Modular Invariant Inflation and Reheating

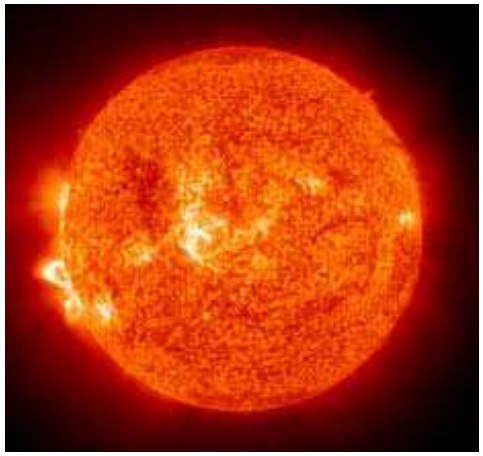
Wenbin Zhao, ICTP-AP, UCAS

Based on Recent works: 2405.06497, 2411.18603, 2512.21597

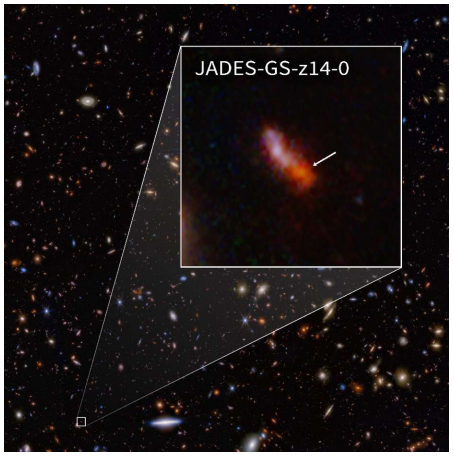
with Gui-Jun Ding, Si-Yi Jiang, Yong Xu

New physics from Cosmology

- Cosmology/Astrophysics has become a strong probe of new physics:

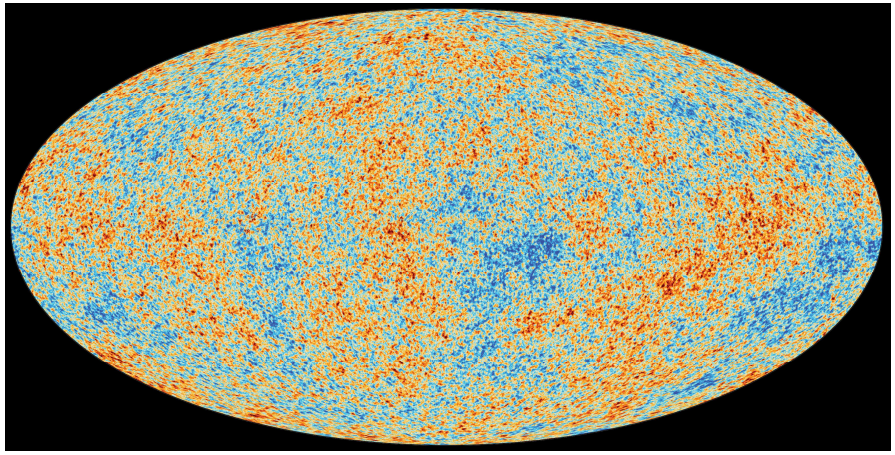


Sun



JADES-GS-z14-0

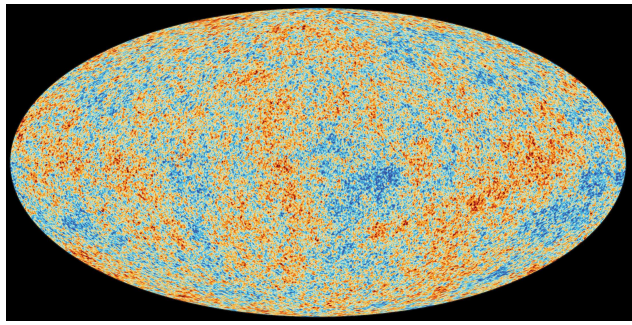
Credit: NASA

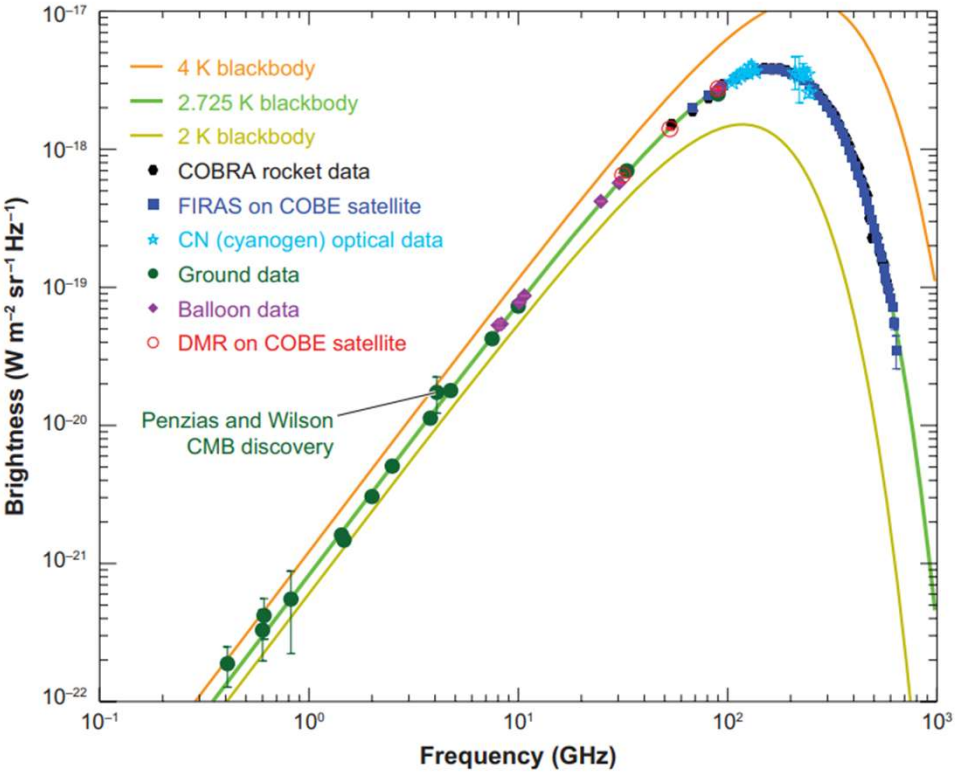


Cosmic microwave background

Credit: ESA

CMB photons





Credit:D Samtleben

- Black Body radiation
- Photons from **All** direction shares the same temperature:

$$T = 2.725\ \mathrm{K}$$

- With a variation

$$\frac{\delta T}{T} \sim 10^{-5}$$

We would like to understand:

- Why the early universe is **isotropic** and **homogeneous**?
- Where the small **fluctuations** comes from?

Inflation as a Solution

- Scalar dynamics in FLRW Universe

$$S = \int \mathrm{d}^4 x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right)$$

- Equation of motion

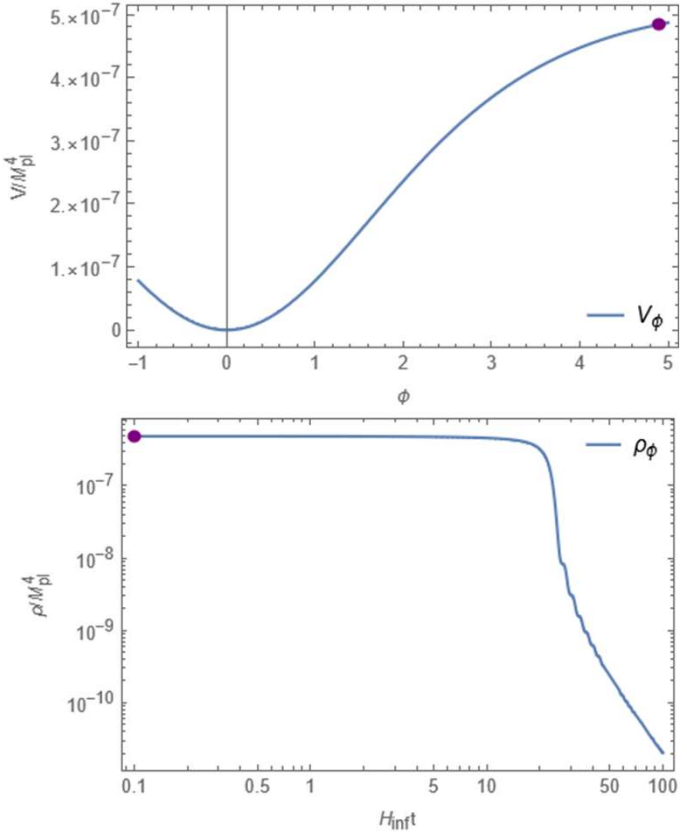
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{1}{3m_{pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

- "slow-roll" solution

$\frac{1}{2}\dot{\phi}^2 \ll V$, $\left|\frac{\ddot{\phi}}{3H\dot{\phi}}\right| \ll 1 \Longrightarrow H^2 \sim$constant,

$a(t) \sim e^{Ht} \Longleftarrow N_e = ln\left(\frac{a_{end}}{a_i}\right) \sim 50 - 60$ CMB



How to test Inflation?

Inflation generates primordial perturbations!

- Scalar perturbations

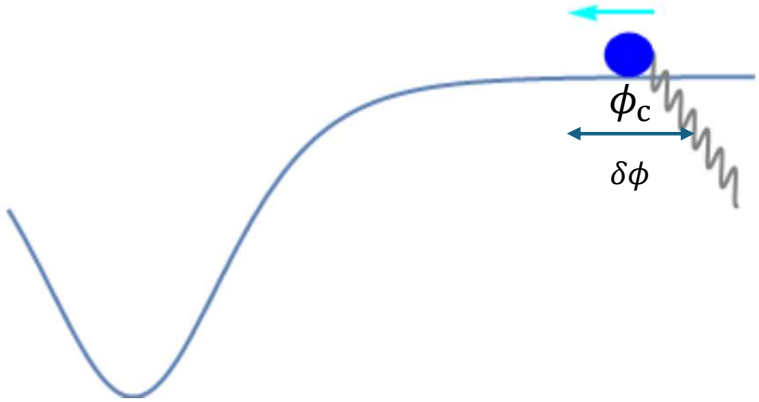
  Temperature anisotropies in the CMB

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

- Tensor perturbations

  Stochastic gravitational wave background

$$\mathcal{P}_t(k) = A_t \left(\frac{k}{k_*}\right)^{n_t}$$

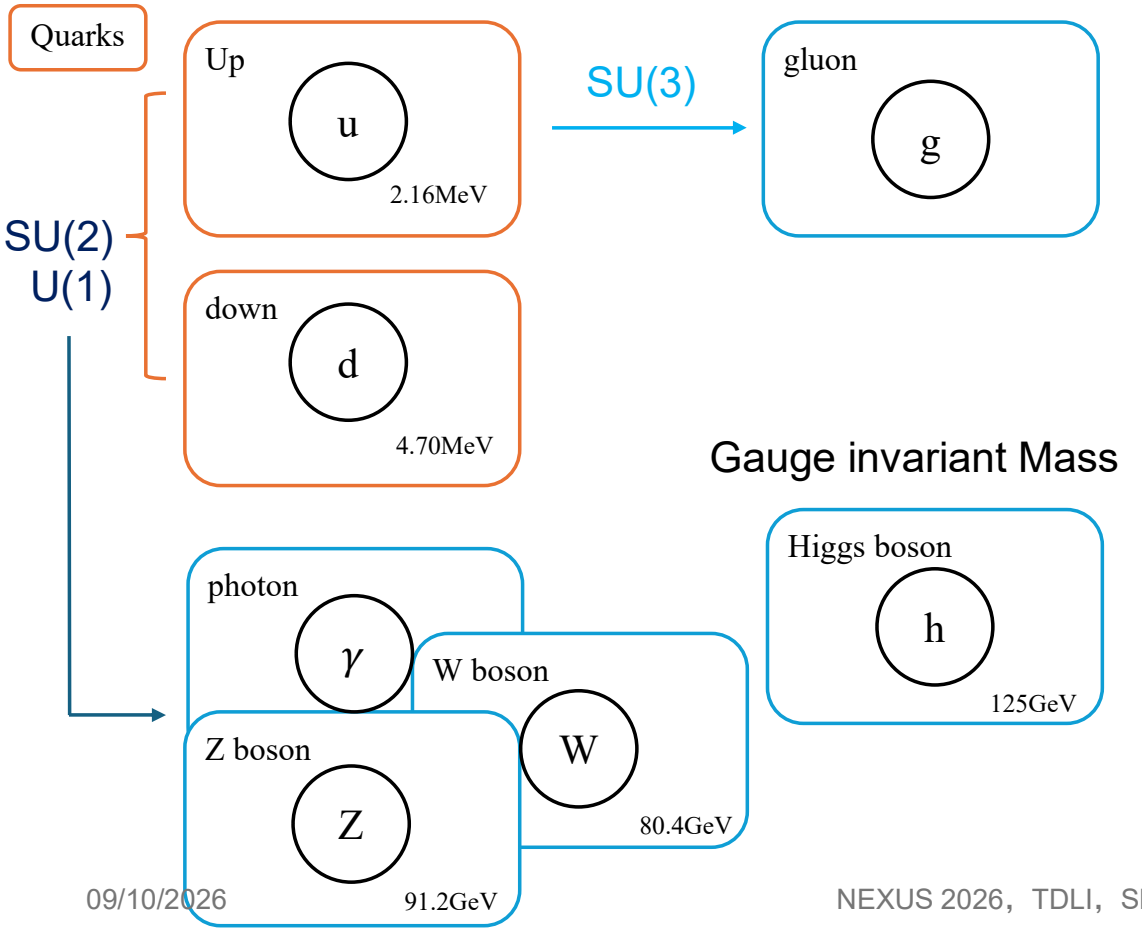


Sensitive to the shape of the potential：

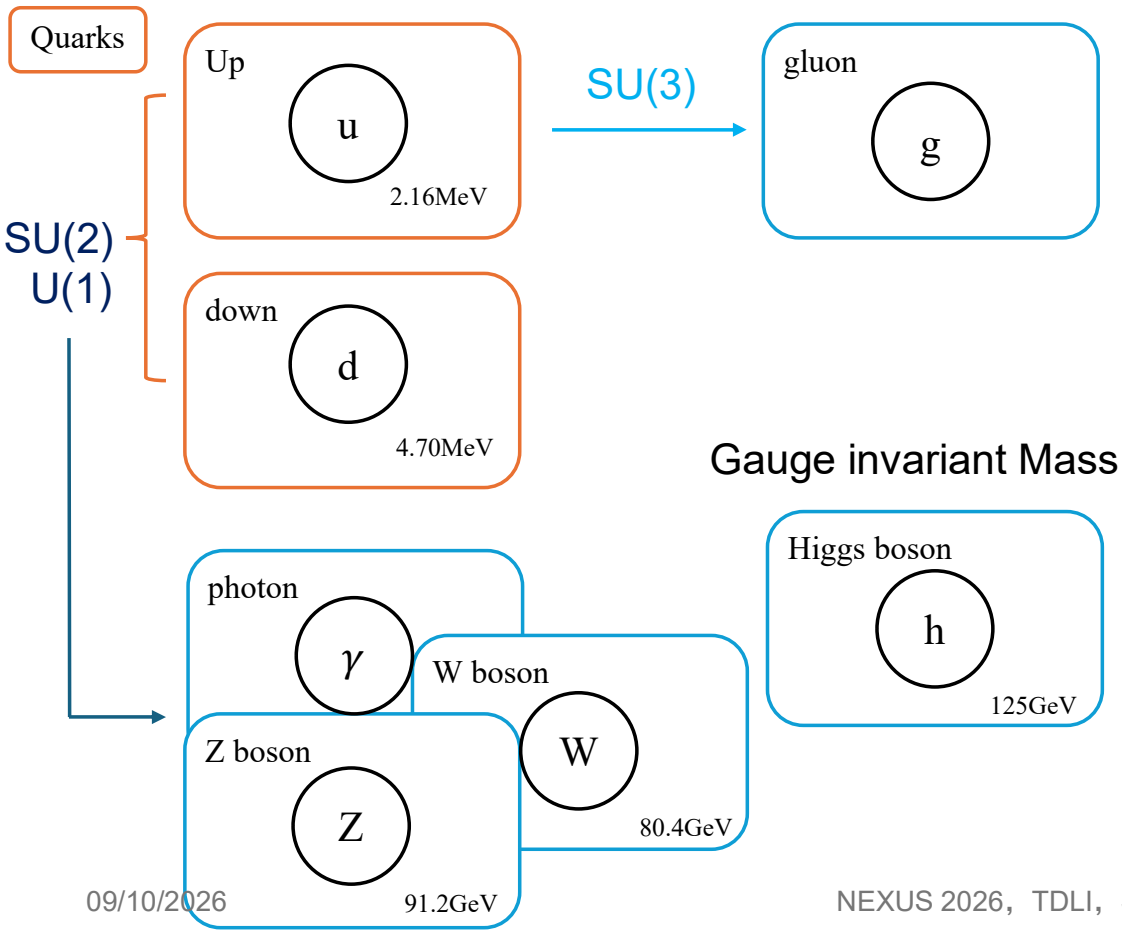
$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \eta = \frac{V''}{V}$$

How to understand the flatness of the potential?
What is the nature of this scalar field?

How to extend the Standard Model?



Supersymmetry



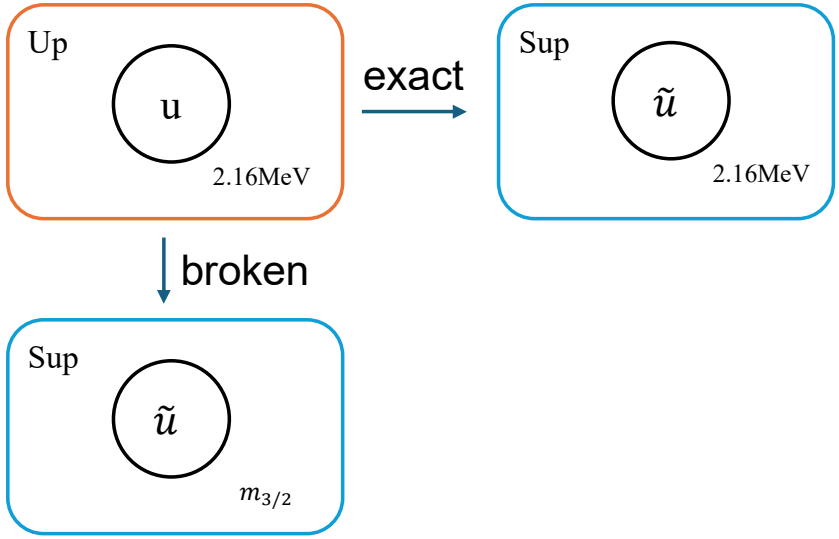
Extend the **symmetry group**

Space-time symmetry× Internal Symmetry

Poincare symmetry × $\boldsymbol{SU(3)} \times \boldsymbol{SU(2)} \times \boldsymbol{U(1)}$

One interesting choice:

Supersymmetry (SUSY)



Supergravity (Gauged Supersymmetry)

- Basic Object:

  Chiral Superfield Φ^{α}, contains both scalar field ϕ^{α} and fermionic field χ^{α}

- Building Blocks:

(Hermitian) Kahler potential $\mathcal{K}(\Phi^{\alpha}, \Phi^{\alpha,*})$ (Holomorphic) Superpotential $\mathcal{W}(\Phi^{\alpha})$

- Relevant Lagrangian:

$$\sqrt{-g}^{-1}\mathcal{L} = \mathcal{K}_{\alpha\bar{\beta}}\left(\partial_{\mu}\phi^{\alpha}\partial^{\mu}\overline{\phi^{\beta}} + i\overline{\chi^{\beta}}\overline{\sigma^{\mu}}\partial_{\mu}\chi^{\alpha}\right) - e^{\mathcal{K}/m_{pl}^2}\left(\mathcal{K}^{\alpha\bar{\beta}}D_{\alpha}\mathcal{W}\overline{D_{\beta}\mathcal{W}} - \frac{3}{m_{pl}^2}|\mathcal{W}|^2\right)$$

Scalar potential, Inflation

$$-\frac{1}{2}e^{\mathcal{K}/(2m_{pl}^2)}\left[\left(D_{\alpha}D_{\beta}\mathcal{W}\right)\chi^{\alpha}\chi^{\beta} + \text{h.c.}\right],$$

Interaction, reheating

$$\mathcal{K}_{\alpha\bar{\beta}} = \frac{\partial^2\mathcal{K}}{\partial\Phi^{\alpha}\partial\overline{\Phi^{\beta}}} = \partial_{\alpha}\partial_{\bar{\beta}}\mathcal{K}$$

$$D_{\alpha}\mathcal{W} = \frac{\partial\mathcal{W}}{\partial\Phi^{\alpha}} + \frac{1}{m_{pl}^2}\frac{\partial\mathcal{K}}{\partial\Phi^{\alpha}}\mathcal{W}$$

Modular Symmetry from String Theory

$D = 10$ Superstring $\longrightarrow$ Dilaton S and other fields

Compactifycation $\longrightarrow$ modulus fields: massless scalar fields with no potential

$\mathcal{M}_{10} = \mathcal{M}_6 \times \mathcal{M}_4$

moduli stabilization: generate potential, positive mass and vev

$\mathcal{M}_6 = T^2 \times T^2 \times T^2$

Kähler modulus τ and complex structure modulus

Modular group is the mapping class group of torus

$D = 4, N = 1$ SUGRA $\longrightarrow$ Modular invariant scalar potential for S and τ

Modular invariant Inflation

- Modular Symmetry SL(2,Z)

$$\tau \to \gamma\tau = \frac{a\tau + b}{c\tau + d},$$

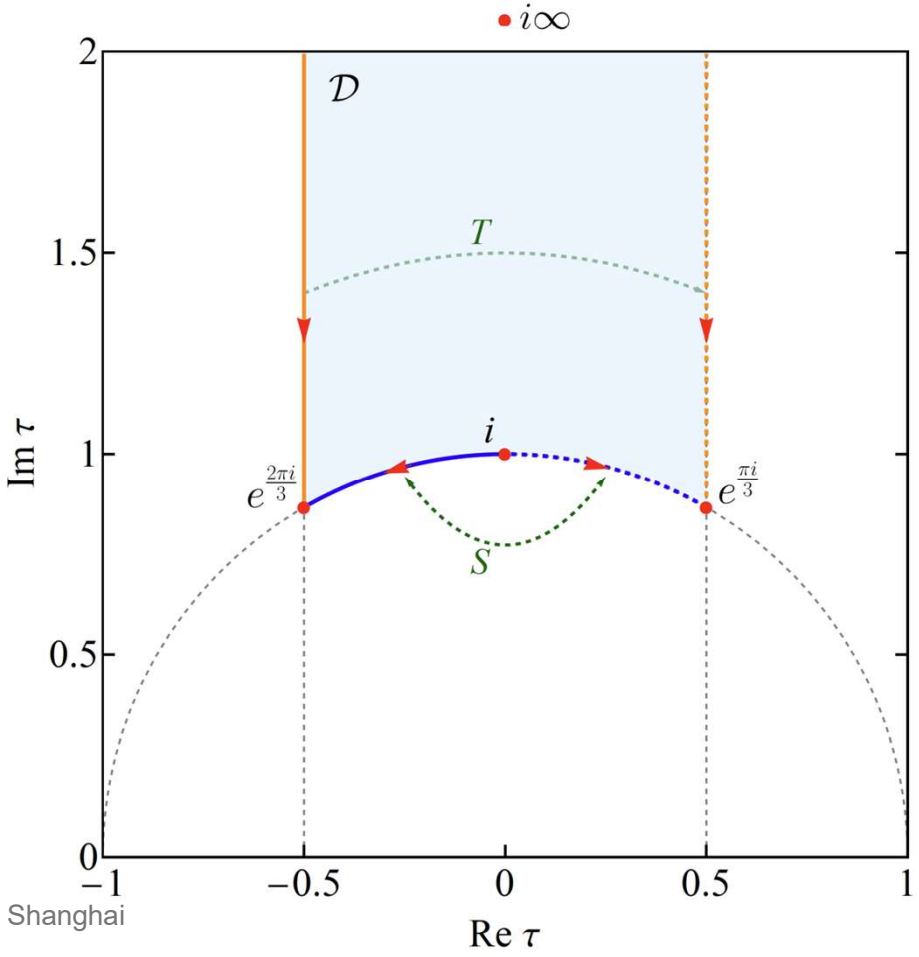
$$\mathrm{Im}(\tau) > 0\,, \mathrm{ad} - \mathrm{bc} = 1$$

- Two generators

$$\mathcal{S}: \tau \to -1/\tau, \quad \mathcal{T}: \tau \to \tau + 1$$

- Three fix points:

$$i, \omega = e^{\frac{2\pi i}{3}}, i\infty$$



Modular invariant Inflation

- Modular forms:

$$f(\gamma\tau) = (c\tau + \mathrm{d})^k f(\tau)$$

- The derivative of a weight k modular form satisfies:

$$f'(\gamma\tau) = (c\tau + d)^{k+2} f'(\tau) + \frac{k}{2\pi i} c(c\tau + d)^{k+1} f(\tau)$$

- For a weight 0 modular form, its derivative is a weight 2 modular form, which should vanish at fix points.
- Half of the problem solved for inflation!

Modular invariant Inflation

$$\mathcal{K}(\tau,\bar{\tau},S,\bar{S}) = K(S,\bar{S}) - 3\ln(-i(\tau-\bar{\tau}))\,,$$
$$\mathcal{W}(S,\tau) = \Lambda_W^3 \frac{\Omega(S)H(\tau)}{\eta^6(\tau)}\,,$$

M. Cvetic, A. Font, L. E. Ibanez, D. Lust, and F. Quevedo

- We assume dilaton S is stabilized.
- η is the Dedekind eta function with a modular *weight* $1/2$:

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n), \qquad q \equiv e^{2\pi i\tau}\,,$$

- Under Modular transformation,

$$-3\ln\left[-i(\tau-\bar{\tau})\right] \to -3\ln\left[-i(\tau-\bar{\tau})\right] + 3\ln(c\tau+d) + 3\ln(c\bar{\tau}+d)\,.$$
$$\mathcal{W} \to e^{i\delta(\gamma)}(c\tau+d)^{-3}\mathcal{W}\,,$$

Modular invariant Inflation

- The most general form without singularity inside the fundamental domain:

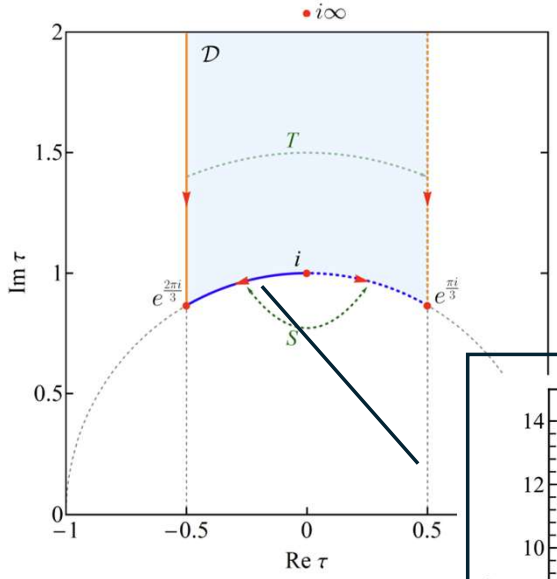
$$H(\tau) = (j(\tau) - 1728)^{m/2} j(\tau)^{n/3} \mathcal{P}(j(\tau)) \,, \quad m, n \in \mathbb{N} \,,$$

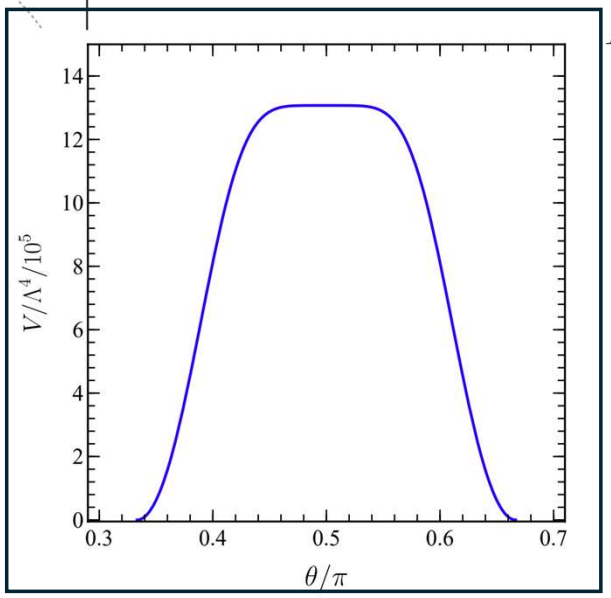
where j is called Klein j invariant.

$$j(i\infty) = +\infty \,, \qquad j(\omega) = 0 \,, \qquad j(i) = 1728 = 12^3 \,.$$

- m, n determine vacuum of the potential
  - $m = 0, n \geq 2$, slow roll from i (saddle point) to ω (Minkowski minimum) along the arc.
  - $m \geq 2, n \geq 2$, slow roll from $i\infty$ to ω(Minkowski minimum) along the left boundary.
  - $m = n = 0,$ slow roll from i (saddle point) to ω (dS minimum)along the arc (King, Wang, 2405.08924).

Modular invariant Inflation





$$\mathcal{K}(\tau,\bar{\tau},S,\bar{S}) = K(S,\bar{S}) - 3\ln(-i(\tau-\bar{\tau}))\,,$$

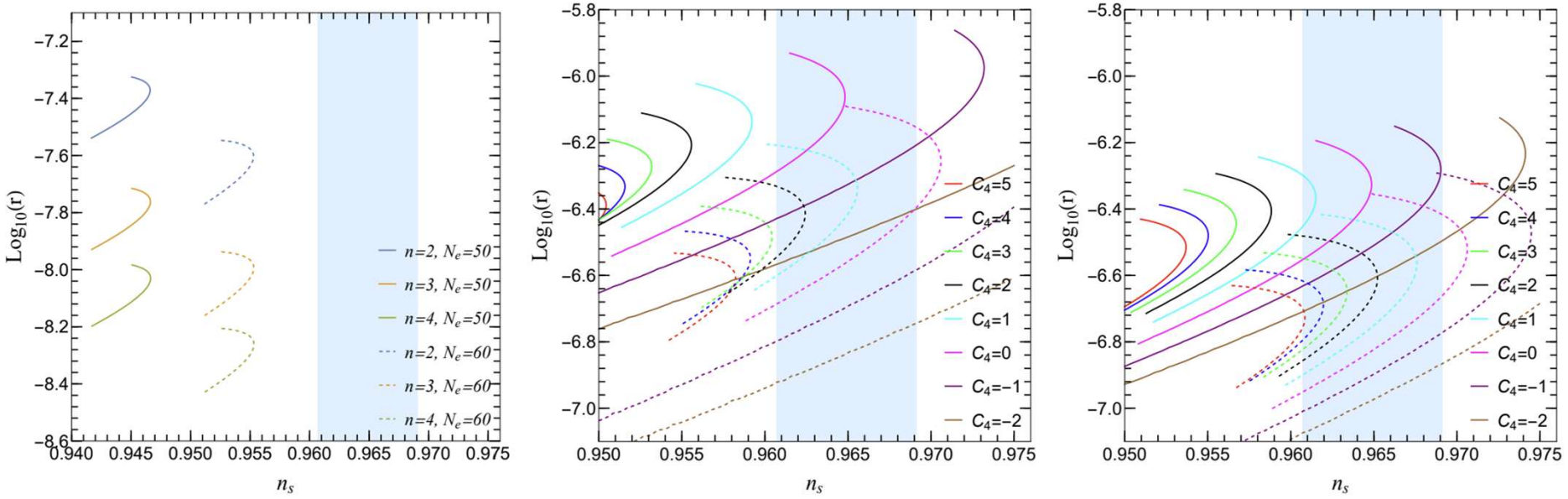
$$\mathcal{W}(S,\tau) = \Lambda_W^3 \frac{\Omega(S)H(\tau)}{\eta^6(\tau)}\,,$$

$$H(\tau) = (j(\tau)-1728)^{m/2} j(\tau)^{n/3} \mathcal{P}(j(\tau))\,, \quad m,n \in \mathbb{N}\,,$$

$$\mathcal{P}(j(\tau)) = 1 + \beta\left(1-\frac{j(\tau)}{1728}\right) + \gamma\left(1-\frac{j(\tau)}{1728}\right)^2$$

- S for positivity of the potential.
- (m,n) controls the vacuum:
  $m \geq 2, n = 0, V(\omega) = 0$
- Inflation around i along the arc of τ
  $\tau = \rho e^{i\theta}$

Prediction of the model



(a) $\mathcal{P}(j) = 1$. (b) with β. (c) with γ.

- Can be approximated by $V(\phi) = \sum_{n\geq 2} C_n \phi^n$
- Prediction: $r < 10^{-6}, \alpha \sim 10^{-3}$. Testable in the next CMB experiments.

Another approach

- A series of work in realizing α attractors with SL(2, Z) symmetry.

2408.05203, 2411.0752, 2503.13682, 2503.14904

- At large $Im(\tau)$, $|j(\tau)|^2 \approx e^{4\pi\,\mathrm{Im}(\tau)}$

One can define

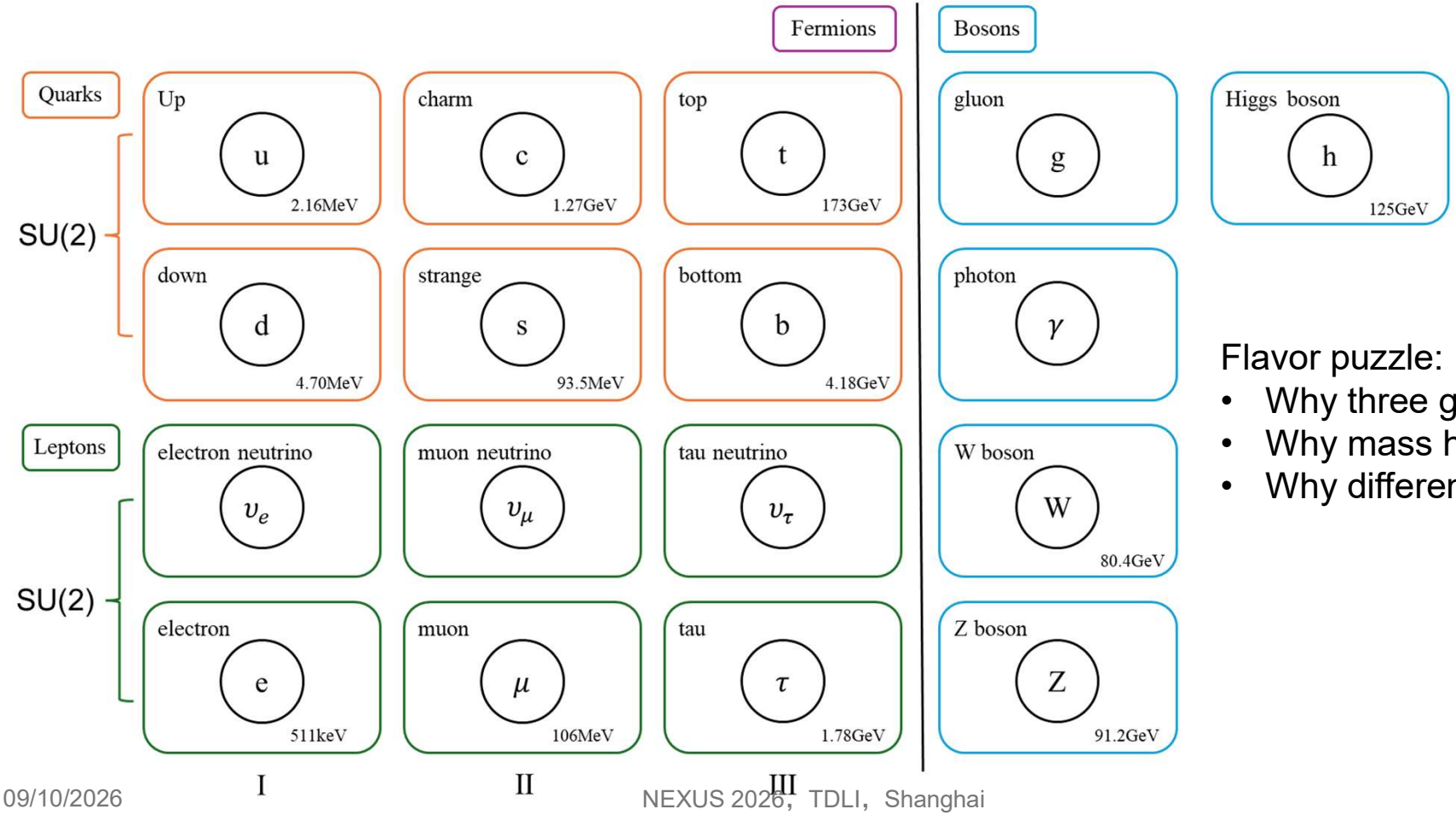
$$I(\tau,\bar{\tau}) = \frac{\ln\left(|j(\tau)|^2+\beta^2\right)}{\ln\beta^2}\,, \quad I(\tau,\bar{\tau})\Big|_{\mathrm{Im}(\tau)\to\infty} \approx \frac{4\pi}{\ln(\beta^2)}\,\mathrm{Im}(\tau) \equiv c\,\mathrm{Im}(\tau)\,,$$

and construct an inflation potential:

$$V(\tau,\bar{\tau}) = V_0\left(\frac{I(\tau,\bar{\tau})-1}{I(\tau,\bar{\tau})+1}\right)^2\,, \quad V(\tau,\bar{\tau})\Big|_{\mathrm{Im}(\tau)\to\infty} \approx V_0\left(1-\frac{8}{c\,\mathrm{Im}(\tau)}+\ldots\right)$$

- Similar construction exist in Sp(4,$\mathbb{Z}$) group.

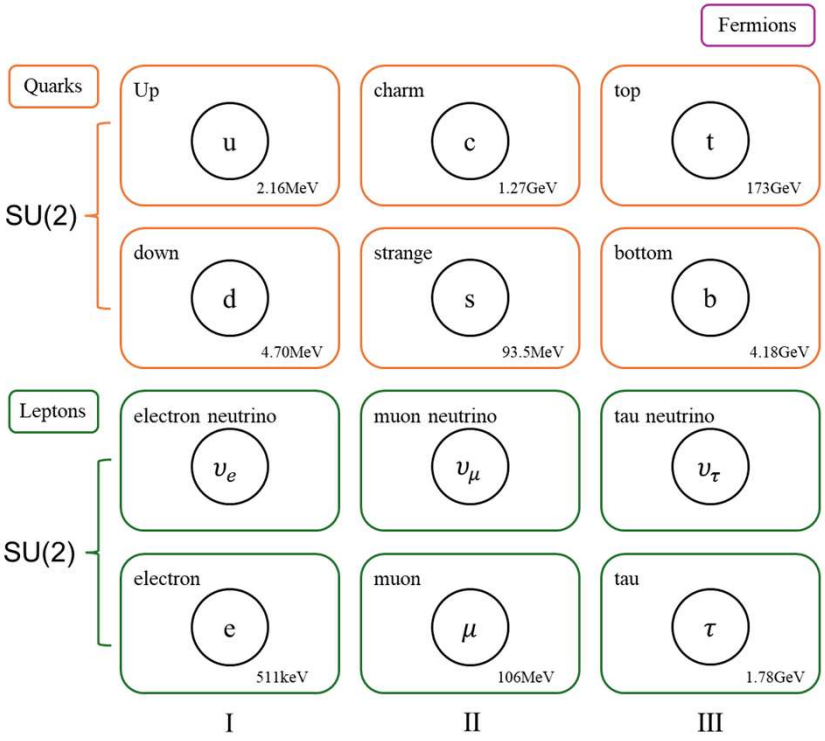
The Standard Model



Flavor puzzle:

- Why three generations?
- Why mass hierarchies?
- Why different mixing?

The Standard Model



Flavor symmetry:

$$
\begin{aligned}
(\nu_i, l_i)_L &\sim (\underline{3}, 1), \\
l_{1R} &\sim (\underline{1}, 1), \\
l_{2R} &\sim (\underline{1}', 1), \\
l_{3R} &\sim (\underline{1}'', 1), \\
N_{iR} &\sim (\underline{3}, 0), \\
\Phi_i = (\phi_i^+, \phi_i^0) &\sim (\underline{3}, 0), \\
\eta = (\eta^+, \eta^0) &\sim (\underline{1}, -1).
\end{aligned}
$$

A_4

Ernest Ma and G. Rajasekaran

$$\frac{1}{2} M N_{iR}^2 + f \bar{N}_{iR} (\nu_{iL} \eta^0 - l_{iL} \eta^+) + h_{ijk} \overline{(\nu_i, l_i)}_L l_{jR} \Phi_k + h.c.$$

Modular symmetry can be a flavor symmetry.

Ferruccio Feruglio

Modular approach to lepton flavor problem

- vev of τ in various models generally, **deviates** from fix points

  Feruglio, Ferruccio, 2211.00659

- A small deviation of fixed point can be used as the Froggatt-Nielsen charge.

  P. P. Novichkov, J. T. Penedo, S. T. Petcov, 2102.07488

- Shifting the minima:

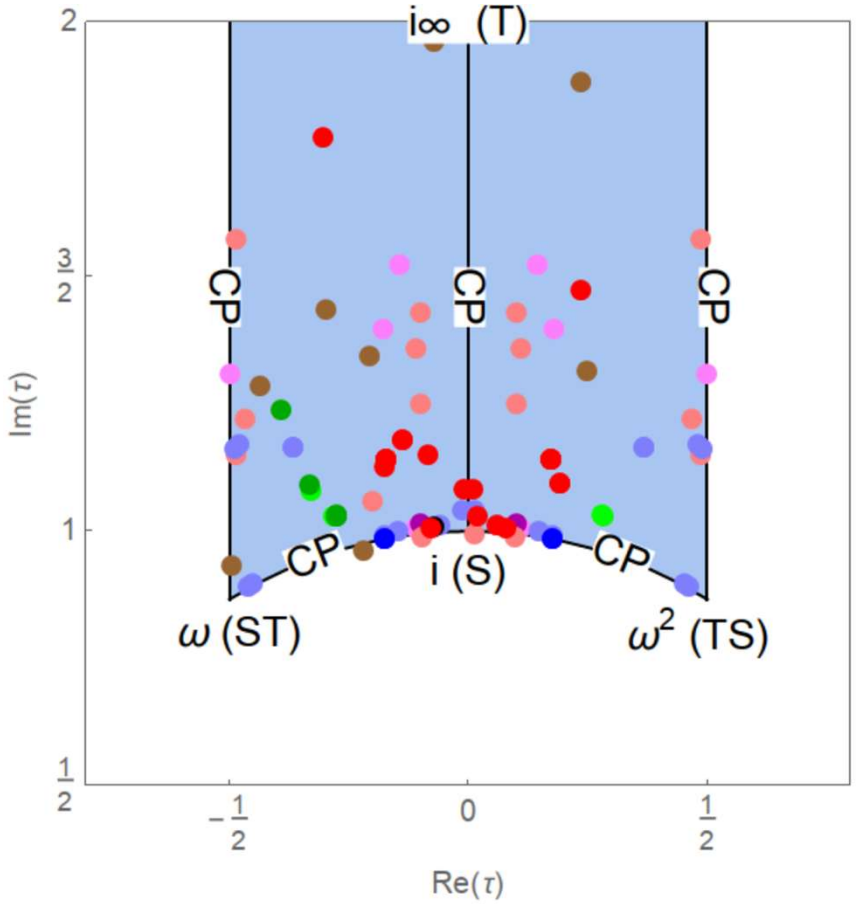
Vacuum of the potential:

$$H(\tau) = (j(\tau) - 1728)^{m/2} j(\tau)^{n/3} \mathcal{P}(j(\tau)), \quad m, n \in \mathbb{N},$$

Make an analogy:

$$H(\tau) = (j(\tau) - j(\tau_0))^2 \left(1 + \beta\left(1 - \frac{j(\tau)}{1728}\right) + \gamma\left(1 - \frac{j(\tau)}{1728}\right)^2\right)$$

For large field: $I(\tau, \bar{\tau}) = \frac{\ln\left(|j(\tau)|^2 + \beta^2\right)}{\ln\left(|j(\tau_0)|^2 + \beta^2\right)}$



Modular approach to lepton flavor problem

To reduce the Free parameters in the SM.

$$g_1\left((N^cL)_{3_S}Y_3^{(2)}\right)_1 H_u + g_2\left((N^cL)_{3_A}Y_3^{(2)}\right)_1 H_u + \frac{1}{2}\Lambda_N\left((N^cN^c)_{3_S}Y_3^{(2)}\right)_1$$

Matter superfield are representations of finite subgroup of modular group.

A_4 $\quad L \sim \mathbf{3}, \quad N^c \equiv \{N_1^c, N_2^c, N_3^c\} \sim \mathbf{3}, \quad Y_{\mathbf{3}}^{(2)}(\tau)$

When fixing $\tau_0 = \langle\tau\rangle = -0.4847 + 0.8747i$, expand τ around τ_0

Zero Order: RHN mass, Yukawa coupling…

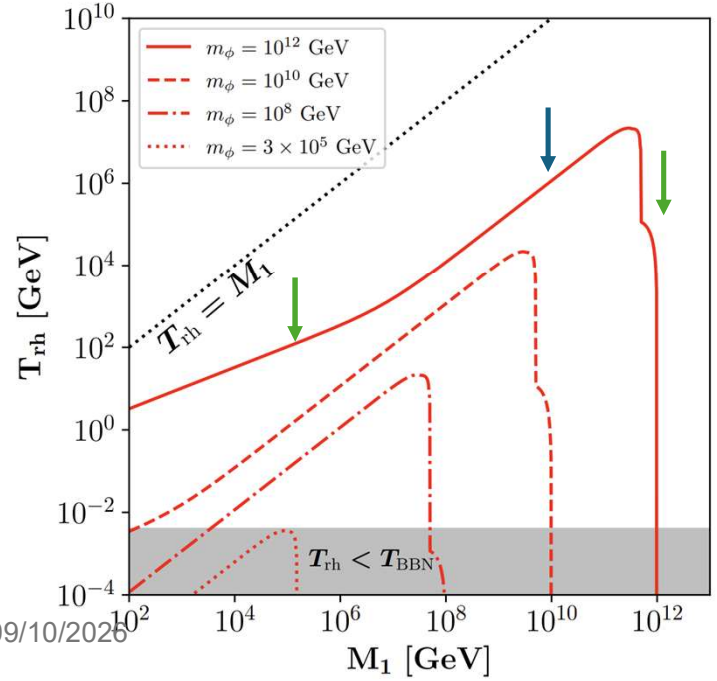
First Order: Planck Suppressed Interactions between Inflaton and other fields.

$$\Gamma(\phi \to N_i^c N_j^c) \propto \left(\frac{\Lambda_N}{M_{pl}}\right)^2 \frac{\left(m_\phi^2 - 4m_{N_i}^2\right)^{3/2}}{8\pi m_\phi^2}$$

Non-thermal Leptogenesis

- If we assume they are the only decay channels

$$\Gamma_{\rm tot} = \Gamma(\phi \to N_i^c N_j^c) + \Gamma(\phi \to \tilde{N}_i^c \tilde{N}_j^c) + \Gamma(\phi \to L_i H N_j^c)$$



Reheating temperature is below RHN mass.
Non-thermal RHN decay →Lepton Asymmetry.

$$\frac{n_L}{s} \simeq \frac{3}{4}\frac{T_{\rm rh}}{m_\phi}\sum_i \epsilon_i \times (2{\rm BR}_2 + {\rm BR}_1) \simeq 2.5 \times 10^{-10}$$

Depends only on inflaton, RHN mass

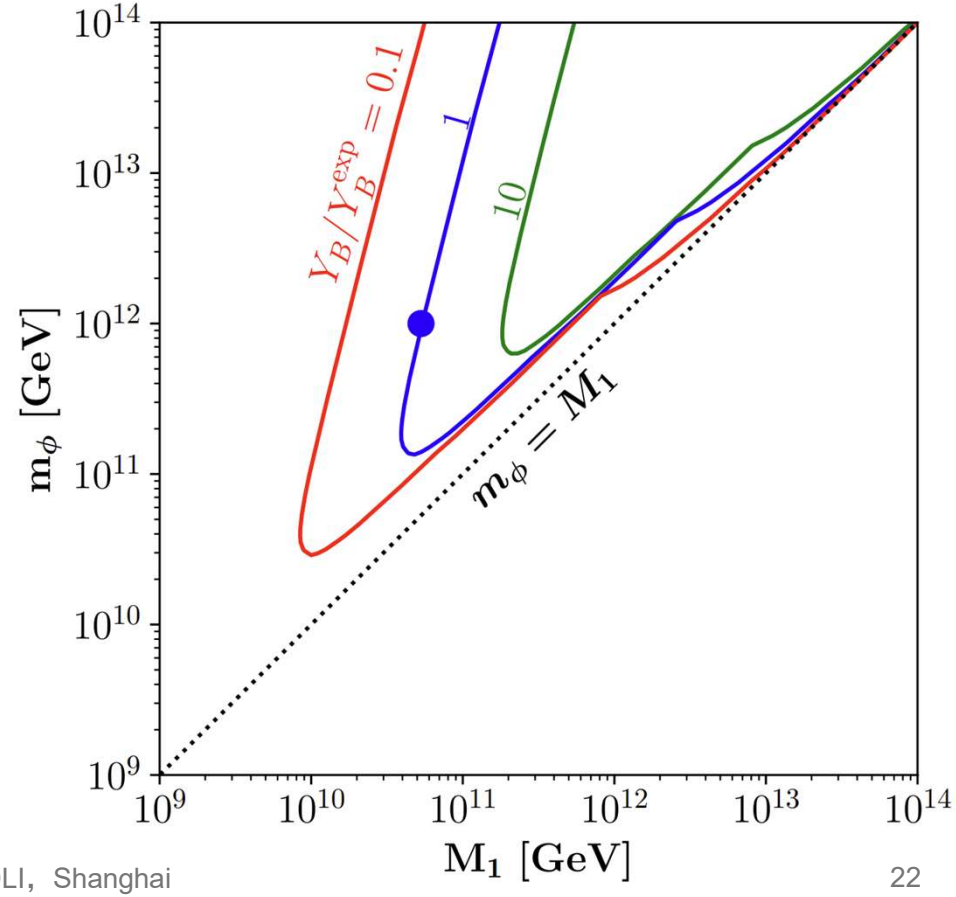
$$\epsilon_i = \frac{\Gamma(N_i \to H_u + L) - \Gamma(N_i \to \overline{H}_u + \overline{L})}{\Gamma(N_i \to H_u + L) + \Gamma(N_i \to \overline{H}_u + \overline{L})}$$

Non-thermal Leptogenesis

Lepton Asymmetry yields BAU through Anomalous B + L Violation processes.

$$\frac{n_B}{s} \simeq -\frac{8}{23}\frac{n_L}{s} \simeq 8.6 \times 10^{-11}$$

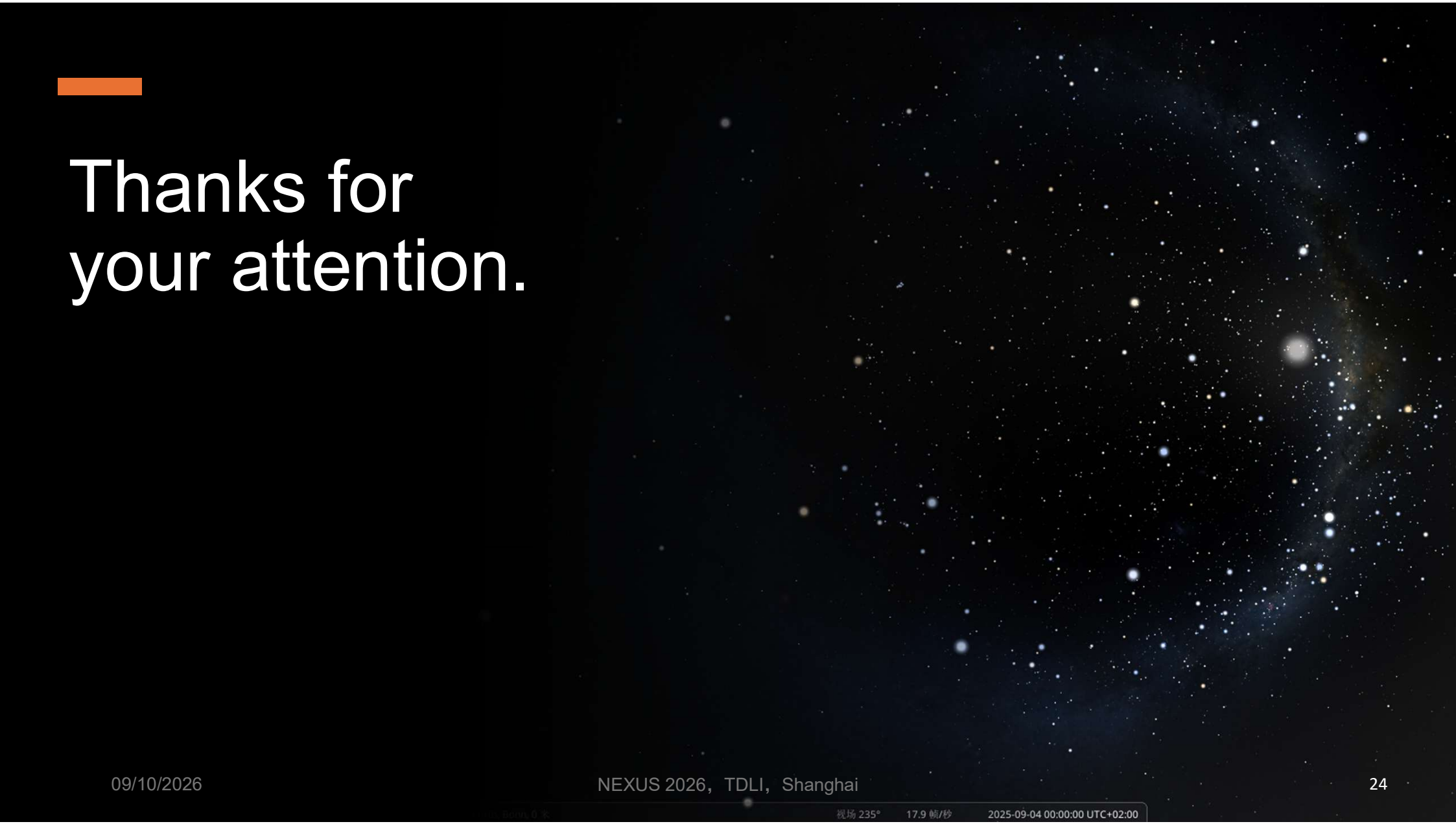
Requires $m_\phi > 10^{11}$ GeV.



Summary

- Modular symmetry can be a good guiding principle to explain:
  - Inflation
  - Flavor puzzle
  - Baryon Asymmetry
- More to explore:
  - Maybe fine-tuned. A more natural way?
  - Dynamics of dilaton field?
  - DM? 1904.03937, 2108.09984, 2409.02178
  - CP-problem? 2305.08908, 2404.08032, 2406.01689, 2504.03506

Thanks for your attention.



Inflation Observables

- Primordial fluctuations

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

Scalar perturbations

$$\mathcal{P}_t(k) = A_t \left(\frac{k}{k_*}\right)^{n_t}$$

Gravitational waves

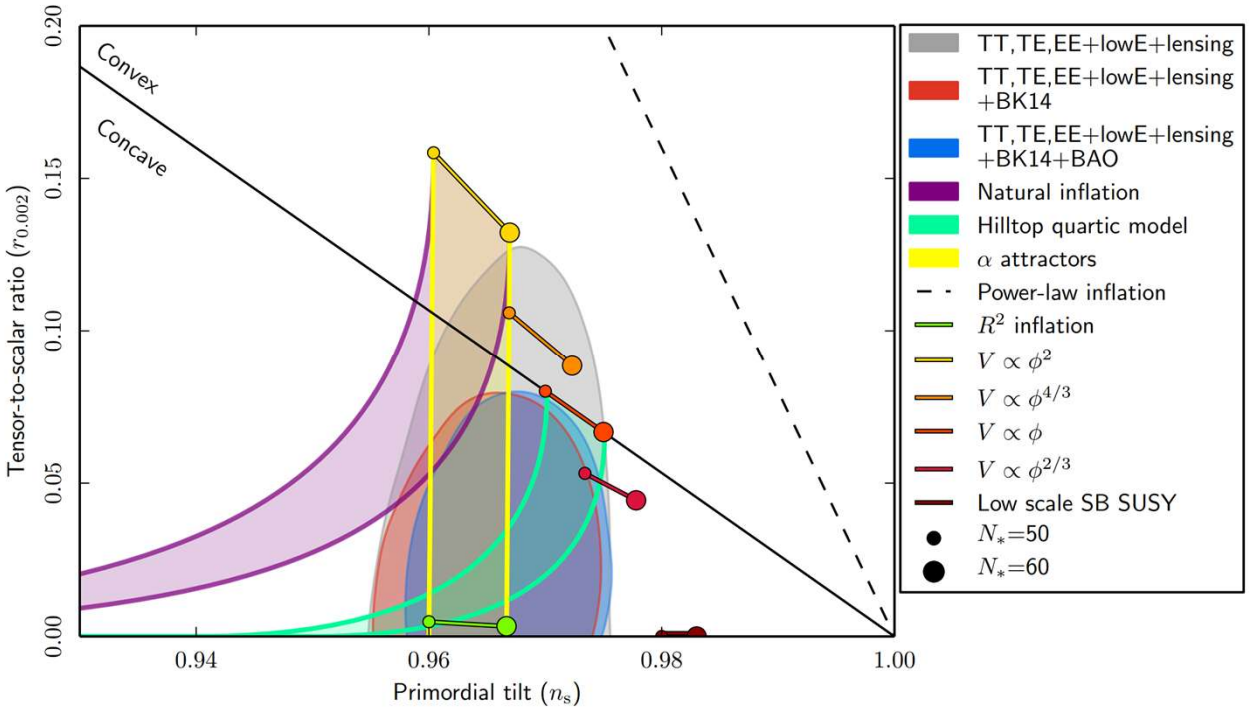
- Sensitive to the Shape of the potential:

$$n_s = 1 + 2\eta_V(\phi_*) - 6\epsilon_V(\phi_*)$$

$$r = A_t/A_s = 16\epsilon_V(\phi_*),$$

$$n_t = -r/8 = -2\epsilon_V(\phi_*),$$

$$\alpha = \frac{\mathrm{dn_s}}{d\log k} = 16\epsilon_V(\phi_*)\eta_V(\phi_*) - 24\epsilon_V(\phi_*)^2 - 2\xi_V(\phi_*)^2$$



Credit:Planck

Modular Scalar potential

We choose the following polynomial:

$$\mathcal{P}(j(\tau)) = 1 + \beta\left(1 - \frac{j(\tau)}{1728}\right) + \gamma\left(1 - \frac{j(\tau)}{1728}\right)^2 ,$$

and the full potential reads:

$$V(\tau) = \frac{\Lambda_S^4}{i(\tau-\bar{\tau})^3|\eta(\tau)|^{12}}\left[(A(S,\bar{S})-3)|H(\tau)|^2 + \hat{V}(\tau,\bar{\tau})\right] ,$$

$$A(S,\bar{S}) = \frac{K^{S\bar{S}} D_S W D_{\bar{S}}\bar{W}}{|W|^2} = \frac{K^{S\bar{S}}|\Omega_S + K_S\Omega|^2}{|\Omega|^2} ,$$

$$\hat{V}(\tau,\bar{\tau}) = \frac{-(\tau-\bar{\tau})^2}{3}\left|H_\tau(\tau) - \frac{3i}{2\pi}H(\tau)\widehat{G}_2(\tau,\bar{\tau})\right|^2 ,$$

$$Z(\tau,\bar{\tau}) = \frac{1}{i(\tau-\bar{\tau})^3|\eta(\tau)|^{12}} ,$$

In short, 3 parameter sets: $(m,n), (\beta,\gamma), A(S,\bar{S})$

Full Superpotential

$$
\begin{aligned}
\eta^6 \mathcal{W}_E &= y_1 e^c (L Y^{(2)}_{\mathbf{3}})_{\mathbf{1}''} H_d + y_2 \mu^c (L Y^{(6)}_{\mathbf{3}I})_{\mathbf{1}''} H_d + y_3 \mu^c (L Y^{(6)}_{\mathbf{3}II})_{\mathbf{1}''} H_d \\
&\quad + y_4 \tau^c (L Y^{(6)}_{\mathbf{3}I})_{\mathbf{1}'} H_d + y_5 \tau^c (L Y^{(6)}_{\mathbf{3}II})_{\mathbf{1}'} H_d \,, \\
\eta^6 \mathcal{W}_\nu &= g_1 \left((N^c L)_{\mathbf{3}_S} Y^{(2)}_{\mathbf{3}} \right)_{\mathbf{1}} H_u + g_2 \left((N^c L)_{\mathbf{3}_A} Y^{(2)}_{\mathbf{3}} \right)_{\mathbf{1}} H_u + \frac{1}{2} \Lambda_N \left((N^c N^c)_{\mathbf{3}_S} Y^{(2)}_{\mathbf{3}} \right)_{\mathbf{1}} .
\end{aligned}
$$

- 10 free parameters

$$
\tau_0 = \langle \tau \rangle = -0.4847 + 0.8747 i \,,
$$

$$
y_2/y_1 = 5.1844 \times 10^2 \,, \quad y_3/y_1 = 1.4659 e^{-2.4143 i} \times 10^2 \,,
$$

$$
y_4/y_1 = 2.3952 \times 10^4 \,, \quad y_5/y_1 = 1.2117 e^{-0.5024 i} \times 10^3 \,,
$$

$$
g_2/g_1 = 0.2465 \,, \quad y_1 v_d = 0.2499 \text{ MeV} \,, \quad \frac{(g_1 v_u)^2}{\Lambda_N} = 19.9673 \text{ meV} \,.
$$

- CP breaking is put by hand.
- Degenerate RHN masses.

$$
M_N = \Lambda_N \mathcal{Y}_N(\tau) = \begin{pmatrix} 2Y^{(2)}_{\mathbf{3},1} & -Y^{(2)}_{\mathbf{3},3} & -Y^{(2)}_{\mathbf{3},2} \\ -Y^{(2)}_{\mathbf{3},3} & 2Y^{(2)}_{\mathbf{3},2} & -Y^{(2)}_{\mathbf{3},1} \\ -Y^{(2)}_{\mathbf{3},2} & -Y^{(2)}_{\mathbf{3},1} & 2Y^{(2)}_{\mathbf{3},3} \end{pmatrix} \frac{\Lambda_N \eta_0^6}{\eta^6}
$$

$$
\sin^2 \theta_{12} = 0.307 \,, \quad \sin^2 \theta_{13} = 0.022 \,, \quad \sin^2 \theta_{23} = 0.454 \,,
$$

$$
\delta_{CP}/\pi = 0.855 \,, \quad \alpha_{21}/\pi = 0.939 \,, \quad \alpha_{31}/\pi = 0.271 \,,
$$

$$
m_e/m_\mu = 0.00474 \,, \quad m_\mu/m_\tau = 0.0588 \,, \quad \frac{\Delta m^2_{21}}{\Delta m^2_{31}} = 0.0296 \,,
$$

$$
m_1 = 25.725 \text{ meV} \,, \quad m_2 = 27.127 \text{ meV} \,, \quad m_3 = 56.274 \text{ meV} \,,
$$

$$
m_{\beta\beta} = 9.615 \text{ meV} \,, \quad (M_1, M_2, M_3) = \Lambda_N (1.372, 1.447, 2.818) \,,
$$

Lepton asymmetry to Baryon asymmetry

- Asymmetry measured by chemical potential:

$$n_i - \overline{n}_i = \frac{gT^3}{6} \begin{cases} \beta\mu_i + \mathcal{O}\left((\beta\mu_i)^3\right) , & \text{fermions} , \\ 2\beta\mu_i + \mathcal{O}\left((\beta\mu_i)^3\right) , & \text{bosons} . \end{cases}$$

- Sphaleron process:

  Transition between different vacuum states.

$$O_{B+L} = \prod_{i=1\ldots3} (q_{Li} q_{Li} q_{Li} l_{Li})$$

Effective interactions

$$u^c + d^c + c^c \to d + 2s + 2b + t + \nu_e + \nu_\mu + \nu_\tau$$

$$\Delta B = \Delta L = 3$$

Lepton asymmetry to Baryon asymmetry

- Constrained equations:

$$\sum_i (3\mu_{qi} + \mu_{li}) = 0 \ .$$

SU(2) instantons

$$\sum_i (2\mu_{qi} - \mu_{ui} - \mu_{di}) = 0 \ .$$

SU(3) instantons

$$\sum_i \left(\mu_{qi} + 2\mu_{ui} - \mu_{di} - \mu_{li} - \mu_{ei} + \frac{2}{N_f}\mu_H \right) = 0 \ .$$

Vanishing hypercharges

$$\mu_{qi} - \mu_H - \mu_{dj} = 0 \ , \quad \mu_{qi} + \mu_H - \mu_{uj} = 0 \ ,$$
$$\mu_{li} - \mu_H - \mu_{ej} = 0 \ .$$

SM interactions

$$B = \sum_i (2\mu_{qi} + \mu_{ui} + \mu_{di}) \ ,$$
$$L_i = 2\mu_{li} + \mu_{ei} \ , \quad L = \sum_i L_i \ .$$

$$B = c_s(B-L); \quad L = (c_s - 1)(B-L)$$

Initial B-L from inflaton decay