Dark Matter Inelastic Scattering with Nuclei

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Based on 2609.xxxxx

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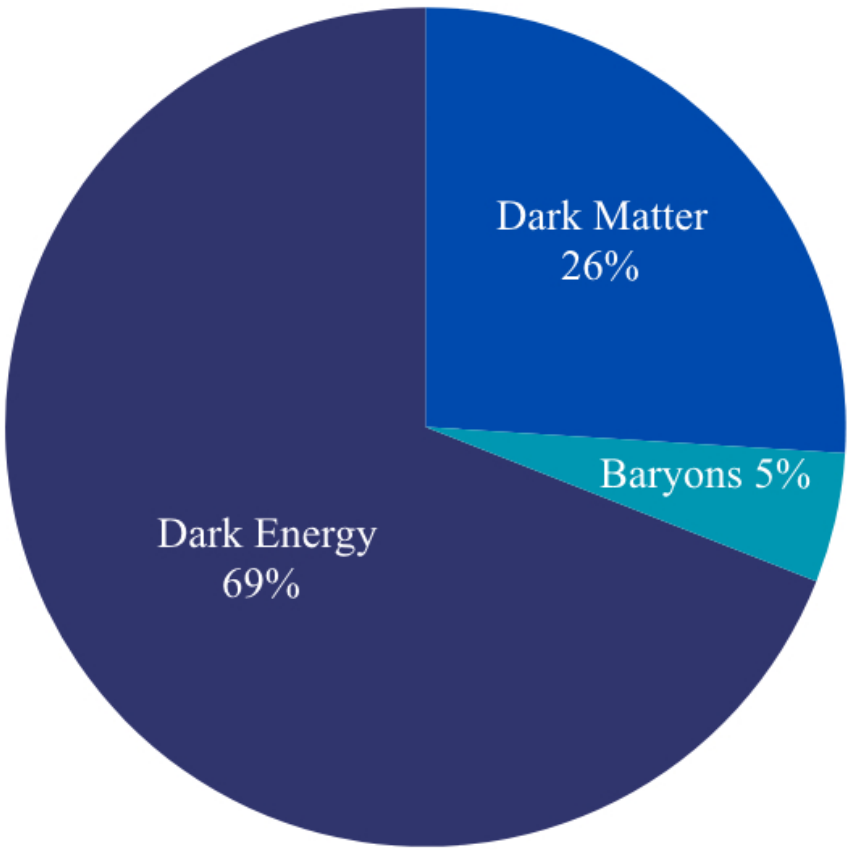
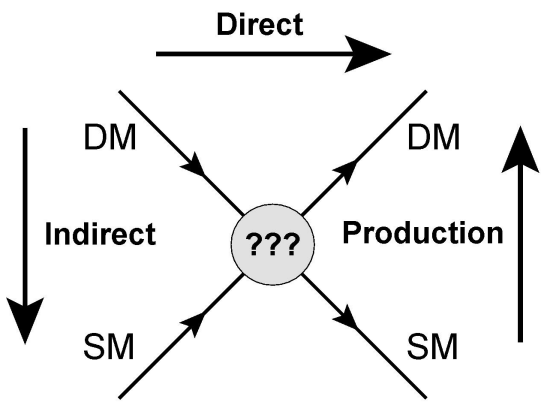
Dark Matter

Energy density of the Universe



Possible particle DM

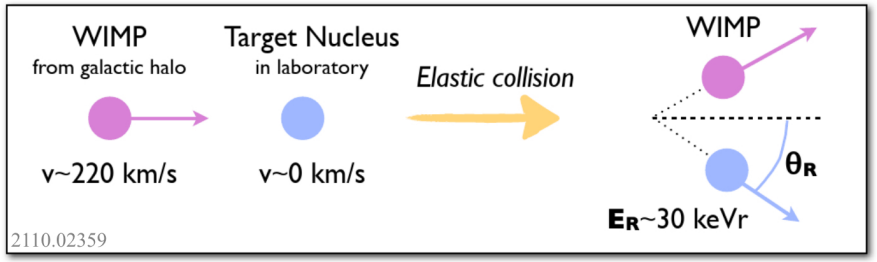
- Neutrinos
- **Weakly interacting massive particles (WIMPs)**
- Axions & axion-like particles
- Sterile neutrinos
- Dark photons
- Dark sectors
- ...



WIMP Inelastic Scattering

Elastic scattering: "standard" case

- Final nucleus in ground state
- Signal: nuclear recoil



Inelastic scattering (on nuclear side)

- Final nucleus in excited state
- Signal: nuclear recoil + de-excitation

Isotope	Abundance (%)	E^* (keV)	J^π	$m_\chi^{\min}$ (GeV)
$^{129}_{54}$Xe	26.4	0	$1/2^+$	0
		39.5774	$3/2^+$	14
		236.14	$11/2^-$	182
		274.29	$(9/2^-)$	279
		318.1787	$3/2^+$	512
		321.711	$5/2^+$	543
$^{131}_{54}$Xe	21.2	0	$3/2^+$	0
		80.1854	$1/2^+$	31
		163.93	$11/2^-$	86
		341.144	$9/2^-$	718
		364.49	$5/2^+$	1279

$$|\boldsymbol{q}|^{(\text{min/max})} = \mu v \left(1 \pm \sqrt{1 - \frac{2E^*}{\mu v^2}}\right)$$

$$E_R^{(\text{min/max})} = \frac{\mu^2 v^2}{2m_A}\left(1 \mp \sqrt{1 - \frac{2E^*}{\mu v^2}}\right)^2$$

$$v \geq v_{\min} \equiv \frac{|\boldsymbol{q}|}{2\mu} + \frac{E^*}{|\boldsymbol{q}|}, \quad \mu \equiv \frac{m_\chi m_A}{m_\chi + m_A}$$

$$m_\chi \geq m_\chi^{\min} \equiv \frac{m_A E^*}{m_A v_{\max}^2/2 - E^*} \geq 0$$

Operators & Nuclear Form Factors

$$\frac{dR}{dE_R} = \frac{\rho_\chi}{32\pi m_A^2 m_\chi^3} \int_{v_{\min}}^{v_{\max}} d^3\boldsymbol{v}\, \frac{f_E(\boldsymbol{v})}{v}\, \overline{|\mathcal{M}|^2}$$

Non-relativistic approximation

$$\mathcal{L}(x) = g\overline{\Psi}_\chi(x)\Gamma_\chi\Psi_\chi(x)\overline{\Psi}_N(x)\Gamma_N\Psi_N(x)$$

$v_\chi \sim 10^{-3}$, $v_N \lesssim 10^{-1}$ $\Rightarrow$ expand $\mathcal{L}(x)$
Lowest order:

- SI: $\Gamma_\chi \to 1_\chi$, $\Gamma_N \to 1_N$
- SD: $\Gamma_\chi \to \boldsymbol{S}_\chi$, $\Gamma_N \to \boldsymbol{S}_N$

Point nucleus ($\Leftrightarrow |\boldsymbol{q}| = 0$) & elastic

- SI: $\overline{|\mathcal{M}|^2} \propto A^2\overline{|\mathcal{M}_0|^2}$
- SD: $\overline{|\mathcal{M}|^2} \propto \overline{|\mathcal{M}_0|^2}$

Finite $|\boldsymbol{q}|$ effects

- $m_\chi = 1\,\text{TeV}$ + Xe target
  $\Rightarrow$ $|\boldsymbol{q}| \lesssim 200\,\text{MeV}$ - not small!
- $1/|\boldsymbol{q}| \sim 1\,\text{fm} < R_A \approx 6\,\text{fm}$
  $\Rightarrow$ see nuclear structure
  $\Rightarrow$ form factors $\overline{|\mathcal{M}|^2} \propto F^2(\boldsymbol{q})\overline{|\mathcal{M}_0|^2}$
- Higher-order NR operators arise

Naive form factors

- Same parameters for all targets

$$F_{\text{Helm}}(\boldsymbol{q}^2) \equiv 3\frac{j_1(|\boldsymbol{q}|r_n)}{|\boldsymbol{q}|r_n} e^{-(|\boldsymbol{q}|s)^2/2}$$

  $\Rightarrow$ cannot reproduce fine details for individual nuclei
- Probe only SI/SD
- Elastic only

Non-Relativistic Effective Field Theory

NR EFT formalism (Haxton et al., 1203.3542, 1308.6288)

- Expand $\mathcal{L}$ in powers of $v \sim 10^{-3}$, $|\boldsymbol{q}|/m_N \sim 10^{-1}$ up to cubic terms
- Can build a set of Galilean-invariant & Hermitian operators from $\boldsymbol{S}_N$, $\boldsymbol{S}_\chi$, $i\boldsymbol{q}$, $\boldsymbol{v}^\perp \equiv \frac{1}{2}\left(\frac{\boldsymbol{p}'_\chi + \boldsymbol{p}_\chi}{m_\chi} - \frac{i\overleftarrow{\nabla}_N - i\overrightarrow{\nabla}_N}{m_N}\right)$

$\mathcal{O}_{i\in 1\sim 4}$	$\mathcal{O}_{i\in 5\sim 8}$	$\mathcal{O}_{i\in 9\sim 12}$	$\mathcal{O}_{i\in 13\sim 16}$
$1_\chi 1_N$	$i\boldsymbol{S}_\chi\cdot\left(\frac{\boldsymbol{q}}{m_N}\times\boldsymbol{v}^\perp\right)$	$i\boldsymbol{S}_\chi\cdot\left(\boldsymbol{S}_N\times\frac{\boldsymbol{q}}{m_N}\right)$	$i(\boldsymbol{S}_\chi\cdot\boldsymbol{v}^\perp)\left(\boldsymbol{S}_N\cdot\frac{\boldsymbol{q}}{m_N}\right)$
$(\boldsymbol{v}^\perp)^2$	$\left(\boldsymbol{S}_\chi\cdot\frac{\boldsymbol{q}}{m_N}\right)\left(\boldsymbol{S}_N\cdot\frac{\boldsymbol{q}}{m_N}\right)$	$i\boldsymbol{S}_N\cdot\frac{\boldsymbol{q}}{m_N}$	$i\left(\boldsymbol{S}_\chi\cdot\frac{\boldsymbol{q}}{m_N}\right)(\boldsymbol{S}_N\cdot\boldsymbol{v}^\perp)$
$i\boldsymbol{S}_N\cdot\left(\frac{\boldsymbol{q}}{m_N}\times\boldsymbol{v}^\perp\right)$	$\boldsymbol{S}_N\cdot\boldsymbol{v}^\perp$	$i\boldsymbol{S}_\chi\cdot\frac{\boldsymbol{q}}{m_N}$	$-\left(\boldsymbol{S}_\chi\cdot\frac{\boldsymbol{q}}{m_N}\right)\left[(\boldsymbol{S}_N\times\boldsymbol{v}^\perp)\cdot\frac{\boldsymbol{q}}{m_N}\right]$
$\boldsymbol{S}_\chi\cdot\boldsymbol{S}_N$	$\boldsymbol{S}_\chi\cdot\boldsymbol{v}^\perp$	$\boldsymbol{S}_\chi\cdot(\boldsymbol{S}_N\times\boldsymbol{v}^\perp)$	$-\left[(\boldsymbol{S}_\chi\times\boldsymbol{v}^\perp)\cdot\frac{\boldsymbol{q}}{m_N}\right]\left(\boldsymbol{S}_N\cdot\frac{\boldsymbol{q}}{m_N}\right)$

Matching:

$$\mathcal{L}(x) = g\overline{\Psi}_\chi(x)\Gamma_\chi\Psi_\chi(x)\overline{\Psi}_N(x)\Gamma_N\Psi_N(x)$$

$$\rightarrow \mathcal{H}_{\text{NR EFT}} \equiv \psi^\dagger_\chi(\boldsymbol{x})\psi^\dagger_N(\boldsymbol{x})\left[\sum_{\tau=0,1}\sum_{i=1}^{15} c_i^\tau \mathcal{O}_i t^\tau\right]\psi_\chi(\boldsymbol{x})\psi_N(\boldsymbol{x})$$

Amplitude: Separation of DM & Nuclear Parts

Desirable factorization of amplitude:

$$\mathcal{M} \propto \langle \chi' | \mathcal{O}_\chi | \chi \rangle \times \langle A' | \mathcal{O}_N | A \rangle \quad \Rightarrow \quad \text{can separate BSM from nuclear physics}$$

Cannot be done right away!

- Not all $\mathcal{O}_i = \mathcal{O}_\chi \otimes \mathcal{O}_N$ ($\boldsymbol{v}^\perp$ acts on both DM and nucleon)
- Need to evaluate nuclear matrix element in nuclear CM rest frame $\Rightarrow$ separate nuclear CM motion as $|A\rangle = |\boldsymbol{p}_A\rangle \otimes |A_{\text{int}}\rangle$

$$\Rightarrow \quad \boldsymbol{v}^\perp = \underbrace{\boldsymbol{v}_T^\perp}_{\text{CM}} - \underbrace{\boldsymbol{v}_N}_{\text{int}}, \qquad \boldsymbol{v}_T^\perp \equiv \frac{1}{2}\left(\frac{\boldsymbol{p}'_\chi + \boldsymbol{p}_\chi}{m_\chi} - \frac{\boldsymbol{p}'_A + \boldsymbol{p}_A}{m_A}\right), \qquad \boldsymbol{v}_N \equiv \frac{i\overleftarrow{\boldsymbol{\nabla}}_{N,\,\text{int}} - i\overrightarrow{\boldsymbol{\nabla}}_{N,\,\text{int}}}{2m_N}$$

$$\begin{aligned}
\mathcal{M} = \sum_{\tau=0,1} \int d^3\boldsymbol{x}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \Bigg[& \left\langle \chi' \middle| \psi_\chi^\dagger(0) \ell_0^\tau \psi_\chi(0) \middle| \chi \right\rangle \left\langle A'_{\text{int}} \middle| \psi_N^\dagger(\boldsymbol{x}) 1_N t^\tau \psi_N(\boldsymbol{x}) \middle| A_{\text{int}} \right\rangle \\
& + \left\langle \chi' \middle| \psi_\chi^\dagger(0) \boldsymbol{\ell}_5^\tau \psi_\chi(0) \middle| \chi \right\rangle \cdot \left\langle A'_{\text{int}} \middle| \psi_N^\dagger(\boldsymbol{x}) \boldsymbol{\sigma}_N t^\tau \psi_N(\boldsymbol{x}) \middle| A_{\text{int}} \right\rangle \\
& + \left\langle \chi' \middle| \psi_\chi^\dagger(0) \boldsymbol{\ell}_M^\tau \psi_\chi(0) \middle| \chi \right\rangle \cdot \left\langle A'_{\text{int}} \middle| \psi_N^\dagger(\boldsymbol{x}) \boldsymbol{v}_N t^\tau \psi_N(\boldsymbol{x}) \middle| A_{\text{int}} \right\rangle \\
& + \left\langle \chi' \middle| \psi_\chi^\dagger(0) \ell_0^{A\tau} \psi_\chi(0) \middle| \chi \right\rangle \left\langle A'_{\text{int}} \middle| \psi_N^\dagger(\boldsymbol{x}) (\boldsymbol{\sigma}_N \cdot \boldsymbol{v}_N) t^\tau \psi_N(\boldsymbol{x}) \middle| A_{\text{int}} \right\rangle \\
& + \left\langle \chi' \middle| \psi_\chi^\dagger(0) \boldsymbol{\ell}_E^\tau \psi_\chi(0) \middle| \chi \right\rangle \cdot \left\langle A'_{\text{int}} \middle| \psi_N^\dagger(\boldsymbol{x}) i[\boldsymbol{\sigma}_N \times \boldsymbol{v}_N] t^\tau \psi_N(\boldsymbol{x}) \middle| A_{\text{int}} \right\rangle \Bigg]
\end{aligned}$$

Amplitude

DM currents:

$$\ell_0^\tau \equiv c_1^\tau + ic_5^\tau \left[\frac{\boldsymbol{q}}{m_N} \times \boldsymbol{v}_T^\perp\right] \cdot \boldsymbol{S}_\chi + c_8^\tau (\boldsymbol{v}_T^\perp \cdot \boldsymbol{S}_\chi) + ic_{11}^\tau \left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_\chi\right),$$

$$\ell_0^{A\tau} \equiv -\frac{1}{2}\left[c_7^\tau + ic_{14}^\tau \left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_\chi\right)\right],$$

$$\boldsymbol{\ell}_5^\tau \equiv \frac{1}{2}\Big\{ic_3^\tau \left[\frac{\boldsymbol{q}}{m_N} \times \boldsymbol{v}_T^\perp\right] + c_4^\tau \boldsymbol{S}_\chi + c_6^\tau \frac{\boldsymbol{q}}{m_N}\left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_\chi\right) + c_7^\tau \boldsymbol{v}_T^\perp + ic_9^\tau \left[\frac{\boldsymbol{q}}{m_N} \times \boldsymbol{S}_\chi\right] + ic_{10}^\tau \frac{\boldsymbol{q}}{m_N}$$
$$+ c_{12}^\tau \left[\boldsymbol{v}_T^\perp \times \boldsymbol{S}_\chi\right] + ic_{13}^\tau \frac{\boldsymbol{q}}{m_N}\left(\boldsymbol{v}_T^\perp \cdot \boldsymbol{S}_\chi\right) + ic_{14}^\tau \boldsymbol{v}_T^\perp \left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_\chi\right) + c_{15}^\tau \left[\frac{\boldsymbol{q}}{m_N} \times \boldsymbol{v}_T^\perp\right]\left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_\chi\right)\Big\},$$

$$\boldsymbol{\ell}_M^\tau \equiv ic_5^\tau \left[\frac{\boldsymbol{q}}{m_N} \times \boldsymbol{S}_\chi\right] - c_8^\tau \boldsymbol{S}_\chi,$$

$$\boldsymbol{\ell}_E^\tau \equiv \frac{1}{2}\Big\{c_3^\tau \frac{\boldsymbol{q}}{m_N} + ic_{12}^\tau \boldsymbol{S}_\chi - c_{13}^\tau \left[\frac{\boldsymbol{q}}{m_N} \times \boldsymbol{S}_\chi\right] - ic_{15}^\tau \frac{\boldsymbol{q}}{m_N}\left(\frac{\boldsymbol{q}}{m_N} \cdot \boldsymbol{S}_\chi\right)\Big\}.$$

Nuclear states: $|A_{\rm int}\rangle = |J_i\, M_i\rangle$, $|A'_{\rm int}\rangle = |J_f\, M_f\rangle \Rightarrow$ multipole decomposition of amplitude:

$$\mathcal{M} = \sum_{\tau=0,1} \Big\langle J_f\, M_f \Big| \Big\{ \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}(-i)^J \left[\langle \ell_0^\tau \rangle M_{J0;\tau}(|\boldsymbol{q}|) - i\langle \ell_0^{A\tau}\rangle \frac{|\boldsymbol{q}|}{m_N} \tilde{\Omega}_{J0;\tau}(|\boldsymbol{q}|)\right]$$
$$+ \sum_{J=1}^{\infty} \sqrt{2\pi(2J+1)}(-i)^J \sum_{\lambda=\pm 1} (-1)^\lambda \Big\{ \langle \ell_{5\lambda}^\tau \rangle \left[\lambda \Sigma_{J,-\lambda;\tau}(|\boldsymbol{q}|) + i\Sigma'_{J,-\lambda;\tau}(q)\right]$$
$$- i\frac{|\boldsymbol{q}|}{m_N} \langle \ell_{M\lambda}^\tau \rangle \left[\lambda \Delta_{J,-\lambda;\tau}(|\boldsymbol{q}|) + i\Delta'_{J,-\lambda;\tau}(|\boldsymbol{q}|)\right] - i\frac{|\boldsymbol{q}|}{m_N} \langle \ell_{E\lambda}^\tau \rangle \left[\lambda \tilde{\Phi}_{J,-\lambda;\tau}(|\boldsymbol{q}|) + i\tilde{\Phi}'_{J,-\lambda;\tau}(|\boldsymbol{q}|)\right] \Big\}$$
$$+ \sum_{J=0}^{\infty} \sqrt{4\pi(2J+1)}(-i)^J \left[i\langle \ell_{50}^\tau \rangle \Sigma''_{J0;\tau}(|\boldsymbol{q}|) + \frac{|\boldsymbol{q}|}{m_N} \langle \ell_{M0}^\tau \rangle \tilde{\Delta}''_{J0;\tau}(|\boldsymbol{q}|) + \frac{|\boldsymbol{q}|}{m_N} \langle \ell_{E0}^\tau \rangle \Phi''_{J0;\tau}(|\boldsymbol{q}|)\right] \Big\} \Big| J_i\, M_i \Big\rangle.$$

Nuclear Multipole Operators

$$M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x}) \equiv j_J(|\boldsymbol{q}||\boldsymbol{x}|)Y_{JM}(\Omega_{\boldsymbol{x}}), \qquad \boldsymbol{M}^L_{JM}(\boldsymbol{q}\cdot\boldsymbol{x}) \equiv j_L(|\boldsymbol{q}||\boldsymbol{x}|)\boldsymbol{Y}^L_{JM}(\Omega_{\boldsymbol{x}}),$$

$$M_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x}) t^\tau \psi_N(\boldsymbol{x}), \quad \Sigma_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\cdot\boldsymbol{\sigma}\, t^\tau \psi_N(\boldsymbol{x}),$$

$$\Sigma'_{JM;\tau}(|\boldsymbol{q}|) \equiv -i\int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \left[\frac{\boldsymbol{\nabla}}{|\boldsymbol{q}|}\times \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right]\cdot\boldsymbol{\sigma}\, t^\tau \psi_N(\boldsymbol{x}),$$

$$\Sigma''_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \frac{\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}}{|\boldsymbol{q}|} M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x}) t^\tau \psi_N(\boldsymbol{x}),$$

$$\Delta_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\cdot\frac{\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} t^\tau \psi_N(\boldsymbol{x}),$$

$$\Delta'_{JM;\tau}(|\boldsymbol{q}|) \equiv -i\int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \left[\frac{\boldsymbol{\nabla}}{|\boldsymbol{q}|}\times \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right]\cdot\frac{\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} t^\tau \psi_N(\boldsymbol{x}),$$

$$\tilde{\Delta}''_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \left\{\left[\frac{\boldsymbol{\nabla}}{|\boldsymbol{q}|} M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right]\cdot\frac{\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} - \frac{1}{2} M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right\} t^\tau \psi_N(\boldsymbol{x}),$$

$$\tilde{\Phi}_{JM;\tau}(|\boldsymbol{q}|) \equiv i\int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \left\{\boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\cdot\frac{\boldsymbol{\sigma}\times\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} + \left[\frac{\boldsymbol{\nabla}}{2|\boldsymbol{q}|}\times \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right]\cdot\boldsymbol{\sigma}\right\} t^\tau \psi_N(\boldsymbol{x}),$$

$$\tilde{\Phi}'_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \left\{\left[\frac{\boldsymbol{\nabla}}{|\boldsymbol{q}|}\times \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right]\cdot\frac{\boldsymbol{\sigma}\times\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} + \boldsymbol{M}^J_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\cdot\frac{\boldsymbol{\sigma}}{2}\right\} t^\tau \psi_N(\boldsymbol{x}),$$

$$\Phi''_{JM;\tau}(|\boldsymbol{q}|) \equiv i\int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \frac{\boldsymbol{\nabla}}{|\boldsymbol{q}|} M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\cdot\frac{\boldsymbol{\sigma}\times\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} t^\tau \psi_N(\boldsymbol{x}),$$

$$\tilde{\Omega}_{JM;\tau}(|\boldsymbol{q}|) \equiv \int d^3\boldsymbol{x}\, \psi^\dagger_N(\boldsymbol{x}) \left[M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\boldsymbol{\sigma}\cdot\frac{\overrightarrow{\boldsymbol{\nabla}}}{|\boldsymbol{q}|} + \boldsymbol{\sigma}\cdot\frac{\boldsymbol{\nabla}}{|2\boldsymbol{q}|} M_{JM}(\boldsymbol{q}\cdot\boldsymbol{x})\right] t^\tau \psi_N(\boldsymbol{x}).$$

NME Reduction & Selection Rules

Wigner-Eckart theorem: $\langle J_f,\, M_f|\, \mathcal{T}_{JM}\, |J_i,\, M_i\rangle = \frac{1}{\sqrt{2J_f+1}} C^{J_f M_f}_{J_i M_i\, JM}\, \langle J_f \| \mathcal{T}_J \| J_i\rangle$

Angular momentum

- $|J_f - J_i| \leq J \leq J_f + J_i$
- $M_i + M = M_f$
- $J \geq 1$ for transverse operators

Parity: $\pi_i \pi_f = \pi_{\mathcal{T}}$

- $\pi_{\mathcal{T}} = (-1)^J$,
  $\mathcal{T} = M, \Sigma, \Delta', \widetilde{\Delta}'', \widetilde{\Phi}', \Phi''$
- $\pi_{\mathcal{T}} = (-1)^{J+1}$, $\mathcal{T} = \widetilde{\Omega}, \Sigma', \Sigma'', \Delta, \widetilde{\Phi}$

Time reversal

Inelastic:

$$\langle J_f \| \mathcal{T}_J \| J_i\rangle = (-1)^{J_f - J_i - J}\, \eta_{\mathcal{T}}\, \langle J_i \| \mathcal{T}_J \| J_f\rangle$$

Elastic ($J_i = J_f$): $\eta_{\mathcal{T}} = (-1)^J$

- $\eta_{\mathcal{T}} = +1$, $\mathcal{T} = M, \widetilde{\Omega}, \widetilde{\Phi}, \widetilde{\Phi}', \Phi''$
- $\eta_{\mathcal{T}} = -1$, $\mathcal{T} = \Sigma, \Sigma', \Sigma'', \Delta, \Delta, \widetilde{\Delta}''$

Elastic vs Inelastic: Selection Rules

Matrix elements limited by P & T symmetries $\Rightarrow$ some contributions vanish

$\mathcal{O}_i$	DM current	Multipole	Multipole (Elastic)	Response	Response (Elastic)
$\mathcal{O}_1$	ℓ_0	M	M	W_M, $W_{\Phi''M}$	W_M, $W_{\Phi''M}$
$\mathcal{O}_3$	ℓ_5	Σ, Σ'	Σ'	W_Σ, $W_{\Sigma'}$	$W_{\Sigma'}$
	ℓ_E	Φ''	$\tilde{\Phi}''$	$W_{\Phi''}$, $W_{\Phi''M}$	$W_{\Phi''}$, $W_{\Phi''M}$
$\mathcal{O}_4$	ℓ_5	Σ, Σ', Σ''	Σ', Σ''	W_Σ, $W_{\Sigma'}$, $W_{\Sigma''}$, $W_{\Delta\Sigma'}$, $W_{\Delta'\Sigma}$	$W_{\Sigma'}$, $W_{\Sigma''}$, $W_{\Delta\Sigma'}$
$\mathcal{O}_5$	ℓ_0	M	M	W_M	W_M
	ℓ_M	Δ, Δ'	Δ	W_Δ, $W_{\Delta'}$, $W_{\Delta\Sigma'}$, $W_{\Delta'\Sigma}$	W_Δ, $W_{\Delta\Sigma'}$
$\mathcal{O}_6$	ℓ_5	Σ''	Σ', Σ''	$W_{\Sigma''}$	$W_{\Sigma''}$
$\mathcal{O}_7$	ℓ_0^A	$\tilde{\Omega}$	-	$W_{\tilde{\Omega}}$	-
	ℓ_5	Σ, Σ'	Σ'	W_Σ, $W_{\Sigma'}$	$W_{\Sigma'}$
$\mathcal{O}_8$	ℓ_0	M	M	W_M	W_M
	ℓ_M	Δ, Δ', $\tilde{\Delta}''$	Δ	W_Δ, $W_{\Delta'}$, $W_{\tilde{\Delta}''}$, $W_{\Delta\Sigma'}$, $W_{\Delta'\Sigma}$	W_Δ, $W_{\Delta\Sigma'}$
$\mathcal{O}_9$	ℓ_5	Σ, Σ'	Σ'	W_Σ, $W_{\Sigma'}$, $W_{\Delta\Sigma'}$, $W_{\Delta'\Sigma}$	$W_{\Sigma'}$, $W_{\Delta\Sigma'}$
$\mathcal{O}_{10}$	ℓ_5	Σ''	Σ''	$W_{\Sigma''}$	$W_{\Sigma''}$
$\mathcal{O}_{11}$	ℓ_0	M	M	W_M, $W_{\Phi''M}$	W_M, $W_{\Phi''M}$
$\mathcal{O}_{12}$	ℓ_5	Σ, Σ', Σ''	Σ', Σ''	W_Σ, $W_{\Sigma'}$, $W_{\Sigma''}$	$W_{\Sigma'}$, $W_{\Sigma''}$
	ℓ_E	$\tilde{\Phi}$, $\tilde{\Phi}'$, Φ''	$\tilde{\Phi}'$, Φ''	$W_{\tilde{\Phi}}$, $W_{\tilde{\Phi}'}$, $W_{\Phi''}$	$W_{\tilde{\Phi}'}$, $W_{\Phi''}$
$\mathcal{O}_{13}$	ℓ_5	Σ''	Σ''	$W_{\Sigma''}$	$W_{\Sigma''}$
	ℓ_E	$\tilde{\Phi}$, $\tilde{\Phi}'$	$\tilde{\Phi}'$	$W_{\tilde{\Phi}}$, $W_{\tilde{\Phi}'}$	$W_{\tilde{\Phi}'}$
$\mathcal{O}_{14}$	ℓ_0^A	$\tilde{\Omega}$	-	$W_{\tilde{\Omega}}$	-
	ℓ_5	Σ, Σ'	Σ'	W_Σ, $W_{\Sigma'}$	$W_{\Sigma'}$
$\mathcal{O}_{15}$	ℓ_5	Σ, Σ'	Σ'	W_Σ, $W_{\Sigma'}$	$W_{\Sigma'}$
	ℓ_E	Φ''	Φ''	$W_{\Phi''}$	$W_{\Phi''}$

Squared Amplitude & Response Functions

$$\overline{|\mathcal{M}|^2} = \frac{4\pi}{2J_i+1}\sum_{\tau,\tau'=0,1}\Bigg\{ R_M^{\tau\tau'} W_M^{\tau\tau'} + R_{\Sigma''}^{\tau\tau'} W_{\Sigma''}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\Phi''}^{\tau\tau'} W_{\Phi''}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}$$

$$+ R_{\Sigma'}^{\tau\tau'} W_{\Sigma'}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\Delta}^{\tau\tau'} W_{\Delta}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'}^{\tau\tau'} \quad \text{(agree with 1308.6288)}$$

$$+ \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\tilde{\Omega}}^{\tau\tau'} W_{\tilde{\Omega}}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\tilde{\Delta}''}^{\tau\tau'} W_{\tilde{\Delta}''}^{\tau\tau'} + R_{\Sigma}^{\tau\tau'} W_{\Sigma}^{\tau\tau'} \quad \text{(differences with 1906.10466)}$$

$$+ \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\Delta'}^{\tau\tau'} W_{\Delta'}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\Delta'\Sigma}^{\tau\tau'} W_{\Delta'\Sigma}^{\tau\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} R_{\tilde{\Phi}}^{\tau\tau'} W_{\tilde{\Phi}}^{\tau\tau'} \Bigg\}$$

Nuclear responses: $W_{\mathcal{T}^{(1)}\mathcal{T}^{(2)}}^{\tau\tau'} \equiv \sum_J \left\langle J_f \left\| \mathcal{T}_{J;\tau}^{(1)} \right\| J_i \right\rangle \left\langle J_f \left\| \mathcal{T}_{J';\tau'}^{(2)} \right\| J_i \right\rangle$

DM Responses:

$$R_M^{\tau\tau'} = c_1^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(\frac{|\boldsymbol{q}|^2}{m_N^2} v_T^{\perp 2} c_5^\tau c_5^{\tau'} + v_T^{\perp 2} c_8^\tau c_8^{\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} c_{11}^\tau c_{11}^{\tau'}\right),$$

$$R_{\Phi''}^{\tau\tau'} = \frac{|\boldsymbol{q}|^2}{4m_N^2} c_3^\tau c_3^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau - \frac{|\boldsymbol{q}|^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{|\boldsymbol{q}|^2}{m_N^2} c_{15}^{\tau'}\right),$$

$$R_{\Phi''M}^{\tau\tau'} = c_3^\tau c_1^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\left(c_{12}^\tau - \frac{|\boldsymbol{q}|^2}{m_N^2} c_{15}^\tau\right) c_{11}^{\tau'},$$

$$R_{\tilde{\Phi}'}^{\tau\tau'} = R_{\tilde{\Phi}}^{\tau\tau'} = \frac{j_\chi(j_\chi+1)}{12}\left(c_{12}^\tau c_{12}^{\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2} c_{13}^\tau c_{13}^{\tau'}\right),$$

$$R_{\Sigma''}^{\tau\tau'} = \frac{|\boldsymbol{q}|^2}{4m_N^2} c_{10}^\tau c_{10}^{\tau'} + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + \frac{|\boldsymbol{q}|^2}{m_N^2}(c_4^\tau c_6^{\tau'} + c_6^\tau c_4^{\tau'}) + \frac{|\boldsymbol{q}|^4}{m_N^4} c_6^\tau c_6^{\tau'} + c_{12}^\tau c_{12}^{\tau'} v_T^{\perp 2} + \frac{|\boldsymbol{q}|^2}{m_N^2} v_T^{\perp 2} c_{13}^\tau c_{13}^{\tau'}\right],$$

$$R_{\Sigma'}^{\tau\tau'} = R_{\Sigma}^{\tau\tau'} = \frac{1}{8}\left(\frac{|\boldsymbol{q}|^2}{m_N^2} v_T^{\perp 2} c_3^\tau c_3^{\tau'} + v_T^{\perp 2} c_7^\tau c_7^{\tau'}\right) + \frac{j_\chi(j_\chi+1)}{12}\left[c_4^\tau c_4^{\tau'} + c_9^\tau c_9^{\tau'}\frac{|\boldsymbol{q}|^2}{m_N^2} + \frac{v_T^{\perp 2}}{2}\left(c_{12}^\tau - \frac{|\boldsymbol{q}|^2}{m_N^2} c_{15}^\tau\right)\left(c_{12}^{\tau'} - \frac{|\boldsymbol{q}|^2}{m_N^2} c_{15}^{\tau'}\right) + \frac{|\boldsymbol{q}|^2}{2m_N^2} v_T^{\perp 2} c_{14}^\tau c_{14}^{\tau'}\right],$$

$$R_{\Delta}^{\tau\tau'} = R_{\Delta'}^{\tau\tau'} = \frac{j_\chi(j_\chi+1)}{3}\left(\frac{|\boldsymbol{q}|^2}{m_N^2} c_5^\tau c_5^{\tau'} + c_8^\tau c_8^{\tau'}\right),$$

$$R_{\Delta\Sigma'}^{\tau\tau'} = -R_{\Delta'\Sigma}^{\tau\tau'} = \frac{j_\chi(j_\chi+1)}{3}(c_5^\tau c_4^{\tau'} - c_8^\tau c_9^{\tau'}),$$

$$R_{\tilde{\Omega}}^{\tau\tau'} = \frac{1}{4}\left(c_7^\tau c_7^{\tau'} + \frac{j_\chi(j_\chi+1)}{3}\frac{|\boldsymbol{q}|^2}{m_N^2} c_{14}^\tau c_{14}^{\tau'}\right),$$

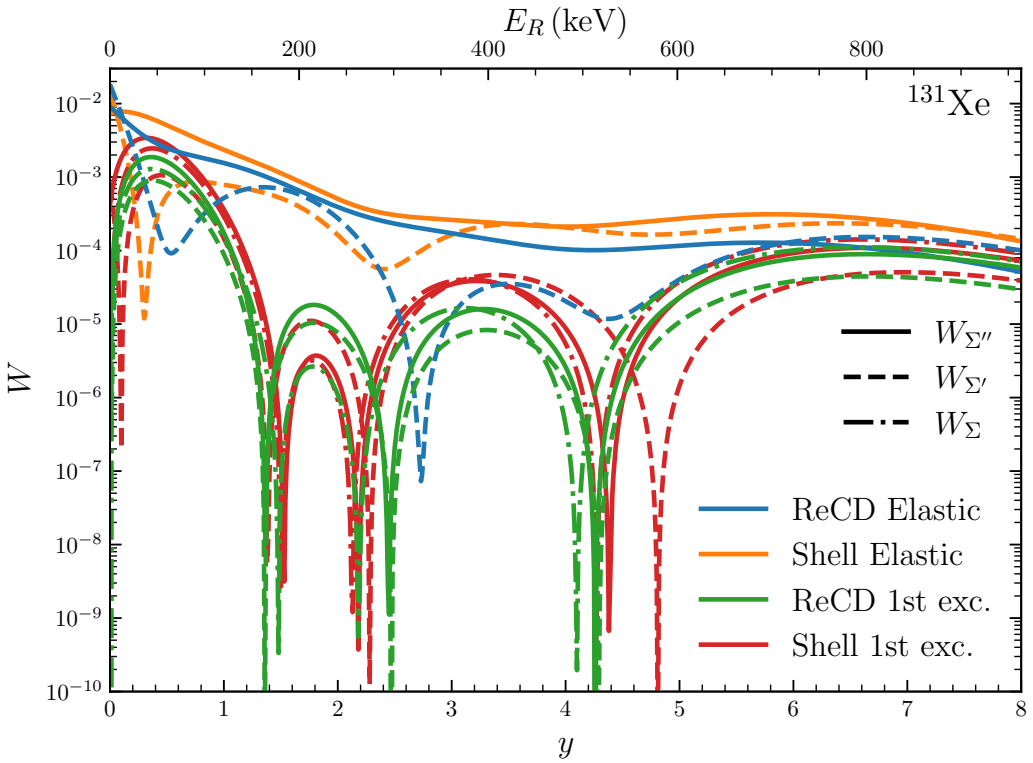
$$R_{\Delta''}^{\tau\tau'} = \frac{j_\chi(j_\chi+1)}{3} c_8^\tau c_8^{\tau'}.$$

Evaluation

$$\frac{dR}{dE_R} = \frac{1}{32\pi m_A^2 m_\chi^2} \frac{\rho_\chi}{m_\chi} \int_{v_{\min}}^{v_{\max}} d^3\boldsymbol{v}\, \frac{f_E(\boldsymbol{v})}{v} \overline{|\mathcal{M}|^2}$$

Ingredients

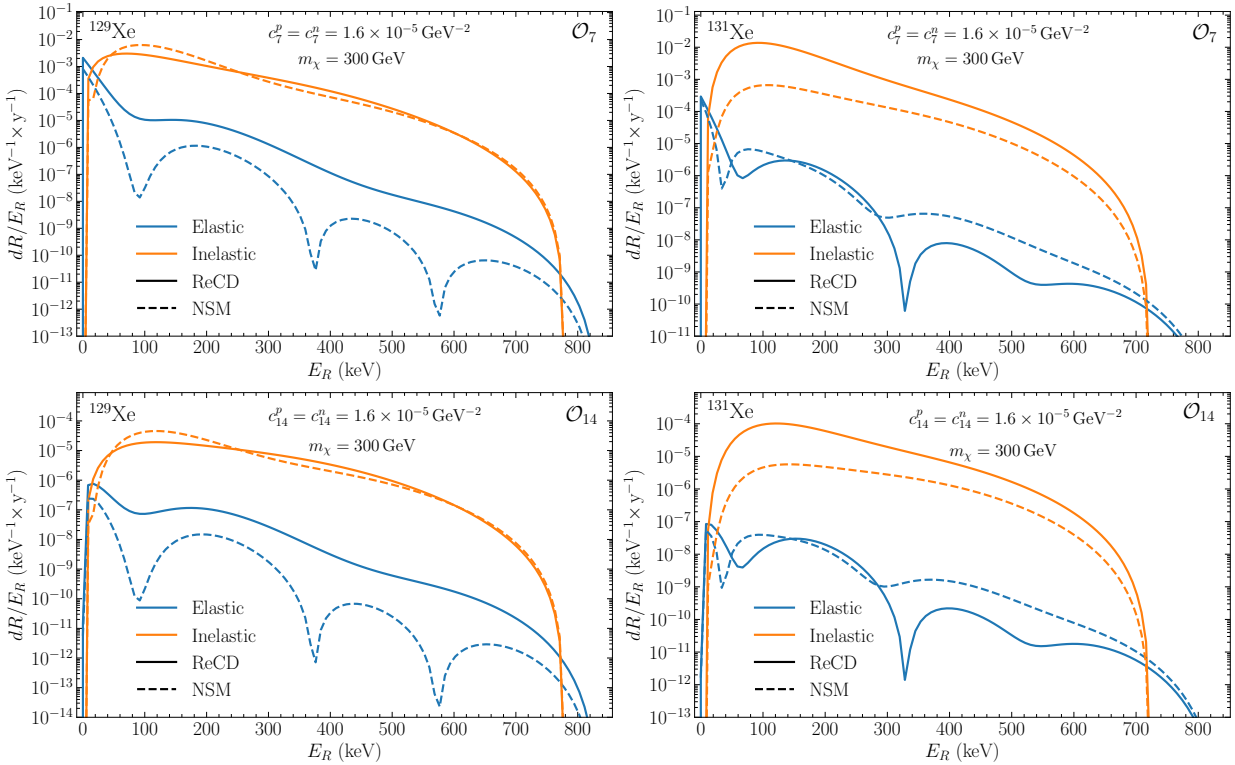
- **DMFormFactor** (Anand *et al.*, 1308.6288)
- Kinematical limits
- Implemented our $\overline{|\mathcal{M}|^2}$
- Extra nuclear responses from **Mu2E** (Haxton *et al.*, 2208.07945)
- Inputs for $W^{\tau\tau'}_{\mathcal{T}^{(1)}\mathcal{T}^{(2)}}$: Relativistic Configuration-interaction Density functional (ReCD) theory



Results: First Excited State

Target: natural xenon $\Rightarrow$ inelastic scattering on ^{129}Xe & ^{131}Xe

	$R^i_{\text{inel}}/R^i_{\text{el}}$	
$\mathcal{O}_i$	$m_\chi = 300\,\text{GeV}$	$m_\chi = 10\,\text{TeV}$
$\mathcal{O}_1$	1.1×10^{-5}	2.2×10^{-5}
$\mathcal{O}_3$	5.4×10^{-4}	1.4×10^{-3}
$\mathcal{O}_4$	9.8×10^{-2}	0.26
$\mathcal{O}_5$	8.5×10^{-2}	0.29
$\mathcal{O}_6$	0.13	0.24
$\mathcal{O}_7$	67	340
$\mathcal{O}_8$	1.3×10^{-2}	3.2×10^{-2}
$\mathcal{O}_9$	0.21	0.39
$\mathcal{O}_{10}$	0.19	0.37
$\mathcal{O}_{11}$	4.4×10^{-5}	1.0×10^{-4}
$\mathcal{O}_{12}$	2.3×10^{-4}	5.1×10^{-4}
$\mathcal{O}_{13}$	1.2	3.3
$\mathcal{O}_{14}$	4.0×10^2	1.9×10^3
$\mathcal{O}_{15}$	1.2×10^{-3}	3.3×10^{-3}



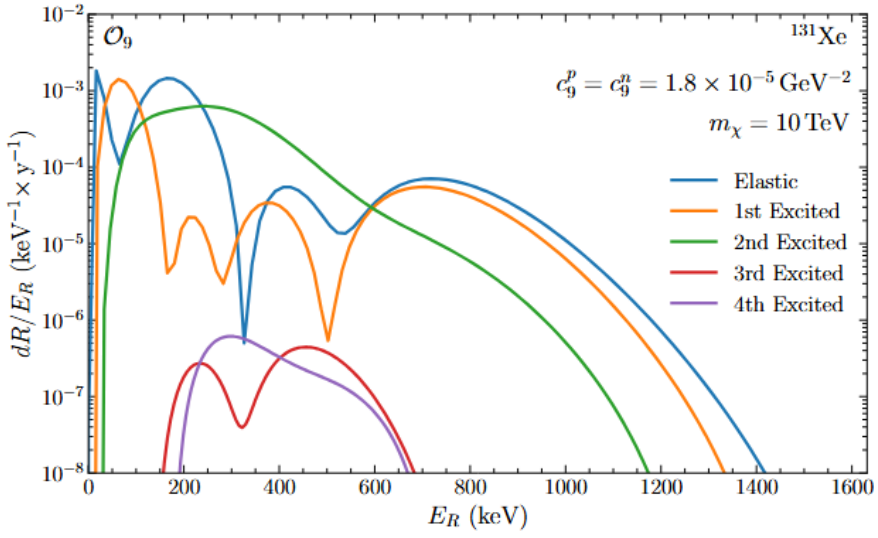
Inelastic contributions dominate for

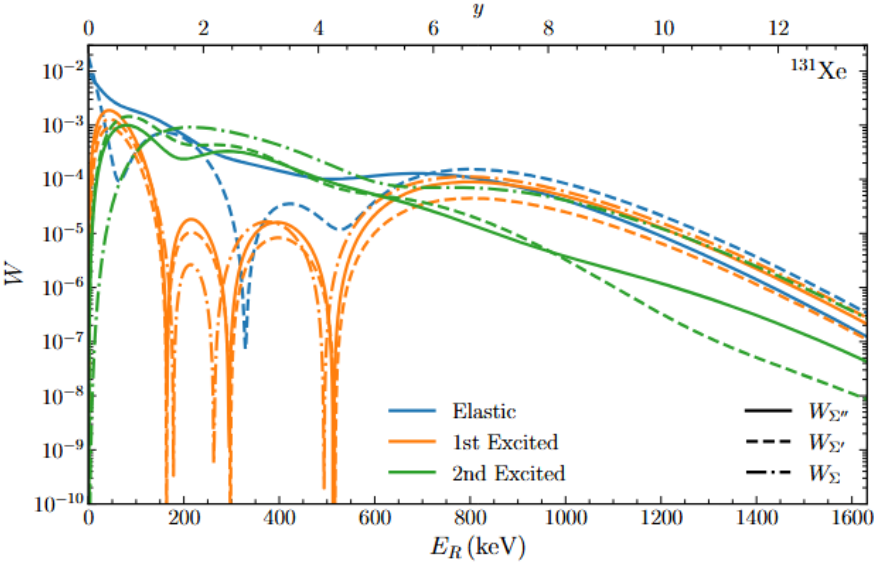
- $O_{13} = i(\boldsymbol{S}_\chi \cdot \boldsymbol{v}^\perp)\left(\boldsymbol{S}_N \cdot \frac{\boldsymbol{q}}{m_N}\right)$
- $O_7 = \boldsymbol{S}_N \cdot \boldsymbol{v}^\perp$
- $O_{14} = i\left(\boldsymbol{S}_\chi \cdot \frac{\boldsymbol{q}}{m_N}\right)(\boldsymbol{S}_N \cdot \boldsymbol{v}^\perp)$

$$\frac{R_{14\,\text{inel}}}{R_{14\,\text{el}}} \sim \frac{\sum_A \frac{|\boldsymbol{q}|^2_{\text{inel}}}{m_N^2}\left\{(v_T^\perp)^2\left[W^{(\text{inel})}_{\Sigma'} + W^{(\text{inel})}_{\Sigma}\right] + 2\frac{|\boldsymbol{q}|^2_{\text{inel}}}{m_N^2} W^{(\text{inel})}_{\tilde{\Omega}}\right\}}{\sum_A (v_T^\perp)^2 \frac{q^2_{\text{el}}}{m_N^2} W^{(\text{el})}_{\Sigma'}}$$

$$\sim \frac{2}{(v_T^\perp)^2}\frac{|\boldsymbol{q}|^4_{\text{inel}}}{m_N^4}\frac{m_N^2}{|\boldsymbol{q}|^2_{\text{el}}}\frac{W^{(\text{inel})}_{\tilde{\Omega}}(^{131}\text{Xe})}{W^{(\text{el})}_{\Sigma'}(^{131}\text{Xe})} \sim 10^6 \times 10^{-3} \times 10^2 \times 10^{-2} = 10^3$$

Results: Higher Excited States





$\mathcal{O}_i$	$R^i_{\text{inel, 1}}/R^i_{\text{el}}$	$R^i_{\text{inel, 2}}/R^i_{\text{el}}$
$\mathcal{O}_6$	0.13	0.13
$\mathcal{O}_9$	0.45	0.69

$$\mathcal{O}_9 = i\boldsymbol{S}_\chi \cdot \left(\boldsymbol{S}_N \times \frac{\boldsymbol{q}}{m_N}\right)$$

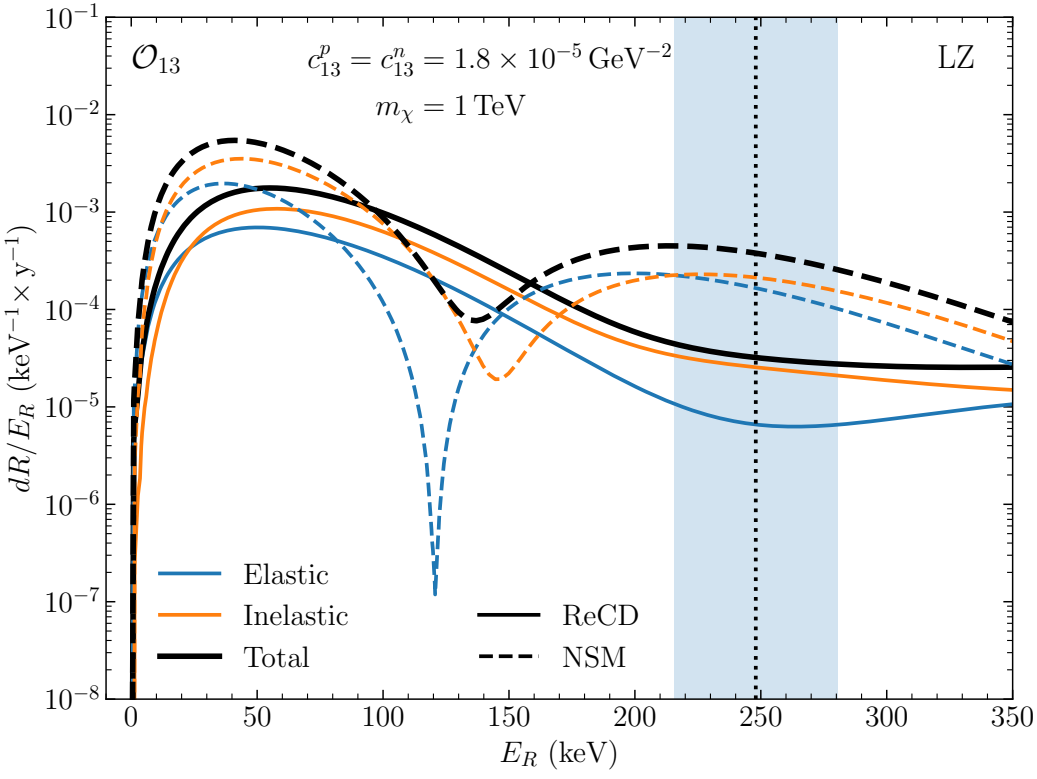
$$\mathcal{O}_6 = \left(\boldsymbol{S}_\chi \cdot \frac{\boldsymbol{q}}{m_N}\right)\left(\boldsymbol{S}_N \cdot \frac{\boldsymbol{q}}{m_N}\right)$$

- For ^{131}Xe, 2nd excited state may contribute more than 1st
- Higher excited states are suppressed

LZ Excess

Recent LZ result (2609.02823)

- Event at $E_R = 248 \pm 23\,(\text{stat}) \pm 23\,(\text{sys})\,\text{keV}$
- Need spectrum with maximum around $E_R = 250\,\text{keV}$
- Analysis: elastic nuclear scattering & NSM
- Example: $\mathcal{O}_{13}$, $m_\chi = 1\,\text{TeV} \Rightarrow 3.4\sigma$



Our ReCD-based prediction:

- Elastic spectrum has a minimum around $250\,\text{keV}$, $(dR^{\text{NSM}}/dE_R)/(dR^{\text{ReCD}}/dE_R) = 25$
- Inelastic spectrum is comparable to elastic
- ReCD $<$ NSM elastic at $250\,\text{keV}$
- Expect smaller significance

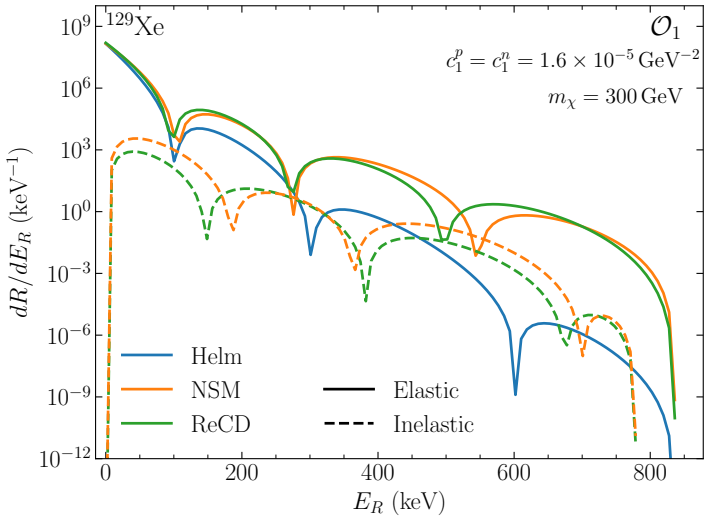
Summary

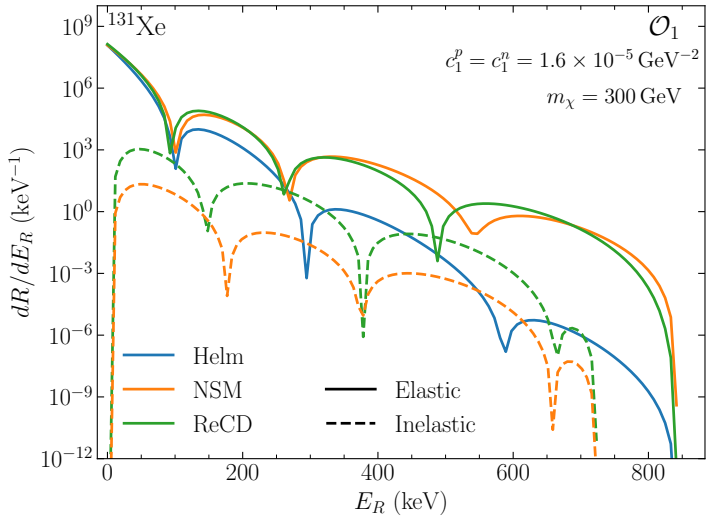
- Inelastic DM-nucleus scattering can improve the sensitivity of direct detection experiments.
- In natural xenon, for operators $\mathcal{O}_7$, $\mathcal{O}_{13}$ and $\mathcal{O}_{14}$, inelastic scattering dominates.
- For scattering on ^{131}Xe via operators $\mathcal{O}_6$ and $\mathcal{O}_9$, the contribution of the second excited state becomes significant.
- LZ excess: sensitive to nuclear inputs, need more careful analysis.

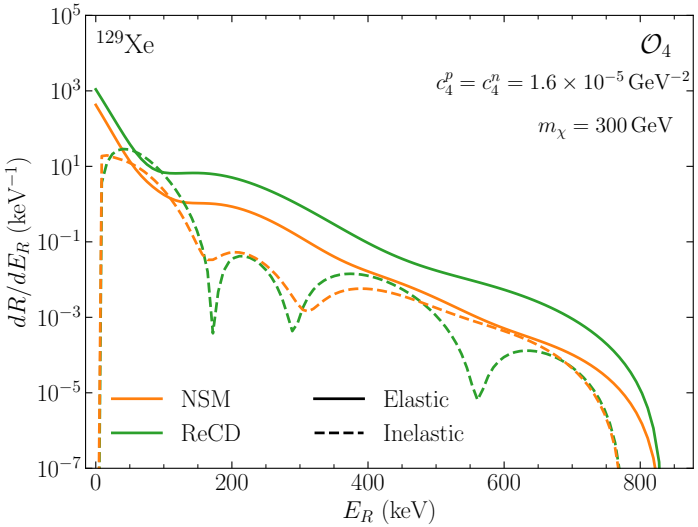
Thank you!

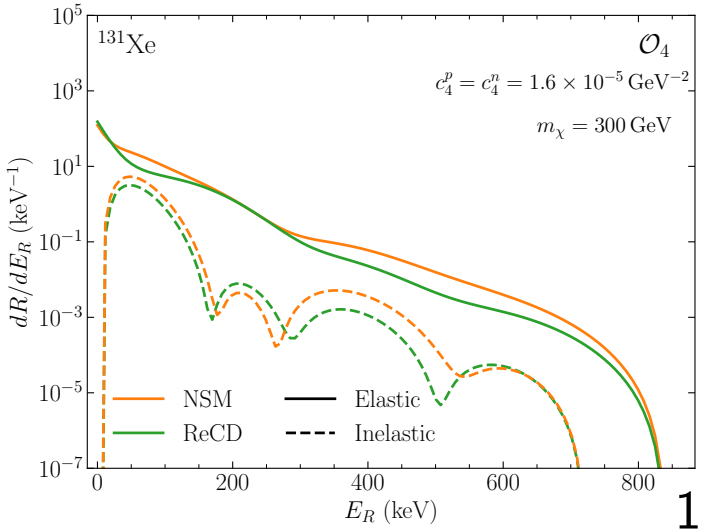
Backup

$\mathcal{O}_1$ & $\mathcal{O}_4$









Selection Rules for Xenon

				$^{129}_{54}$Xe						$^{131}_{54}$Xe				
E^* (keV)				0	39.58	236.1	274.3	318.2	321.7	0	80.19	163.9	341.1	364.5
$J_i^{\pi_i}$ (ground only) or $J_f^{\pi_f}$				$1/2^+$	$3/2^+$	$11/2^-$	$9/2^-$	$3/2^+$	$5/2^+$	$3/2^+$	$1/2^+$	$11/2^-$	$9/2^-$	$5/2^+$
	$\eta_{\mathcal{T}}$	$\pi_{\mathcal{T}}$	$\pi_i\pi_f$	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$-$	$-$	$+$
M	$+$	$(-)^J$	J	0	2	5	5	2	2	0, 2	2	5, 7	3, 5	2, 4
$\widetilde{\Omega}$	$+$	$(-)^{J+1}$	J	$\times$	1	6	4	1	3	$\times$	1	4, 6	4, 6	1, 3
Σ	$-$	$(-)^J$	J	$\times$	2	5	5	2	2	$\times$	2	5, 7	3, 5	2, 4
Σ'	$-$	$(-)^{J+1}$	J	1	1	6	4	1	3	1, 3	1	4, 6	4, 6	1, 3
Σ''	$-$	$(-)^{J+1}$	J	1	1	6	4	1	3	1, 3	1	4, 6	4, 6	1, 3
Δ	$-$	$(-)^{J+1}$	J	1	1	6	4	1	3	1, 3	1	4, 6	4, 6	1, 3
Δ'	$-$	$(-)^J$	J	$\times$	2	5	5	2	2	$\times$	2	5, 7	3, 5	2, 4
$\widetilde{\Delta}''$	$-$	$(-)^J$	J	1	1	6	4	1	3	1, 3	1	4, 6	4, 6	1, 3
$\widetilde{\Phi}$	$+$	$(-)^{J+1}$	J	$\times$	1	6	4	1	3	$\times$	1	4, 6	4, 6	1, 3
$\widetilde{\Phi}'$	$+$	$(-)^J$	J	$\times$	2	5	5	2	2	2	2	5, 7	3, 5	2, 4
Φ''	$+$	$(-)^J$	J	0	2	5	5	2	2	0, 2	2	5, 7	3, 5	2, 4

Velocities in Inelastic Scattering

Original formalism for elastic scattering

$$\boldsymbol{v}^\perp = \underbrace{\boldsymbol{v}_T^\perp}_{\text{CM}} - \underbrace{\boldsymbol{v}_N}_{\text{int}}, \quad \boldsymbol{v}_T^\perp \equiv \frac{1}{2}\left(\frac{\boldsymbol{p}'_\chi + \boldsymbol{p}_\chi}{m_\chi} - \frac{\boldsymbol{p}'_A + \boldsymbol{p}_A}{m_A}\right), \quad \boldsymbol{v}_N \equiv \frac{i\overleftarrow{\boldsymbol{\nabla}}_{N,\,\text{int}} - i\overrightarrow{\boldsymbol{\nabla}}_{N,\,\text{int}}}{2m_N}$$

Inelastic kinematics:

$$\boldsymbol{v}_T^\perp \cdot \boldsymbol{q} = \boldsymbol{v}_T \cdot \boldsymbol{q} + \frac{|\boldsymbol{q}|^2}{2\mu_A} = -\frac{\boldsymbol{q}^2}{2\mu_A} - E^* + \frac{|\boldsymbol{q}|^2}{2\mu_A} = -E^*.$$

Modification for inelastic scattering: $\boldsymbol{v}_T^\perp \to \widetilde{\boldsymbol{v}}_T^\perp$

$$\boldsymbol{v}^\perp \equiv \widetilde{\boldsymbol{v}}_T^\perp - \boldsymbol{v}_N^\perp, \quad \widetilde{\boldsymbol{v}}_T^\perp \equiv \boldsymbol{v}_T^\perp + \boldsymbol{q}\frac{E^*}{|\boldsymbol{q}|^2}, \quad \boldsymbol{v}_N^\perp \equiv \boldsymbol{v}_N + \boldsymbol{q}\frac{E^*}{|\boldsymbol{q}|^2}$$

Numerically, effect is $\lesssim 0.1\%$