$\mu - e$ conversion in R-parity violating SUSY

Rong-Rong Zhang

In collaboration with Yu-Qi Xiao, Xiao-Gang He, Hong-Yi Niu

JHEP 05(2026) 296

2026.9.13

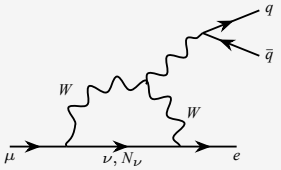
Ouline

- Introduction
- Model
- EFT Approach
- Numerical Results
- Conlusions and Prospects

Introduction

Can charged laptons change to another one like neutrinos?

*S. Weinberg, G. Feinberg, Phys. Rev. Lett. 3 (1959) 111.



• SM contribution on $\mu N \to eN$

There is no tree-level $\mu - e$ conversion because lepton flavor is exactly conserved in the SM.

$$\Gamma_{\text{conv.}} \propto \frac{3\alpha}{32\pi}\Big|\sum_l (V_{PMNS})^*_{\mu i}\,(V_{PMNS})_{ei}\,\frac{m^2_{\nu_i}}{M^2_W}\Big|^2$$

Highly Suppressed

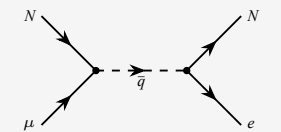
• New Physics contribution on $\mu N \to eN$

In type-I seesaw model, RH neutrino contributes via one-loop.

$$\Gamma_{\text{conv.}} \propto 10^{-5}\Big(\sum_i K^*_{\mu i}K_{ei}\; G\big(\frac{m^2_{N_k}}{m^2_W}\big)\Big)^2 \sim O(0.1)$$

In RPV-SUSY, there exists tree-level contributions.

*R. Alonso, M. Dhen, M. B. Gavela, T. Hambye. arXiv:1209.2679



Observation of cLFV would indicate a clear signal of New physics.

Introduction: Experiment

Decay modes	Limits on Br.	Exp.
$\mu + \mathrm{Ti} \to e + \mathrm{Ti}$	$< 6.1 \times 10^{-13}$	SINDRUM II [12] (1998)
$\mu + \mathrm{Au} \to e + \mathrm{Au}$	$< 7 \times 10^{-13}$	SINDRUM II [13] (2006)
$\mu^+ \to e^+\gamma$	$< 1.5 \times 10^{-13}$	MEG-II [14] (2025)
$\mu^+ \to e^+e^+e^-$	$< 1.0 \times 10^{-12}$	SINDRUM [15] (1987)
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 3 \times 10^{-15}$	COMET (Phase I) [16]
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 3 \times 10^{-17}$	COMET (Phase II)
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 6.2 \times 10^{-16}$	Mu2e (Run I) [17]
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 3 \times 10^{-17}$	Mu2e (Run II) [18]
$\mu^+ \to e^+\gamma$	$\sim 6.0 \times 10^{-14}$	MEG-II [14]
$\mu^+ \to e^+e^+e^-$	$\sim 2.0 \times 10^{-15}$	Mu3e (Phase I) [19]
$\mu^+ \to e^+e^+e^-$	$\sim 10^{-16}$	Mu3e (Phase II)

- $\mu - e$ conversion and $\mu \to 3e$ decay have not updated in 20 years.

- COMET (Phase I) and Mu2e (Run I) will improve the sensitivity by 2-3 orders of magnitude, which will enable a more comprehensive examination of new physics models

Model: RPV-SUSY

• R-parity: $R_p = (-1)^{3(B-L)+2S}$
(+1 for ordinary particles; -1 for their superpartners)
If imposed: forbids B/L violating superpotential terms.

• Minimal SUSY model superpotential

$$W_{MSSM} = y_u U^c Q H_u - y_d D^c Q H_d - y_e E^c L H_d + \mu H_u H_d$$

• **Trilinear R-parity Violation superpotential**

$$W \supset \tfrac{1}{2}\lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \tfrac{1}{2}\lambda''_{ijk} U^c_i D^c_j D^c_k$$

$$\mathcal{L}_\lambda = -\lambda_{ijk}\left(\overline{\nu^c_{Li}} E_{Lj} \tilde{E}^*_{Rk} + \overline{\nu^c_{Li}} \tilde{E}_{Lj} E^c_{Rk} + \tilde{\nu}_{Li}\overline{E^c_{Lj}} E^c_{Rk}\right) + \text{h.c.}$$

$$\mathcal{L}_{\lambda'} = -\lambda'_{ijk}\Big(\overline{\nu^c_{Li}} D_{Lj} \tilde{D}^*_{Rk} + \overline{\nu^c_{Li}} \tilde{D}_{Lj} D^c_{Rk} + \tilde{\nu}_{Li}\overline{D^c_{Lj}} D^c_{Rk}$$
$$- \overline{E^c_{Li}} U_{Lj} \tilde{D}^*_{Rk} - \overline{E^c_{Li}} \tilde{U}_{Lj} D^c_{Rk} - \tilde{E}_{Li}\overline{U^c_{Lj}} D^c_{Rk}\Big) + \text{h.c.}$$

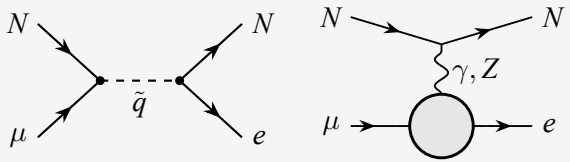
Name		spin-0	spin-1/2	$SU(3)_C \times SU(2)_L \times U(1)_Y$
sqaurk, quark	Q	$(\tilde{U}_L, \tilde{D}_L)$	(U_L, D_L)	$(3, 2, 1/6)$
	U	$\tilde{U}_R$	U_R	$(3, 1, 2/3)$
	D	$\tilde{D}_R$	D_R	$(3, 1, -1/3)$
slepton, lepton	L	$(\tilde{\nu}, \tilde{E}_L)$	(ν, E_L)	$(1, 2, -1/2)$
	E	$\tilde{E}_R$	E_R	$(1, 1, -1)$
Higgs, higgsino	H_u	(H^+_u, H^0_u)	$(\tilde{H}^+_u, \tilde{H}^0_u)$	$(1, 2, 1/2)$
	H_d	(H^0_d, H^-_d)	$(\tilde{H}^0_u, \tilde{H}^-_u)$	$(1, 2, -1/2)$

Name	spin-1	spin-1/2	$SU(3)_C \times SU(2)_L \times U(1)_Y$
gluon, gluino	g	$\tilde{g}$	$(8, 1, 0)$
W bosons, winos	$W^\pm, W^0$	$\tilde{W}^\pm, \tilde{W}^0$	$(1, 3, 0)$
B boson,bino	B^0	$\tilde{B}^0$	$(1, 1, 0)$

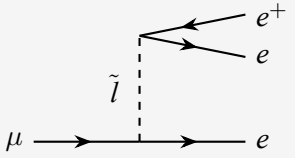
To avoid rapid proton decay, the λ and λ' terms cannot coexist with the λ'' term simultaneously. Due to the main contribution from the λ'' term is one loop-level, here we only focus on the λ and λ' terms.

cLFV Processes

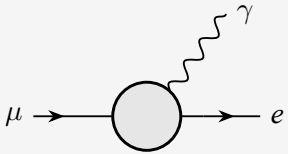
$\mu N \to eN$

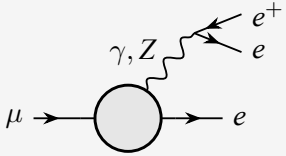


$\mu \to 3e$



$\mu \to e\gamma$





cLFV Processes: EFT Approach

$$\mathcal{L}_{\text{eff}}^{\text{SMEFT}} = \mathcal{L}_{SM,(4)} + \sum_i \mathcal{C}_{i,(5)}\mathcal{O}_{i,(5)} + \sum_i \mathcal{C}_{i,(6)}\mathcal{O}_{i,(6)} + \ldots$$

$$\mathcal{L}_{\text{eff,d5}}^{\text{LEFT}} = [\mathcal{C}_{e\gamma}]_{pr}\left(\overline{E_{Lp}}\sigma^{\mu\nu}E_{Rr}\right)F_{\mu\nu}$$

$$\mathcal{L}_{\text{eff, d6}}^{\text{LEFT}} = \left[\mathcal{C}_{eu}^{V,LL}\right]_{prst}\left(\overline{E_{Lp}}\gamma^{\mu}E_{Lr}\right)\left(\overline{U_{Ls}}\gamma_{\mu}U_{Lt}\right) + \left[\mathcal{C}_{ed}^{V,LL}\right]_{prst}\left(\overline{E_{Lp}}\gamma^{\mu}L_{Lr}\right)\left(\overline{D_{Ls}}\gamma_{\mu}D_{Lt}\right) + \left[\mathcal{C}_{eu}^{V,LR}\right]_{prst}\left(\overline{E_{Lp}}\gamma^{\mu}E_{Lr}\right)\left(\overline{U_{Rs}}\gamma_{\mu}U_{Rt}\right) + \ldots$$

	Energy	RGEs
SUSY model	$(\sim \Lambda_{\text{NP}})$	
SMEFT	$[\Lambda_{\text{EW}}, \Lambda_{\text{NP}}]$	$\frac{d\mathcal{C}_i}{dln\mu} = \frac{1}{16\pi^2}\sum_j \gamma_{ji}\mathcal{C}_i$
LEFT	$[\Lambda_{\chi}, \Lambda_{\text{EW}}]$	$\frac{d\mathcal{C}_i}{dln\mu} = \frac{g_s^2}{16\pi^2}\sum_j \gamma_{ji}^s\mathcal{C}_i + \frac{e^2}{16\pi^2}\sum_j \gamma_{ji}^e\mathcal{C}_i$

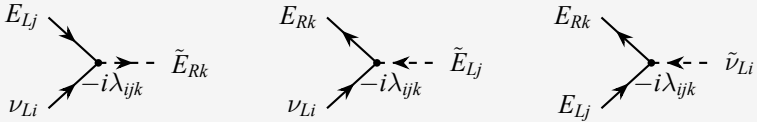
$\Lambda_{\chi}(\sim$ 2GeV) is the energy of the progresses

*Elizabeth E. Jenkins, Aneesh V. Manohar, Michael Trott. arXiv:1310.4838v3 [hep-ph]

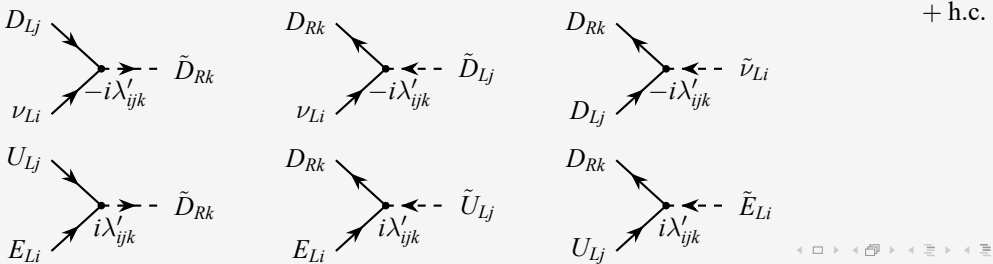
*Elizabeth E. Jenkins, Aneesh V. Manohar, Peter Stoffer. arXiv:1711.05270v3 [hep-ph]

EFT Approach: Lagrangian

$$\mathcal{L}_\lambda = -\lambda_{ijk}\left(\overline{\nu^c_{Li}}E_{Lj}\tilde{E}^*_{Rk} + \overline{\nu^c_{Li}}\tilde{E}_{Lj}E^c_{Rk} + \tilde{\nu}_{Li}\overline{E^c_{Lj}}E^c_{Rk}\right) + \text{h.c.}$$



$$\mathcal{L}_{\lambda'} = -\lambda'_{ijk}\left(\overline{\nu^c_{Li}}D_{Lj}\tilde{D}^*_{Rk} + \overline{\nu^c_{Li}}\tilde{D}_{Lj}D^c_{Rk} + \tilde{\nu}_{Li}\overline{D^c_{Lj}}D^c_{Rk} - \overline{E^c_{Li}}U_{Lj}\tilde{D}^*_{Rk} - \overline{E^c_{Li}}\tilde{U}_{Lj}D^c_{Rk} - \tilde{E}_{Li}\overline{U^c_{Lj}}D^c_{Rk}\right) + \text{h.c.}$$

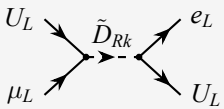


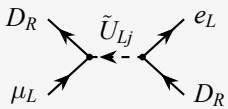
EFT Approach: λ' term tree-level($\lambda = 0$)

New Physics

$$\mathcal{L}_{\text{eff}} \supset \frac{\lambda'_{rtk}\lambda'^{*}_{psk}}{2m^2_{\tilde{D}^*_{Rk}}} \left(\overline{E_{Lp}}\gamma_\mu \overline{E_{Lr}}\right)\left(\overline{U_{Ls}}\gamma^\mu \overline{U_{Lt}}\right)$$

$$- \frac{\lambda'_{rjt}\lambda'^{*}_{pjs}}{2m^2_{\tilde{D}^*_{Lj}}} \left(\overline{E_{Lp}}\gamma_\mu \overline{E_{Lr}}\right)\left(\overline{D_{Rs}}\gamma^\mu \overline{D_{Rt}}\right)$$





SMEFT

$$\mathcal{L}_{\text{eff}}^{\text{SMEFT}} = \mathcal{C}_{\ell q}^{(1)}\mathcal{O}_{\ell q}^{(1)} + \mathcal{C}_{\ell q}^{(3)}\mathcal{O}_{\ell q}^{(3)} + \mathcal{C}_{\ell d}\mathcal{O}_{\ell d}$$

Operators	
$\mathcal{O}_{\ell q}^{(1)}$	$\left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{Q_{Ls}}\gamma^\mu Q_{Lt}\right)$
$\mathcal{O}_{\ell q}^{(3)}$	$\left(\overline{L_{Lp}}\gamma_\mu \tau^I L_{Lr}\right)\left(\overline{Q_{Ls}}\gamma^\mu \tau^I Q_{Lt}\right)$
$\mathcal{O}_{\ell d}$	$\left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{D_{Rs}}\gamma^\mu D_{Rt}\right)$

*τ^I are Pauli matrixes.

EFT Approach: λ' term tree-level($\lambda = 0$)

E.g. Expression of SMEFT Operators

$$\mathcal{O}_{\ell q}^{(1)} = \left(\overline{L_{Lp}}\gamma_\mu \overline{L_{Lr}}\right)\left(\overline{Q_{Ls}}\gamma^\mu \overline{Q_{Lt}}\right) = \begin{pmatrix}\overline{\nu}_{Lp} & \overline{E}_{Lp}\end{pmatrix}\gamma_\mu \begin{pmatrix}\nu_{Lr} \\ E_{Lr}\end{pmatrix} \times \begin{pmatrix}\overline{U}_{Ls} & \overline{D}_{Ls}\end{pmatrix}\gamma^\mu \begin{pmatrix}U_{Lt} \\ D_{Lt}\end{pmatrix}$$
$$= \left(\overline{\nu_{Lp}}\gamma_\mu \nu_{Lr}\right)\left(\overline{U_{Ls}}\gamma^\mu U_{Lt}\right) + \left(\overline{\nu_{Lp}}\gamma_\mu \nu_{Lr}\right)\left(\overline{D_{Ls}}\gamma^\mu D_{Lt}\right) + \dots$$

Effective Lagrangian from SUSY

$$\mathcal{L}_{\text{eff}} \supset \frac{\lambda'_{rtk}\lambda'^{*}_{psk}}{2m^2_{\tilde{D}^*_{Rk}}}\left[\left(\overline{E_{Lp}}\gamma_\mu \overline{E_{Lr}}\right)\left(\overline{U_{Ls}}\gamma^\mu \overline{U_{Lt}}\right) + \left(\overline{\nu_{Lp}}\gamma_\mu \overline{\nu_{Lr}}\right)\left(\overline{D_{Ls}}\gamma^\mu \overline{D_{Lt}}\right)\right] + \dots$$

SMEFT

$$\mathcal{L}_{\text{eff}}^{\text{SMEFT}} = \mathcal{C}_{\ell q}^{(1)}\mathcal{O}_{\ell q}^{(1)} + \mathcal{C}_{\ell q}^{(3)}\mathcal{O}_{\ell q}^{(3)} + \mathcal{C}_{\ell d}\mathcal{O}_{\ell d}$$
$$= \left(\mathcal{C}_{\ell q}^{(1)} - \mathcal{C}_{\ell q}^{(3)}\right)\left[\left(\overline{E_{Lp}}\gamma_\mu \overline{E_{Lr}}\right)\left(\overline{U_{Ls}}\gamma^\mu \overline{U_{Lt}}\right) + \left(\overline{\nu_{Lp}}\gamma_\mu \overline{\nu_{Lr}}\right)\left(\overline{D_{Ls}}\gamma^\mu \overline{D_{Lt}}\right)\right] + \dots$$

$$\Rightarrow \quad \mathcal{C}_{\ell q,prst}^{(1)} = \frac{1}{4}\frac{\lambda'_{rtk}\lambda'^{*}_{psk}}{m^2_{\tilde{D}_{Rk}}}, \quad \mathcal{C}_{\ell q,prst}^{(3)} = -\frac{1}{4}\frac{\lambda'_{rtk}\lambda'^{*}_{psk}}{m^2_{\tilde{D}_{Rk}}}, \quad \mathcal{C}_{\ell d,prst} = -\frac{1}{2}\frac{\lambda'_{rjt}\lambda'^{*}_{pjs}}{m^2_{\tilde{D}_{Lj}}}$$

EFT Approach: λ' term tree-level($\lambda = 0$)

SMEFT

Operators	
$\mathcal{O}_{\ell q}^{(1)}$	$\left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{Q_{Ls}}\gamma^\mu Q_{Lt}\right)$
$\mathcal{O}_{\ell q}^{(3)}$	$\left(\overline{L_{Lp}}\gamma_\mu \tau^I L_{Lr}\right)\left(\overline{Q_{Ls}}\gamma^\mu \tau^I Q_{Lt}\right)$
$\mathcal{O}_{\ell d}$	$\left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{D_{Rs}}\gamma^\mu D_{Rt}\right)$

LEFT

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{LEFT}} = &\left[\mathcal{C}_{eu}^{V,LL}\right]_{prst}\left(\overline{E_{Lp}}\gamma^\mu E_{Lr}\right)\left(\overline{U_{Ls}}\gamma_\mu U_{Lt}\right)\\ &+\left[\mathcal{C}_{ed}^{V,LL}\right]_{prst}\left(\overline{E_{Lp}}\gamma^\mu L_{Lr}\right)\left(\overline{D_{Ls}}\gamma_\mu D_{Lt}\right)\\ &+\left[\mathcal{C}_{ed}^{V,LR}\right]_{prst}\left(\overline{E_{Lp}}\gamma^\mu E_{Lr}\right)\left(\overline{D_{Rs}}\gamma_\mu D_{Rt}\right)\end{aligned}$$

The matching results from SMEFT to LEFT

$$\left[\mathcal{C}_{eu}^{V,LL}\right]_{prst}(\Lambda_{\text{EW}}) = \left[\mathcal{C}_{\ell q}^{(1)} - \mathcal{C}_{\ell q}^{(3)}\right]_{prst}(\Lambda_{\text{EW}}) + \mathcal{C}(g_Z^2/m_Z^2)$$

$$\left[\mathcal{C}_{ed}^{V,LL}\right]_{prst}(\Lambda_{\text{EW}}) = \left[\mathcal{C}_{\ell q}^{(1)} + \mathcal{C}_{\ell q}^{(3)}\right]_{prst}(\Lambda_{\text{EW}}) + \mathcal{C}(g_Z^2/m_Z^2)$$

$$\left[\mathcal{C}_{ed}^{V,LR}\right]_{prst}(\Lambda_{\text{EW}}) = \left[\mathcal{C}_{\ell d}\right]_{prst}(\Lambda_{\text{EW}}) + \mathcal{C}(g_Z^2/m_Z^2)$$

The $\mathcal{C}(g_Z^2/m_Z^2)$ represents the Z boson contributions.

Approach: Match Results – From NP to SMEFT

Operators

- λ term: tree-level

$$\mathcal{O}_{\ell\ell} \quad (\overline{L_{Lp}}\gamma_\mu L_{Lr})\ (\overline{L_{Ls}}\gamma^\mu L_{Lt})$$
$$\mathcal{O}_{\ell e} \quad (\overline{L_{Lp}}\gamma_\mu L_{Lr})\ (\overline{E_{Rs}}\gamma^\mu E_{Rt})$$

- λ' term: one loop-level

$$\mathcal{O}_{eW} \quad (\overline{L_{Lp}}\sigma^{\mu\nu} E_{Rr})\ \tau^I \phi W^I_{\mu\nu}$$
$$\mathcal{O}_{eB} \quad \left(\overline{\ell_p}\sigma^{\mu\nu} E_{Rr}\right) \phi B_{\mu\nu}$$
$$\mathcal{O}^{(1)}_{\ell q} \quad (\overline{L_{Lp}}\gamma_\mu L_{Lr})\ \left(\overline{Q_{Ls}}\gamma^\mu Q_{Lt}\right)$$

…

$m_{\tilde{U}_R} = m_{\tilde{D}_R} = m_{\tilde{Q}_L} \equiv m_{\tilde{q}} \sim \mathcal{O}(TeV)$

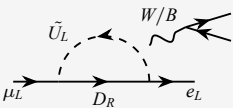
$F_{2,5}$ are loop functions,
$F_2 \sim 1/6, F_2' \sim -1/9, F_5 \sim 1/3$

Coefficients Relations

- λ term: tree-level

$$[\mathcal{C}_{\ell\ell}]_{prst} = \frac{\lambda_{rtk}\lambda^*_{psk}}{4m^2_{\tilde{E}_{Rk}}}, [\mathcal{C}_{\ell e}]_{prst} = -\frac{\lambda_{rjs}\lambda^*_{pjt}}{2m^2_{\tilde{L}_j}}$$

- λ' term: one loop-level



$$\mathcal{C}^{12}_{eW} = \frac{-3g_2 Y_e^{*2}}{128\pi^2 m^2_{\tilde{q}}}\lambda'_{spr}\lambda'^*_{1pr}\left(\frac{1}{3}F_2^{(1)} + \frac{2}{3}F_2^{(1)} - \frac{2}{3}F_5^{(1)} - \frac{1}{3}F_5^{(1)}\right)$$

$$\mathcal{C}^{12}_{eB} = \frac{3g_1 Y_e^{*2}}{128\pi^2 m^2_{\tilde{q}}}\lambda'_{spr}\lambda'^*_{1pr}\left(\frac{1}{3}F_2^{(1)} + \frac{2}{3}F_2^{(1)} - \frac{2}{3}F_5^{(1)} - \frac{1}{3}F_5^{(1)}\right)$$

$$\mathcal{C}^{(1)}_{\ell q,12st} = \frac{-3g_1^2 c_W^2}{192\pi^2 m^2_{\tilde{q}}}\lambda'_{2pr}\lambda'^*_{1pr}\left(\tfrac{1}{3}F_2'^{(1)} + \tfrac{2}{3}F_2'^{(1)} - \tfrac{2}{3}F_5'^{(1)} - \tfrac{1}{3}F_5'^{(1)}\right)$$

Loop Functions

…

Approach: Match Results – From NP to SMEFT

Operators

- λ term: one loop-level

$$\mathcal{O}_{eW} \quad \left(\overline{L_{Lp}}\sigma^{\mu\nu}E_{Rr}\right)\tau^I\phi W^I_{\mu\nu}$$
$$\mathcal{O}_{eB} \quad \left(\overline{\ell_p}\sigma^{\mu\nu}E_{Rr}\right)\phi B_{\mu\nu}$$
$$\mathcal{O}^{(1)}_{\ell q} \quad \left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{Q_{Ls}}\gamma^\mu Q_{Lt}\right)$$
$$\mathcal{O}^{(3)}_{\ell q} \quad \left(\overline{L_{Lp}}\gamma_\mu \tau^I L_{Lr}\right)\left(\overline{Q_{Ls}}\gamma^\mu \tau^I Q_{Lt}\right)$$
$$\mathcal{O}_{\ell u} \quad \left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{U_{Rs}}\gamma^\mu U_{Rt}\right)$$
$$\mathcal{O}_{\ell d} \quad \left(\overline{L_{Lp}}\gamma_\mu L_{Lr}\right)\left(\overline{D_{Rs}}\gamma^\mu D_{Rt}\right)$$

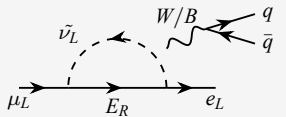
...

$m_{\tilde{E}_R} = m_{\tilde{L}} \equiv m_{\tilde{l}} \sim \mathcal{O}(TeV)$

$F_{2,5}$ are loop functions,
$F_2 \sim 1/6, F_2' \sim -1/9, F_5 \sim 1/3$

Coefficients Relations

- λ term: one loop-level



$$\mathcal{C}^{12}_{eW} = \frac{g_2 Y_e^2}{128\pi^2 m_{\tilde{\ell}}^2}\lambda_{3sk}\lambda^*_{31k}\left(F_2^{(1)} - F_5^{(2)}\right)$$

$$\mathcal{C}^{12}_{eB} = -\frac{g_1 Y_e^2}{128\pi^2 m_{\tilde{\ell}}^2}\lambda_{3sk}\lambda^*_{31k}\left(F_2^{(1)} - F_5^{(2)}\right)$$

$$\mathcal{C}^{(1)}_{\ell q,12st} = \frac{g_1^2 c_W^2}{6}\frac{\lambda_{ij1}\lambda^*_{ij2}}{32\pi^2 m_{\tilde{\ell}}}\left(-F_2'^{(1)} + F_5'^{(2)}\right)$$

$$\mathcal{C}^{(3)}_{\ell q,12st} = -\frac{g_1^2 c_W^2}{2}\frac{\lambda_{ij1}\lambda^*_{ij2}}{32\pi^2 m_{\tilde{\ell}}}\left(-F_2'^{(1)} + F_5'^{(2)}\right)$$

Loop Functions

...

Branch of $\mu - e$ conversion

$$
\begin{aligned}
\mathrm{Br} = \frac{1}{\Gamma_{\mathrm{cap.}}} \Big(& \left| (D\mathcal{C}^{*}_{e\gamma,12})/(2m_\mu) \right. \\
& + \left(2\mathcal{C}^{V,LL}_{eu,1211} + 2\mathcal{C}^{V,LL}_{eu,1211} + \mathcal{C}^{V,LL}_{ed,1211} + \mathcal{C}^{V,LR}_{ed,1211} \right) V^{(p)} \\
& + \left(\mathcal{C}^{V,LL}_{eu,1211} + \mathcal{C}^{V,LR}_{eu,1211} + 2\mathcal{C}^{V,LL}_{ed,1211} + 2\mathcal{C}^{V,LR}_{ed,1211} \right) V^{(n)} \Big|^2 \\
& + \Big| \left(2\mathcal{C}^{V,RR}_{eu,1211} + 2\mathcal{C}^{V,LR}_{ue,1112} + \mathcal{C}^{V,RR}_{ed,1211} + \mathcal{C}^{V,LR}_{de,1112} \right) V^{(p)} \\
& + \left(\mathcal{C}^{V,RR}_{eu,1211} + \mathcal{C}^{V,LR}_{ue,1112} + 2\mathcal{C}^{V,RR}_{ed,1211} + 2\mathcal{C}^{V,LR}_{de,1112} \right) V^{(n)} \\
& \left. + (D\mathcal{C}_{e\gamma,21})/(2m_\mu) \right|^2 \Big)
\end{aligned}
$$

Branch of $\mu \to e\gamma$

$$
\mathrm{Br}(\mu \to e\gamma) = \tau_\mu \times \frac{\left(m_\mu^2 - m_e^2\right)^3}{4\pi m_\mu^3} \left(|\mathcal{C}_{e\gamma,12}|^2 + |\mathcal{C}_{e\gamma,21}|^2 \right)
$$

Branch of $\mu \to 3e$

$$
\begin{aligned}
\mathrm{Br}(\mu \to 3e) = & \frac{1}{64G_F^2} \left(\left| \mathcal{C}^{S,RR}_{ee,1121} \right|^2 + \left| \mathcal{C}^{S,RR}_{ee,1112} \right|^2 \right) \\
& + \frac{\alpha_{\mathrm{em}}}{\pi G_F^2 m_\mu^2} \left(\ln \frac{m_\mu^2}{m_e^2} - \frac{17}{4} \right) \left(|\mathcal{C}_{e\gamma,12}|^2 + |\mathcal{C}_{e\gamma,21}|^2 \right) \\
& + \frac{1}{8G_F^2} \left(2 \left| \mathcal{C}^{V,RR}_{ee,1112} + \frac{4e}{m_\mu} \mathcal{C}^{*}_{e\gamma,12} \right|^2 \right. \\
& + 2 \left| \mathcal{C}^{V,LL}_{ee,1112} + \frac{4e}{m_\mu} \mathcal{C}_{e\gamma,21} \right|^2 \\
& \left. + \left| \mathcal{C}^{V,LR}_{ee,1112} + \frac{4e}{m_\mu} \mathcal{C}^{*}_{e\gamma,12} \right|^2 + \left| \mathcal{C}^{V,LR}_{ee,1211} + \frac{4e}{m_\mu} \mathcal{C}_{e\gamma,21} \right|^2 \right)
\end{aligned}
$$

Connecting with the experimental branch data, we can get the upper limits of λ and λ'. In calculation, we set only one λ or λ' nonzero once.

Phenomena

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty dr\, r^2 \left[-E(r)\right] \left(g_e^- f_\mu^- + f_e^- g_\mu^-\right),$$

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 Z\, \rho^{(p)} \left(g_e^- g_\mu^- - f_e^- f_\mu^-\right),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 (A-Z)\, \rho^{(n)} \left(g_e^- g_\mu^- - f_e^- f_\mu^-\right),$$

$$V^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 Z\, \rho^{(p)} \left(g_e^- g_\mu^- + f_e^- f_\mu^-\right),$$

$$V^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 (A-Z)\, \rho^{(n)} \left(g_e^- g_\mu^- + f_e^- f_\mu^-\right)$$

Density distributions of proton and neutron.

Isotopes	D $[m_\mu^{5/2}]$	$S^{(p)}$ $[m_\mu^{5/2}]$	$V^{(p)}$ $[m_\mu^{5/2}]$	$S^{(n)}$ $[m_\mu^{5/2}]$	$V^{(n)}$ $[m_\mu^{5/2}]$	$\Gamma_{\text{cap.}}$ $[10^6 \text{s}^{-1}]$
^{27}Al	0.0359	0.0159	0.0165	0.0172	0.0178	0.7054
^{48}Ti	0.0859	0.0379	0.0407	0.0448	0.0481	2.590
^{197}Au	0.166	0.0523	0.0866	0.0781	0.129	13.07

$D, S^{(p,n)}, V^{(p,n)}$:
the overlap integrals

The integral uncertainties:
$2\% \sim 5\%$ for Al, Ti
$8\% \sim 10\%$ for Au

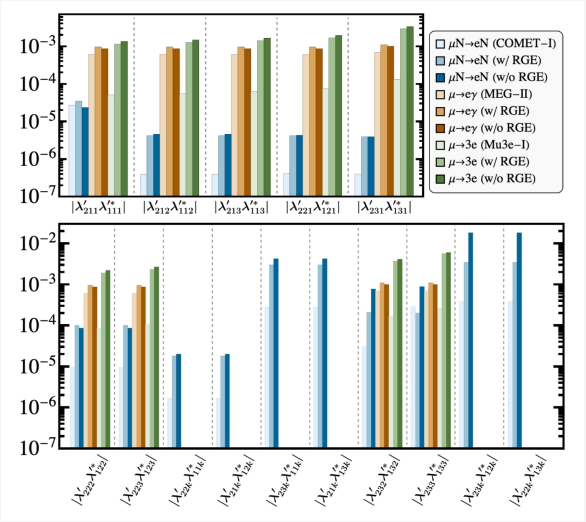
*Julian Heeck, Robert Szafron, Yuichi Uesaka. arXiv:2203.00702 [hep-ph]

Phenomena

Decay modes	Limits on Br.	Exp.
$\mu + \mathrm{Ti} \to e + \mathrm{Ti}$	$< 6.1 \times 10^{-13}$	SINDRUM II [12] (1998)
$\mu + \mathrm{Au} \to e + \mathrm{Au}$	$< 7 \times 10^{-13}$	SINDRUM II [13] (2006)
$\mu^+ \to e^+\gamma$	$< 1.5 \times 10^{-13}$	MEG-II [14] (2025)
$\mu^+ \to e^+e^+e^-$	$< 1.0 \times 10^{-12}$	SINDRUM [15] (1987)
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 3 \times 10^{-15}$	COMET (Phase I) [16]
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 3 \times 10^{-17}$	COMET (Phase II)
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 6.2 \times 10^{-16}$	Mu2e (Run I) [17]
$\mu + \mathrm{Al} \to e + \mathrm{Al}$	$\sim 3 \times 10^{-17}$	Mu2e (Run II) [18]
$\mu^+ \to e^+\gamma$	$\sim 6.0 \times 10^{-14}$	MEG-II [14]
$\mu^+ \to e^+e^+e^-$	$\sim 2.0 \times 10^{-15}$	Mu3e (Phase I) [19]
$\mu^+ \to e^+e^+e^-$	$\sim 10^{-16}$	Mu3e (Phase II)

We can get the branch of $\mu - e$ conversion, $\mu \to e\gamma$ and $\mu \to 3e$ from the experiments.

Numerical Results

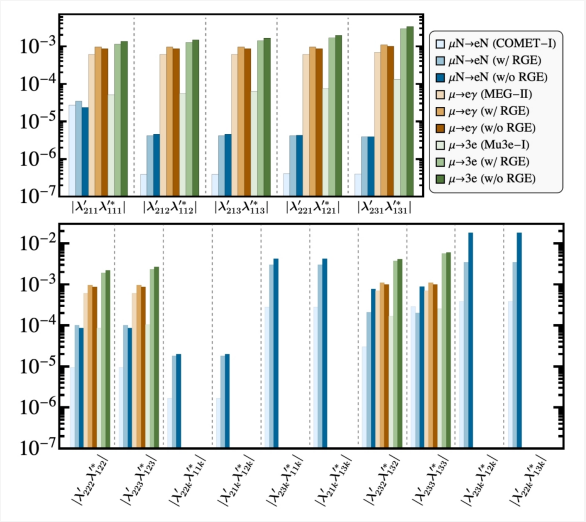


- For $\mu \to e\gamma$, $\mu \to 3e$, in some cases contributions are GIM suppressed.
- For $\mu N \to eN$, in certain cases the RG running can give 80% improvement of the limit.
- For certain cases, neutrino mass will give much stronger limits than cLFV.

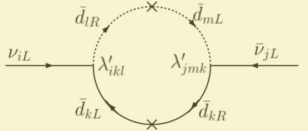
$$|\lambda'_{133}\lambda'_{233}| < \mathcal{O}(10^{-7} \sim 10^{-8}) \times (\tilde{m}\ \mathrm{TeV}^{-1})$$

*Aneesh Manohar, Howard Georgi. Nuclear Physics B 231(1984)194

Numerical Results



PRD 26, 287 (1982)



$$M_{\nu,ij} \simeq \frac{3}{8\pi^2}\,\lambda'_{i33}\lambda'_{j33}\,\frac{m_b^2}{\tilde{m}} + \ldots$$

• I … ce can be …

• I … $e\gamma$ and μ … n near fut… most of the…

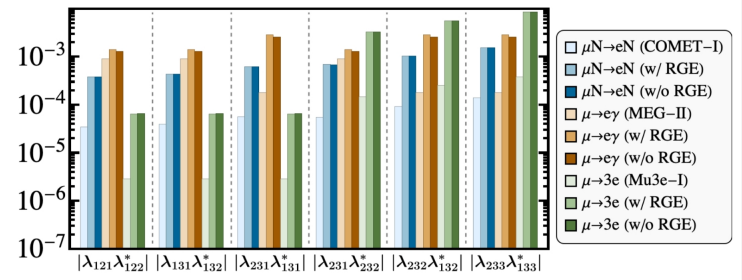
• … more than one nonzero at a time.

• For certain cases, neutrino mass will give much stronger limits than cLFV.

$$|\lambda'_{133}\lambda'_{233}| < \mathcal{O}(10^{-7} \sim 10^{-8}) \times (\tilde{m}\ \mathrm{TeV}^{-1})$$

*Aneesh Manohar, Howard Georgi. Nuclear Physics B 231(1984)194

Numerical Results



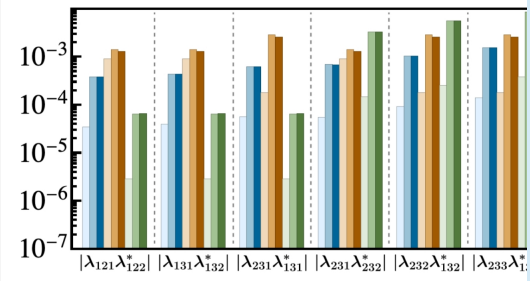
For the first three cases:

- Tree-level contribution to $\mu \to 3e$
- Future $\mu \to 3e$ exp. will give the most stringent limits on the para.

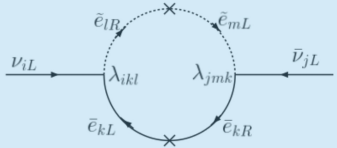
For the last three cases:

- No tree-level contribution to $\mu \to 3e$
- Future $\mu N \to eN$ exp. will give the stronger limits.

Numerical Results



PRD 26, 287 (1982)



$$M_{\nu,ij} \simeq \frac{1}{8\pi^2}\lambda_{i33}\lambda_{j33}\frac{m_\tau^2}{\tilde{m}} + \ldots$$

$$|\lambda_{233}\lambda_{133}| < 3 \times 10^{-6}$$

For the first three cases:

- Tree-level contribution to $\mu \to 3e$
- Future $\mu \to 3e$ exp. will give the most stringent limits on the para.

For the last three cases:

- No tree-level contribution to $\mu \to 3e$
- Future $\mu N \to eN$ exp. will give the stronger limits.

Conclusions and Prospects

• We find that the $\mu N \to eN$ can give much stronger limits than $\mu \to e\gamma$, $\mu \to 3e$ in most cases for λ' terms, also the same for specific cases of λ terms.

• In certain cases, $\mu N \to eN$ conversion is the only process that can provide constraints.

• The RGE running effects can play an important role and cannot be neglected in certain cases.

• Combined analysis of all the possible phenomena will lead to a much more comprehensive examination on the parameter space.

• As there will be cLFV experimental improvements in the near future, it is necessary to pay more attention to how they can play roles in NP models.

Thank you!

Back up

$\mathcal{C}(g_Z^2/m_Z^2)$

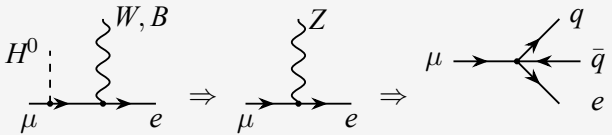
SMEFT Operators

$$O_{\varphi\ell}^{(1)} \quad \left(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi\right)\left(L_{Lp}\gamma^\mu L_{Lr}\right)$$

$$O_{\varphi\ell}^{(3)} \quad \left(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi\right)\left(L_{Lp}\tau^I\gamma^\mu L_{Lr}\right)$$

$$O_{\varphi e} \quad \left(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi\right)\left(E_{Rp}\gamma^\mu E_{Rr}\right)$$

$\Lambda_{\rm NP} \sim$ TeV $\quad \Lambda_{\rm EW} > m_Z \quad \Lambda_\chi \sim 2$GeV



$$D_\mu\Phi = \left(\partial_\mu - ig\frac{\tau_i}{2}W_\mu^i - ig'\frac{1}{2}B_\mu\right)\Phi$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \partial_\mu - \frac{i}{2}\left(gW_\mu^3 + g'B_\mu\right) & -\frac{ig}{2}\left(W_\mu^1 - iW_\mu^2\right) \\ -\frac{ig}{2}\left(W_\mu^1 + iW_\mu^2\right) & \partial_\mu + \frac{i}{2}\left(gW_\mu^3 - g'B_\mu\right) \end{pmatrix}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$Z_\mu = \frac{1}{g_Z}(gW_\mu^3 - g'B_\mu)$$

$$g_Z = \sqrt{g^2 + g'^2}$$

These three operators would also contribute to tree-level $\mu - e$ conversion.

Back up

SMEFT

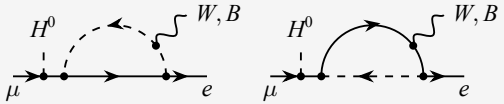
$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{SMEFT}} &= \mathcal{O}_{eW} + \mathcal{O}_{eB} \\ &= \left(\overline{L_{Lp}}\sigma^{\mu\nu}E_{Rr}\right)\tau^I\phi W^I_{\mu\nu} \\ &+ \left(\overline{\ell_p}\sigma^{\mu\nu}E_{Rr}\right)\phi B_{\mu\nu}\end{aligned}$$

NP to SMEFT

$$\mathcal{C}^{12}_{eW} = \frac{g_2 Y_e^{s2}}{128\pi^2 m^2_{\tilde{\ell}}}\lambda_{3sk}\lambda^*_{31k}\left(F_2^{(1)} - F_5^{(2)}\right)$$

$$\mathcal{C}^{12}_{eB} = -\frac{g_1 Y_e^{s2}}{128\pi^2 m^2_{\tilde{\ell}}}\lambda_{3sk}\lambda^*_{31k}\left(F_2^{(1)} - F_5^{(2)}\right)$$

which can be derived through the Mathematica package Matchete. F_2, F_5 are loop functions.



LEFT

$$\mathcal{L}_{\text{eff}}^{\text{LEFT}} = \left[\mathcal{C}_{e\gamma}\right]_{pr}\left(\overline{E_{Lp}}\sigma^{\mu\nu}E_{Rr}\right)F_{\mu\nu}$$

SMEFT to LEFT

$$\mathcal{C}_{e\gamma,12} = -83.57\mathcal{C}^{12}_{eW} + 152.7\mathcal{C}^{12}_{eB}$$

$$\mathcal{C}_{e\gamma,21} = -83.57\mathcal{C}^{21}_{eW} + 152.7\mathcal{C}^{21}_{eB}$$

* RGEs are solved by Wilson package

*Elizabeth E. Jenkins, Aneesh V. Manohar, Peter Stoffer, arXiv:1709.04486

Back up

SMEFT

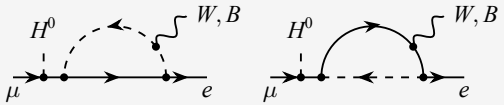
$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{SMEFT}} &= \mathcal{O}_{eW} + \mathcal{O}_{eB} \\ &= \left(\overline{L_{Lp}}\sigma^{\mu\nu}E_{Rr}\right)\tau^I\phi W^I_{\mu\nu} \\ &+ \left(\overline{\ell_p}\sigma^{\mu\nu}E_{Rr}\right)\phi B_{\mu\nu}\end{aligned}$$

NP to SMEFT

$$\mathcal{C}_{eW}^{pr} = \frac{g_2 Y_e^{sr}}{128\pi^2 m_{\tilde{\ell}}^2}\lambda_{3sk}\lambda_{3pk}^*\left(F_2^{(1)} - F_5^{(2)}\right)$$

$$\mathcal{C}_{eB}^{pr} = -\frac{g_1 Y_e^{sr}}{128\pi^2 m_{\tilde{\ell}}^2}\lambda_{3sk}\lambda_{3pk}^*\left(F_2^{(1)} - F_5^{(2)}\right)$$

which can be derived through the Mathematica package Matchete. F_2, F_5 are loop functions.



LEFT

$$\mathcal{L}_{\text{eff}}^{\text{LEFT}} = \left[\mathcal{C}_{e\gamma}\right]_{pr}\left(\overline{E_{Lp}}\sigma^{\mu\nu}E_{Rr}\right)F_{\mu\nu}$$

SMEFT to LEFT

$$\mathcal{C}_{e\gamma,12} = -83.57\mathcal{C}_{eW}^{12} + 152.7\mathcal{C}_{eB}^{12}$$

$$\mathcal{C}_{e\gamma,21} = -83.57\mathcal{C}_{eW}^{21} + 152.7\mathcal{C}_{eB}^{21}$$

* RGEs are solved by Wilson package

*Elizabeth E. Jenkins, Aneesh V. Manohar, Peter Stoffer, arXiv:1709.04486

Back up

$$\mathrm{Br}(\mu N \to eN) = \frac{\Gamma_{\text{conv.}}}{\Gamma_{\text{cap.}}}$$

$\Gamma_{\text{cap.}}$: the capture rate of muons.

$\Gamma_{\text{conv.}} = |\mathcal{M}|^2 = -|\langle e, N| \mathcal{L}_{\text{eff}} |\mu, N\rangle|^2$

Effective Lagrangian relating to $\mu - e$ conv.(∼2GeV)

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & -\frac{4G_F}{\sqrt{2}} \left(m_\mu A_R \bar{\mu}\sigma^{\mu\nu} P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu}\sigma^{\mu\nu} P_R e F_{\mu\nu} + \text{h.c.}\right) \\
& - \frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} \Big[\left(g^q_{LS} \bar{e} P_R \mu + g^q_{RS} \bar{e} P_L \mu\right) \bar{q} q + \left(g^q_{LP} \bar{e} P_R \mu + g^q_{RP} \bar{e} P_L \mu\right) \bar{q} \gamma_5 q \\
& + \left(g^q_{LV} \bar{e} \gamma^\mu P_L \mu + g^q_{RV} \bar{e} \gamma^\mu P_R \mu\right) \bar{q} \gamma_\mu q + \left(g^q_{LA} \bar{e} \gamma^\mu P_L \mu + g^q_{RA} \bar{e} \gamma^\mu P_R \mu\right) \bar{q} \gamma_\mu \gamma_5 q \\
& + \frac{1}{2} \left(g^q_{LT} \bar{e} \sigma^{\mu\nu} P_R \mu + g^q_{RT} \bar{e} \sigma^{\mu\nu} P_L \mu\right) \bar{q} \sigma_{\mu\nu} q + \text{h.c.} \Big]
\end{aligned}$$

*Julian Heeck, Robert Szafron, Yuichi Uesaka. arXiv:2203.00702 [hep-ph]

Back up

$$\Gamma_{\text{conv.}} = |\mathcal{M}|^2 = |\langle e, N| \mathcal{L}_{\text{eff}} |\mu, N\rangle|^2$$

$$\mathcal{M} = \langle e, N| -\mathcal{L}_{\text{eff}} |\mu, N\rangle = \langle e| (-\mathcal{L}_\ell) |\mu\rangle \langle N| (-\mathcal{L}_N) |N\rangle$$

Lepton Part

$$\langle e| A_R \overline{\mu} \sigma^{\mu\nu} P_L e |\mu\rangle = \int d^3x A_R^* \bar{\psi}^{\mu(e)}_{\kappa,W} \sigma^{\alpha\beta} P_R \bar{\psi}^{(\mu)}_{1s}$$

$$\langle e| \bar{e} P_R \mu |\mu\rangle = \int d^3x \bar{\psi}^{\mu(e)}_{\kappa,W} P_R \bar{\psi}^{(\mu)}_{1s}$$

$\dots$

Quark Part

$$\langle N| F_{\mu\nu} |N\rangle \to E(r) = \frac{Ze}{r^2} \int_0^r r'^2 \rho^{(p)}(r') dr'$$

$$\langle N| \bar{q} q |N\rangle = Z G_S^{(q,p)} \rho^{(p)} + (A - Z) G_S^{(q,n)} \rho^{(n)}$$

$$\langle N| \bar{q} \gamma^0 q |N\rangle = Z G_V^{(q,p)} \rho^{(p)} + (A - Z) G_V^{(q,n)} \rho^{(n)}$$

Others are all zero.

$\rho^{(p,n)}$ are proton and neutron densities. $\psi^{(e,\mu)}$ are derived from the Dirac equations. κ, μ and W are quantum numbers. $E(r)$ is the electric field derived from the Maxwell equations. $1s$ means the initial state is $1s$ state of the muonic atom.

*Julian Heeck, Robert Szafron, Yuichi Uesaka. arXiv:2203.00702 [hep-ph]

Back up

The Dirac Equation in Nucleus

$$W\psi = \left[-i\gamma_5\sigma_r\left(\frac{\partial}{\partial r} + \frac{1}{r} - \frac{\beta}{r}K\right) + V(r) + m_i\beta\right]\psi, \quad V(r) = -e\int_r^\infty E(r')\,dr'$$

Solutions

$$\psi_{1s}^{(\mu)}(r,\theta,\phi) = \begin{pmatrix} g_\mu^-(r)\,\chi_{-1}^{\pm 1/2}(\theta,\phi) \\ if_\mu^-(r)\,\chi_1^{\pm 1/2}(\theta,\phi) \end{pmatrix}$$

$$\psi_{\kappa=-1,W}^{\mu=\pm 1/2(e)}(r,\theta,\phi) = \begin{pmatrix} g_e^-(r)\chi_{-1}^{\pm 1/2}(\theta,\phi) \\ if_e^-(r)\chi_1^{\pm 1/2}(\theta,\phi) \end{pmatrix}, \quad \psi_{\kappa=1,W}^{\mu=\pm 1/2(e)}(r,\theta,\phi) = \begin{pmatrix} g_e^+(r)\chi_1^{\pm 1/2}(\theta,\phi) \\ if_e^+(r)\chi_{-1}^{\pm 1/2}(\theta,\phi) \end{pmatrix}$$

Neglecting the electron mass, we have $g_e^+ = if_e^-$ and $g_e^- = if_e^+$. So we can take these two electric wave functions as one.

*Julian Heeck, Robert Szafron, Yuichi Uesaka. arXiv:2203.00702 [hep-ph]

Back up

The $\mu - e$ conversion rate

$$\Gamma_{\text{conv.}} = 2G_F^2 \left| A_R D + \tilde{g}_{LS}^p S^{(p)} + \tilde{g}_{LS}^n S^{(n)} + \tilde{g}_{LV}^p V^{(p)} + \tilde{g}_{LV}^n V^{(n)} \right|^2 + 2G_F^2 \left| A_L D + \tilde{g}_{RS}^p S^{(p)} + \tilde{g}_{RS}^n S^{(n)} + \tilde{g}_{RV}^p V^{(p)} + \tilde{g}_{RV}^n V^{(n)} \right|^2$$

$$\tilde{g}_{LS}^p = G_S^{p,u} g_{LS}^u + G_S^{p,d} g_{LS}^d + G_S^{p,s} g_{LS}^s,$$
$$\tilde{g}_{RS}^p = G_S^{p,u} g_{RS}^u + G_S^{p,d} g_{RS}^d + G_S^{p,s} g_{RS}^s, \ldots$$

$G_V^{p,u} = G_V^{n,d} = 2$, $G_V^{p,d} = G_V^{n,u} = 1$,
$G_V^{p,s} = G_V^{n,s} = 0$.

The values of G_S factors are derived from arXiv:2411.04268.

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty dr\, r^2 \left[-E(r)\right] \left(g_e^- f_\mu^- + f_e^- g_\mu^- \right),$$
$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty dr\, r^2 Z \rho^{(p)} \left(g_e^- g_\mu^- - f_e^- f_\mu^- \right), \ldots$$

Wava function $\chi(\theta, \phi)$ are vanished through

$$\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, \chi_\kappa^{\mu *} \chi_{\kappa'}^{\mu'} = \delta_{\mu\mu'} \delta_{\kappa\kappa'}$$

*Julian Heeck, Robert Szafron, Yuichi Uesaka. arXiv:2203.00702 [hep-ph]

Back up

LEFT operators relating to $\mu - e$ conv.(∼2GeV)

$$\mathcal{L}^{\text{LEFT}}_{\text{eff,d5}} = \left[\mathcal{C}_{e\gamma}\right]_{pr} \left(\overline{E_{Lp}}\sigma^{\mu\nu} E_{Rr}\right) F_{\mu\nu}$$

$$\begin{aligned}
\mathcal{L}^{\text{LEFT}}_{\text{eff, d6}} = & \left[\mathcal{C}^{T,RR}_{eu}\right]_{prst} \left(\overline{E_{Lp}}\sigma^{\mu\nu} E_{Rr}\right) \left(\overline{U_{Ls}}\sigma_{\mu\nu} U_{Rt}\right) + \left[\mathcal{C}^{T,RR}_{ed}\right]_{prst} \left(\overline{E_{Lp}}\sigma^{\mu\nu} E_{Rr}\right) \left(\overline{U_{Ls}}\sigma_{\mu\nu} U_{Rt}\right) \\
& + \left[\mathcal{C}^{V,LL}_{eu}\right]_{prst} \left(\overline{E_{Lp}}\gamma^{\mu} E_{Lr}\right) \left(\overline{U_{Ls}}\gamma_{\mu} U_{Lt}\right) + \left[\mathcal{C}^{V,LL}_{ed}\right]_{prst} \left(\overline{E_{Lp}}\gamma^{\mu} E_{Lr}\right) \left(\overline{D_{Ls}}\gamma_{\mu} D_{Lt}\right) \\
& + \left[\mathcal{C}^{V,LR}_{eu}\right]_{prst} \left(\overline{E_{Lp}}\gamma^{\mu} E_{Lr}\right) \left(\overline{U_{Rs}}\gamma_{\mu} U_{Rt}\right) + \left[\mathcal{C}^{V,LR}_{ed}\right]_{prst} \left(\overline{E_{Lp}}\gamma^{\mu} E_{Lr}\right) \left(\overline{D_{Rs}}\gamma_{\mu} D_{Rt}\right) \\
& + \left[\mathcal{C}^{V,LR}_{ue}\right]_{prst} \left(\overline{U_{Ls}}\gamma_{\mu} U_{Lt}\right) \left(\overline{E_{Rp}}\gamma^{\mu} E_{Rr}\right) + \left[\mathcal{C}^{V,LR}_{de}\right]_{prst} \left(\overline{D_{Ls}}\gamma_{\mu} D_{Lt}\right) \left(\overline{E_{Rp}}\gamma^{\mu} E_{Rr}\right) \\
& + \left[\mathcal{C}^{V,RR}_{eu}\right]_{prst} \left(\overline{E_{Rp}}\gamma^{\mu} E_{Rr}\right) \left(\overline{U_{Rs}}\gamma_{\mu} U_{Rt}\right) + \left[\mathcal{C}^{V,RR}_{ed}\right]_{prst} \left(\overline{E_{Rp}}\gamma^{\mu} E_{Rr}\right) \left(\overline{D_{Ls}}\gamma_{\mu} D_{Rt}\right) \\
& + \left[\mathcal{C}^{S,RR}_{eu}\right]_{prst} \left(\overline{E_{Lp}} E_{Rr}\right) \left(\overline{U_{Ls}} U_{Rt}\right) + \left[\mathcal{C}^{S,RR}_{ed}\right]_{prst} \left(\overline{E_{Lp}} E_{Rr}\right) \left(\overline{D_{Ls}} D_{Rt}\right) \\
& + \left[\mathcal{C}^{S,RL}_{eu}\right]_{prst} \left(\overline{E_{Lp}} E_{Rr}\right) \left(\overline{U_{Rs}} U_{Lt}\right) + \left[\mathcal{C}^{S,RL}_{ed}\right]_{prst} \left(\overline{E_{Lp}} E_{Rr}\right) \left(\overline{D_{Rs}} D_{Lt}\right)
\end{aligned}$$

Back up

Effective Lagrangian

$$\mathcal{L}_{\text{eff}} \supset -\frac{4G_F}{\sqrt{2}} m_\mu A_R \overline{\mu}\sigma^{\mu\nu} P_L e F_{\mu\nu} - \frac{G_F}{\sqrt{2}} g_{LS}^q \bar{e} P_R \mu \bar{q} q$$

LEFT Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{eff}}^{\text{LEFT}} \supset & \mathcal{C}_{e\gamma,pr} \left(\overline{E_{Lp}}\sigma^{\mu\nu} E_{Rr}\right) F_{\mu\nu} \\ & + \mathcal{C}_{eu,1211}^{S,RR} \left(\overline{E_{L1}} E_{R2}\right) \left(\overline{U_{L1}} U_{R1}\right) \\ & + \mathcal{C}_{eu,1211}^{S,RL} \left(\overline{E_{L1}} E_{R2}\right) \left(\overline{U_{R1}} U_{L1}\right)\end{aligned}$$

Example 1.

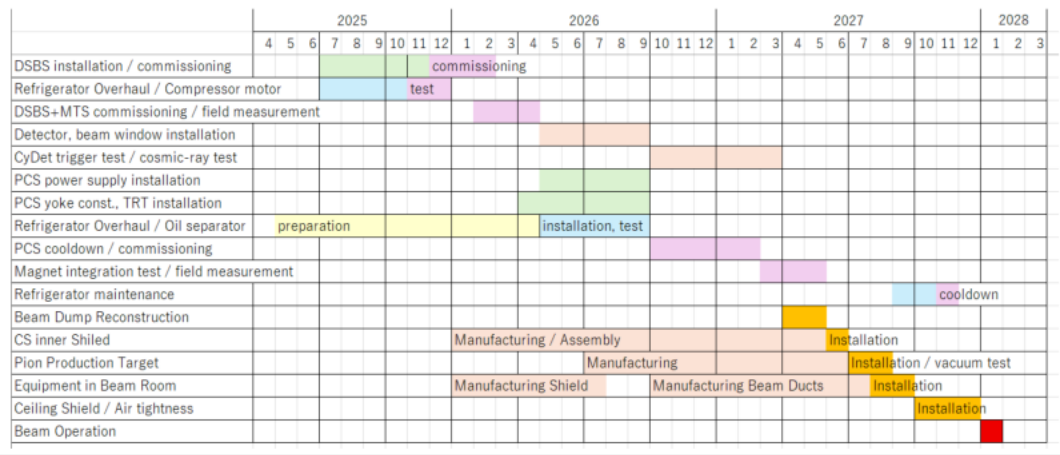
$$\begin{aligned}& \left[\mathcal{C}_{e\gamma,12} \left(\overline{E_{L1}}\sigma^{\mu\nu} E_{R2}\right) F_{\mu\nu}\right]^\dagger \\ &= \mathcal{C}_{e\gamma,21}^* \left(\overline{\mu_R}\sigma^{\mu\nu} e_L\right) F_{\mu\nu} \\ &= \mathcal{C}_{e\gamma,21}^* \left(\overline{\mu}\sigma^{\mu\nu} P_L e\right) F_{\mu\nu}, \\ \Rightarrow & A_R = -\frac{\mathcal{C}_{e\gamma,12}^*}{2\sqrt{2} G_F m_\mu}\end{aligned}$$

Example 2.

$$\begin{aligned}& \mathcal{C}_{eu,1211}^{S,RR} \left(\overline{E_{L1}} E_{R2}\right) \left(\overline{U_{L1}} U_{R1}\right) = \mathcal{C}_{eu,1211}^{S,RR} \left(\bar{e} P_R \mu\right) \left(\bar{u} P_R u\right) \\ & \mathcal{C}_{eu,1211}^{S,RL} \left(\overline{E_{L1}} E_{R2}\right) \left(\overline{U_{R1}} U_{L1}\right) = \mathcal{C}_{eu,1211}^{S,RL} \left(\bar{e} P_R \mu\right) \left(\bar{u} P_L u\right) \\ \Rightarrow & g_{LS}^u = -\frac{1}{\sqrt{2} G_F} [\mathcal{C}_{eu,1211}^{S,RR} + \mathcal{C}_{eu,1211}^{S,RL}]\end{aligned}$$

*Julian Heeck, Robert Szafron, Yuichi Uesaka, arXiv:2203.00702 [hep-ph]

Back up


2025
2026
2027
2028
4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12 1 2 3
DSBS installation / commissioning
commissioning
Refrigerator Overhaul / Compressor motor
test
DSBS+MTS commissioning / field measurement
Detector, beam window installation
CyDet trigger test / cosmic-ray test
PCS power supply installation
PCS yoke const., TRT installation
Refrigerator Overhaul / Oil separator
preparation
installation, test
PCS cooldown / commissioning
Magnet integration test / field measurement
Refrigerator maintenance
cooldown
Beam Dump Reconstruction
CS inner Shiled
Manufacturing / Assembly
Installation
Pion Production Target
Manufacturing
Installation / vacuum test
Equipment in Beam Room
Manufacturing Shield
Manufacturing Beam Ducts
Installation
Ceiling Shield / Air tightness
Installation
Beam Operation

Back up

The RG running effects can be solved by the wilson package in python, which would give the new match results from SMEFT to LEFT after running. For example,

$$\begin{aligned}\mathcal{C}^{V,LL}_{eu,1211} = \quad & 0.9899\mathcal{C}^{(1)}_{\ell q,1211} + 0.2288\mathcal{C}^{(1)}_{\ell q,1221} + 0.2288\mathcal{C}^{(1)}_{\ell q,1212} \\ & - 0.2489\mathcal{C}^{(3)}_{\ell q,1212} - 0.2489\mathcal{C}^{(3)}_{\ell q,1221} + ...\end{aligned}$$

In the above effective Lagrangian, the $U_{L\alpha}$ should be replaced by $(V_{\rm CKM})^{\dagger}_{\alpha i}U_{Li}$, where the $V_{\rm CKM}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix with the standard parameterization, and the values are chosen from the python package wilson. Because we should use mass eigenstates in low energy effective theory, but in RPV-SUSY we used flavor eigenstates.

$$V_{\rm CKM} = \begin{pmatrix} 0.9745 & 0.2243 & (1.236 - 3.402i) \times 10^{-3} \\ -0.2242 - 1.399 \times 10^{-4}i & 0.9736 - 3.221 \times 10^{-4}i & 0.04220 \\ (8.262 - 3.313i) \times 10^{-3} & -0.04140 - 7.625 \times 10^{-4}i & 0.9991 \end{pmatrix}$$

Back up

Para.	$\mathcal{C}^{V,LL}_{eu,1211}$		$\mathcal{C}^{V,LL}_{ed,1211}$		$\mathcal{C}^{V,LR}_{eu,1211}$		$\mathcal{C}^{V,LR}_{ed,1211}$	
	w/ RG effects	w/o RG effects	w/ RG effects	w/o RG effects	w/ RG effects	w/o RG effects	w/ RG effects	w/o RG effects
$\lambda'_{211}\lambda'^{*}_{111}$	0.5130	0.4748	2.768×10^{-3}	0	-5.735×10^{-3}	0	-0.5045	-0.5
$\lambda'_{212}\lambda'^{*}_{112}$	0.5130	0.4748	2.768×10^{-3}	0	-5.735×10^{-3}	0	2.908×10^{-3}	0
$\lambda'_{213}\lambda'^{*}_{113}$	0.5130	0.4748	2.758×10^{-3}	0	-5.735×10^{-3}	0	2.908×10^{-3}	0
$\lambda'_{221}\lambda'^{*}_{121}$	2.147×10^{-2}	2.515×10^{-2}	2.828×10^{-3}	0	-5.708×10^{-3}	0	-0.5045	-0.5
$\lambda'_{231}\lambda'^{*}_{131}$	-3.352×10^{-2}	0	3.743×10^{-2}	0	1.016×10^{-2}	0	-0.5128	-0.5
$\lambda'_{222}\lambda'^{*}_{122}$	2.147×10^{-2}	2.515×10^{-2}	2.828×10^{-3}	0	-5.708×10^{-3}	0	2.895×10^{-3}	0
$\lambda'_{223}\lambda'^{*}_{123}$	2.148×10^{-2}	2.515×10^{-2}	2.818×10^{-3}	0	-5.710×10^{-3}	0	2.895×10^{-3}	0
$\lambda'_{22k}\lambda'^{*}_{11k}$	0.1195	0.1093	0	0	0	0	0	0
$\lambda'_{21k}\lambda'^{*}_{12k}$	0.1195	0.1093	0	0	0	0	0	0
$\lambda'_{23k}\lambda'^{*}_{11k}$	3.448×10^{-4}	5.215×10^{-4}	2.988×10^{-4}	0	1.370×10^{-4}	0	7.028×10^{-5}	0
$\lambda'_{21k}\lambda'^{*}_{12k}$	3.448×10^{-4}	5.215×10^{-4}	2.988×10^{-4}	0	1.370×10^{-4}	0	7.028×10^{-5}	0
$\lambda'_{23k}\lambda'^{*}_{12k}$	1.301×10^{-3}	1.201×10^{-4}	-1.468×10^{-3}	0	-6.730×10^{-4}	0	3.450×10^{-4}	0
$\lambda'_{22k}\lambda'^{*}_{13k}$	1.301×10^{-3}	1.201×10^{-4}	-1.468×10^{-3}	0	-6.730×10^{-4}	0	3.450×10^{-4}	0

Back up

Para.	$\mathcal{C}^{V,LL}_{eu,1211}$		$\mathcal{C}^{V,LL}_{ed,1211}$		$\mathcal{C}^{V,LR}_{eu,1211}$		$\mathcal{C}^{V,LR}_{ed,1211}$	
	w/ RG effects	w/o RG effects	w/ RG effects	w/o RG effects	w/ RG effects	w/o RG effects	w/ RG effects	w/o RG effects
$\lambda'_{211}\lambda'^{*}_{111}$	0.5130	0.4748	2.768×10^{-3}	0	-5.735×10^{-3}	0	-0.5045	-0.5
$\lambda'_{212}\lambda'^{*}_{112}$	0.5130	0.4748	2.768×10^{-3}	0	-5.735×10^{-3}	0	2.908×10^{-3}	0
$\lambda'_{213}\lambda'^{*}_{113}$	0.5130	0.4748	2.75					
$\lambda'_{221}\lambda'^{*}_{121}$	2.147×10^{-2}	2.515×10^{-2}	2.82					
$\lambda'_{231}\lambda'^{*}_{131}$	-3.352×10^{-2}	0	3.74					
$\lambda'_{222}\lambda'^{*}_{122}$	2.147×10^{-2}	2.515×10^{-2}	2.82					
$\lambda'_{223}\lambda'^{*}_{123}$	2.148×10^{-2}	2.515×10^{-2}	2.81					
$\lambda'_{22k}\lambda'^{*}_{11k}$	0.1195	0.1093						
$\lambda'_{21k}\lambda'^{*}_{12k}$	0.1195	0.1093						
$\lambda'_{23k}\lambda'^{*}_{11k}$	3.448×10^{-4}	5.215×10^{-4}	2.98					
$\lambda'_{21k}\lambda'^{*}_{12k}$	3.448×10^{-4}	5.215×10^{-4}	2.98					
$\lambda'_{23k}\lambda'^{*}_{12k}$	1.301×10^{-3}	1.201×10^{-4}	-1.46					
$\lambda'_{22k}\lambda'^{*}_{13k}$	1.301×10^{-3}	1.201×10^{-4}	-1.468×10^{-3}	0	-6.730×10^{-4}	0	3.450×10^{-4}	0

The table gives some of the final matching results as examples.

E.g. The first column in the table means, with RGEs, the relation between the LEFT coefficient $\mathcal{C}^{V,LL}_{eu,1211}$ and the RPV-SUSY coefficients λ' is

$$\begin{aligned}\mathcal{C}^{V,LL}_{eu,1211} &= 0.5130\lambda'_{211}\lambda'^{*}_{111} + 0.5130\lambda'_{212}\lambda'^{*}_{112} \\ &+ 0.5130\lambda'_{213}\lambda'^{*}_{113} + (2.147\times10^{-2})\lambda'_{221}\lambda'^{*}_{121} \\ &- (3.352\times10^{-2})\lambda'_{231}\lambda'^{*}_{131} + (2.147\times10^{-2})\lambda'_{222}\lambda'^{*}_{122} \\ &+ (2.148\times10^{-2})\lambda'_{223}\lambda'^{*}_{123} + \dots\end{aligned}$$

Back up

$$\mathrm{Br}(\mu N \to eN) = \frac{\Gamma_{\text{conv.}}}{\Gamma_{\text{cap.}}}$$

$$\Gamma_{\text{conv.}} = 2G_F^2 \left| A_R D + \tilde{g}_{LS}^p S^{(p)} + \tilde{g}_{LS}^n S^{(n)} + \tilde{g}_{LV}^p V^{(p)} + \tilde{g}_{LV}^n V^{(n)} \right|^2 + 2G_F^2 \left| A_L D + \tilde{g}_{RS}^p S^{(p)} + \tilde{g}_{RS}^n S^{(n)} + \tilde{g}_{RV}^p V^{(p)} + \tilde{g}_{RV}^n V^{(n)} \right|^2$$

$$A_R = -\frac{\mathcal{C}_{e\gamma,12}^*}{2\sqrt{2}G_F m_\mu}, \quad A_L = -\frac{\mathcal{C}_{e\gamma,21}}{2\sqrt{2}G_F m_\mu},$$

$$g_{RS}^u = -\frac{1}{\sqrt{2}G_F}[\mathcal{C}_{eu,2111}^{S,RR*} + \mathcal{C}_{eu,2111}^{S,RL*}], \quad g_{RP}^u = -\frac{1}{\sqrt{2}G_F}[-\mathcal{C}_{eu,2111}^{S,RR*} + \mathcal{C}_{eu,2111}^{S,RL*}], \quad \ldots$$

Isotopes	D $[m_\mu^{5/2}]$	$S(p)$ $[m_\mu^{5/2}]$	$V(p)$ $[m_\mu^{5/2}]$	$S(n)$ $[m_\mu^{5/2}]$	$V(n)$ $[m_\mu^{5/2}]$	$\Gamma_{\text{cap.}}$ $[10^6 \mathrm{s}^{-1}]$
^{27}Al	0.0359	0.0159	0.0165	0.0172	0.0178	0.7054
^{48}Ti	0.0859	0.0379	0.0407	0.0448	0.0481	2.590
^{197}Au	0.166	0.0523	0.0866	0.0781	0.129	13.07

$D, S^{p,n}, V^{p,n}$:
the overlap integrals

The integral uncertainties:
$2\% \sim 5\%$ for Al, Ti
$8\% \sim 10\%$ for Au

*Julian Heeck, Robert Szafron, Yuichi Uesaka. arXiv:2203.00702 [hep-ph]

Overlap Integral

$$D = \frac{4}{\sqrt{2}} m_\mu \int_0^\infty \mathrm{d}r\, r^2 \left[-E(r)\right] \left(g_e^- f_\mu^- + f_e^- g_\mu^-\right)$$

radial electric field

$$S^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r\, r^2 Z\rho^{(p)} \left(g_e^- g_\mu^- - f_e^- f_\mu^-\right),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r\, r^2 N\rho^{(n)} \left(g_e^- g_\mu^- - f_e^- f_\mu^-\right),$$

radial parts of wavefunctions

$$V^{(p)} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r\, r^2 Z\rho^{(p)} \left(g_e^- g_\mu^- + f_e^- f_\mu^-\right),$$

$$V^{(n)} = \frac{1}{2\sqrt{2}} \int_0^\infty \mathrm{d}r\, r^2 N\rho^{(n)} \left(g_e^- g_\mu^- + f_e^- f_\mu^-\right),$$

contain all the information about the **structure of the nucleus** (spherically symmetric) through the density distributions ρ of protons and neutrons.

$$\int_0^\infty \mathrm{d}r\, 4\pi r^2 \rho^{(c),(p),(n)}(r) = 1 \qquad E(r) = \frac{Ze}{r^2} \int_0^r \mathrm{d}\tilde{r}\, \tilde{r}^2 \rho^{(c)}(\tilde{r})$$

The overlap integrals can be calculated for a given isotope after specifying the **nuclear distributions**.

- Three-parameter Fermi model (3pF)

  $w = 0$ (2pF); $w = 0$, $z = 0.52$ fm (1pF)

  $$\rho^{(c)}(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{z}\right)} \left(1 + w\frac{r^2}{c^2}\right)$$

- Three-parameter Gaussian model (3pG)

  $$\rho^{(c)}(r) = \frac{\rho_0}{1 + \exp\left(\frac{r^2-c^2}{z^2}\right)} \left(1 + w\frac{r^2}{c^2}\right)$$

- Modified-harmonic oscillator model (MHO)

  $$\rho^{(c)}(r) = \rho_0 \left(1 + w\frac{r^2}{a^2}\right) \mathrm{e}^{-r^2/a^2}$$

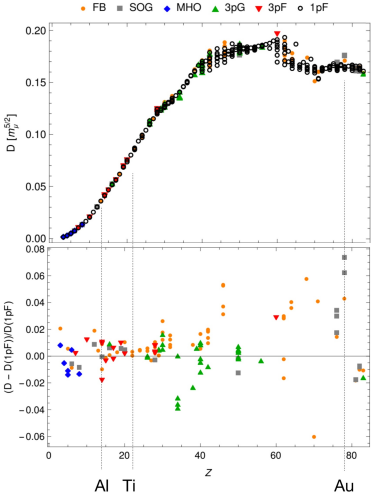
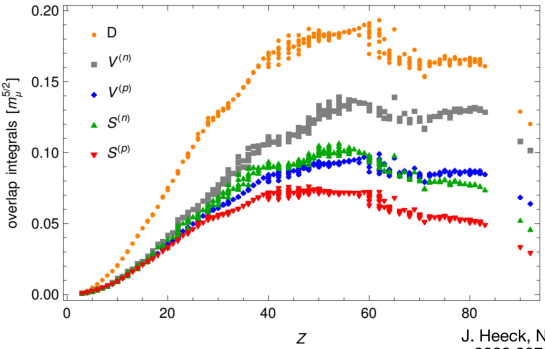
- Fourier–Bessel expansion (FB)

  $$\rho^{(c)}(r) = \begin{cases} \sum_k^n a_k j_0\left(\frac{k\pi r}{R}\right), & r \le R, \\ 0, & r > R. \end{cases}$$

- Sum of Gaussians (SOG)

  $$\rho^{(c)}(r) = \sum_k^n Q_k \frac{\mathrm{e}^{-\frac{(r-R_k)^2}{\gamma^2}} + \mathrm{e}^{-\frac{(r+R_k)^2}{\gamma^2}}}{2\pi^{3/2}\gamma^3(1 + 2R_k^2/\gamma^2)}$$

Back up



Isotopes	D $[m_\mu^{5/2}]$	$S^{(p)}$ $[m_\mu^{5/2}]$	$V^{(p)}$ $[m_\mu^{5/2}]$	$S^{(n)}$ $[m_\mu^{5/2}]$	$V^{(n)}$ $[m_\mu^{5/2}]$	$\Gamma_{\text{cap.}}$ $[10^6\ \text{s}^{-1}]$
^{27}Al	0.0359	0.0159	0.0165	0.0172	0.0178	0.7054
^{48}Ti	0.0859	0.0379	0.0407	0.0448	0.0481	2.590
^{197}Au	0.166	0.0523	0.0866	0.0781	0.129	13.07