北京航空航天大學
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Reviving primordial black hole formation in slow first-order phase transitions

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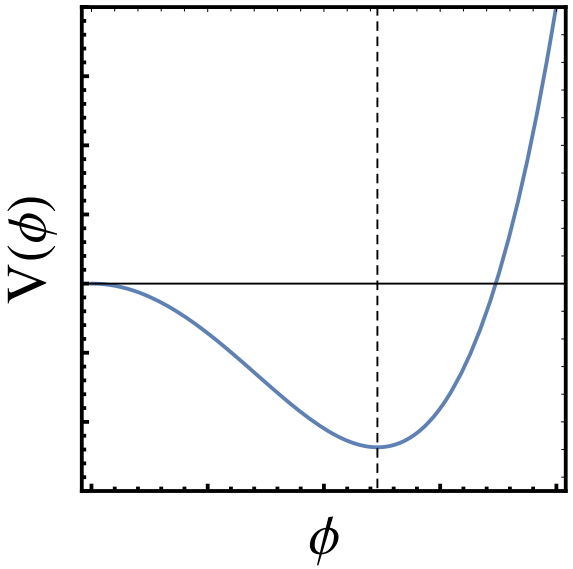
2026.9.11, NEXUS 2026, T. D. Lee Institute, Shanghai

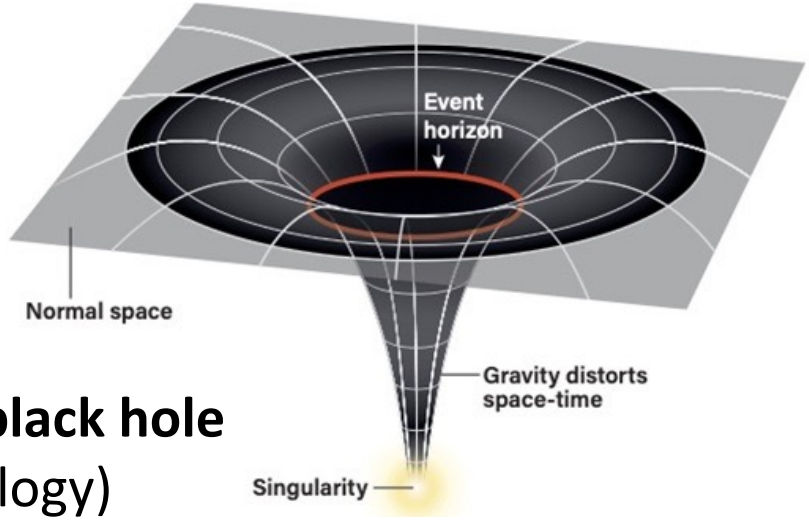
+ Wen-Yuan Ai, arXiv: 2605.11332

Overview



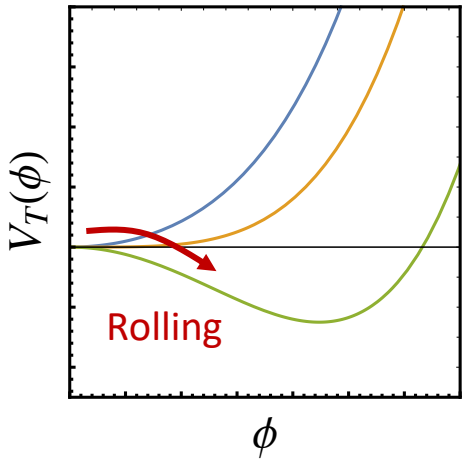
Spontaneous symmetry breaking
(particle physics)

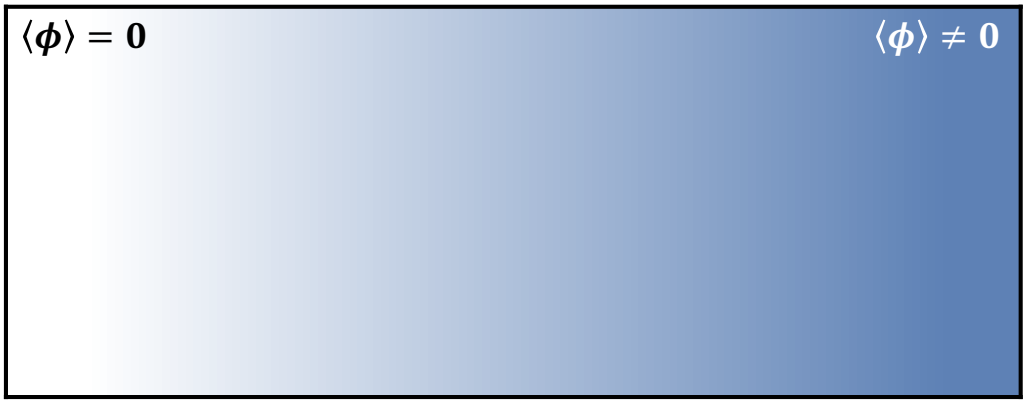
This work



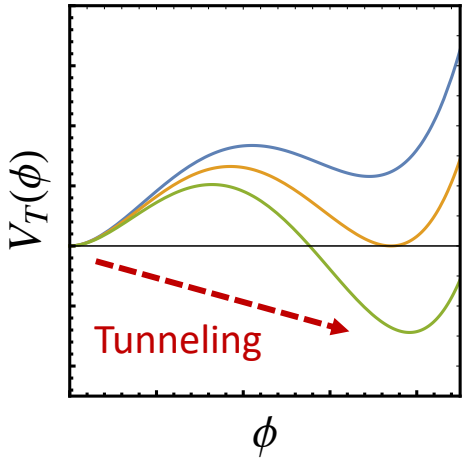
Primordial black hole
(cosmology)

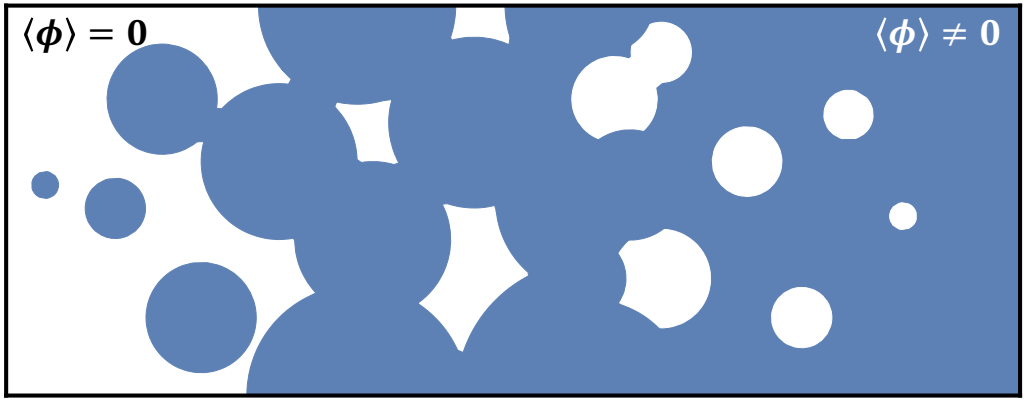
Patterns of symmetry breaking





Second-order phase transition / Crossover

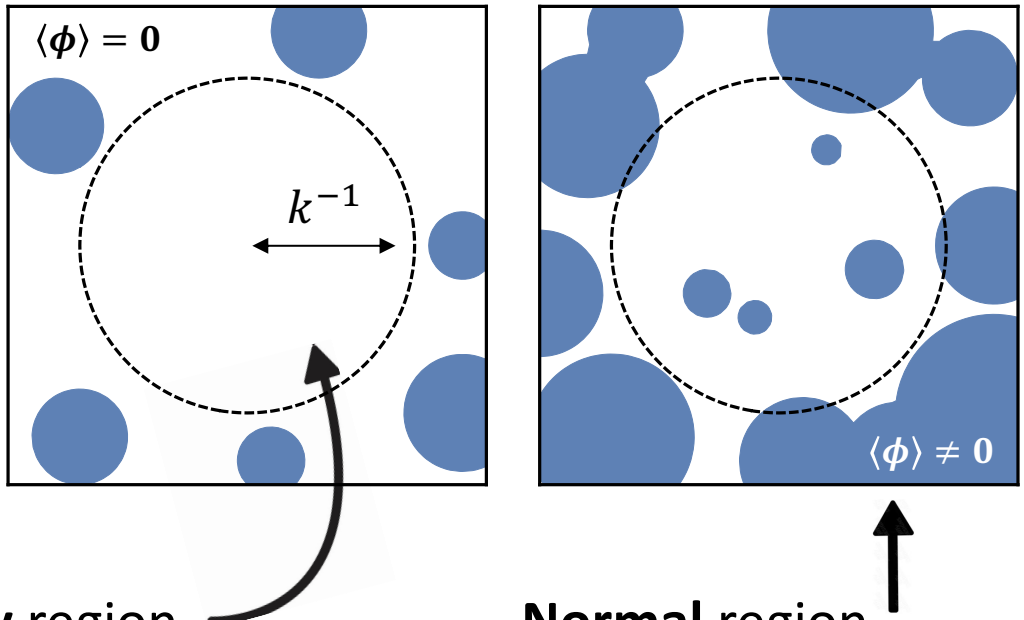




First-order phase transition (FOPT)

Generation of overdensities

Randomness of bubble nucleation

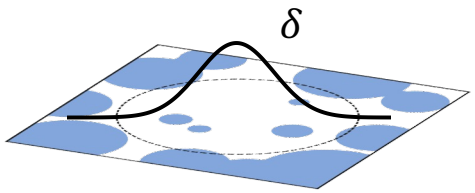


Delayed-decay region
More vacuum energy, $\rho_V \propto a^0$

Normal region
More radiation energy, $\rho_r \propto a^{-4}$

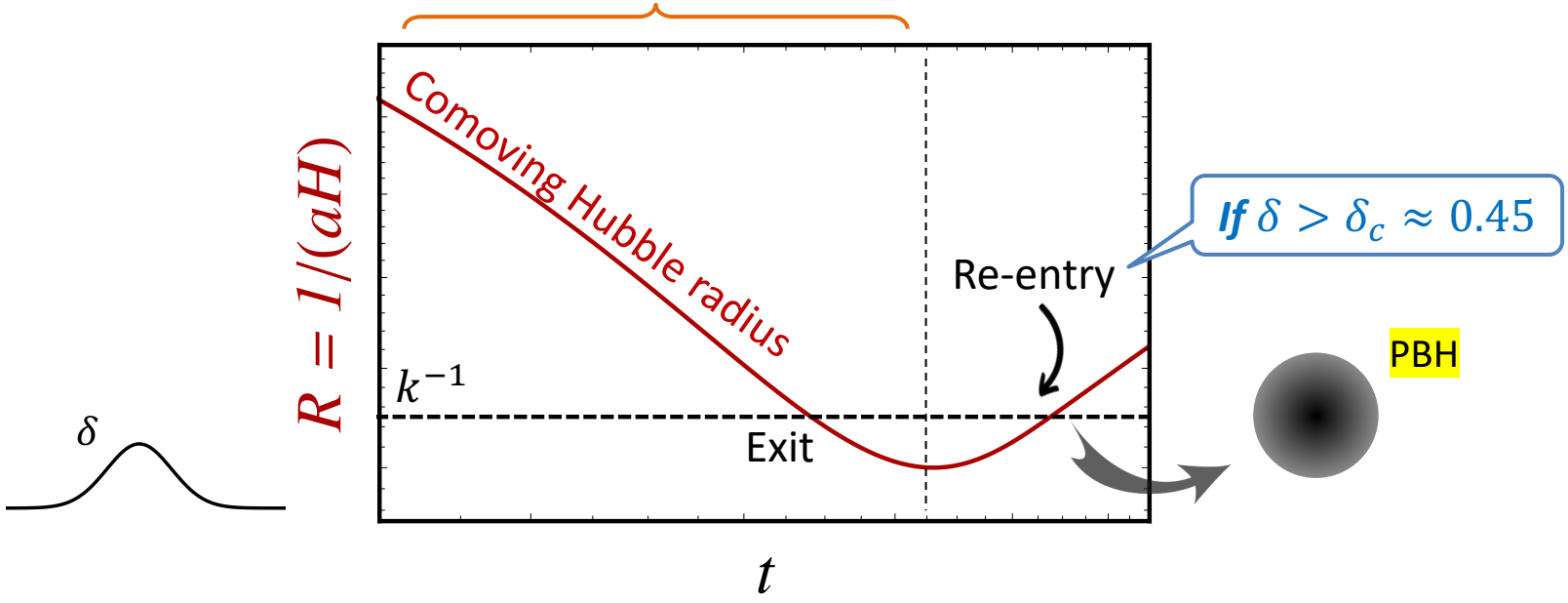
Energy density contrast

$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} > 0$$



Black hole formation

Supercooling: temporary vacuum domination before FOPT



Tracing back to Kodama *et al*, Prog. Theor. Phys. 68 (1982) 1979

Recent **big progress** in Liu, Bian, Cai, Guo, and Wang, Phys.Rev.D 105 (2022) 2, L021303

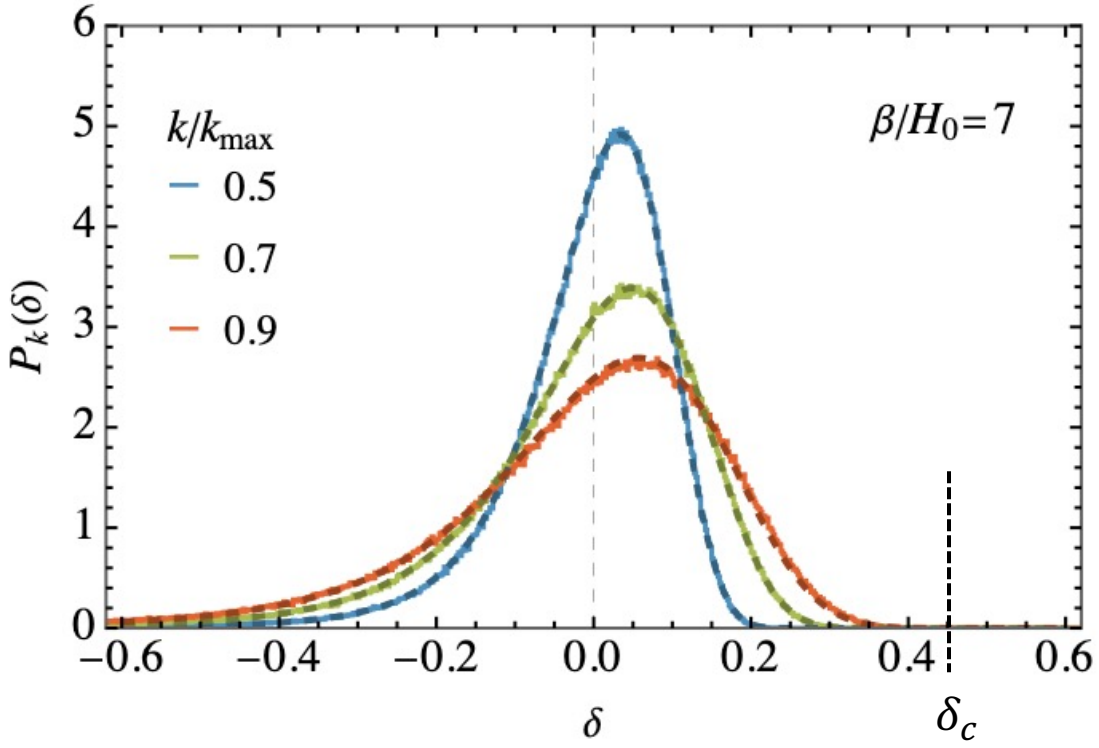
A popular mechanism since 2022

Hashino *et al*, PLB 838 (2023) 137688; Lewicki *et al*, JHEP 09 (2023) 092; He *et al*, SCPMA 67 (2024) 4, 240411; Cai *et al*, SCPMA 67 (2024) 9, 290411; Flores *et al*, PRD 110 (2024) 1, 015005; Kanemura, Tanaka, **KPX**, JHEP 06 (2024) 036; Kawana *et al*, PRD 108 (2023) 103531; Gouttenoire *et al*, PRD 110 (2024) 4, 4; Banerjee *et al*, JCAP 07 (2025) 007; Huang *et al*, PRD 113 (2026) 5, 055013; etc

Calculation framework

Bubble nucleation history simulation Lewicki *et al*, Phys.Rev.Lett. 133 (2024) 22, 221003

- Probability distribution of δ (non-Gaussian)

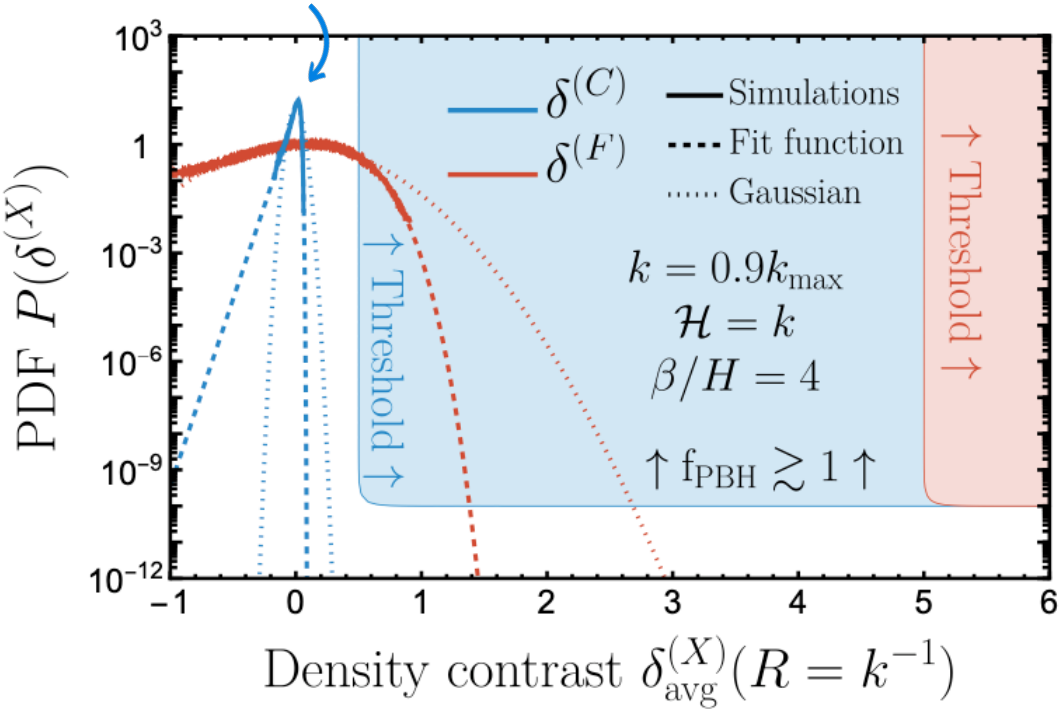


- PBH abundance obtained by integrating $P_k(\delta)$ over $\delta > \delta_c$
- **Slow transitions** with lead to abundant PBH formation

Debates

δ depends on coordinate choice Franciolini *et al*, Phys.Rev.Lett. 136 (2026) 17, 171404

- Bubble simulation → *flat-gauge* value $\delta^{(F)}$
- PBH threshold → *comoving-gauge* value $\delta^{(C)} \gtrsim 0.45$
- $\delta^{(C)}$ **too small** to form PBHs!



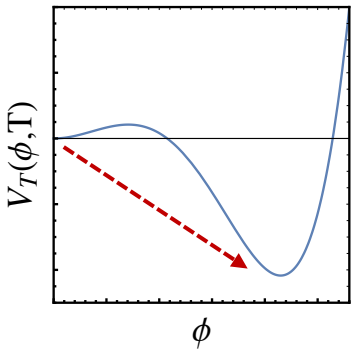
Similar result obtained independently in Wang *et al*, Phys.Rev.D 113 (2026) 12, 123026

Conclusion: **this mechanism is ruled out** *???*

Our insight

A *new* gauge-invariant approach to calculate $\delta^{(C)}$ [this work]

- Confirming $\delta^{(C)} \ll \delta_c \approx 0.45$
- But this does **not** preclude the formation of PBHs



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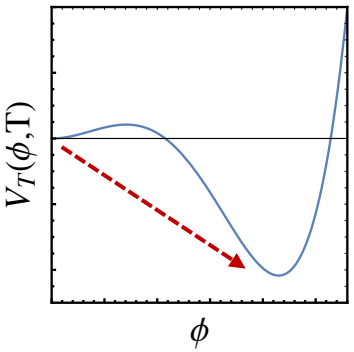
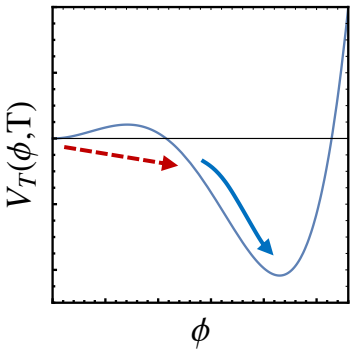
Fast reheating



$\Gamma_\phi / H_* > 1$

More accurately



Key variable:

$\frac{\Gamma_\phi}{H_*}$ Decay width / Hubble at transition

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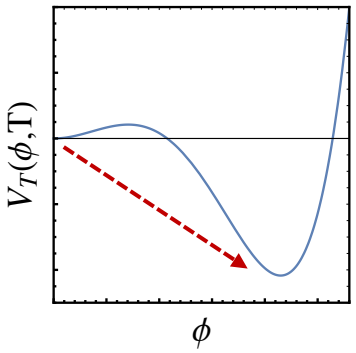

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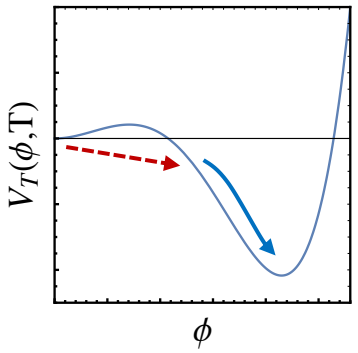
$\Gamma_\phi/H_* < 1$

More accurately



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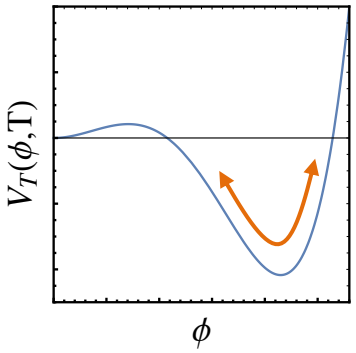
Slow reheating



- ϕ coherently oscillates, decaying to SM particles, reheating the Universe
- **Early matter era** with $\rho_\phi \propto a^{-3}$, pressure ≈ 0
- Small $\delta^{(C)}$ grows and collapses, **forming PBHs** ✓

A unified treatment

	Condition	Energy transfer
Fast reheating	$\Gamma_\phi/H_* > 1$	Vacuum → Radiation
Slow reheating	$\Gamma_\phi/H_* < 1$	Vacuum → ϕ oscillation → Radiation

Generation of δ

Two-fluid system

1. Vacuum, $P_V = -\rho_V$
2. Species f with $P_f = w_f \rho_f$

$$\begin{cases} \text{Fast reheating} \rightarrow f = r \ (w_r = 1/3) \\ \text{Slow reheating} \rightarrow f = \phi \ (w_\phi = 0) \end{cases}$$

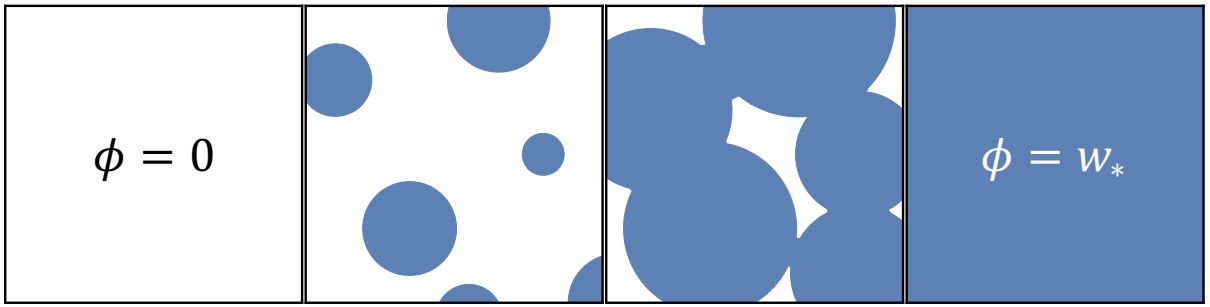
Relativistic perturbation theory

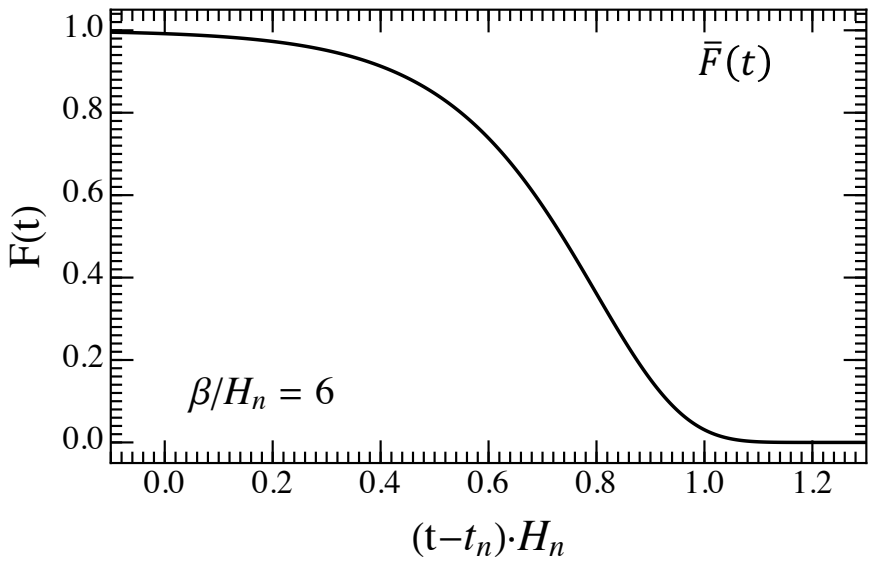
$$\rho(t, \mathbf{k}) = \bar{\rho}(t) + \delta\rho(t, \mathbf{k}); \qquad \delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\delta\rho_V + \delta\rho_f}{\bar{\rho}_V + \bar{\rho}_f}$$

- FOPT yields $\delta\rho_V \rightarrow$ sourcing $\delta \rightarrow$ PBH formation

Vacuum energy

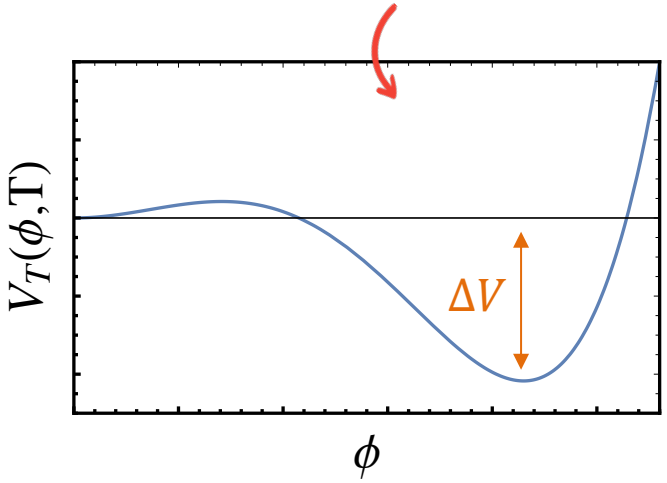
False vacuum volume fraction: the whole Universe





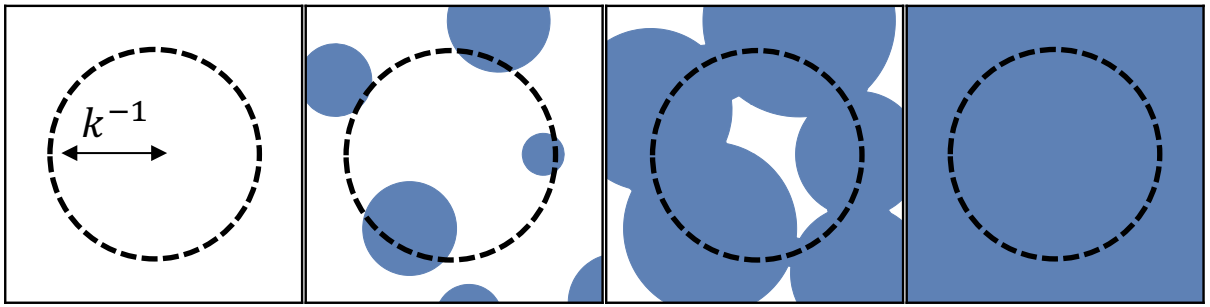
Background vacuum energy

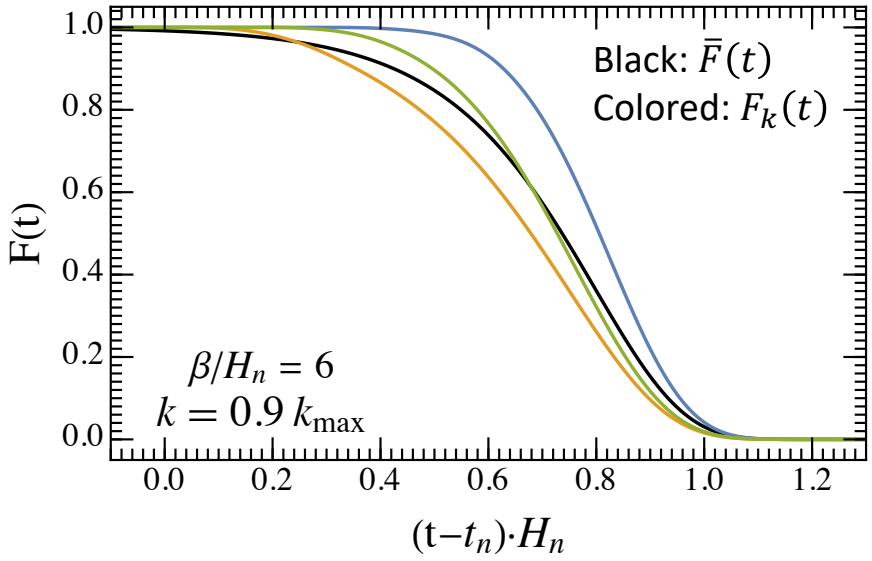
$$\bar{\rho}_V(t) = \Delta V \cdot \bar{F}(t)$$



Vacuum energy perturbation

False vacuum volume fraction: a finite sphere





Comoving volume $4\pi k^{-3}/3$

$$\rho_V(t) = \Delta V \cdot F_k(t)$$

Obtained from **bubble nucleation simulation**

Fluctuation $\delta\rho_V = \rho_V - \bar{\rho}_V = \Delta V \cdot \left(F_k(t) - \bar{F}(t)\right)$

Relativistic perturbation theory

Our method: start from **gauge-invariant** equations

$$\dot{\Delta} - 3H\frac{\bar{P}}{\bar{\rho}}\dot{\Delta} = \left(1+\frac{\bar{P}}{\bar{\rho}}\right)\frac{k^2\mathcal{V}}{a}; \quad \dot{\mathcal{V}} + H\mathcal{V} = -\frac{1}{a(\bar{\rho}+\bar{P})}\left(\delta P_{\rm nad} + \frac{\dot{\bar{P}}}{\dot{\bar{\rho}}}\bar{\rho}\Delta\right) - \frac{\Phi}{a}$$

But evaluate the r.h.s. in **flat gauge**

$$\dot{\mathcal{Q}} + (4+3w_f)H\mathcal{Q} = \frac{1+w_f}{a}\delta\rho_V - \left[\frac{w_f}{a}\bar{\rho} + \frac{aH}{2k^2}(\dot{\bar{\rho}} - 3(1+w_f)\dot{\bar{\rho}}_V)\right]\Delta$$

$\mathcal{Q} = (\bar{\rho}+\bar{P})\mathcal{V}$, still gauge-invariant

From bubble simulations

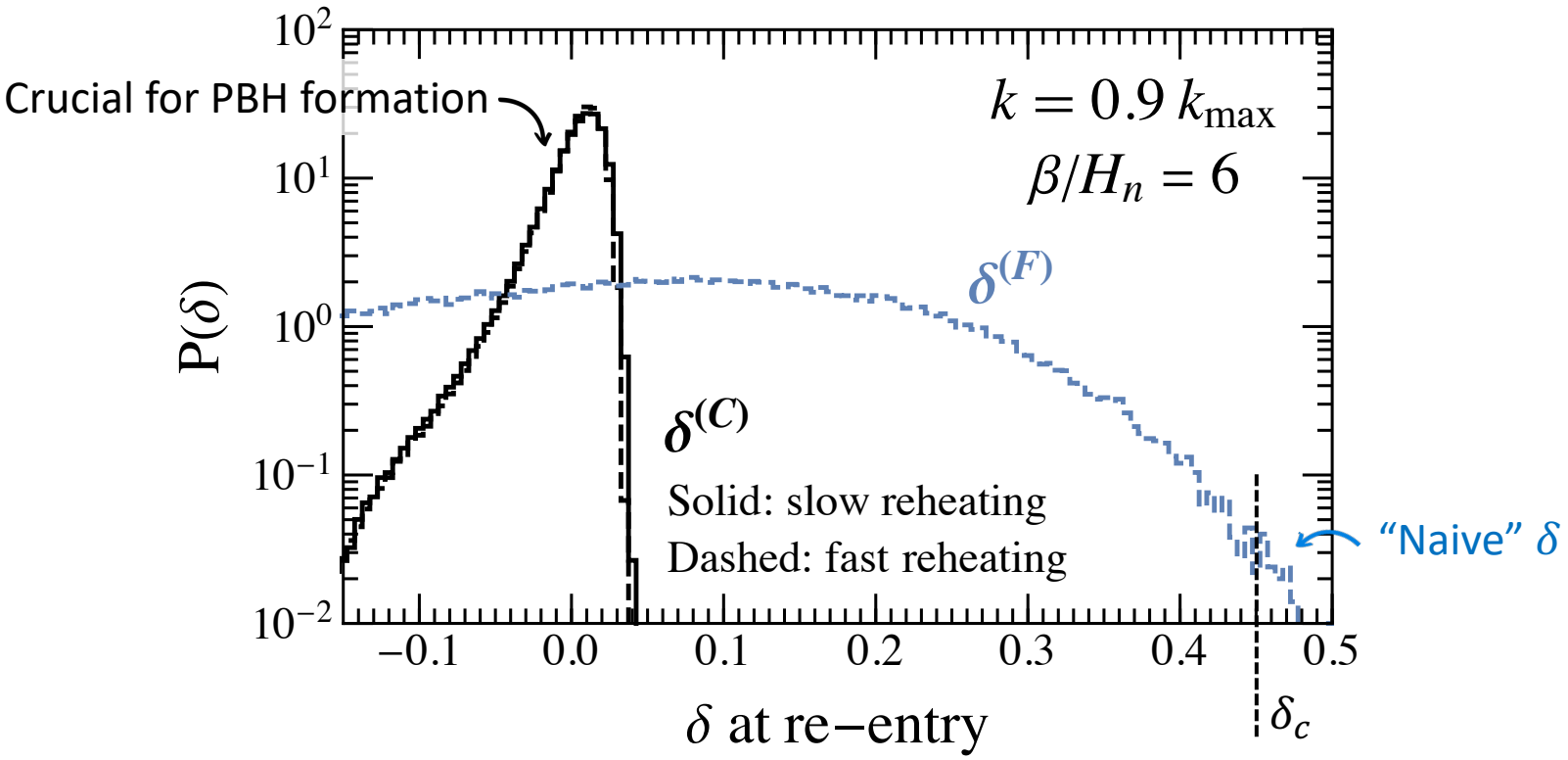
$\Delta \equiv \delta^{(C)}$ [**comoving gauge**] resolved, evolved to horizon re-entry

- Single realization → one $\delta^{(C)}$
- Multiple realizations → probability distribution of $\delta^{(C)}$

Distribution of δ

Very sensitive to gauge choice: $\delta^{(C)} \ll \delta^{(F)}$

Not sensitive to efficiency of reheating: $\delta^{(C)}(f = r) \approx \delta^{(C)}(f = \phi)$

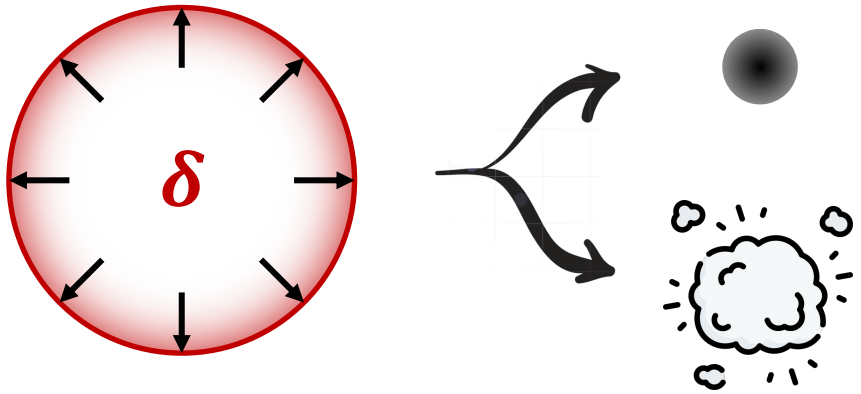


Crucial: the **post-FOPT evolution** of δ

Hereafter, δ solely denotes $\delta^{(C)}$

Fate of the density contrast

Fast reheating: pressure $P_r = \rho_r/3$ resists collapse



$\delta > \delta_c$: instantaneous collapse

Black hole formation

$\delta < \delta_c$: no collapse

Overdensity dissipates

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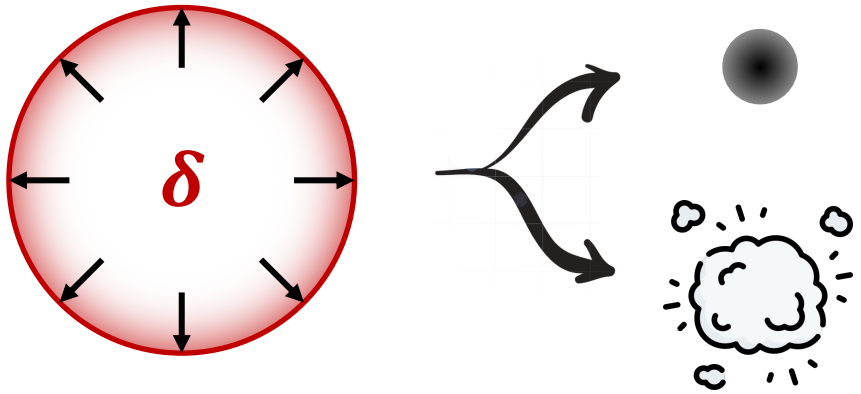
Fast reheating: pressure $P_r = \rho_r/3$ resists collapse



$\delta > \delta_c$: instantaneous collapse

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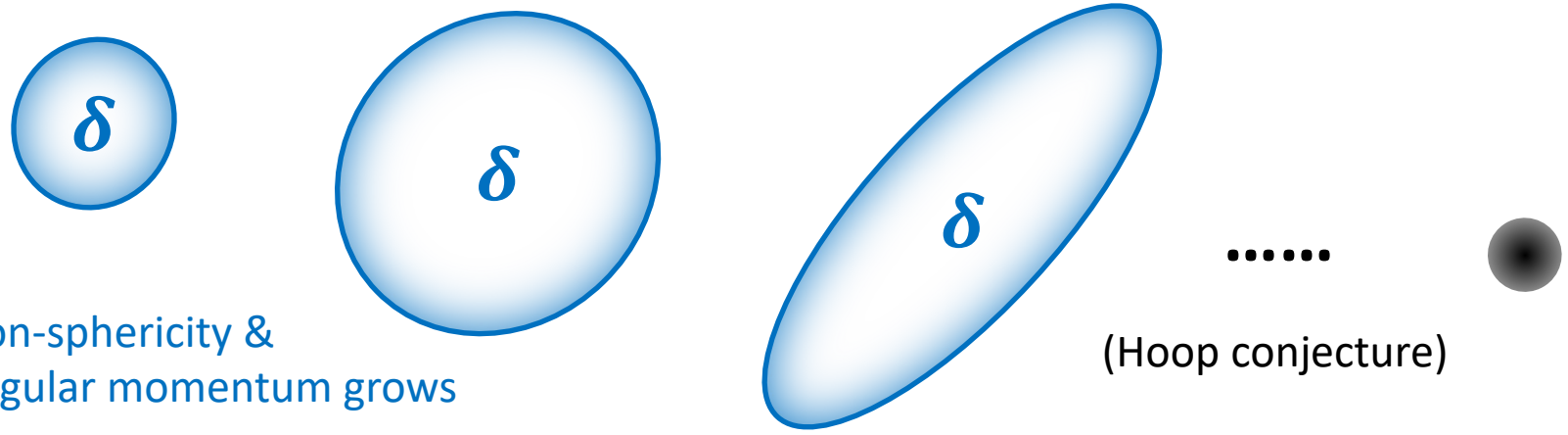
Overdensity dissipates

Slow reheating: $P_\phi \approx 0$, perturbation δ increases with time



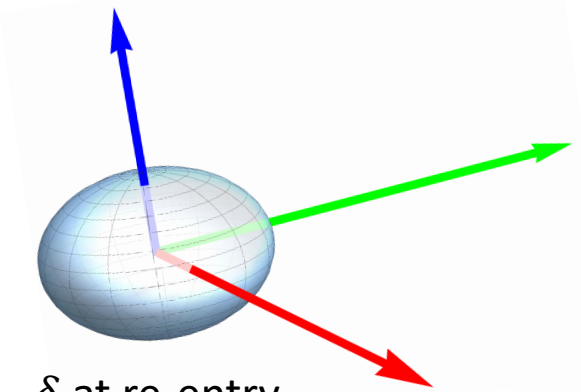
Non-sphericity &
angular momentum grows

(Hoop conjecture)

"Pancake collapse" at late time, **black hole** formation

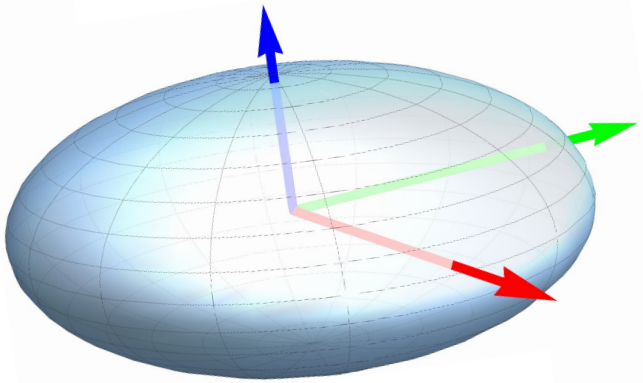
Details of the collapse

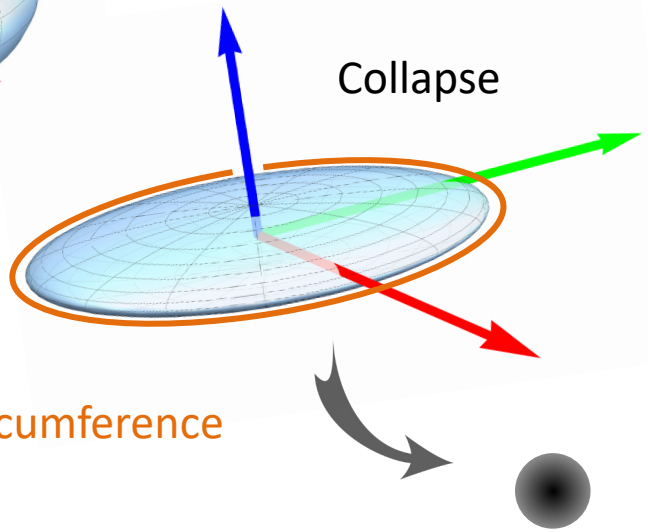
Ellipsoid-shaped density contrast



δ at re-entry

Evolves during matter era ($P_\phi \approx 0$)





Collapse

Hoop conjecture Thorne, 1972

PBH forms *if* and *only if*

Mass $\dfrac{M}{2m_{\rm Pl}^2} > \mathcal{C}$ Circumference

$^*m_{\rm Pl} = (8\pi G)^{-1/2}$

PBH formation in the matter era

Perturbation shape describe by the **deformation tensor $\mathcal{D}_{3\times3}$**

Our crucial assumption

$$P(\mathcal{D}) = P(\delta) \times P_{\widetilde{\mathcal{D}}}\left(\widetilde{\mathcal{D}}_{ij}\right)$$

From our simulation (Non-Gaussian) ← $\delta = -\operatorname{tr}\mathcal{D}$ Trace part

$\widetilde{\mathcal{D}}_{ij} = \mathcal{D}_{ij} + \frac{\delta_{ij}}{3}\delta$ Traceless part → Assumed to be Gaussian

(Calling for future detailed simulations!)

We improve Refs. Harada *et al*, Astrophys. J. 833 (2016) no. 1, 61, PRD 96 (2017) no. 8, 083517

PBH formation criterion: $\Gamma_\phi / H_* \lesssim 10^{-3}$ (long matter era)

Probability

$$\beta_k \approx \int_{\delta_{\rm th}}^{\infty} \mathrm{d}\delta \int_{-\infty}^{\infty} \mathrm{d}y \int_{2|y|}^{\infty} \mathrm{d}z \left[P(\delta) \frac{9\sqrt{2}}{\sqrt{\pi}\sigma_3^5} (z^3 - 4zy^2) e^{-\left(2y^2+3z^2/2\right)/\sigma_3^2} \right]$$

From hoop conjecture, typically $\delta_{\rm th} \ll 0.45$

PBH profile

Key variables $\begin{cases} \text{New physics scale } \Lambda_{\rm NP} \approx \Delta V^{1/4} \\ \boldsymbol{\beta/H_n} = \text{(Hubble time)/(Transition duration)} \\ \boldsymbol{\Gamma_\phi/H_*} = \text{(Hubble time)/(Early matter era duration)} \end{cases}$

PBH mass & dark matter fraction

No accretion Ballesteros *et al*, JCAP 06 (2020) 014

$$M_{\rm min} \sim 10^{20}\ {\rm g} \times \left(\frac{10^6\ {\rm GeV}}{\Lambda_{\rm NP}}\right)^2$$

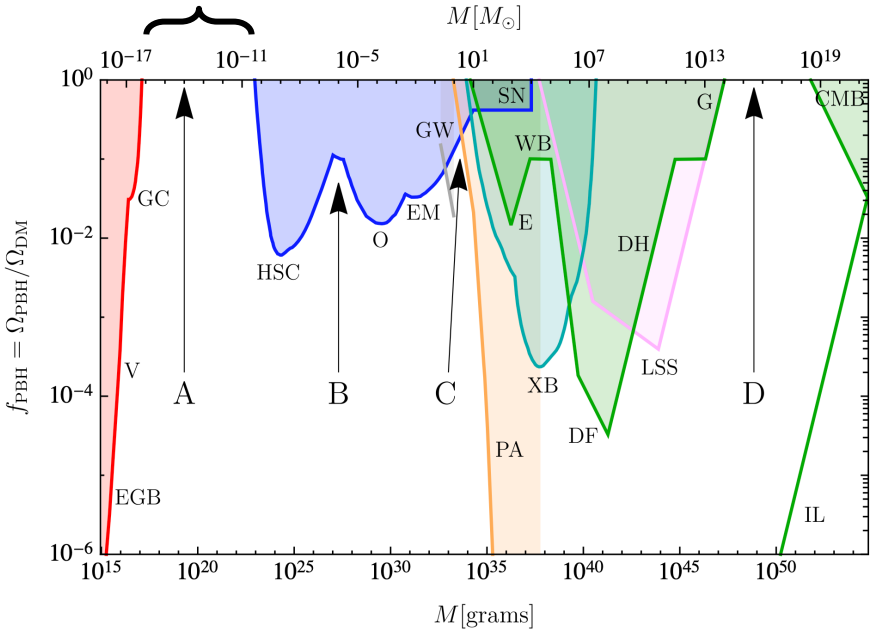
$$f_{\rm min} \sim 10^{0.4\cdot(12-\beta/H_n)} \left(\frac{\Lambda_{\rm NP}}{10^6\ {\rm GeV}}\right) \left(\frac{\Gamma_\phi}{H_*}\right)^{1/2}$$

Maximal accretion de Jong *et al*, JCAP 03 (2022) 03, 029

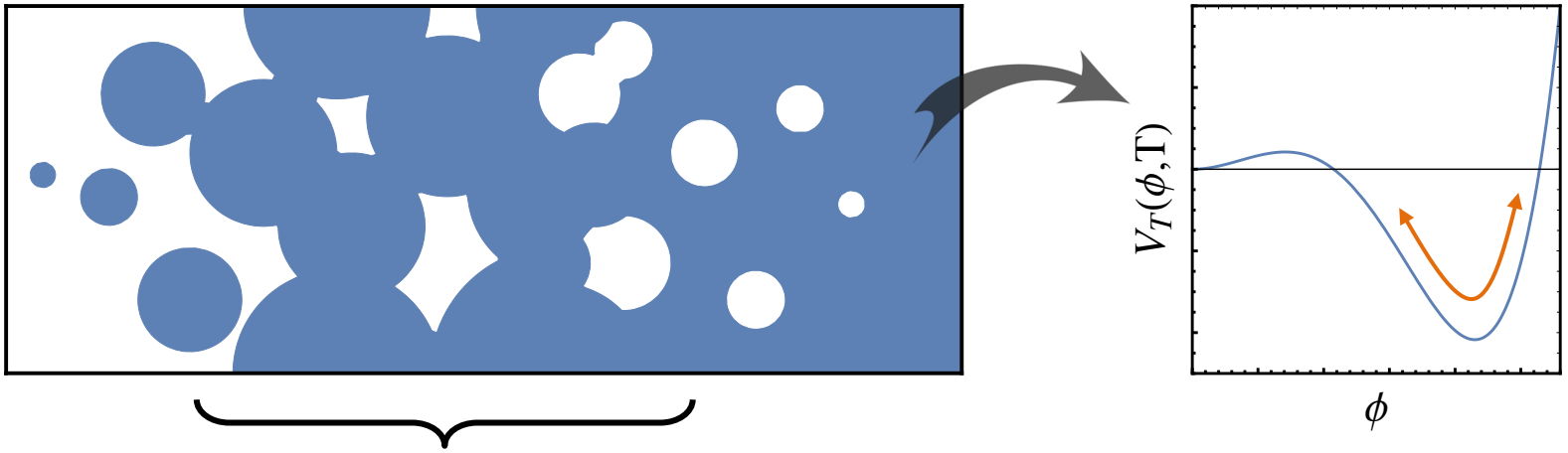
$$M_{\rm max} \sim 10^{20}\ {\rm g} \times \left(\frac{10^6\ {\rm GeV}}{\Lambda_{\rm NP}}\right)^2 \left(\frac{H_*}{\Gamma_\phi}\right)$$

$$f_{\rm max} \sim 10^{0.4\cdot(12-\beta/H_n)} \left(\frac{\Lambda_{\rm NP}}{10^6\ {\rm GeV}}\right) \left(\frac{H_*}{\Gamma_\phi}\right)^{1/2}$$

Asteroid-mass (10^{20} g – 10^{22} g), allowing for 100% **dark matter**



Novelty of our work



During FOPT: δ generation

After FOPT: δ evolution in EMD

New method for calculating $\delta^{(C)}$

- ✓ Gauge-invariant, with no super-horizon approximations
- ✓ Matches previous work
- ✓ Simple (numerical & analytic)

New scenario for PBH formation

- ✓ Natural early matter era
- ✓ The full calculation framework established

A simpler, **gauge-invariant** method, and a natural **EMD-driven** PBH formation scenario

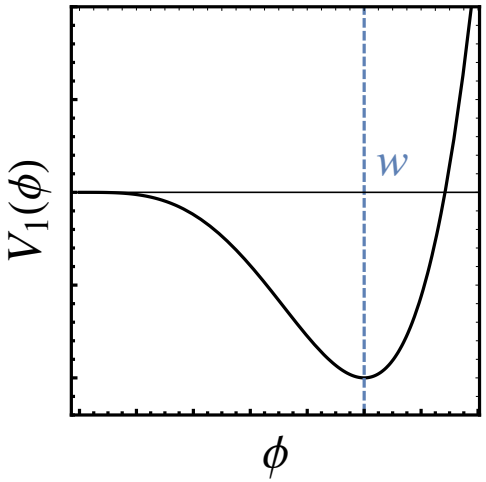
Model-building guide

Supercooled and slow FOPT

- Classically conformal principle

$$V_1(\phi) = \frac{B}{4}\phi^4\left(\log\frac{\phi}{w} - \frac{1}{4}\right)$$

- Motivated by hierarchy problem Bardeen, FERMILAB-CONF-95-391-T; Meissner *et al*, PLB 660 (2008) 260-266
- Leads to supercooling Iso *et al*, PRL 119, 141301 (2017)



Slow (post-FOPT) reheating

- Suppressed $\phi \to$ SM SM decay

Dark $U(1)_X$ model

Babu *et al*, Phys. Rev. D 57 (1998) 6788–6792

- Scalar Φ, boson X
- Kinetic mixing

$$\mathcal{L} \sim -\frac{\epsilon}{2}X_{\mu\nu}B^{\mu\nu}$$

- $\phi \to X^*X^* \to f\bar{f}f'\bar{f}'$, $\Gamma_\phi \propto g_X^{10}\epsilon^4$

Minimal $U(1)_{B-L}$ model

Iso *et al*, Phys.Lett.B 676 (2009) 81-87

- Scalar Φ, gauge boson Z'
- Higgs mixing

$$\theta \approx \frac{v_{\rm ew}}{w}\frac{m_h^2}{m_h^2 - m_\phi^2} \sim g_{B-L}^2\left(\frac{v_{\rm ew}}{m_\phi}\right)^3$$

- $\Gamma_\phi \propto \theta^2$

Conclusion

This work Ai and **KPX**, arXiv: 2605.11332

- Indeed, $\delta^{(C)}$ from FOPT is small, $\ll \delta_C$
- But **PBH formation** is **revived** via slow reheating

Outlook

- Post-FOPT reheating & EMD
- $\mathcal{D}_{3\times3}$ simulations & PBH spin distribution
- Gravitational waves from FOPT + EMD, and non-spherical collapse
- Particle phenomenology: collider, neutrino oscillation, etc

Conclusion

This work Ai and **KPX**, arXiv: 2605.11332

- Indeed, $\delta^{(C)}$ from FOPT is small, $\ll \delta_C$
- But **PBH formation** is **revived** via slow reheating

Outlook

- Post-FOPT reheating & EMD
- $\mathcal{D}_{3\times3}$ simulations & PBH spin distribution
- Gravitational waves from FOPT + EMD, and non-spherical collapse
- Particle phenomenology: collider, neutrino oscillation, etc

"Problems worthy of attack
prove their worth by fighting back."
— Piet Hein

值得攻克的难题
会以反击来证明自身的价值。

Thank you!

Backup: FOPT setup

Bubble nucleation rate

$$\Gamma(t) = H_n^4 e^{\beta(t-t_n) - \frac{\zeta^2}{2}(t-t_n)^2 + \cdots}$$

t_n: nucleation time; $H_n \equiv H(t_n)$

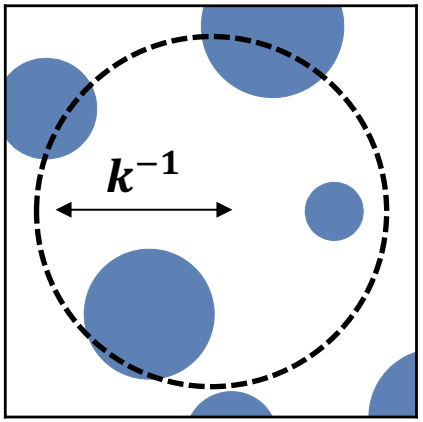
Ratio of Hubble time to FOPT duration: β/H_n

Average false vacuum fraction

$$\bar{F}(t) = \exp\left[-\frac{4\pi}{3}\int_{-\infty}^{t} \mathrm{d}t' \Gamma(t') \left(\int_{t'}^{t} v_w \mathrm{d}t'' \frac{a(t')}{a(t'')}\right)^3\right]$$

$v_w \approx 1$ for a supercooled FOPT

False vacuum fraction $F_k(t)$ in a comoving volume with radius k^{-1} obtained from simulations



Backup: relativistic perturbation theory

$$\mathrm{d}s^2 = (1+2A)\mathrm{d}t^2 - 2a(\partial_i B)\mathrm{d}t\mathrm{d}x^i - a^2\left[\delta_{ij}(1+2C) + 2\partial_{\langle i}\partial_{j\rangle}E\right]\mathrm{d}x^i\mathrm{d}x^j$$

$\delta \to$ gauge-dependent

Bubbles simulation $\to$ **flat gauge** ($C = E = 0$)

PBH criterion $\delta_c \approx 0.45 \to$ **comoving gauge** ($v + B = E = 0$)

Gauge-independent variables & equations

$$\Delta = \frac{\delta\rho + a\dot{\bar{\rho}}(v+B)}{\bar{\rho}}; \quad \mathcal{V} = v + a\dot{E}; \quad \Phi = -C + \frac{1}{3}\nabla^2 E - aH\left(B - a\dot{E}\right)$$

$$\dot{\Delta} - 3H\frac{\bar{P}}{\bar{\rho}}\Delta = \left(1 + \frac{\bar{P}}{\bar{\rho}}\right)\frac{k^2\mathcal{V}}{a}; \quad \dot{\mathcal{V}} + H\mathcal{V} = -\frac{1}{a(\bar{\rho}+\bar{P})}\left(\delta P_{\mathrm{nad}} + \frac{\dot{\bar{P}}}{\dot{\bar{\rho}}}\bar{\rho}\Delta\right) - \frac{\Phi}{a}$$

Our scenario: two-fluid system, under flat gauge:

$$\delta P_{\mathrm{nad}} \to \frac{1+w_f}{\dot{\bar{\rho}}}\left(\delta\rho_f\dot{\bar{\rho}}_V - \delta\rho_V\dot{\bar{\rho}}_f\right); \quad \Phi \to -aHB$$

Backup: comparison

$\delta^{(C)}$ from a supercooled FOPT: three independent approaches

[1] Franciolini *et al*, Phys.Rev.Lett. 136 (2026) 17, 171404

- Explicitly **gauge-dependent**

[2] Wang *et al*, Phys.Rev.D 113 (2026) 12, 123026

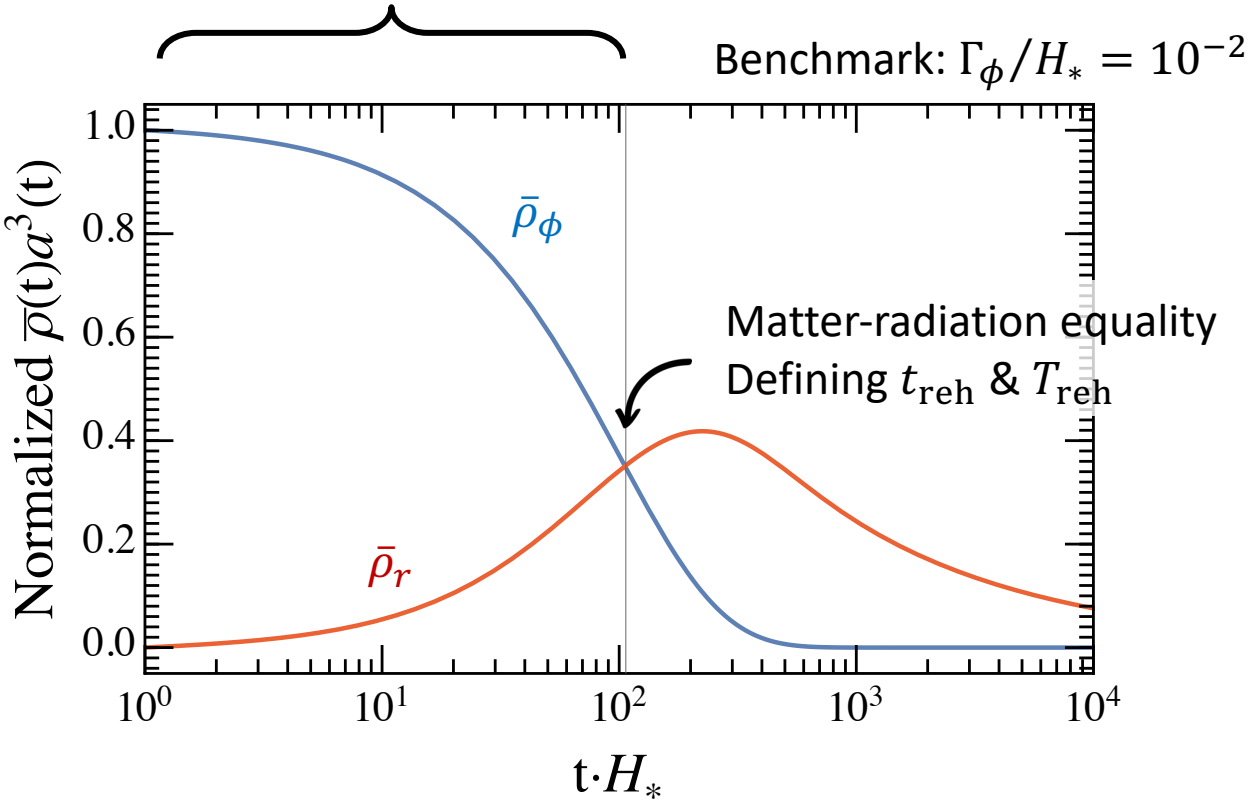
- **Gauge-invariant**, but relies on super-horizon approximations
- Agrees with [1] to order of magnitude

[3] Our work

- **Gauge-invariant**, no super-horizon approximations, and simple to implement (numerically and analytically)
- Almost coincides with [1], agrees with [2] to order of magnitude

Backup: slow reheating

Slow reheating: matter-dominated era



$$\dot{\bar{\rho}}_\phi + 3H\bar{\rho}_\phi = -\Gamma_\phi \bar{\rho}_\phi; \quad \dot{\bar{\rho}}_r + 4H\bar{\rho}_r = \Gamma_\phi \bar{\rho}_\phi$$

Numerically, reheating temperature $T_{\rm reh} \approx 0.64\, T_V \cdot \left(\Gamma_\phi/H_*\right)^{1/2}$

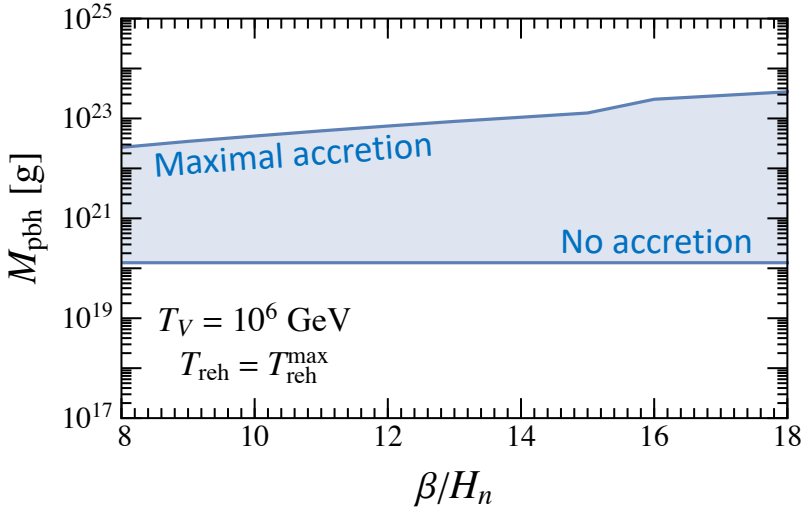
Backup: PBH mass & abundance (1)

Profile most sensitive to $\boldsymbol{\beta/H_n}$ = (Hubble time)/(Transition duration)

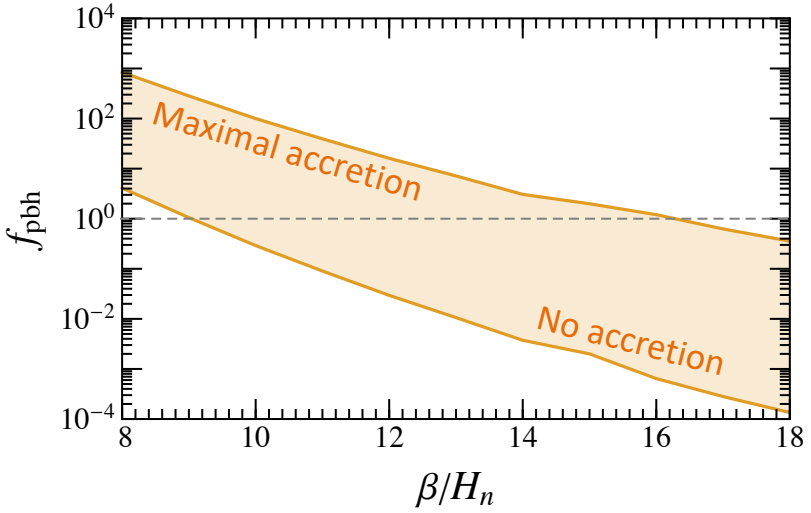
Just one benchmark:

- New physics scale $\Delta V^{1/4} \approx T_V = 10^6$ GeV
- Fix $T_{\rm reh} \approx T_V \cdot \left(\Gamma_\phi/H_*\right)^{1/2}$ as $T_{\rm reh}^{\rm max}$ (equivalent to $\Gamma_\phi/H_* \approx 10^{-3}$)

Mass



Dark matter fraction



- Asteroid-mass (10^{20} g – 10^{22} g), allowing for 100% **dark matter**; detectable in the future (e.g., femto-lensing)

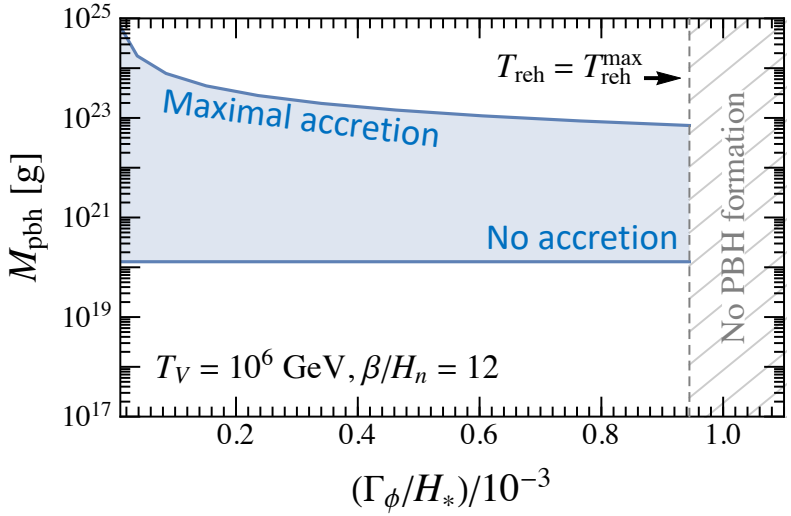
Backup: PBH mass & abundance (2)

Dependence on $\boldsymbol{\Gamma_\phi / H_*}$ = (Hubble time)/(Reheating duration)

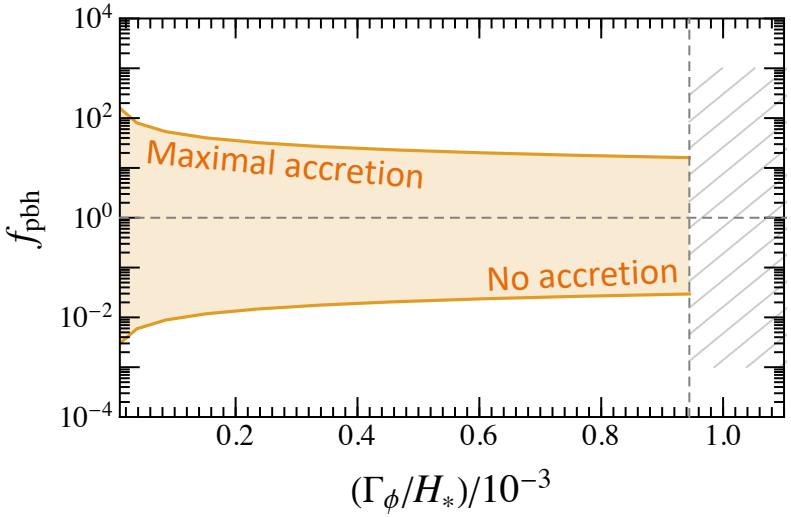
Just one benchmark:

- New physics scale $\Delta V^{1/4} \approx T_V = 10^6$ GeV
- $T_V \cdot \left(\Gamma_\phi / H_*\right)^{1/2} \approx T_{\rm reh} < T_{\rm reh}^{\rm max}$ (PBH formation condition)

Mass



Dark matter fraction



- Asteroid-mass (10^{20} g – 10^{22} g), allowing for 100% **dark matter**; detectable in the future (e.g., femto-lensing)

Backup: details of the illustrative model

Classically conformal gauge $U(1)_X$ model, tree level: $V_0(\Phi) = \lambda_\phi |\Phi|^4$

Coleman-Weinberg potential

$$V_1(\phi) = \frac{3g_X^4}{32\pi^2}\phi^4\left(\log\frac{\phi}{w} - \frac{1}{4}\right)$$

Spectrum: $m_X = g_X w$ and $m_\phi = \sqrt{3/2}\, g_X^2 w/(2\pi)$

Kinetic mixing, $\mathcal{L}_{\rm int} = -(\sin\epsilon/2)X_{\mu\nu}B^{\mu\nu}$

Decay channels:

- 4-body, $\phi \to X^*X^* \to f\bar{f}f'\bar{f}'$
- 3-body, $\phi \to X^*Z \to W^+W^-Z + f\bar{f}Z$
- 2-body, $\phi \to ZZ$

} $\to \Gamma_\phi \propto \epsilon^4$, suppressed

Finite-temperature effective potential

$$V_T(\phi, T) = V_1(\phi) + \frac{3T^4}{2\pi^2}J_B\left(\frac{g_X^2\phi^2}{T^2}\right) - \frac{T}{12\pi}g_X^3\left[\left(\phi^2 + \frac{T^2}{3}\right)^{3/2} - \phi^3\right]$$

Supercooled FOPT, see for example Ayazi *et al*, JHEP 03 (2019) 181

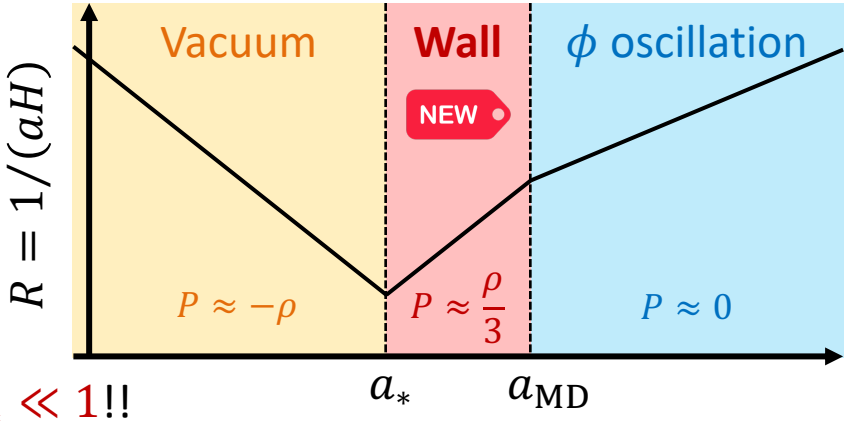
Backup: debates on post-FOPT EMD

Lattice simulation of the FOPT Mansour *et al*, arXiv: 2607.19469

Can the universe be matter-dominated after a supercooled first-order phase transition?

Wall (**gradient**)
radiation-like

Bulk (**oscillation**)
matter-like



Wall-era always exists, even though $\Gamma_\phi/H_* \ll 1$!!

Efficiency of "gradient → bulk":

- *If* free-streaming: $a_{\rm MD}/a_* \sim \gamma_w \gg 1$, **no PBH at all**!
- *If* prompt thermalization: $a_{\rm MD}/a_* \sim \mathcal{O}(1)$, **little impact on PBH**

Classically conformal models: nonlinear, definitely not free-streaming, **hopefully** prompt thermalization; but still *an open question*