NEXUS 2026

Ultraviolet sensitive axion potential from small-instantons

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Based on 2608.08602 [hep-ph]

Naturalness and Strong CP

Why are fundamental parameters so hierarchical and what protects their stability?

$$g_1,\ g_2,\ g_3 \qquad \mu_H^2,\ \lambda_H \qquad Y_u,\ Y_d,\ Y_e \qquad \bar{\theta} \qquad \rho_\Lambda$$

- *Technical naturalness* asks whether small values remain stable under quantum corrections; $(g_1, g_2, g_3, Y_{u,d,e})$
- Symmetries can protect small parameters.
- *Anthropic selection* restricts observations to environments that permit observers; (vev, ρ_Λ)

$$\mathscr{L} \supset \bar{\theta}\,\frac{g_s^2}{32\pi^2} G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} \qquad \bar{\theta} = \theta + \mathrm{Arg\,det}\big(\big(\mathrm{M_u\,M_d}\big)\big) \qquad |\bar{\theta}| \lesssim 10^{-10}$$

Known naturalness criteria or anthropic selection rules does not constrain strong CP and may be explained dynamically.

The KSVZ axion and QCD potential

- A heavy coloured vector-like quark Q and a complex singlet scalar Φ, charged under a global $U(1)_{\mathrm{PQ}}$.

$$\mathscr{L} \supset y_Q \overline{Q} Q \Phi \qquad \Phi = \frac{v+\sigma}{\sqrt{2}} e^{ia/v}, \qquad M_Q = \frac{y_Q v}{\sqrt{2}} .$$

- The VEV $\langle \Phi \rangle = v/\sqrt{2}$ spontaneously breaks PQ symmetry. The angular mode a has a continuous shift symmetry.

$$\mathscr{L} \supset \frac{\alpha_s}{8\pi} \left(\bar{\theta} + \frac{a}{f_a} \right) G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} . \qquad a \to a + 2\pi f$$

- Nonperturbative QCD generates a periodic potential and reduces the continuous shift symmetry to a discrete symmetry.

$$V_{\mathrm{QCD}}(a) = E_{\mathrm{QCD}} \left(\bar{\theta} + \frac{a}{f_a} \right) .$$

- The minimum conserves CP and the potential's curvature determines the axion mass.

- Instantons describe tunnelling between topologically distinct gauge vacua and provide a semiclassical description of QCD topological effects.

- Small instantons probe shorter distances, where semiclassical calculations can apply.

Instanton NDA

2311.09285 [hep-ph]

$$C_{N_c} d^4x \frac{d\rho}{\rho^5} \left(\frac{2\pi}{\alpha_s(\mu)}\right)^{2N_c} \exp\left[-\frac{2\pi}{\alpha_s(\mu)}\right] (\mu\rho)^{b_3} \Pi_i \overline{\psi}\,\psi$$

$$\chi_{\rm SI}^{(k,r)} \sim 2C_3 Y_{\rm SM} \int_{1/M_{\rm UV}}^{1/\Lambda} \frac{d\rho}{\rho^5} \left(\frac{2\pi}{\alpha_s(1/\rho)}\right)^6 e^{-2\pi/\alpha_s(1/\rho)} (M_Q\rho)^r \left(\frac{y_Q^2}{16\pi^2}\right)^k$$

Instanton NDA

2311.09285 [hep-ph]

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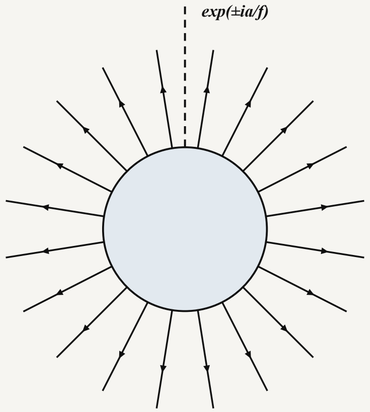
$$Q = (15,1,-1/3), \qquad T(15) = 10,$$

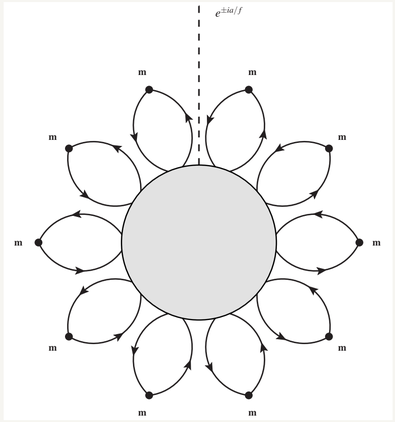
Index theorem relates zero modes $\quad \mathcal{O}_I \propto (\bar{Q}_R Q_L)^{20} \qquad 20 = 2k + r, \qquad \Delta = b_3 + r - 4 = \frac{29}{3} - 2k$

Open 't Hooft vertex

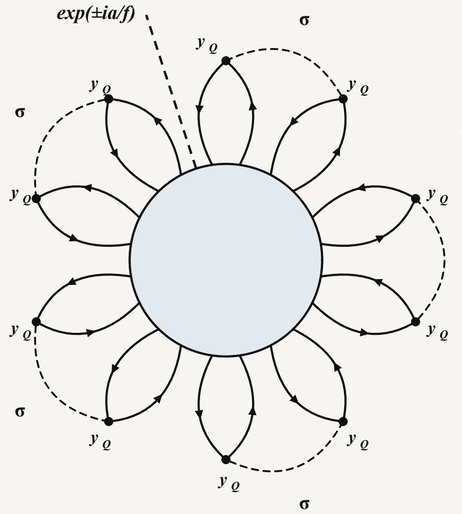


Mass insertions

$$(k,r) = (0,20): \quad (M_Q\rho)^{20}$$



Scalar–Yukawa closures



$$(k,r) = (10,0): \quad \left(\frac{y_Q^2}{16\pi^2}\right)^{10}$$

Instanton NDA-II

2311.09285 [hep-ph]

$$\chi_{\rm SI}^{(k,r)} \sim 2C_3 Y_{\rm SM} \int_{1/M_{\rm UV}}^{1/\Lambda} \frac{d\rho}{\rho^5} \left(\frac{2\pi}{\alpha_s(1/\rho)}\right)^6 e^{-2\pi/\alpha_s(1/\rho)} (M_Q \rho)^r \left(\frac{y_Q^2}{16\pi^2}\right)^k$$

$$\int d\rho\, \rho^{b_3+r-5}, \qquad \Delta = b_3 + r - 4 \qquad Y_{\rm SM} = \frac{\prod_f y_f}{(16\pi^2)^3}$$

$$b_3 = 7 - \sum_Q \frac{2}{3} T(R_3) \dim(\mathrm{R}_2)$$

$$\Delta < 0: \ \text{UV} \qquad \Delta = 0: \ \log \qquad \Delta > 0: \ \text{IR}$$

$$\chi_{\rm SI} \approx \Lambda_{\rm QCD}^4 \left(\frac{2\pi}{\alpha_s(M_{\rm UV})}\right)^6 \left(\frac{\Lambda_{\rm QCD}}{\Lambda}\right)^3 \left(\frac{M_Q}{\Lambda}\right)^r \left(\frac{y_Q^2}{16\pi^2}\right)^k \frac{1}{-\Delta} \left(\frac{M_{\rm Pl}}{\Lambda}\right)^{-\Delta}$$

$$V(a) = -(\chi_{\rm QCD} + \chi_{\rm SI}) \cos\left(\bar{\theta} + \frac{a}{f_a}\right)$$

- **Ássumption: Equal periodicity and aligned CP-conserving minima.**
- **Small instanton can modify the potential curvature.**

Which vector-like quarks can we study?

- Long-lived VQs can modify the early cosmological history.
- Existing KSVZ catalogues include decay operators up to mass dimension (d=9) .

Label	VQ	δb_1	δb_2	δb_3	$n^{(1)}_{Q,\max}$	$n^{(2)}_{Q,\max}$	E	N	E/N
R_1	$(3,1,-1/3)$	$4/15$	0	$2/3$	28	25	$1/3$	$1/2$	$2/3$
R_2	$(3,1,2/3)$	$16/15$	0	$2/3$	11	11	$4/3$	$1/2$	$8/3$
R_3	$(3,2,1/6)$	$2/15$	2	$4/3$	8	8	$5/3$	1	$5/3$
R_4	$(3,2,-5/6)$	$10/3$	2	$4/3$	3	3	$17/3$	1	$17/3$
R_5	$(3,2,7/6)$	$98/15$	2	$4/3$	1	1	$29/3$	1	$29/3$
R_6	$(3,3,-1/3)$	$4/5$	8	2	2	2	7	$3/2$	$14/3$
R_7	$(3,3,2/3)$	$16/5$	8	2	2	2	10	$3/2$	$20/3$
R_8	$(3,3,-4/3)$	$64/5$	8	2	0	0	22	$3/2$	$44/3$
R_9	$(6,1,-1/3)$	$8/15$	0	$10/3$	5	5	$2/3$	$5/2$	$4/15$
R_{10}	$(6,1,2/3)$	$32/15$	0	$10/3$	5	4	$8/3$	$5/2$	$16/15$
R_{11}	$(6,2,1/6)$	$4/15$	4	$20/3$	2	2	$10/3$	5	$2/3$
R_{12}	$(8,1,-1)$	$32/5$	0	4	1	1	8	3	$8/3$
R_{13}	$(8,2,-1/2)$	$16/5$	$16/3$	8	2	2	8	6	$4/3$
R_{14}	$(10,1,-1)$	8	0	10	1	1	10	$15/2$	$4/3$
R_{15}	$(10,2,-1/2)$	4	$20/3$	20	0	0	10	15	$2/3$
R_{16}	$(15,1,-1/3)$	$4/3$	0	$40/3$	1	1	$5/3$	10	$1/6$
R_{17}	$(15,1,2/3)$	$16/3$	0	$40/3$	1	1	$20/3$	10	$2/3$

TABLE I. Gauge contributions, copy-number bounds and anomaly coefficients for the different VQ candidates at $\Lambda = 5 \times 10^{11}\,\mathrm{GeV}$. The two-loop values are obtained by using $y_Q = 1$ and the perturbativity condition of $\alpha_i(M_{\mathrm{Pl}}) < 1$. The entries of E and N are obtained by using $\Delta X = 1$.

Taken from: arXiv:

1610.07593, 1705.05370, 2107.12378, 2310.16087, 2412.17896

The spectrum

Single VQ

- 15 allowed representations
- The zero modes is fixed by Dynkin index of the SU(3) of the respective VQ.
- Color anomaly is proportional to the Dynkin index.

Identical copies

- Identical copies added at same scale
- Zero modes gets enhanced by the identical copies
- Instanton UV sensitivity increases
- E/N remains fixed

Distinct VQs

- Distinct VQs added at same scale
- Zero modes are increased, increasing the UV sensitivity.
- E/N varies

Common matching scale $\Lambda = 5 \times 10^{11}\ \text{GeV}, \qquad y_Q = 1$

Gauge perturbativity at two loops; $\alpha_1(\mu), \alpha_2(\mu), \alpha_3(\mu) < 1, \qquad \Lambda < \mu < M_{\text{Pl}}$

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R_{10}	$(6,1,2/3)$	$32/15$	0	$10/3$	5	4	$8/3$	$5/2$	$16/15$
R_{11}	$(6,2,1/6)$	$4/15$	4	$20/3$	2	2	$10/3$	5	$2/3$
R_{12}	$(8,1,-1)$	$32/5$	0	4	1	1	8	3	$8/3$
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R_{14}	$(10,1,-1)$	8	0	10	1	1	10	$15/2$	$4/3$
R_{15}	$(10,2,-1/2)$	4	$20/3$	20	0	0	10	15	$2/3$
R_{16}	$(15,1,-1/3)$	$4/3$	0	$40/3$	1	1	$5/3$	10	$1/6$
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n	S_n	$2N_S$	(k,r)	$-\Delta$
2	$\{R_{11}, R_{13}\}$	22	$(11,0)$	$35/3$
3	$\{R_3, R_{11}, R_{13}\}$	24	$(12,0)$	13
4	$\{R_1, R_2$ $R_{11}, R_{13}\}$	24	$(12,0)$	13
5	$\{R_1, R_3, R_6$ $R_{12}, R_{13}\}$	24	$(12,0)$	13
6	$\{R_1, R_2, R_3$ $R_6, R_9, R_{13}\}$	24	$(12,0)$	13
7	$\{R_1, R_2, R_3, R_4$ $R_6, R_9, R_{11}\}$	24	$(12,0)$	13

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R_8	$(3,3,-4/3)$	$64/5$	8	2					
R_9	$(6,1,-1/3)$	$8/15$	0	$10/$					
R_{10}	$(6,1,2/3)$	$32/15$	0	$10/$					
R_{11}	$(6,2,1/6)$	$4/15$	4	$20/$					
R_{12}	$(8,1,-1)$	$32/5$	0	4					
R_{13}	$(8,2,-1/2)$	$16/5$	$16/3$	8					
R_{14}	$(10,1,-1)$	8	0	1					
R_{15}	$(10,2,-1/2)$	4	$20/3$	2					
R_{16}	$(15,1,-1/3)$	$4/3$	0	$40/3$	1	1	$5/3$	10	$1/6$
R_{17}	$(15,1,2/3)$	$16/3$	0	$40/3$	1	1	$20/3$	10	$2/3$

- We cannot add higher than 15-dimensional VQ charged under SU(3).
- For some VQs number of copies changes from one loop to two loops
- For some VQ, even one copy is not allowed
- We cannot add more than seven distinct VQs

* at $\Lambda = 5 \times 10^{11}$ GeV and $\alpha_i < 1\Big|_{M_{pl}}$

		$2N_S$	(k,r)	$-\Delta$
	$1, R_{13}\}$	22	$(11,0)$	$35/3$
	$, R_{11}, R_{13}\}$	24	$(12,0)$	13
	$, R_2$ $, R_{13}\}$	24	$(12,0)$	13
	$, R_3, R_6$ $, R_{13}\}$	24	$(12,0)$	13
	$, R_2, R_3$ $R_6, R_9, R_{13}\}$	24	$(12,0)$	13
7	$\{R_1, R_2, R_3, R_4$ $R_6, R_9, R_{11}\}$	24	$(12,0)$	13

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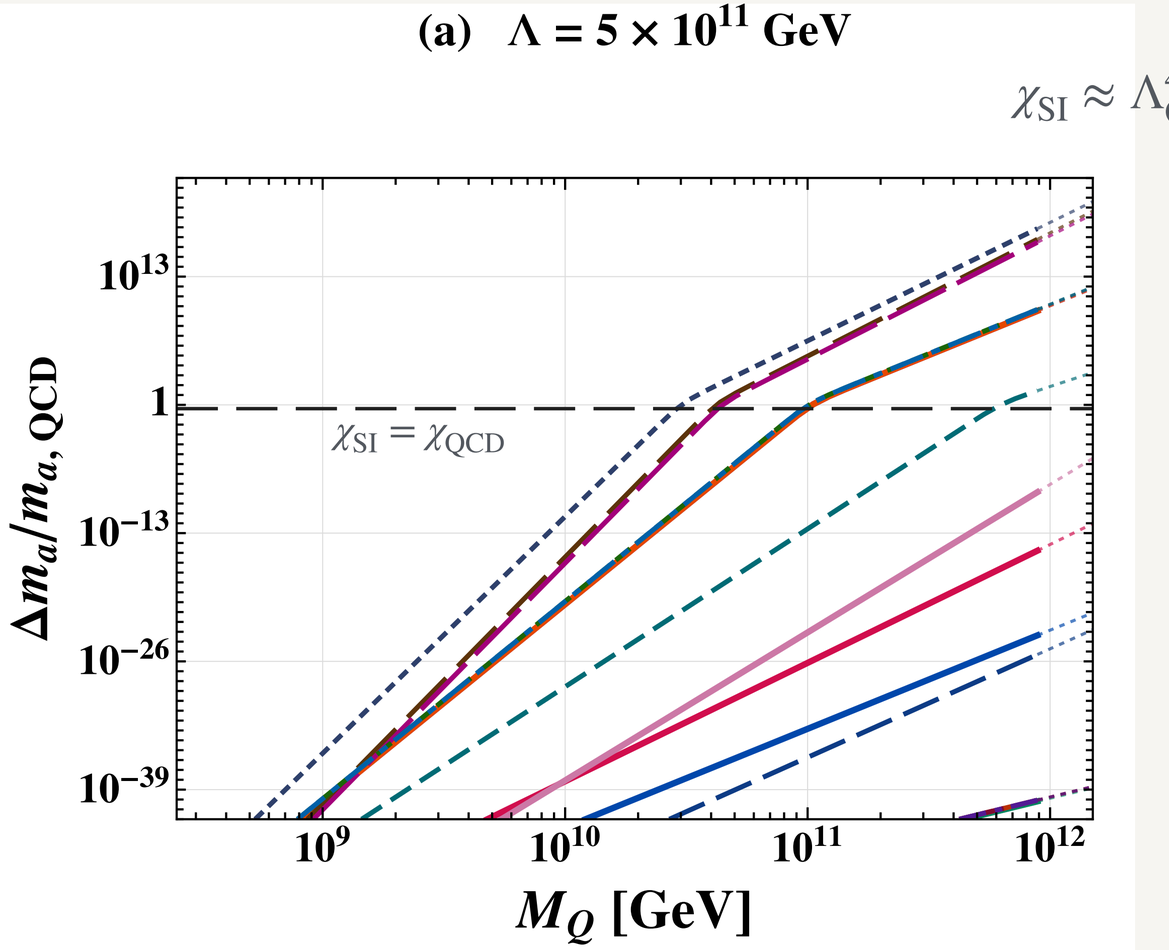
Implication on axion-mass



$$\chi_{\mathrm{SI}} \approx \Lambda_{\mathrm{QCD}}^4 \left(\frac{2\pi}{\alpha_s(M_{\mathrm{UV}})}\right)^6 \left(\frac{\Lambda_{\mathrm{QCD}}}{\Lambda}\right)^3 \left(\frac{M_Q}{\Lambda}\right)^r \left(\frac{y_Q^2}{16\pi^2}\right)^k \frac{1}{-\Delta} \left(\frac{M_{\mathrm{Pl}}}{\Lambda}\right)^{-\Delta}$$

$$\frac{\chi_{\mathrm{SI}}}{\chi QCD} = \frac{m_a^2 - m_{a,\mathrm{QCD}}^2}{m_{a,\mathrm{QCD}}^2} \qquad y_Q = 2\frac{M_Q}{\Lambda}$$

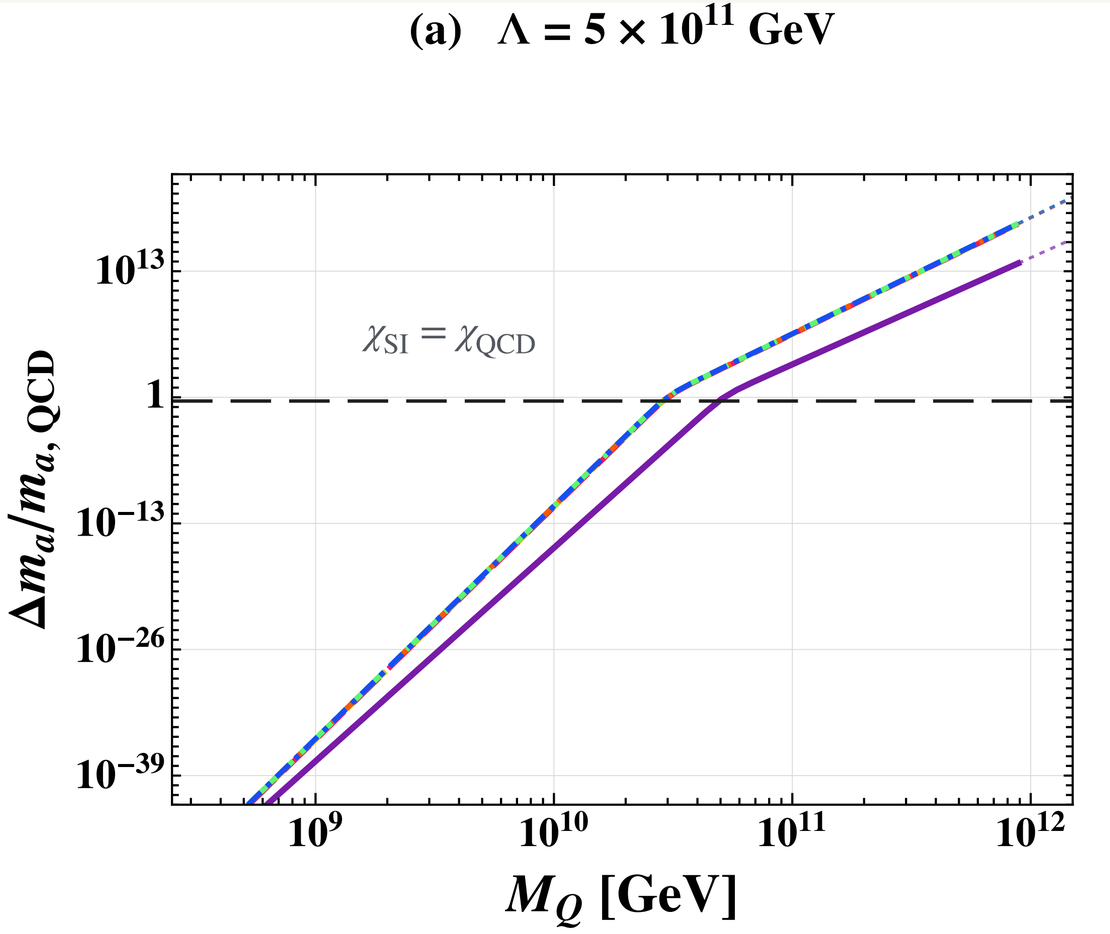
$$\frac{m_a}{m_{a,\mathrm{QCD}}} = \sqrt{1 + \frac{\chi_{\mathrm{SI}}}{\chi_{\mathrm{QCD}}}} \qquad \Delta m_a = m_a - m_{\mathrm{QCD}}$$

One VQ R_{16} or R_{17} : $m_a = 3.43$ eV

Two identical copies $R_{13}^{(2)}$: $m_a = 71.2$ MeV

$y_Q > \sqrt{4\pi}$. The one-VQ curves are R_{11} (—), R_{12} (—), R_{13} (—), R_{14} (—), R_{16} (—), and R_{17} (—). The maximal identical-copy spectra are $R_1^{(25)}$ (– –), $R_2^{(11)}$ (– –), $R_3^{(8)}$ (– –), $R_4^{(3)}$ (– –), $R_6^{(2)}$ (– –), $R_7^{(2)}$ (– –), $R_9^{(5)}$ (– –), $R_{10}^{(4)}$ (– –), $R_{11}^{(2)}$ (– –), and $R_{13}^{(2)}$ (– –). The black dashed horizontal line marks (– –) the equal susceptibilities of $\chi_{\mathrm{SI}} = \chi_{\mathrm{QCD}}$.

Implication on axion mass from distinct VQ sets

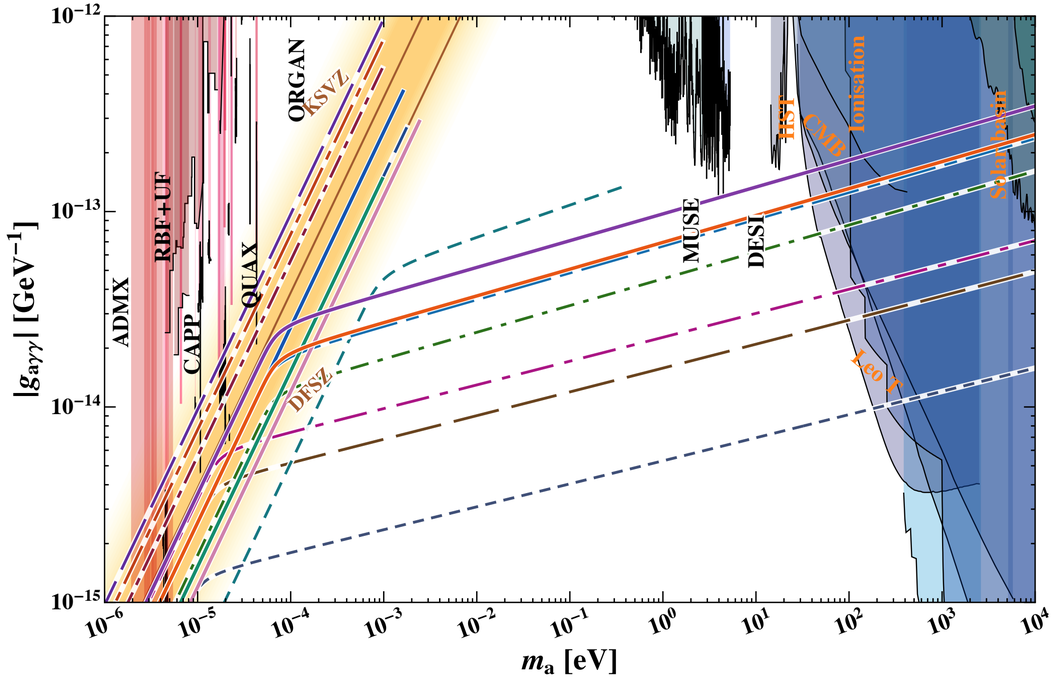


n	S_n	$2N_S$	(k,r)	$-\Delta$	$C_3(S_n)$	$(E/N, N_{\rm DW})_{\rm min}$	$(E/N, N_{\rm DW})_{\rm max}$
2	$\{R_{11}, R_{13}\}$	22	$(11,0)$	$35/3$	1.75	$(14/3, 2)$	$(34/33, 22)$
3	$\{R_3, R_{11}, R_{13}\}$	24	$(12,0)$	13	3.13	$(19/6, 4)$	$(13/12, 24)$
4	$\{R_1, R_2$ $R_{11}, R_{13}\}$	24	$(12,0)$	13	3.13	$(11/3, 2)$	$(31/33, 22)$
5	$\{R_1, R_3, R_6$ $R_{12}, R_{13}\}$	24	$(12,0)$	13	4.14	$(5/3, 6)$	$(-5/3, 6)$
6	$\{R_1, R_2, R_3$ $R_6, R_9, R_{13}\}$	24	$(12,0)$	13	4.14	$(-1/3, 2)$	$(7/24, 16)$
7	$\{R_1, R_2, R_3, R_4$ $R_6, R_9, R_{11}\}$	24	$(12,0)$	13	4.14	$(5/3, 4)$	$(-4/3, 12)$

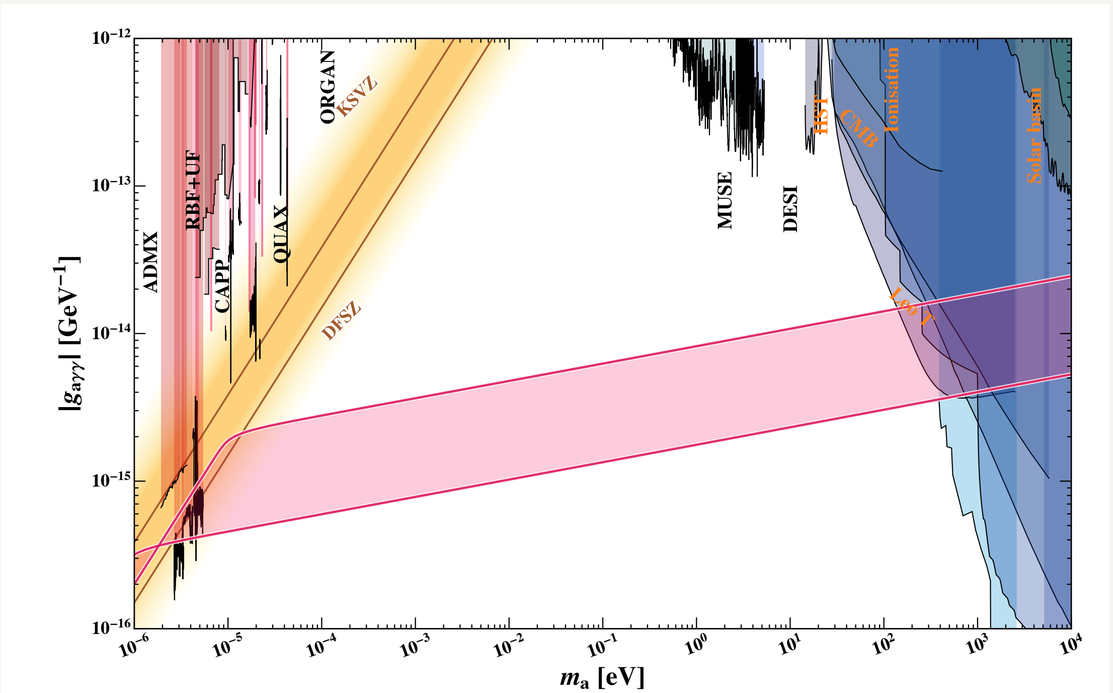
Impact on axion-photon coupling

$$g_{a\gamma\gamma} = \frac{\alpha_{\rm em}}{2\pi f_a}\left(\frac{E}{N} - 1.92\right) \qquad m_a^2 = \frac{\chi_{\rm QCD} + \chi_{\rm SI}}{f_a^2} \qquad f_a = \frac{\Lambda}{N_{\rm DW}}$$

Single VQ and identical copies



Distinct spectrum S_4



- At fixed $y_q = 1$, $g_{a\gamma\gamma}$ and m_a are computed and Λ is varied. Moving from left to right Λ decreases, axion mass increases and affecting axion-photon coupling plane

Summary

- Comprehensively analysed small-instanton effects across KSVZ models with single, identical and distinct vector-like quark spectra.
- Their magnitude depends on the matter content, gauge running and interactions that saturate the fermionic zero modes.
- Quantifying these ultraviolet contributions is essential for reliable predictions of axion properties in realistic models.

Outlook

- Apply the framework to realistic grand unified scenarios (SUSY), incorporating complete particle spectra, interactions and non-degenerate thresholds.
- Develop precision calculations beyond the present NDA estimates.

For physical axion, ultraviolet matter matters