Detecting RG-stable relations in multi-Higgs models

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Several Higgs generations

Higgses can come in generations $\rightarrow$ *N*-Higgs-doublet models (NHDMs).

- T.D. Lee, 1973: 2HDM as a new source of *CP*-violation (CPV);
- Weinberg, 1976: 3HDM with natural flavor conservation and CPV;
- Intense activity in the 1970s–1980s aiming to deduce the hierarchical flavor sector from symmetries and their breaking;
- 1990s–2000s: MSSM requires two Higgs doublets;
- Cosmological consequences: scalar dark matter candidates protected by residual symmetries; strong first-order phase transitions $\rightarrow$ baryogenesis and GW signals.
- In total, $\mathcal{O}(10^4)$ papers over 40 years [Branco et al, 1106.0034; Ivanov, 1702.03776]

Symmetries as anchor structures

The general renormalizable potential with N Higgs doublets has $N^2(N^2+3)/2$ free parameters, many more in the Yukawa sector.

Symmetries guide exploration of the rich NHDM phenomenology.

Strategy:

- identify all symmetry options (= all groups, all irrep assignments, all symmetry breaking options) available in the scalar and Yukawa sectors of a given multi-Higgs model;
- exact vs. spontaneously broken vs. softly broken symmetry group;
- pheno consequences for each symmetry option or in its vicinity.

Symmetries in the 2HDM

$$
\begin{aligned}
V &= m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2) - m_{12}^{2\,*}(\phi_2^\dagger\phi_1) \\
&+ \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
&+ \left[\frac{1}{2}\lambda_5(\phi_1^\dagger\phi_2)^2 + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \text{h.c.}\right]
\end{aligned}
$$

It contains $4 + 10 = 14$ free parameters.

- $\mathbb{Z}_2$ symmetry: invariance under $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$ leads to

$$m_{12}^2 = 0\,, \quad \lambda_6 = \lambda_7 = 0\,.$$

- Softly broken $\mathbb{Z}_2$ ($m_{12}^2 \neq 0$) + NFC in the Yukawa sector $\to$ the popular 2HDM framework.

Symmetries in the 2HDM

Six classes of 2HDMs with symmetric scalar sectors [Ivanov, 2005; Ferreira, Haber, Silva, 2009]:

- CP1: the usual CP, $\phi_a \to \phi_a^*$.
- $\mathbb{Z}_2$: $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$.
- CP2: generalized CP, $\phi_1 \to \phi_2^*$, $\phi_2 \to -\phi_1^*$.
- U(1): Peccei-Quinn, $\phi_2 \to e^{i\alpha}\phi_2$.
- CP3: generalized CP, $\phi_1 \to c_\alpha \phi_1^* + s_\alpha \phi_2^*$.
- SO(3): any unitary rotation of (ϕ_1, ϕ_2).

Symmetry groups $\Rightarrow$ relations among m_{ab}^2 and λ's.

Spontaneous symmetry breaking patterns [Ivanov, 2007].
Extensions to the Yukawa section classified in [Ferreira, Silva, 2008; Ivanov, Nishi, 2013].

RG evolution stability

Relations among parameters driven by unbroken symmetries are stable under RGE.

Is the converse true?

That is, does every RG-stable relation necessarily imply a symmetry?

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The naturalness argument ['t Hooft, 1979] as "folk wisdom"; no formal proof.

What about $V = V_1(\phi_1) + V_2(\phi_2) + \lambda|\phi_1|^2|\phi_2|^2$, where $\lambda = 0$ is RG stable to all orders?

RG stability does not necessarily imply the presence of a usual symmetry.

Since RG-stable relations are important phenomenologically, it makes perfect sense to search for RG-stable relations irrespective of whether or not they follow from symmetries.

GOOFy symmetries

A 2HDM surprise

Ferreira, Grzadkowski, Ogreid, Osland, EPJC 84, 234 (2024): in 2HDM, the set of relations

$$m_{11}^2 + m_{22}^2 = 0\,, \quad \lambda_1 = \lambda_2\,, \quad \lambda_7 = -\lambda_6$$

is RG-stable to all orders in the scalar/gauge couplings and at least two loops in the Yukawas.

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These relations cannot be imposed by any traditional symmetry acting on ϕ_a. Indeed, writing

$$V = \frac{1}{2}(m_{11}^2 + m_{22}^2)(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) + \dots\,,$$

we see that it's impossible to forbid this term by any traditional symmetry argument because $m_{11}^2 + m_{22}^2$ is invariant under any transformation that keeps the kinetic term invariant.

So, is there any structural feature behind these relations?

Bilinears in 2HDM

Bilinear space construction [Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008].

V is built of the bilinears $\phi_a^\dagger \phi_b$. Let's organize them as $r^\mu = (r_0, r_i)$, where

$$r_0 = \frac{1}{2}\phi_a^\dagger \phi_a \equiv \frac{1}{2}\left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2\right), \quad r_i = \frac{1}{2}\phi_a^\dagger \sigma_{ab}^i \phi_b \equiv \begin{pmatrix} \mathrm{Re}(\phi_1^\dagger \phi_2) \\ \mathrm{Im}(\phi_1^\dagger \phi_2) \\ (\phi_1^\dagger \phi_1 - \phi_2^\dagger \phi_2)/2 \end{pmatrix}.$$

The orbit space: $r_0 \geq 0$ and $r^\mu r_\mu = r_0^2 - r_i^2 \geq 0 \Rightarrow$ vectors r^μ fill the "forward cone" in $\mathbb{R}^{1+3}$.

Basis change: r_0 invariant, r_i transforms by an $SO(3)$ rotation.
General CP transformation: r_0 invariant, r_i transforms by an improper rotation (reflection + rotation).

2HDM Higgs potential in the bilinear form

The Higgs potential of the general 2HDM becomes

$$V = M_\mu r^\mu + \frac{1}{2}\Lambda_{\mu\nu} r^\mu r^\nu = M_0 r_0 - M_i r_i + \Lambda_0 r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j \,.$$

All 14 parameters reside inside M_0, M_i, Λ_0, L_i, and Λ_{ij}.

- Symmetries easily formulated in terms of M and L with respect to the eigenvectors of Λ.
- Basis-independent criteria and many other features come out naturally.

RGE on m^2_{ab} and λ's can be rewritten in terms of M_0, M_i, Λ_0, L_i, and Λ_{ij}.
Explicit β-functions up to 3 loops, structural arguments to all orders [Bednyakov, JHEP 11 (2018) 154].

r_0-symmetry

Back to Ferreira, Grzadkowski, Ogreid, Osland, EPJC 84, 234 (2024). The relations

$$m_{11}^2 + m_{22}^2 = 0\,, \quad \lambda_1 = \lambda_2\,, \quad \lambda_7 = -\lambda_6$$

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Let's look again:

$$V = M_0 r_0 - M_i r_i + \Lambda_0 r_0^2 + L_i r_0 r_i + \Lambda_{ij} r_i r_j\,, \quad \text{with} \quad r_0 = \frac{1}{2}\left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2\right)\,.$$

This transformation in the bilinear space cannot be induced by any transformation in the field space.

Interpretations

How to explain this feature?

- "GOOFy" symmetry proposed by [Ferreira, Grzadkowski, Ogreid, Osland] $\rightarrow$ FGOO $\rightarrow$ GOOF. Symmetric under:

$$\phi_1 \rightarrow -\phi_2^* , \quad \phi_1^* \rightarrow \phi_2 , \qquad ,$$

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$$\phi_1 \to -\phi_2^*\,, \quad \phi_1^* \to \phi_2\,, \quad x^\mu \to i x^\mu\,, \quad B^\mu \to i B^\mu\,,$$

  which indeed send $r_0 \to -r_0$ and preserves the gauge-kinetic term. Further developments in 2506.21145, 2602.20849, 2604.07458, 2608.05304.

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- Haber, Ferreira, 2502.11011: take a model with RG-stable relations but no symmetry, complexify it, write its RGEs, and observe that they contain the old RGEs as a subset. In the complexified theory, new symmetries can be imposed, leading to the original RG-stable relations.

- Additional insights: outer automorphisms [Trautner, 2505.00099, 2508.02646, 2603.12318] or scaling + non-overlapping symmetries [Pilaftsis, 2605.18341] as the origin of RG stable relations.

Looking for RG-stable relations

Looking for RG-stable relations

The problem we address:

> Given a set of beta-functions, what relations among the parameters are RG-stable, irrespective of symmetries?

- RG-stable relations coming from conserved symmetries.
- RG-stable relations coming from GOOFy "symmetries".
- RG-stable relations not linked to any "symmetry".

Ideally, we want to develop a systematic procedure applicable to any set of coefficients at any order.

Pragmatic approach: start with one-loop beta functions and search for linear relations.

Generic procedure: a single linear relation

Consider quartic couplings λ_i, $i = 1, \ldots, N$, so that $\lambda_i \in V = \mathbb{R}^N$. One-loop RGEs:

$$\beta_{\lambda_i} \equiv \frac{d\lambda_i}{dt} = \beta_{ijk}\lambda_j\lambda_k\,, \quad t = (\ln \mu^2)/16\pi^2\,.$$

All structures, including RG-stable relations, are in principle determined by β_{ijk}.

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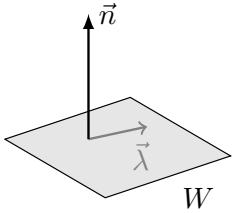
A single linear relation is parametrized by a vector $\vec{n} \in \mathbb{R}^N$:

$$f(\lambda_i) \equiv n_i\lambda_i = 0\,.$$

Define $W \perp \vec{n}$. Then, a RG stable linear relation implies W is stable:

$$\frac{df(\lambda_i)}{dt} = n_i\frac{d\lambda_i}{dt} = n_i\beta_{ijk}\lambda_j\lambda_k = B_{jk}\lambda_j\lambda_k = 0 \quad \text{for all } \vec{\lambda} \in W.$$

That is, if $\vec{\lambda} \in W$, it stays inside W under RGE.



Generic procedure: a single linear relation

The condition $B_{jk}\lambda_j\lambda_k = 0$ for all $\vec{\lambda} \in W$ can be recast into

$$B_{j'k'}\left(\delta_{j'j} - \frac{n_{j'}n_j}{\vec{n}^2}\right)\left(\delta_{k'k} - \frac{n_{k'}n_k}{\vec{n}^2}\right)\tilde{\lambda}_j\tilde{\lambda}_k = 0 \quad \text{for any } \tilde{\lambda}_j.$$

Therefore, the matrix itself is zero:

$$PBP \equiv \left(\delta_{jj'} - \frac{n_j n_{j'}}{\vec{n}^2}\right) B_{j'k'}\left(\delta_{k'k} - \frac{n_{k'}n_k}{\vec{n}^2}\right) = 0_{jk}\,.$$

As a single scalar condition:

$$\|PBP\|^2 \equiv \mathrm{Tr}(PBP \cdot PBP) = \mathrm{Tr}(BPBP) = 0\,.$$

This is a single scalar condition of 6th order in components of $\vec{n}$.

A polynomial equation of an even order $\rightarrow$ no guarantee that non-trivial solutions exist.

Generic procedure: k linear relations

We have k linear relations: $\vec{n}^{(\alpha)}\lambda_i = 0$ for all $\alpha = 1, \ldots, k$.

Vectors $\vec{n}_i^{(\alpha)}$ span a k-dimensional subspace $U \subset V$, with $U \oplus W = V$, and projectors $P^{(U)}$ and $P^{(W)}$. Then, the search for a set of k linear relations is equivalent to search of non-trivial solutions to

$$P^{(U)}_{ii'} P^{(W)}_{jj'} P^{(W)}_{kk'} \cdot \beta_{ijk}\beta_{i'j'k'} = 0\,, \quad \text{where} \quad P^{(U)} + P^{(W)} = \mathbf{1}\,.$$

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The most constrained case: $k = N - 1$, all λ_i are proportional to each other. RG-stability implies $\beta_i \propto \lambda_i$. In other words, there exists p_i that is an eigenvector of β_{ijk}:

$$\beta_{ijk} p_j p_k \propto p_i\,.$$

Generic procedure: RG-stable linear relations

Given N quartic couplings λ_i, $i = 1, \ldots, N$, and their β-functions encoded via tensor β_{ijk}, in order to find all RG-stable sets of k relations, we need to find k-dimensional subspaces $U \subset V$ satisfying

$$P^{(U)}_{ii'} P^{(W)}_{jj'} P^{(W)}_{kk'} \cdot \beta_{ijk} \beta_{i'j'k'} = 0\,, \quad \text{where} \quad P^{(U)} + P^{(W)} = \mathbf{1}\,.$$

This task is challenging:

- Analytical solutions exist only in particular cases $\rightarrow$ numerical search.
- Notorious basis-choice dependence: the same model may look differently in different bases $\rightarrow$ differently looking solutions can correspond to the same physical situation.

Generic procedure: adding quadratic terms

Quadratic coefficients M_a, $a = 1, \ldots, q$ are dimensionful: their RGEs are given by

$$\beta_{M_a} \equiv \frac{dM_a}{dt} = C_{ab} M_b \,,$$

where C_{ab} depend linearly on the quartic parameters λ_i.

For each set of RG-stable relations among quartic coefficients, check the space spanned by tensors C_{ab} and verify if it has invariant subspaces.

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The generic outcome of the strategy:

- For each $k = 1, N-1$, we should (numerically) obtain models A, B, C, ... for quartic coefficients with an RG-stable set k relations.
- Together with quadratic terms, we get models A1, A2, ...; B1, B2, ...; ....
- Once all RG-stable sets of relations are found, check which of them emerge from previously established symmetries, either normal or "GOOFy", and which are not symmetry-generated.

Preliminary results

Toy model: real ϕ_1 and ϕ_2

Toy model with two real scalar fields ϕ_1 and ϕ_2 and

$$V = \frac{1}{2}m_{11}^2\phi_1^2 + \frac{1}{2}m_{22}^2\phi_2^2 + m_{12}^2\phi_1\phi_2 + \frac{1}{4!}\lambda_1\phi_1^4 + \frac{1}{4!}\lambda_2\phi_2^4 + \frac{1}{4}\lambda_3\phi_1^2\phi_2^2 + \frac{1}{3!}\lambda_4\phi_1^3\phi_2 + \frac{1}{3!}\lambda_5\phi_2^3\phi_1 \,.$$

Beta functions:

$$\begin{aligned}
\beta_{\lambda_1} &= 3\lambda_1^2 + 3\lambda_3^2 + 6\lambda_4^2 \\
\beta_{\lambda_2} &= 3\lambda_2^2 + 3\lambda_3^2 + 6\lambda_5^2 \\
\beta_{\lambda_3} &= \lambda_1\lambda_3 + \lambda_2\lambda_3 + 4\lambda_3^2 + 2\lambda_4^2 + 2\lambda_5^2 + 2\lambda_4\lambda_5 \\
\beta_{\lambda_4} &= 3\lambda_1\lambda_4 + 6\lambda_3\lambda_4 + 3\lambda_3\lambda_5 \\
\beta_{\lambda_5} &= 3\lambda_2\lambda_5 + 6\lambda_3\lambda_5 + 3\lambda_3\lambda_4 \,,
\end{aligned}$$

together with a set of quadratic parameters β-functions.

NB: the model possesses the $O(2)$ group of basis-change freedom.

Symmetry groups available: $\mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_2$, and $O(2)$-symmetric models.

Toy model: real ϕ_1 and ϕ_2

- Maximally constrained quartic sector: 4 relations among 5 λ's, can be worked out analytically.

  Three classes of quartic sectors: model A, model B, and $O(2)$-symmetric model.

  Models A and B cannot be obtained via symmetries.

  Together with quadratic terms, we get A1 ($\mathbb{Z}_2$), A1', A2, A2' (can be obtained via "GOOFy" symmetry), B1, B2.
- Less-constrained quartic sectors: 3 or 2 relations. Situations found via numerical search.
- No solutions found for a single linear relation.

The 2HDM scalar sector

Bilinear formalism: 1-loop beta-functions for Λ_0, L_i, Λ_{ij}, M_0, and M_i (expressions up to three loops in [Bednyakov, JHEP 11 (2018) 154]).

$$
\begin{aligned}
\beta_{\Lambda_0} &= 4\Lambda_0^2 + 6L_iL_i + \mathrm{Tr}(\Lambda^2) + \mathrm{Tr}(\Lambda)\Lambda_0, \\
\beta_{L_i} &= 6(\Lambda_0 L_i + \Lambda_{ij}L_j), \\
\beta_{\Lambda_{ij}} &= [3\Lambda_0 - \mathrm{Tr}(\Lambda)]\Lambda_{ij} + 4\Lambda_{ik}\Lambda_{kj} + 6L_iL_j, \\
\beta_{M_0} &= \frac{1}{2}M_0[\mathrm{Tr}(\Lambda) + 5\Lambda_0] + 3L_iM_i, \\
\beta_{M_i} &= \frac{1}{2}M_i[\Lambda_0 - \mathrm{Tr}(\Lambda)] + 3(M_0L_i + \Lambda_{ij}M_j).
\end{aligned}
$$

Quartic sector: flat 10D coupling vectors

$$q = (\Lambda_0, L_1, L_2, L_3, \Lambda_{11}, \Lambda_{12}, \Lambda_{13}, \Lambda_{22}, \Lambda_{23}, \Lambda_{33})$$

and 4D quadratic coupling vector (M_0, M_i).

Numerical search for an RG stable set of k linear relations

- **Candidate subspaces.** For each k, search for k-tuples of linear relations $n^{(\alpha)} \in \mathbb{R}^{10}$, $\alpha = 1, \ldots, k$, which span the subspace of linear constraints U. Orthonormalize them via SVD.
- **Numerical optimization.** The RG-stability conditions: $E^{(\alpha)}_{ml} = P^{(W)}_{mj} \beta_{ijk} n^{(\alpha)}_i P^{(W)}_{kl} = 0_{ml} \rightarrow$ nonlinear least squares with the Levenberg-Marquardt algorithm.
- 2^{15} quasi-Monte-Carlo **seed points** via either uniform or quasi-random distribution.
- **Classification.** The output is a set of $10 - k$ vectors $q^{(a)}_i \in \mathbb{R}^{10}$, $a = 1, \ldots, 10 - k$. To avoid basis-dependence redundancy, convert each $q^{(a)}$ into a set basis invariants and compare solutions via these sets.

RG-stable 2HDM scalar sectors

- Maximally constrained quartic sector: 9 relations among 10 λ's $\rightarrow$ analytically via bilinears $\rightarrow$ 3 classes of RG-stable situations.
- Less-than-maximally constrained quartic sector: $3 \leq k < 9$ relations $\rightarrow$ analytical derivation + numerical search $\rightarrow$ 16 RG-stable quartic sectors in total. The classification is basis-independent by construction.
- Each quartic sector can be accompanied with several matching RG-stable quadratic terms.
- There are RG-stable scalar sectors coming from normal symmetries, or "GOOFy symmetries", or no symmetry at all.

Inequivalent relation classes of 2HDM in basis independent way

k	Class	Physical-coordinate basis $\mathbf{q}_a$	N_k
9	$\mathcal{A}_1$	$\vec{L}=0, I_1=\Lambda_0, I_2=0, I_3=0$	3
	$\mathcal{A}_2$	$\vec{L}^2=\Lambda_0^2, I_1=\Lambda_0, I_2=0, I_3=0$	
	$\mathcal{A}_3$	$\vec{L}=0, I_1=0, I_2=0, I_3=0$	
8	$\mathcal{B}_1$	$\vec{L}=0, I_3=0, \Delta=0, Q\neq 0$	4
	$\mathcal{B}_2$	$E_k=0, I_1=\Lambda_0, I_2=0, I_3=0$	
	$\mathcal{B}_3$	$\vec{L}=0, I_3=0, I_2=0$	
	$\mathcal{B}_4$	$\vec{L}=0, Q=0$	
7	$\mathcal{C}_1$	$E_k=0, I_2=0, I_3=0$	3
	$\mathcal{C}_2$	$\vec{L}=0, \Delta=0, Q\neq 0$	
	$\mathcal{C}_3$	$\vec{L}=0, I_3=0$	

k	Class	Physical-coordinate basis $\mathbf{q}_a$	N_k
6	$\mathcal{D}_1$	$E_k=0, I_3=0$	4
	$\mathcal{D}_2$	$\vec{L}=0, I_3=0$	
	$\mathcal{D}_3$	$\vec{L}=0$	
	$\mathcal{D}_4$	$E_k=0, d=0$	
5	$\mathcal{E}_1$	$\vec{L}=0$	2
	$\mathcal{E}_2$	$E_k=0$	
4	$\mathcal{F}_1$	$\varepsilon_{ijk}E_i\Lambda_{jm}E_m=0, \lambda_E=0$	2
	$\mathcal{F}_2$	$E_k=0$	
3	$\mathcal{G}_1$	$\vec{L}=0$	2
	$\mathcal{G}_2$	$\varepsilon_{ijk}E_i\Lambda_{jm}E_m=0$	

Conclusions

- RG-stable relations in multi-Higgs models serve as anchor structures for phenomenological exploration $\rightarrow$ all such scenarios must be appreciated.
- Regular global symmetries or the recently proposed "Goofy symmetries" reveal only some of all RG-stable situations. A systematic method for identification of all RG-stable sets of relations, irrespective of the presence of a symmetry, is needed.
- We developed a generic procedure to detect all such situations at least at one loop and applied it to a toy model or the 2HDM. We are finalizing the classification of all the cases.
- It will be intriguing to apply the approach to more elaborate cases such as the 3HDM, where we expect to observe novel situations not induced by the "r_0-symmetry".

Seven new "symmetries"

Combining the r_0 "symmetry" with all six usual symmetries generates new RG-stable 2HDMs, Ferreira, Grzadkowski, Ogreid, Osland, EPJC 84, 234 (2024)

Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
r_0		$-m_{11}^2$			λ_1					$-\lambda_6$
0CP1		$-m_{11}^2$	real		λ_1			real	real	$-\lambda_6$
$0Z_2$		$-m_{11}^2$	0		λ_1				0	0
0U(1)		$-m_{11}^2$	0		λ_1			0	0	0
0CP2	0	0	0		λ_1					$-\lambda_6$
0CP3	0	0	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$	0	0
$0SO(3)$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0