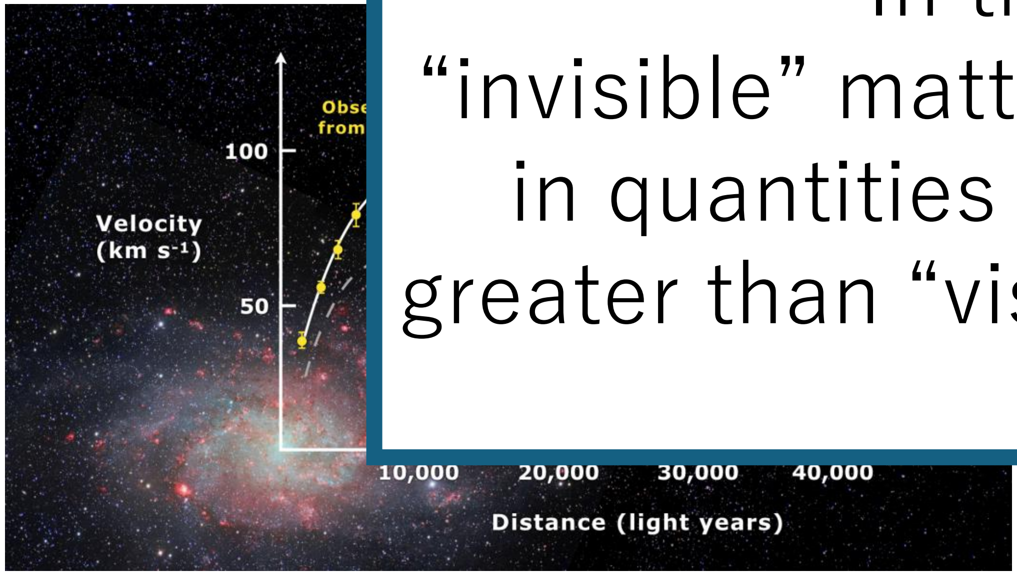
Based on HF, Moroi, Nitta et al. 2510.01816

HF, Moroi, Sichanugrist 2511.03253

Introduction

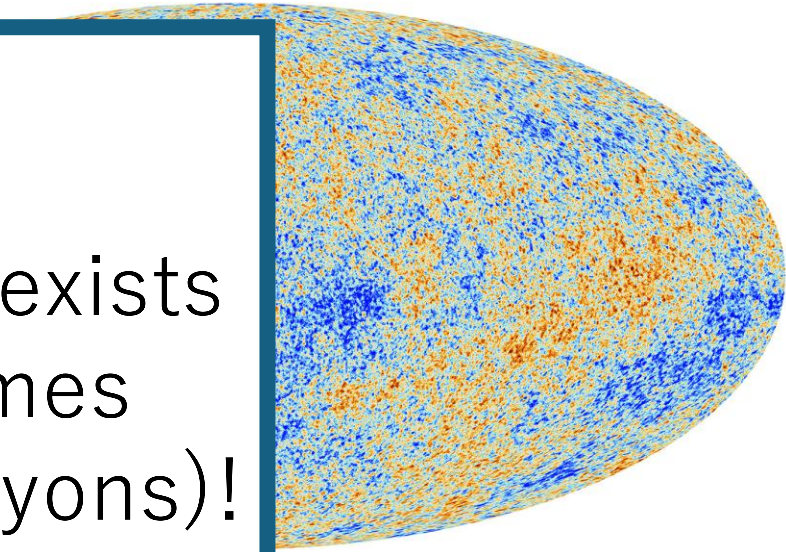
Dark Matter in the Universe

In the universe,
"invisible" matter (**dark matter**) exists
in quantities more than five times
greater than "visible" matter (baryons)!



Rotation velocity of galaxy
From Wikimedia Commons

Pink: X-ray observation
Blue: Gravitational lensing



n, From ESA

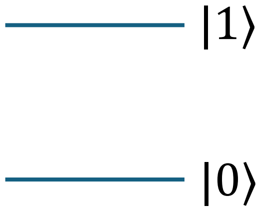
See T. Nitta's talk

Use of Quantum Sensors

- In this talk, I focus on wave-like dark matter, $m \lesssim \text{eV}$
- **Quantum sensors** are useful for such light dark matter

  Quantum sensor: an artificial two-level systems such as a **qubit**

  (Our results can be used to cavities if the states are read-out to qubits)



- Why?
  - Qubits typically have very small energy gaps, $\Delta E \ll \text{eV}$
  - As we will see soon, qubits are insensitive to the unknown phase of the dark matter: $\phi(x) \sim \phi_0 \cos(mt + \varphi)$
  - Use of entanglement of qubits

DM-Sensor Interaction

- We want to know DM-SM interaction strength, e.g.,
  - $\frac{a}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu}$ for axion-like particles, $-\frac{1}{2}\varepsilon F_{\mu\nu}F'^{\mu\nu}$ for dark photons
- The interaction is reduced to an effective potential
  - Since qubits are two-level systems, it is written using Pauli matrices
- Of course, the details vary, but let us assume the potential is proportional to σ_X
- The qubit Hamiltonian is

$$H = H_0 + \Delta H, \qquad H_0 = -\frac{1}{2}\omega\sigma_z, \qquad \Delta H = 2\eta\sigma_X \sin mt$$

All the information we need is in η!
→We want to estimate η using qubits

Time Evolution of Qubits in DM Wave

- We first move to the interaction picture

$$
\begin{aligned}
H_I &= e^{iH_0 t} \Delta H e^{-iH_0 t} \\
&= \eta \sigma_X \cos(m-\omega)t + \text{(high freq mode)} \simeq \eta \sigma_X
\end{aligned}
$$

Here, we assume DM is resonant to the qubit frequency

e.g., terms proportional to $\cos(m+\omega)t$ such terms are averaged out to zero even if we numerically integrate

- Starting from $|0\rangle$, $|\psi(t)\rangle \simeq (1 - iH_I t)|0\rangle = |0\rangle - i\eta|1\rangle$
- We measure $|1\rangle$ against $|\psi(t)\rangle$ to extract η

If we include the phase φ, $|\psi(t)\rangle \simeq |0\rangle - i\eta e^{i\varphi}|1\rangle$

Quantum Noise on Qubits

What are Quantum Noises?

- What is “quantum” noise in the first place?
  - An operation that changes the state of a sensor to another state with a certain classical probability.

$$|\psi(t_1)\rangle \rightarrow \begin{cases} |\psi(t_1)\rangle & \text{probability } 1-p \\ E|\psi(t_1)\rangle & \text{probability } p \end{cases}, E\text{: Noise operator}$$

- Few comments:
  - We may use density matrices for a unified description
  - E need not be Hermitian
  - We can write a time evolution equation with E (Lindblad eq.)
    - It’s a kind of a generalization of Schrödinger eq.

Thermal Relaxation Noise

- We consider the thermal relaxation noise:

$$E = \left\{1, \sigma_Z, a = |0\rangle\langle 1|, a^\dagger = |1\rangle\langle 0|\right\}$$

  - 1: No noise
  - σ_Z: "Decoherence" noise (erasing the phase information)
  - a: De-excitation noise
  - $a^\dagger$: Excitation noise
- Among them, $a^\dagger$ is most annoying, since it mimics the signal
- In this talk, I ask **if we can reduce the excitation noise**

Correcting Quantum Noise

Signal-Noise Distinction

Shu, Xu, Xu 24, HF, Moroi, Nitta et al. 25, Freiman et al, 25; See B. Xu's talk

Q. Can we distinguish noises and signals?

$$-iH_I t|0\rangle \sim \sum_i \sigma_X^i |0\rangle$$

Signal: **All** qubits are excited

$$a_i^\dagger|0\rangle \qquad \left(\rho = \sum a_i^\dagger|0\rangle\langle 0|a_i\right)$$

Noise: A **particular** qubit is excited

Off-diagonal terms are absent → entangled states can distinguish!

	Normal measurements：Do projection measurements with $\lvert 10\ldots 0\rangle, \lvert 01\ldots 0\rangle, \ldots \lvert 00\ldots 1\rangle$ and sum the probability	Projection measurement by $\lvert W\rangle \equiv \frac{1}{\sqrt{N}}(\lvert 10\ldots 0\rangle + \lvert 01\ldots 0\rangle + \cdots \lvert 00\ldots 1\rangle)$
$\eta\sum_i \sigma_X^i \lvert 0\rangle$	$p = \eta^2 + \eta^2 + \ldots\eta^2 = N\eta^2$	$p = \left(\eta\cdot\frac{N}{\sqrt{N}}\right)^2 = N\eta^2$
$\sqrt{\gamma}a_1^\dagger\lvert 0\rangle$	$p = \gamma$	$p = \frac{\gamma}{N}$

When the Excitation Noise is Too Large…

HF, Moroi, Nitta et al. 25

- **W state is no more useful when the noise is too large**
- With a W state, part of the signal is lost under noise:

$$(1 - iH_I t_1)|0\rangle \to a_1^\dagger (1 - iH_I t_1)|0\rangle$$

- W state cannot measure η from these states
  - W state is one-qubit-excited state, but multiple qubits are excited

Signal/Noise Subspace

- Let us focus on the Hilbert space with/without noise

$$(1 - iH_I t_1)|0\rangle, \qquad a_1^\dagger(1 - iH_I t_1)|0\rangle$$

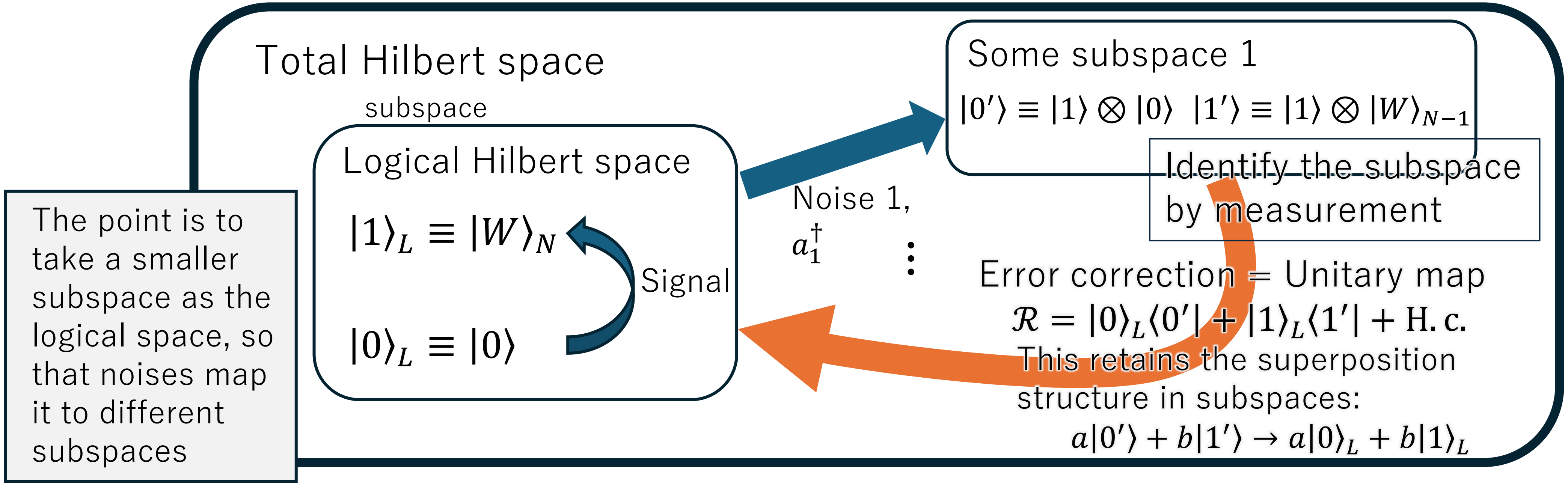
superposition of $|0\rangle$ and $|W\rangle_N$ (almost) superposition of $|1\rangle \otimes |0\rangle$ and $|1\rangle \otimes |W\rangle_{N-1}$

  - $|1\rangle \otimes |0\rangle$ and $|W\rangle_N$ are almost orthogonal：
    - $\langle W|(|1\rangle \otimes |0\rangle) = \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$
  - $|1\rangle \otimes |W\rangle_{N-1}$ is 2-qubit-excited states
- States with/without noise lives almost orthogonal subspaces

$$\{|0\rangle, |W\rangle_N\}, \qquad \{|1\rangle \otimes |0\rangle, |1\rangle \otimes |W\rangle_{N-1}\}$$

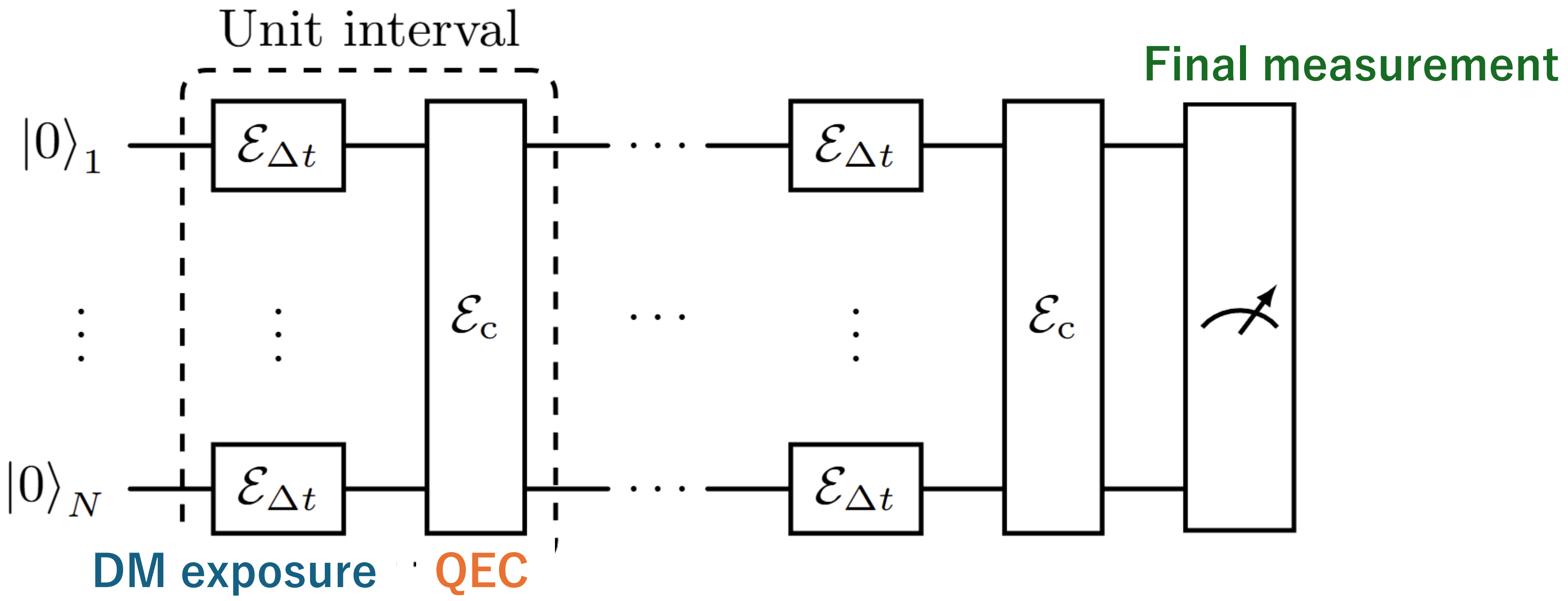
- We may do the **quantum error correction** (QEC)!

Quantum Error Correction

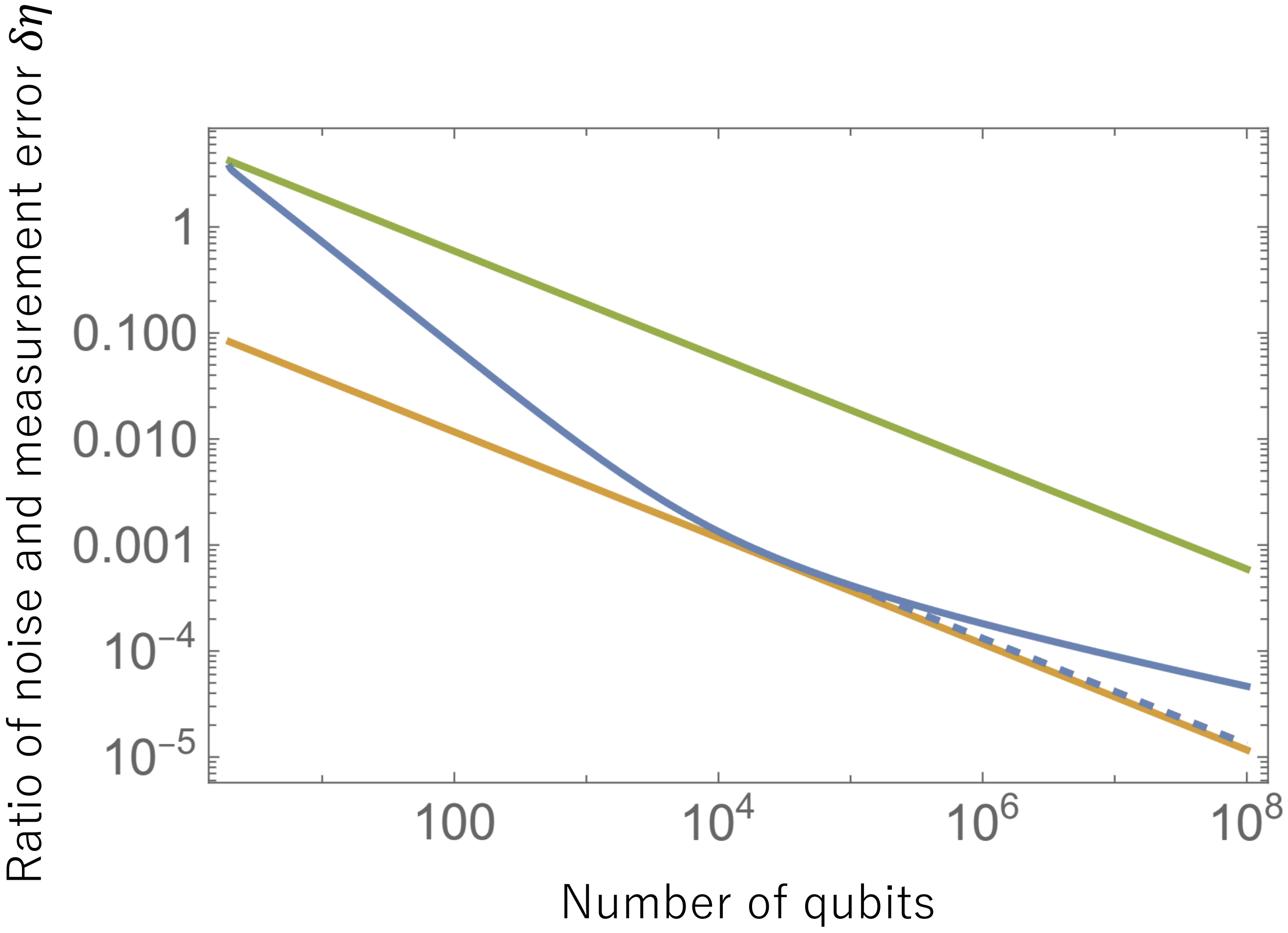


Measurement Protocol

- We want to measure DM while “correcting” errors
- Repeat error “correction” at short intervals



Numerical Result



For simplicity, we consider only excitation noises

Green: No QEC
Orange: The best sensitivity, which is impossible due to unknown phase of DM
Blue, solid: With QEC
Blue, dotted: With QEC and further improvement
(with enough number of qubits, QEC is no more effective and there's more efficient way of measurement)

⇒**With QEC, we achieved the best sensitivity!**

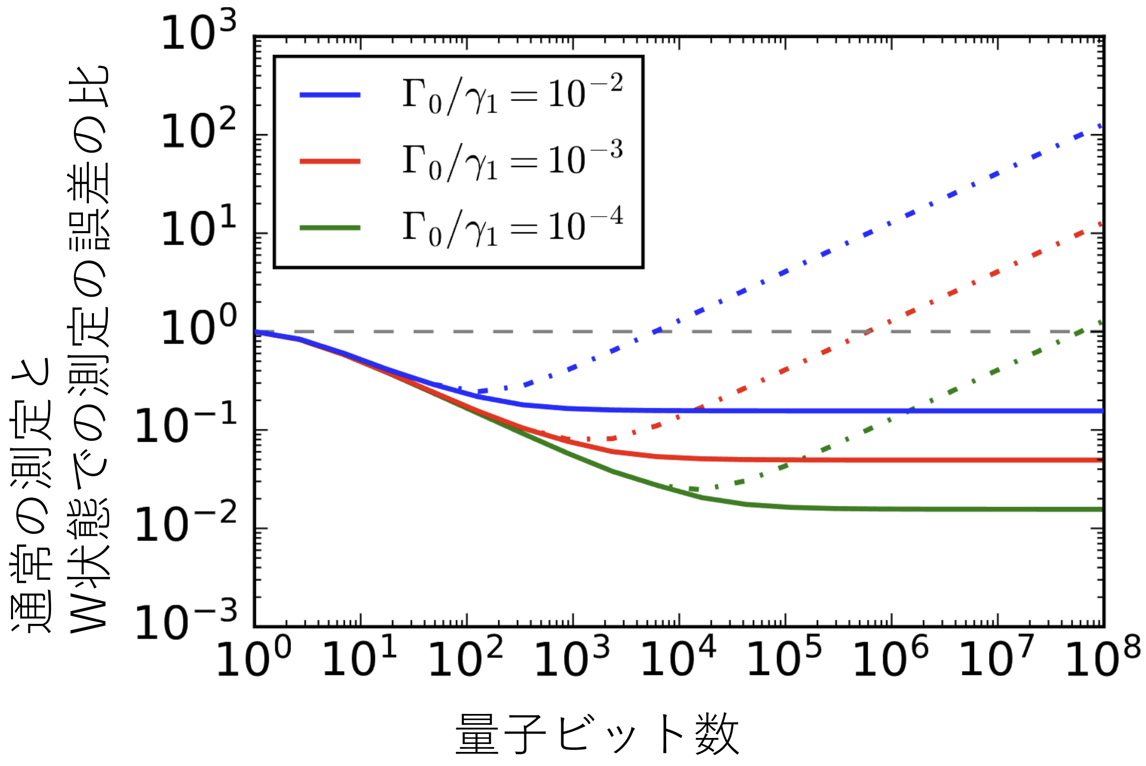
Summary

- Excitation noise are crucial for light DM search with quantum sensors
- We may reduce the noise by using entangled sensors
- By using the QEC-like protocol, we achieve the best sensitivity

Backup

Result for W

- With N qubits, under the thermalization error
- Take the ratio of the standard deviation of estimator of η
  - It is roughly “the ratio of the measurement error of η”



Γ_0: Excitation noise rate
γ_1: Other noise rate
dotted: Measurement with W-state
solid: With some improvement

Dotted line becomes worse from some point
（the sensitivity is limited by signal strength）
With grouping/GHZ-like state/frequency width,
the sensitivity saturates to some value,
stopping to become worse

Error Detection for DM Search

- Signal/Noise subspaces are not strictly orthogonal
- Therefore, the syndrome measurement (detection of the error subspace) should be POVM-measurement
- We propose the following：

Projective meas.：$|\psi\rangle \to P_i|\psi\rangle$, P_i is a projection op. $P_i^2 = P_i$
POVM meas.：$|\psi\rangle \to M_i|\psi\rangle$, M_i is (normalized) "positive op"

$$M_0 \equiv |0\rangle\langle 0| + |W\rangle\langle W| \,,$$

No noise

$$M_i \equiv \sqrt{\frac{N}{N-1}}\, |i\rangle_\perp\langle i|_\perp + \frac{\sqrt{N(N-1)}}{N-2}\, |2,i\rangle_\perp\langle 2,i|_\perp$$

Error on i-th qubit

$$|i\rangle_\perp \equiv |i\rangle - \frac{1}{\sqrt{N}}\, |W\rangle \,,$$

$$|2,i\rangle_\perp \equiv |2,i\rangle - \sqrt{\frac{2}{N}}\, |S\rangle \,.$$

$$M_S \equiv |S\rangle\langle S| \,,$$

Rest

$|S\rangle$: 2 qubit excitation state
$|i\rangle_\perp$: almost i-th excited state, but orthogonal to W
$|2,i\rangle_\perp$: almost $|i\rangle \otimes |W\rangle_{N-1}$, but orthogonal to $|S\rangle$