Specially embedding a Composite Axion Model

Shihwen Hor, Yuichiro Nakai, Motoo Suzuki, Junxuan Xu,
JHEP 08 (2026) 025, arXiv: 2604.26010 [hep-ph].

Shihwen Hor
Tsung-Dao Lee Institute

@ New directions EXploring the Unknown beyond the Standard Model (NEXUS 2026),
13th September 2026.

Outlines

- 1. Introduction
- 2. Original composite axion model
- 3. Specially embedded composite axion
- 4. Axion cosmology
- 5. Summary

1. Introduction

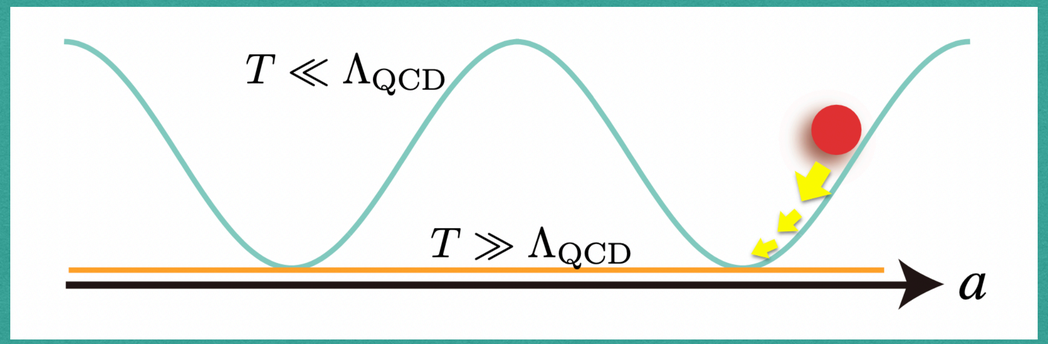
Strong CP problem

- QCD Lagrangian allows $\mathscr{L}_\theta = \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu}$
- The physical strong CP phase $\bar{\theta} = \theta - \arg\det(M_u M_d)$
- The current upper bound on the neutron electric dipole moment $|\bar{\theta}| < 10^{-11}$

Why is $\bar{\theta}$ so small?

Axion solution

[Fuminobu Takahashi's slide]



- Peccei-Quinn mechanism
- $\mathscr{L}_\theta = \left(\theta + \dfrac{a}{f_a}\right) \dfrac{g_s^2}{32\pi^2} G^{a\mu\nu}\tilde{G}^a_{\mu\nu}$
- Axion: a **pNG boson** from the spontaneous breaking of **global $U(1)_{\rm PQ}$ symmetry**
- Non-perturbative QCD effect: explicitly break the $U(1)_{\rm PQ}$

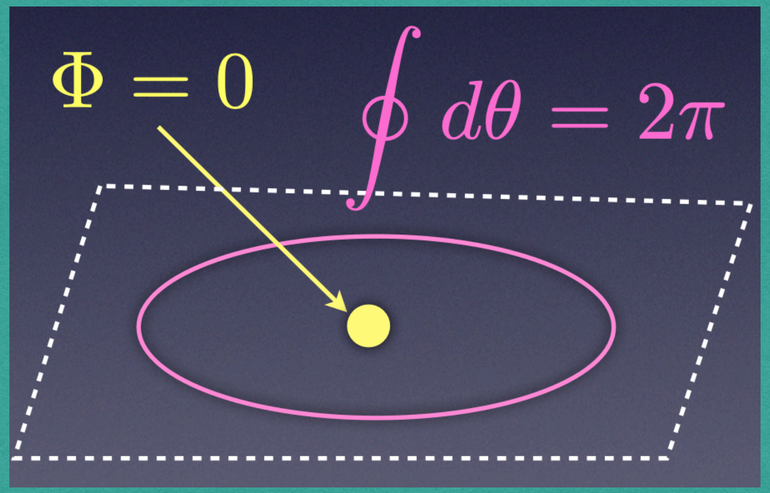
$$V(a) \sim m_\pi^2 f_\pi^2 \cos\left(\theta + \frac{a}{f_a}\right)$$

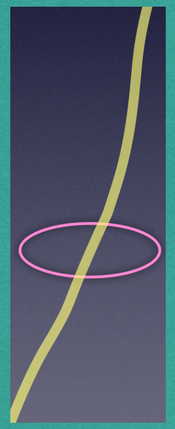
$$10^8\ {\rm GeV} \lesssim f_a \lesssim 10^{12}\ {\rm GeV}$$

- **Dark matter candidate**

Cosmic strings

- ★ $U(1)_{\rm PQ}$ **breaking after inflation —> formation of topological defects**
- ★ $\Phi_{\rm PQ} \sim v_{\rm PQ} e^{i\theta_a}$ (axion: phase direction of the PQ scalar)
- ★ Axion field: spatial variations across the universe





Formation of cosmic strings

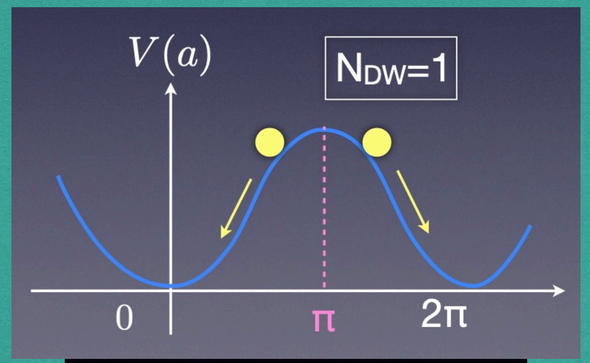
[Masahiro Kawasaki's slide]

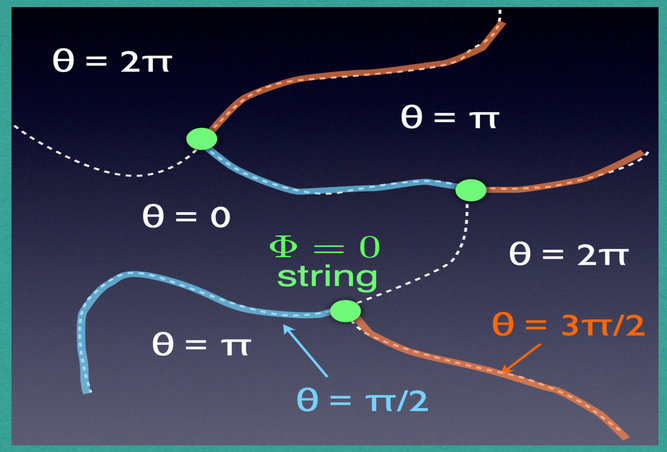
Domain walls

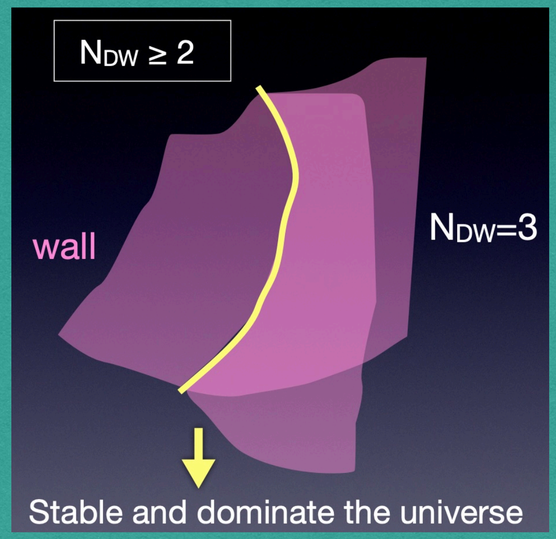
Domain wall number

★ Axion gets a potential with $N_{\rm DW}$ minima

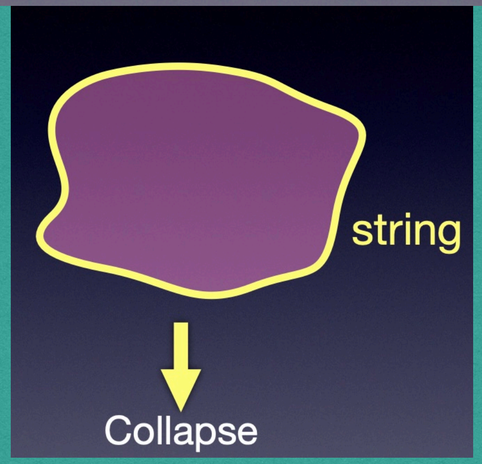
[Masahiro Kawasaki's slide]







$N_{\rm DW} = 1$



$N_{\rm DW} > 1$

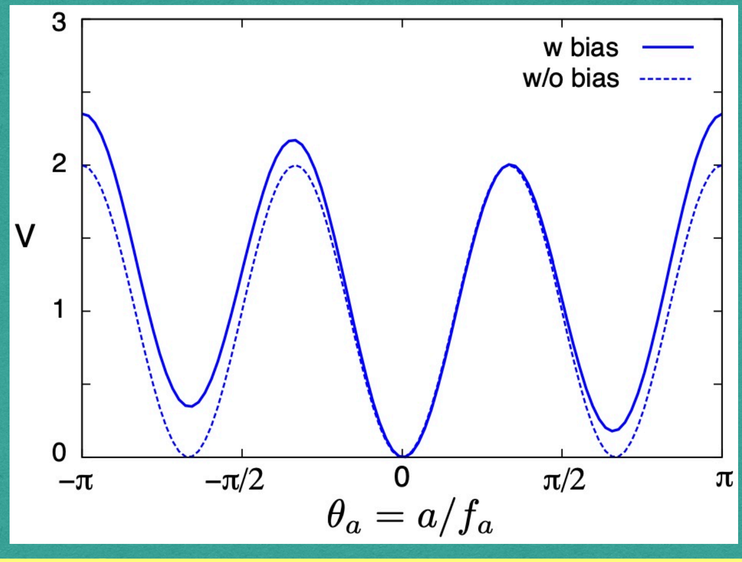
Domain walls form a stable network —> dominates the universe

Domain wall problem

Bias term

- A potential solution: **a small explicit breaking of the $U(1)_{\rm PQ}$ symmetry** (bias term)
- **Bias term: lift the vacuum degeneracy**
- Domain walls collapse before dominating the universe
- Axion DM abundance is changed
- Problem:
  - The introduction of the bias term
  - Axion quality

[Yuichiro Nakai's slide]



Small instantons

- QCD θ-vacuum: superposition of n-vacua: $|\theta> = \sum_{n=-\infty}^{\infty} e^{-in\theta}|n> = \ldots + |0> + e^{-i\theta}|1> + \ldots$
- Instanton: localized objects in Euclidean spacetime

The tunneling effect between degenerate n-vacua

- SU(2) BPST instanton solution with $Q = 1$:
  - $\frac{g^2}{32\pi^2}\int d^4x F^{a\mu\nu}\tilde{F}^a_{\mu\nu}|_{\text{inst.}} = Q \quad (Q \in \mathbb{Z})$
  - $A^a_\mu(x)|_{1-\text{inst.}} = 2\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}$ ← Instanton size
- Instantons contribute to the axion potential $\propto \exp\left(-\frac{8\pi^2}{g^2(1/\rho)}\right)$
- **A hidden gauge sector —> small instanton effects —> bias term —> special embedding**

2. Original composite axion model

Composite axion model

Axion emerges as a pion-like bound state of a new gauge dynamics

Fields	$[SU(3)_c]$	$[SU(N)]$	$U(1)_{\rm PQ}$	$[U(1)_Y]$
Q	$\mathbf{3}$	$\mathbf{N}$	1	a
η	$\mathbf{1}$	$\mathbf{N}$	-3	b
$\bar{Q}$	$\bar{\mathbf{3}}$	$\overline{\mathbf{N}}$	0	$-a$
$\bar{\eta}$	$\mathbf{1}$	$\overline{\mathbf{N}}$	0	$-b$

- An $SU(N)$ **gauge theory**
- **Global** $SU(4)_L \times SU(4)_R$ **flavor symmetry**
  - The diagonal $SU4)_V$ subgroup is partly gauged $SU(4)_V \supset SU(3)_c$
- The global $U(1)_{\rm PQ}$ is embedded in the $SU(4)_L$ flavor symmetry
  - Anomaly coefficients

$$\mathscr{A}(U(1)_{\rm PQ} - SU(3)_c^2) = N\,, \qquad \mathscr{A}(U(1)_{\rm PQ} - SU(N)^2) = 0\,,$$

Composite axion model

- **Below the $SU(N)$ confinement scale $\Lambda_{\rm conf}$, fermion condensates break the flavor symmetry $SU(4)_L \times SU(4)_R \to SU(4)_V$**
  - 15 pNG bosons $\mathbf{15} \to \mathbf{1} + \mathbf{3} + \overline{\mathbf{3}} + \mathbf{8}$
  - Singlet: axion
  - The colored states obtain masses $m_{\mathbf{r}}^2 \approx \dfrac{g_c^2(F_a)}{16\pi^2} C_2(\mathbf{r}) \Lambda_{\rm conf}^2$,
- **Axion emerges as a pion-like bound state of a new confining gauge dynamics**
- Baryons and mesons

Two difficulties

- **Domain problem**
  - Domain wall number $N_{\mathrm{DW}} = N$
  - Special embedding
  - Small instanton effects —> bias term
- **Charged massive particles (CHAMPs)**
  - Charged under $SU(3)_c$
  - The hadrons couple with SM quarks around the QCD phase transition and acquire electromagnetic charges
  - Solved by higher dimensional operators —> decay into the SM sector

3. Specially embedded composite axion

[S. Hor, Y. Nakai, M. Suzuki, J. Xu, JHEP 09 (2025) 047.]

Special embedding

- The identify the IR gauge group as a special subgroup of a UV gauge group —> small instanton effects resolving the vacuum degeneracy of the axion potential
- Gauge symmetry breaking: $SU(MN) \to SU(N)$
  - $T^m_{\rm UV} \quad (m = 1,...,(MN)^2 - 1), \qquad T^a_{\rm IR} \quad (a = 1,...,N^2 - 1)$
  - $T^a_{\rm IR} = \mathcal{O}^{am} T^m_{\rm UV}$
- $\mathbf{r}$ rep. of $SU(N)$: embedded into the fund. rep. of $SU(MN)$
  - $\mathrm{tr}(T^m_{\rm UV} T^n_{\rm UV}) = \frac{1}{2}\delta^{mn}, \qquad \mathrm{tr}(T^a_{\rm IR} T^b_{\rm IR}) = T_{\rm IR}(\mathbf{r})\delta^{\mathbf{ab}},$ Dynkin index
  - $\mathcal{O}^{am}\mathcal{O}^{bn}\delta^{ma} = c\delta^{ab}, \quad c \equiv \frac{T_{\rm IR}(\mathbf{r})}{1/2}$

Special embedding: $c > 1$

[S. Hor, Y. Nakai, M. Suzuki, J. Xu, JHEP 09 (2025) 047.]

Special embedding

- **A Weyl fermion**: transforming as the **fund. rep. of** $SU(MN)$, but the **r rep. of** $SU(N)$ subgroup

$$\mathscr{D}_\mu\psi = \partial_\mu\psi - ig_{\rm UV}A^m_{{\rm UV},\mu}(T^m_{\rm UV})\psi$$

$$\supset \partial_\mu\psi - ig_{\rm IR}A^a_{{\rm IR},\mu}(T^a_{\rm IR})\psi$$

- Canonically normalized kinetic term $A^\ell_{{\rm UV},\mu} = \dfrac{g_{\rm IR}}{g_{\rm UV}}A^a_{{\rm IR},\mu}(\mathscr{O})^{a\ell}$

=> $g_{\rm IR} = g_{\rm UV}/\sqrt{c}$

- θ-term: $\displaystyle\int \frac{g^2_{\rm UV}}{8\pi^2}{\rm tr}(F_{\rm UV}\wedge F_{\rm UV}) = \int \frac{cg^2_{\rm IR}}{8\pi^2}{\rm tr}(F_{\rm IR}\wedge F_{\rm IR})$

Specially embedded composite axion model

Fields	$[SU(4N)]$	$[SU(3)_2]$	$[SU(N)_2]$	$[U(1)_X]$	$U(1)_A$	$U(1)_B$	$SU(4N)_L$	$SU(4N)_R$
Ψ	$\mathbf{4N}$	$\mathbf{1}$	$\mathbf{1}$	x	1	1	$\mathbf{4N}$	$\mathbf{1}$
$\bar{\Psi}$	$\overline{\mathbf{4N}}$	$\mathbf{1}$	$\mathbf{1}$	$-x$	0	-1	$\mathbf{1}$	$\overline{\mathbf{4N}}$
Φ	$\mathbf{(4N)^2-1}$	$\mathbf{8}$	$\mathbf{1}$	0	0	0	$\mathbf{1}$	$\mathbf{1}$
Φ'	$\mathbf{(4N)^2-1}$	$\mathbf{1}$	$\mathbf{N^2-1}$	0	0	0	$\mathbf{1}$	$\mathbf{1}$
Φ''	$\mathbf{4N}$	$\mathbf{1}$	$\overline{\mathbf{N}}$	$3z$	0	0	$\mathbf{1}$	$\mathbf{1}$

- Field decomposition under

$$SU(4N)\times SU(3)_2\times SU(N)_2\times U(1)_X \to [SU(4)_1\times SU(N)_1]_{4N}\times SU(3)_2\times SU(N)_2\times U(1)_X$$

- $\Phi = (\mathbf{(4N)^2-1, 8, 1}, 0) \;\to\; (\mathbf{15, N^2-1, 8, 1}, 0)\oplus(\mathbf{15, 1, 8, 1}, 0)\oplus(\mathbf{1, N^2-1, 8, 1}, 0)$,
- $\Phi' = (\mathbf{(4N)^2-1, 1, N^2-1}, 0) \;\to\; (\mathbf{15, N^2-1, 1, N^2-1}, 0)\oplus(\mathbf{15, 1, 1, N^2-1}, 0)\oplus(\mathbf{1, N^2-1, 1, N^2-1}, 0)$,
- $\Phi'' = (\mathbf{4N, 1, \overline{N}}, 3z) \to (\mathbf{4, N, 1, \overline{N}}, 3z)$.

Specially embedded composite axion model

- Spontaneous symmetry breaking by the VEVs of Φ, Φ', Φ''
  - $(\Phi^A_B)^c_d = (\Phi)^I_i (T^I_{SU(4)})^A_B (T^i_{SU(3)})^c_d$, $\quad \langle \Phi^I_i \rangle = \begin{pmatrix} v\mathbf{1}_{8\times 8} \\ \mathbf{0}_{7\times 8} \end{pmatrix}_{15\times 8}$,
  - $\langle (\Phi')^J_K \rangle = v' \delta^J_K$.
  - $\langle \Phi'' \rangle = \langle (\mathbf{1}, \mathbf{1}, -3, 3z) \rangle = v''$,
- Symmetry breaking
  - $SU(4N) \times SU(3)_2 \times SU(N)_2 \times U(1)_X \xrightarrow{\langle \Phi \rangle} SU(3)_V \times U(1)_Z \times SU(N)_{4N} \times SU(N)_2 \times U(1)_X$,
  - $SU(4N) \times SU(3)_2 \times SU(N)_2 \times U(1)_X \xrightarrow{\langle \Phi' \rangle} SU(N)_V \times SU(4)_{4N} \times SU(3)_2 \times U(1)_X$.
  - $SU(N)_V \times SU(3)_c \times U(1)_Z \times U(1)_X \xrightarrow{\langle \Phi'' \rangle} SU(N)_V \times SU(3)_c \times U(1)_Y$.
    - $Q_Y = \sqrt{24}\, z\, T^{15} + Q_X = z\, Q_Z + Q_X$, where $T^{15} = 1/(2\sqrt{6})\mathrm{diag}(1,1,1,-3)$
- $SU(4N) \times SU(3)_2 \times SU(N)_2 \times U(1)_X \to SU(N)_V \times SU(3)_c \times U(1)_Y$.

Specially embedded composite axion model

- Under $SU(N)_V \times SU(3)_c \times U(1)_Y$ symmetry

  $\Psi : (\mathbf{N}, \mathbf{3}, z+x) \oplus (\mathbf{N}, \mathbf{1}, -3z+x)\ , \quad \bar{\Psi} : (\overline{\mathbf{N}}, \overline{\mathbf{3}}, -z-x) \oplus (\overline{\mathbf{N}}, \mathbf{1}, 3z-x)\ ,$

- Axion appears in the phase of the quark condensates after the confinement

- $$\langle \Psi\bar{\Psi} \rangle \simeq \Lambda_{\rm conf}^3 \Sigma(x) = \Lambda_{\rm conf}^3 \exp\left(\frac{2i}{F_a} \begin{pmatrix} \pi_8(x) + \frac{a(x)}{2\sqrt{6}} \mathbf{1}_{3\times 3} & K(x) \\ K^\dagger(x) & -\frac{3a(x)}{2\sqrt{6}} \end{pmatrix} \right) ,$$

- $f_{\rm PQ}/\sqrt{6} = F_a \sim \Lambda_{\rm conf}/(4\pi)\ .$

Matching conditions

- At the cut-off scale, gauge coupling matching conditions
  - $$\frac{1}{g_c^2(\Lambda_{\rm cut})} = \frac{1}{g^2_{SU(3)_2}(\Lambda_{\rm cut})} + \frac{N}{g^2_{SU(4N)}(\Lambda_{\rm cut})} \,,$$
  - $$\frac{1}{g_Y^2(\Lambda_{\rm cut})} = \frac{1}{g^2_{U(1)_X}(\Lambda_{\rm cut})} + \frac{24z^2N}{g^2_{SU(4N)}(\Lambda_{\rm cut})} \,,$$
  - $$\frac{1}{g_N^2(\Lambda_{\rm cut})} = \frac{1}{g^2_{SU(N)_2}(\Lambda_{\rm cut})} + \frac{4}{g^2_{SU(4N)}(\Lambda_{\rm cut})} \,.$$
- Below the cut-off scale $\Lambda_{\rm cut}$ and above the $SU(N)_V$ confinement scale $\Lambda_{\rm conf}$
  - $$\frac{1}{\alpha_c(\mu)} = \frac{1}{\alpha_c(\Lambda_{\rm conf})} + \frac{1}{2\pi}\left(7 - \frac{2N}{3}\right)\log(\mu/\Lambda_{\rm conf}) \,,$$
  - $$\frac{1}{\alpha_N(\mu)} = \frac{1}{\alpha_N(\Lambda_{\rm conf})} + \frac{1}{2\pi}\left(\frac{11}{3}N - \frac{8}{3}\right)\log(\mu/\Lambda_{\rm conf}) \,,$$

Matching conditions

- θ-terms
  - UV : $\int \frac{\theta_{4N} g^2_{SU(4N)}}{8\pi^2} \mathrm{tr}(F_{4N} \wedge F_{4N}) + \frac{\theta_3 g^2_{SU(3)_2}}{8\pi^2} \mathrm{tr}(F_3 \wedge F_3) + \frac{\theta_{N_2} g^2_{SU(N)_2}}{8\pi^2} \mathrm{tr}(F_{N_2} \wedge F_{N_2})$
  - IR : $\int \frac{N\theta_{4N} g_c^2}{8\pi^2} \mathrm{tr}(F_c \wedge F_c) + \frac{\theta_3 g_c^2}{8\pi^2} \mathrm{tr}(F_c \wedge F_c) + \int \frac{4\theta_{4N} g_N^2}{8\pi^2} \mathrm{tr}(F_N \wedge F_N) + \frac{\theta_{N_2} g_N^2}{8\pi^2} \mathrm{tr}(F_N \wedge F_N)$,
  - $\theta_c = N\theta_{4N} + \theta_3$, $\theta_N = 4\theta_{4N} + \theta_{N_2}$,
- Anomaly coefficients
  - For $U(1)_A$: $\mathscr{A}\left(U(1)_A - SU(3)_c^2\right) = N$, $\mathscr{A}\left(U(1)_A - SU(N)_V^2\right) = 4$,
  - For $U(1)_{\mathrm{PQ}}$: $\mathscr{A}\left(U(1)_{\mathrm{PQ}} - SU(3)_c^2\right) = N$, $\mathscr{A}\left(U(1)_{\mathrm{PQ}} - SU(N)_V^2\right) = 0$.

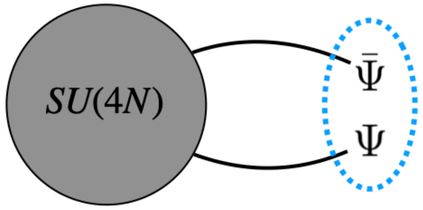
Axion

- After $SU(N)_V$ confinement, $U(1)_A$ and $U(1)_{\rm PQ}$ are spontaneously broken
  - 2 pNG bosons: $\theta_a = a/f_{\rm PQ}$ and $\theta_\eta = \eta/f_{\rm PQ}$
- $\int (N\theta_a + N\theta_\eta + \theta_c)\frac{g_c^2}{8\pi^2}\mathrm{tr}(F_c \wedge F_c)$,
- $\int (4\theta_\eta + \theta_N)\frac{g_N^2}{8\pi^2}\mathrm{tr}(F_N \wedge F_N)$.
  - θ_a : axion, solve the strong CP
  - θ_η remove θ_N of the $SU(N)_V$ strong dynamics
- **Axion potential (non-perturbation QCD effects) :** $V_{\rm QCD} \approx -\frac{m_u m_d}{(m_u+m_d)^2} m_\pi^2 f_\pi^2 \cos\left(N\theta_a + \bar{\theta}_c\right),$
  - $\bar{\theta}_c \equiv \theta_c + N\langle\theta_\eta\rangle - \arg\det\left(M_u M_d\right)$

$f_a = f_{\rm PQ}/N$

Small instanton effects

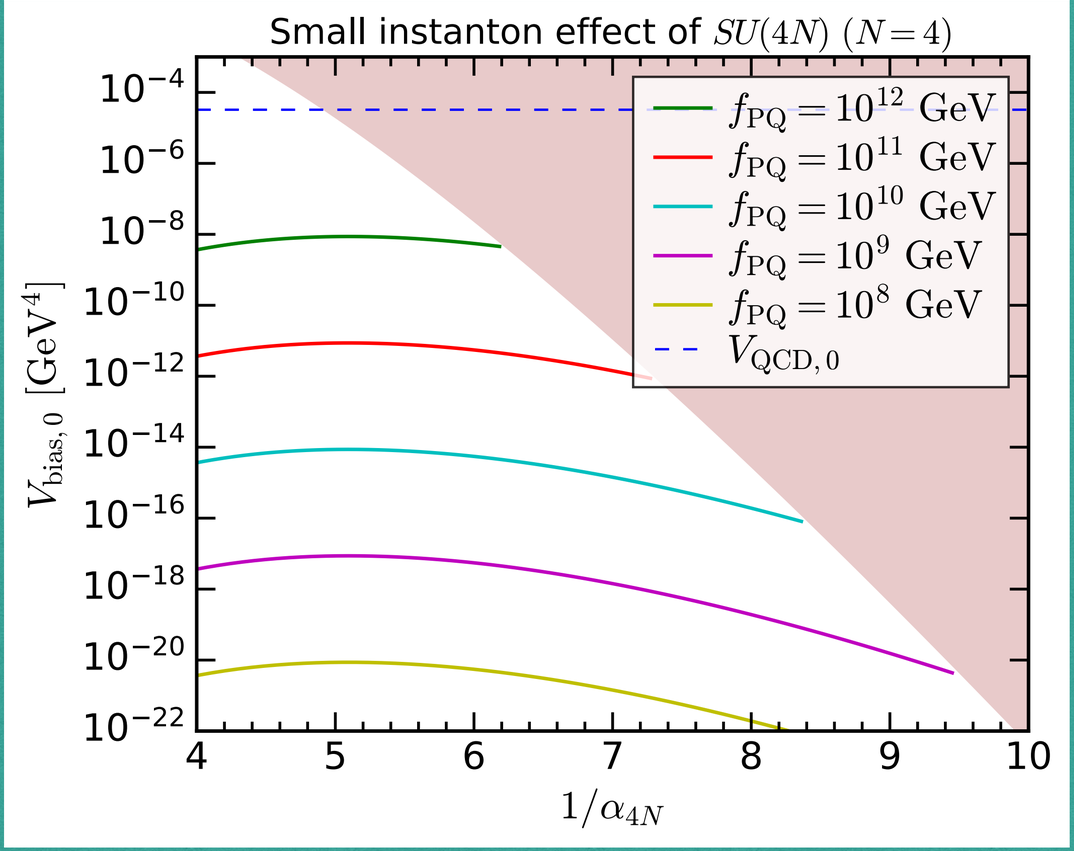
The vacuum degeneracy is lifted by small instant effects (a PQ-violating bias term)

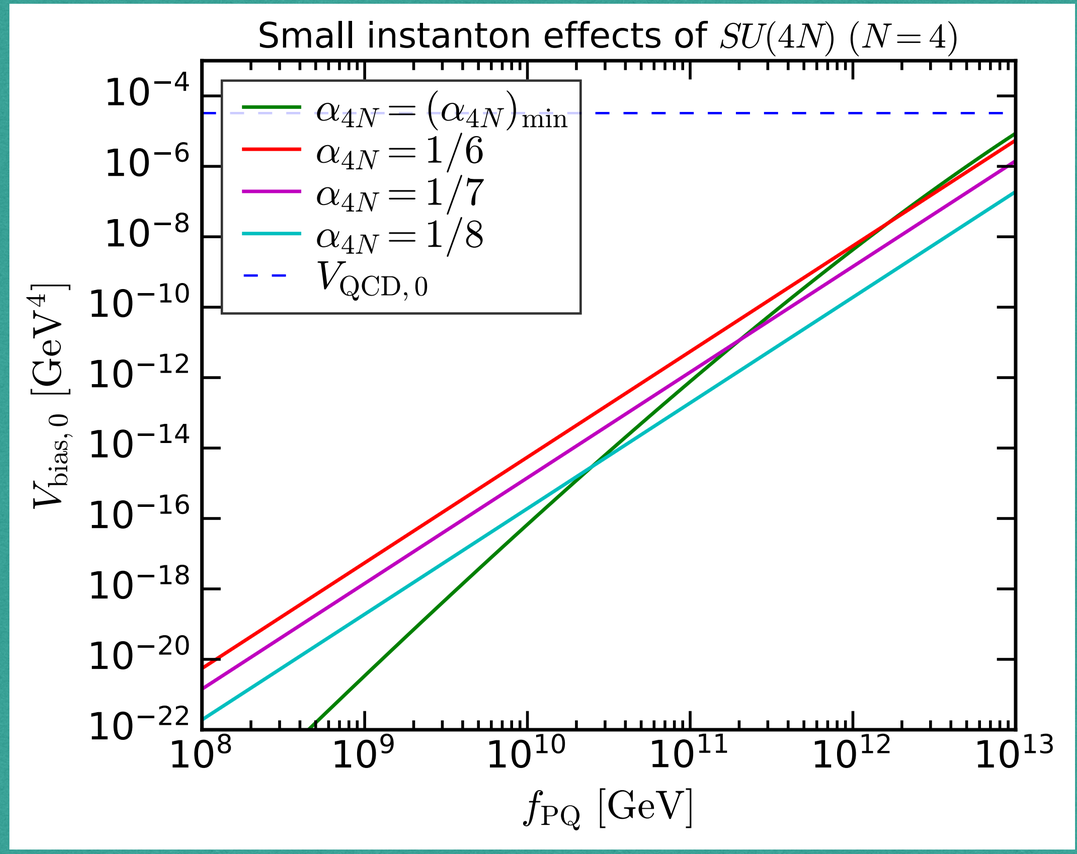


$$\sim \exp\left(\frac{2i}{F_a}\begin{pmatrix}\pi_8(x)+\frac{a(x)}{2\sqrt{6}}\mathbf{1}_{3\times3} & K(x)\\ K^\dagger(x) & -\frac{3a(x)}{2\sqrt{6}}\end{pmatrix}\right)$$

- **Small instanton effect of the $SU(4N)$**
  - $V_I = M_I \Psi\bar{\Psi} \sim C\Lambda_{\rm cut}\Psi\bar{\Psi}e^{-8\pi^2/(g_{SU(4N)}(\Lambda_{\rm cut}))^2}$,
  - $V_{\rm bias} \approx -2C_{4N}\left(\frac{2\pi}{\alpha_{4N}(\Lambda_{\rm cut})}\right)^{2\times 4N}\frac{1}{(4\pi)^2}\Lambda_{\rm cut}\Lambda_{\rm conf}^3 e^{-4\pi^2}e^{-\frac{2\pi}{\alpha_{4N}(\Lambda_{\rm cut})}}\left(3\cos\theta_a+\cos 3\theta_a\right)$,
  - Instanton density $C_{4N} = \frac{K_1 e^{-(S^{(1/2)}-F^{(1/2)})\alpha(1/2)-(S^{(1)}-F^{(1)})\alpha(1)}}{(4N-1)!(4N-2)!}e^{-4NK_2}$,
- **Combining $V_{\rm QCD}$ and $V_{\rm bias}$ leads to $N_{\rm DW}=1$** **Solve the domain wall problem**

Small instanton effects





4. Axion cosmology

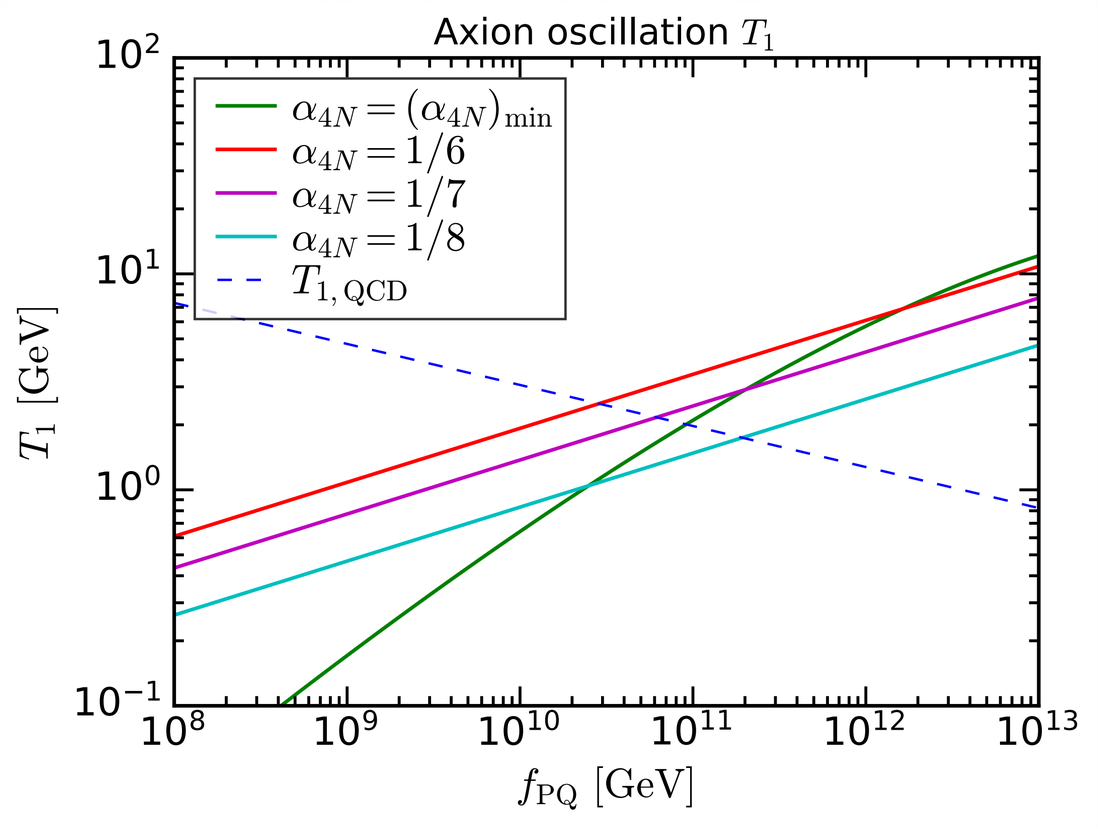
Post-inflation axion

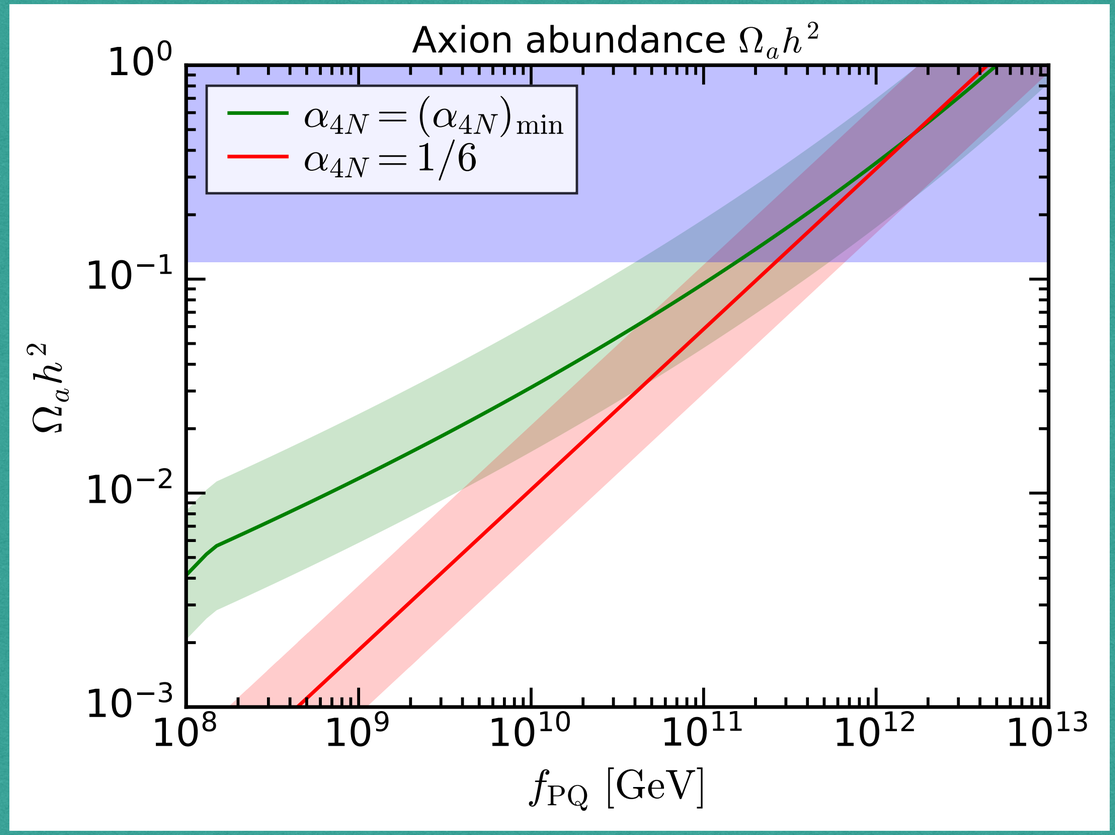
- Axion potential $V(\theta_a) = V_{\rm QCD} + V_{\rm bias}$
- PQ symmetry: spontaneously broken by the $SU(N)_V$ strong dynamics (confinement) after reheating
- **Axion field starts to oscillate due to the axion mass from the bias term**
  - $m_a(T_1) \approx m_{\rm bias} = 3H(T_1)\,, \quad m_{\rm bias}^2 \equiv \dfrac{1}{f_{\rm PQ}} \dfrac{\partial^2 V_{\rm bias}}{\partial \theta_a^2}$
- Temperature when the oscillation starts:
  - $H(T_1)^2 \approx \dfrac{\pi^2}{90 M_{\rm Pl}^2} g_* T_1^4\,.$
  - $T_1 > T_{1,\rm QCD} \equiv 0.98\,{\rm GeV} \left(\dfrac{v_{\rm PQ}/3}{10^{12}\,{\rm GeV}} \right)^{-0.19}\,,$ ← The temperature when the axion would start to oscillate with non-perturbative QCD effects it there was no bias term
- **Domain walls decay in a similar way as the $N_{\rm DW} = 1$ standard case**

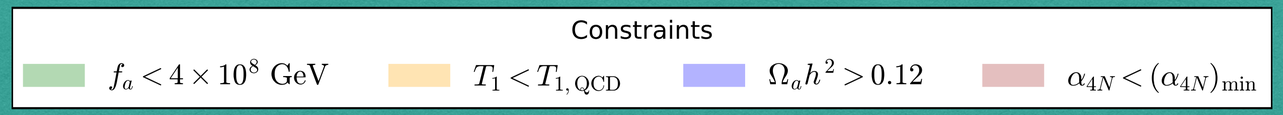
Post-inflation axion

- **Axion abundance** $\Omega_a h^2 \approx \left(2^{+2}_{-1}\right) \times 10^{-12} \dfrac{f_{\rm PQ}}{T_1}$.
  - $\Omega_a h^2 < 0.12$
- A large bias term shifts the axion potential minimum: $\Delta\bar{\theta} = \dfrac{V_{\rm bias,0}}{V_{\rm QCD,0}}\bar{\theta}_c$
  - Neutron EDM —> $\Delta\bar{\theta} \lesssim 10^{-10}$,

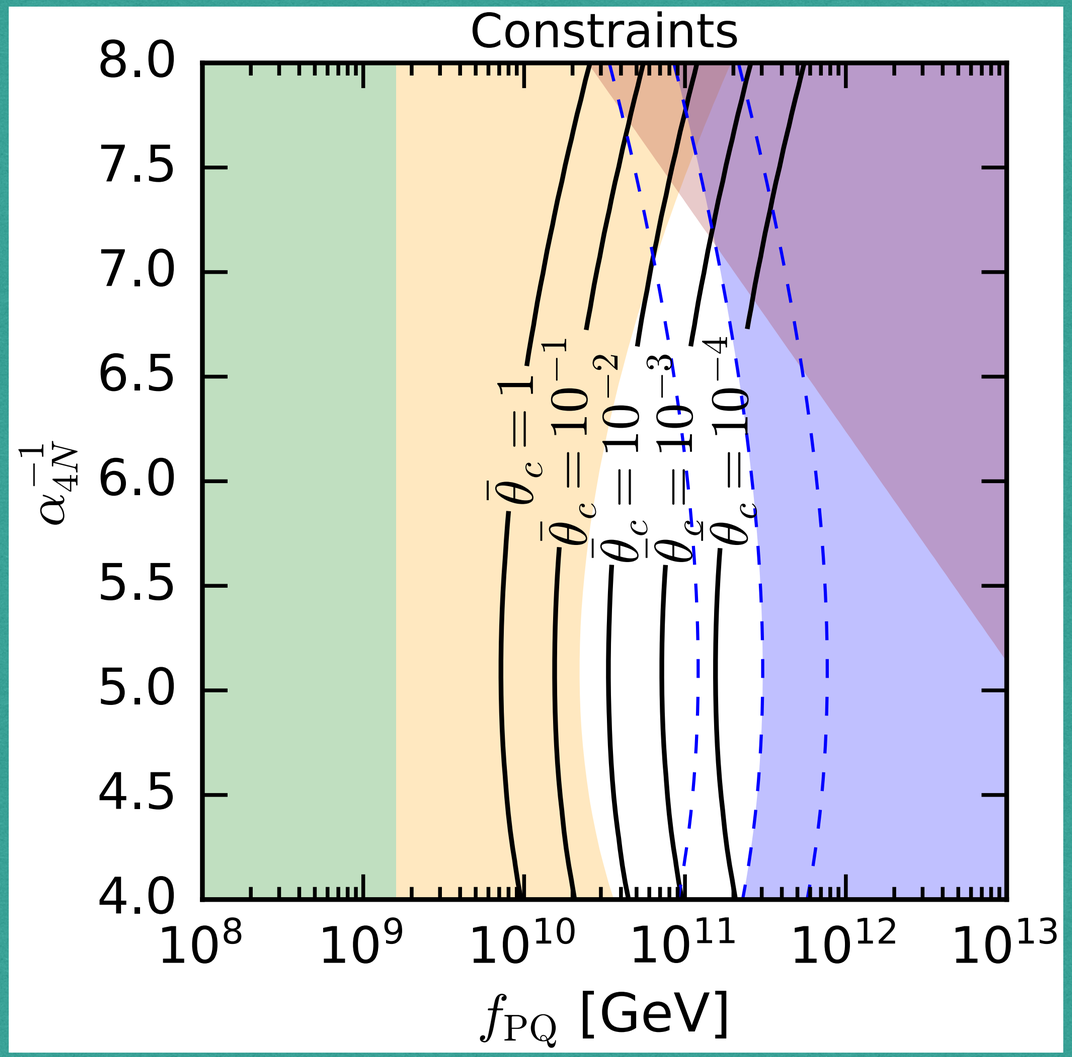
Post-inflation axion







Constraints



5. Summary

Summary

- Specially embedded composite axion model
  - **A special product group (special embedding) with a post-inflationary composite axion**
  - **Small instant effects** on the axion potential : PQ violating bias term and allow the decay of domain walls
  - **A domain-wall-free ($N_{\rm DW} = 1$) UV completion of the IR $N_{\rm DW} = N$ composite axion model**
  - Axion abundance tends to be smaller than the observed DM density