Quantum Fisher Information in Neutrino Oscillation Parameter Estimation

From intrinsic sensitivity to measurement accessibility, matter effects and correlations

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What do we still not know?

The three-flavor framework is well established

Neutrino oscillations are successfully described by three-flavor mixing, but several important properties remain to be determined precisely.

δ_{CP}	θ_{23}	Δm^2_{31}
CP violation	**Octant**	**Mass ordering**
What is the value of the CP-violating phase?	Is θ_{23} in the lower or upper octant?	What is the sign of Δm^2_{31}?

The experimental challenge

These parameters are inferred from changes in neutrino flavor-transition probabilities.

Neutrino oscillation as quantum-state evolution

The flavor state is a coherent superposition of mass eigenstates:

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle.$$

In vacuum, the state evolves as

$$|\nu_\alpha(L)\rangle = e^{-iH_{\rm vac}L} |\nu_\alpha(0)\rangle,$$

where

$$H_{\rm vac} = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{pmatrix} U^\dagger.$$

The experimentally accessible flavor probability is

$$P_{\alpha\beta} = \left| \langle \nu_\beta | \nu_\alpha(L) \rangle \right|^2.$$

The measured probability is derived from an underlying evolving quantum state.

From probability sensitivity to quantum-state sensitivity

Experiments measure flavor-transition probabilities:

$$P_{\alpha\beta} = |\langle \nu_\beta | \nu_\alpha(L) \rangle|^2 .$$

They modify the neutrino quantum state through different features of the oscillation dynamics:

θ_{23}	δ_{CP}	Δm^2_{31}
Mixing amplitude	**Interference**	**Oscillation phase**

$$|\nu_\mu\rangle \xrightarrow{L,E} |\psi(L, E; \Theta)\rangle \xrightarrow{\text{measurement}} P_{\mu\beta}$$

Instead of asking only how a probability changes,
can we quantify how strongly the underlying neutrino quantum state changes when an oscillation parameter changes?

Why Fisher information?

Quantum level

The evolving state depends on an oscillation parameter:

$$|\psi(\lambda)\rangle.$$

How distinguishable are nearby values of λ?

Measurement level

An experiment measures outcomes with probabilities

$$p(x|\lambda).$$

How much information does that particular measurement extract?

QFI: information encoded in the state

$\Downarrow$

CFI: information accessible through a chosen measurement

Quantum Fisher information

For a pure state,

$$F_Q(\lambda) = 4\left[\langle\partial_\lambda\psi|\partial_\lambda\psi\rangle - |\langle\psi|\partial_\lambda\psi\rangle|^2\right].$$

The quantum Cramér–Rao bound is

$$(\Delta\lambda)^2 \geq \frac{1}{MF_Q(\lambda)}.$$

- QFI is measurement independent.
- Larger QFI means larger local distinguishability of nearby parameter values.
- It gives an intrinsic quantum precision limit.

Scope

QFI is not itself a detector-level sensitivity. Event rates, flux, efficiencies, backgrounds and systematic uncertainties require a further experimental analysis.

Analysis setup

- Initial state:

$$\nu_\mu.$$

- Main appearance channel:

$$\nu_\mu \to \nu_e.$$

- Parameters studied:

$$\delta_{\mathrm{CP}}, \qquad \theta_{23}, \qquad \Delta m^2_{31}.$$

- QFI is evaluated as a function of

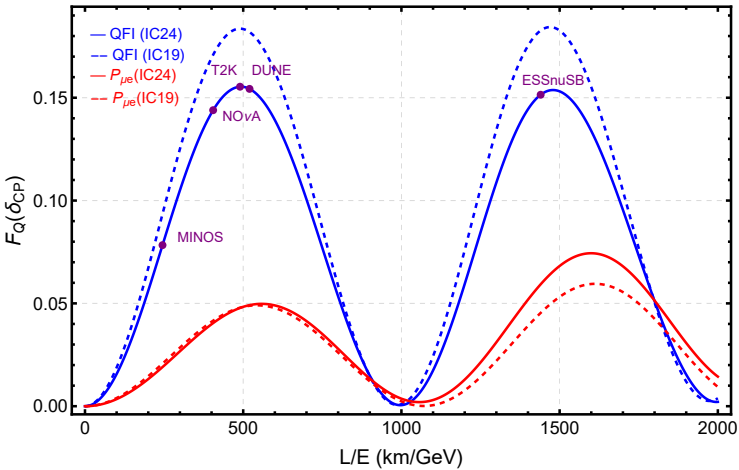
$$L/E.$$

- NuFit-6.0 benchmark values are used.
- IC24 and IC19 benchmark datasets are compared where applicable.
- Experimental reference points include MINOS, T2K, NOνA, DUNE and ESSνSB.

Strategy

First isolate the intrinsic single-parameter sensitivity; then ask how much of it is accessible through flavor measurements and how the picture changes in matter.

Result: QFI for $\delta_{\rm CP}$



- Two prominent maxima:

  $L/E \simeq 500, \quad 1500\,\mathrm{km/GeV}$

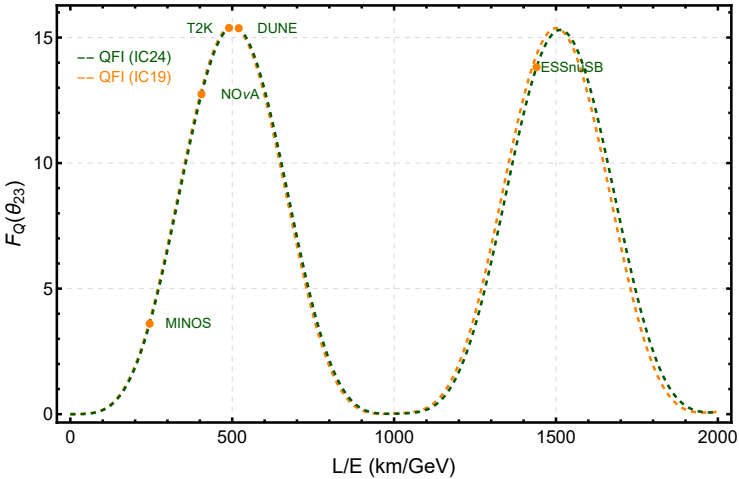
- Maximum value:

  $$F_Q(\delta_{\rm CP}) \sim 0.15.$$

- The maxima closely coincide with the first and second oscillation maxima of $P_{\mu e}$.
- IC24 and IC19 results are very similar.

Message: oscillation maxima are also regions of enhanced intrinsic quantum sensitivity to $\delta_{\rm CP}$.

Result: QFI for θ_{23}



- Again two maxima near

  $L/E \simeq 500, 1500\,\mathrm{km/GeV}.$

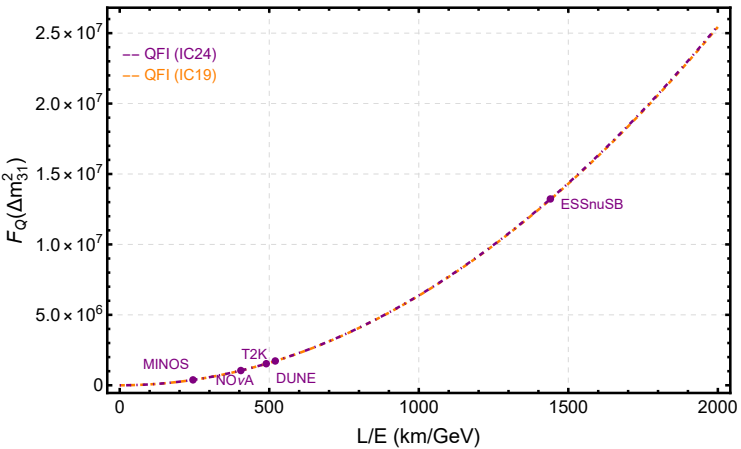
- Much larger QFI:

  $$F_Q(\theta_{23}) \sim 15.$$

- T2K, NOνA and DUNE lie near the first maximum.
- ESSνSB lies near the second maximum.

Message: For the chosen parameterization, θ_{23} shows a larger QFI scale than δ_{CP}.

Result: QFI for Δm^2_{31}



- Qualitatively different behavior.
- QFI increases rapidly with L/E.
- Near $L/E = 2000$ km/GeV:

$$F_Q(\Delta m^2_{31}) \sim 2.5 \times 10^7.$$

- Δm^2_{31} directly controls the oscillation phase.

Why is Δm^2_{31} different?

The oscillation phase contains

$$\phi_{ij} \sim \frac{\Delta m^2_{ij} L}{2E}.$$

Therefore:

Angular parameters

θ_{23} and δ_{CP} mainly modify amplitudes and interference terms.
This gives characteristic sensitivity regions near oscillation maxima.

Mass splitting

Δm^2_{31} directly controls the phase.
Its sensitivity therefore accumulates strongly as L/E increases.

The different QFI trends reflect the different roles of the parameters in the oscillation dynamics.

But can the experiment access all this information?

For the flavor-projection measurement

$$\nu_\mu \to \{\nu_e, \nu_\mu, \nu_\tau\},$$

the classical Fisher information is

$$F_C(\lambda) = \sum_{\beta=e,\mu,\tau} \frac{1}{P_{\mu\beta}} \left(\frac{\partial P_{\mu\beta}}{\partial \lambda} \right)^2 .$$

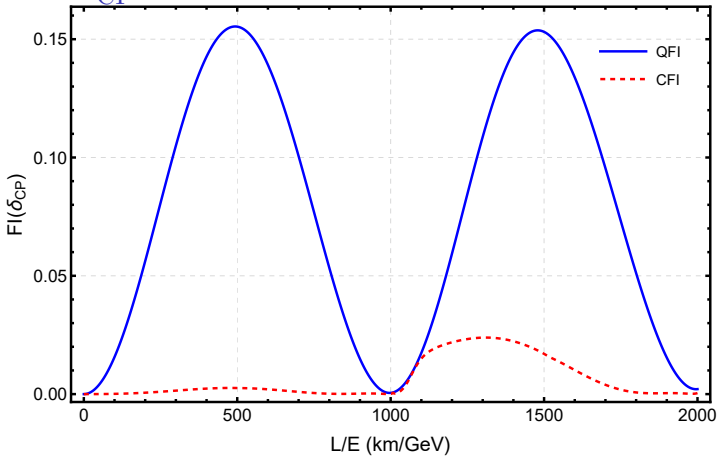
The quantum bound requires

$$\boxed{F_C(\lambda) \leq F_Q(\lambda)}.$$

Physical meaning

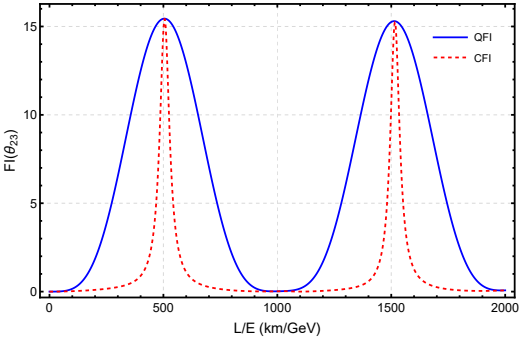
The gap between QFI and CFI quantifies the loss of Fisher information associated with this particular measurement.

QFI versus CFI for δ_{CP}

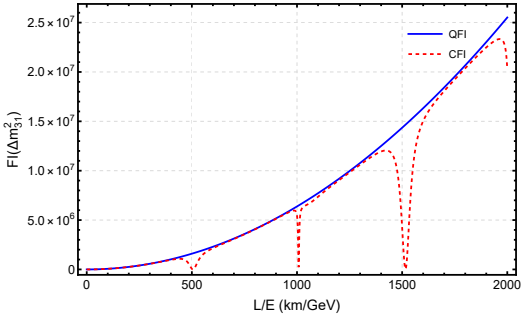


- CFI follows the oscillatory structure of QFI and the maxima are not exactly coincident.
- The flavor measurement therefore does not uniformly extract all the information available in the quantum state.
- Over much of the range, $F_C < F_Q$.

QFI versus CFI for θ_{23} and Δm^2_{31}



For θ_{23}, CFI becomes very close to QFI near the sensitivity maxima.



For Δm^2_{31}, CFI follows QFI over broad regions, with localized departures.

Measurement accessibility

The standard flavor measurement can extract a substantial fraction of the quantum information, but its efficiency is parameter and L/E dependent.

Matter effects: moving to a DUNE-like baseline

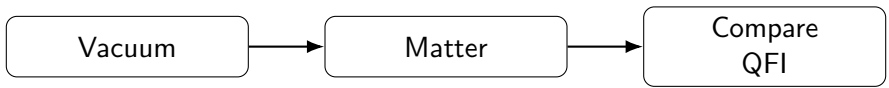
For propagation in matter,

$$H_{\mathrm{mat}} = H_{\mathrm{vac}} + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad V_{CC} = \sqrt{2} G_F N_e.$$

We consider

$$L = 1300 \ \mathrm{km},$$

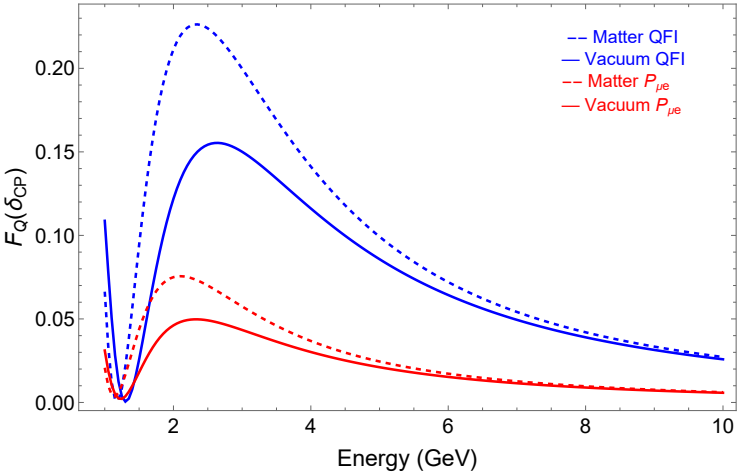
and study the energy dependence.



Question

Does matter change the quantum sensitivity, and is the effect the same for all parameters?

Matter strongly affects δ_{CP} sensitivity



- Strong enhancement of $F_Q(\delta_{\mathrm{CP}})$.
- Most pronounced around

$$E \sim 2 - 3 \text{ GeV}.$$

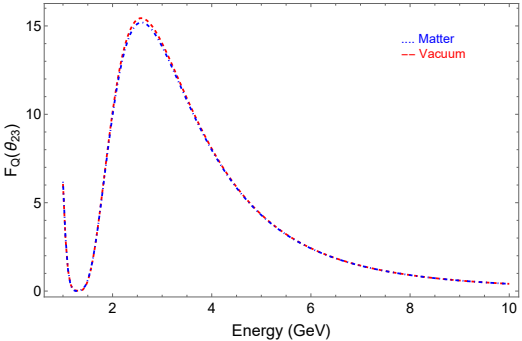
- QFI maximum shifts approximately

$$2.6 \text{ GeV} \rightarrow 2.3 \text{ GeV}.$$

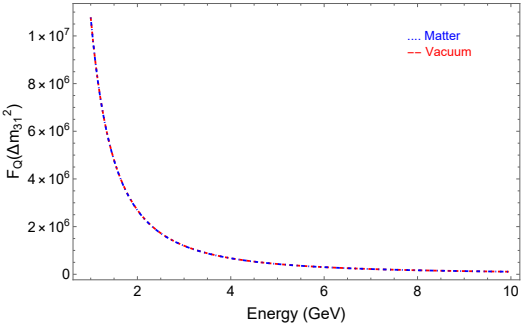
- The QFI maximum does not simply coincide with the probability maximum.

Matter changes both the magnitude and the optimal energy of CP-phase information.

Matter effects on θ_{23} and Δm^2_{31}



$F_Q(\theta_{23})$: vacuum and matter curves nearly overlap.



$F_Q(\Delta m^2_{31})$: vacuum and matter curves nearly overlap.

Result

For the fixed DUNE baseline, matter effects have a strong impact on $\delta_{\rm CP}$ QFI, but a comparatively small impact on θ_{23} and Δm^2_{31} QFI.

From single-parameter estimation to the real multiparameter problem

So far we varied one parameter at a time.
But in a realistic oscillation analysis,

$$\boxed{\theta_{23}, \quad \delta_{\mathrm{CP}}, \quad \Delta m^2_{31}}$$

are estimated simultaneously.
This introduces a new question:

Are the parameter sensitivities independent?

The answer requires the quantum Fisher information matrix (QFIM).

The quantum Fisher information matrix

Define

$$\boldsymbol{\Theta} = (\theta_{23}, \delta_{\mathrm{CP}}, \Delta m^2_{31}).$$

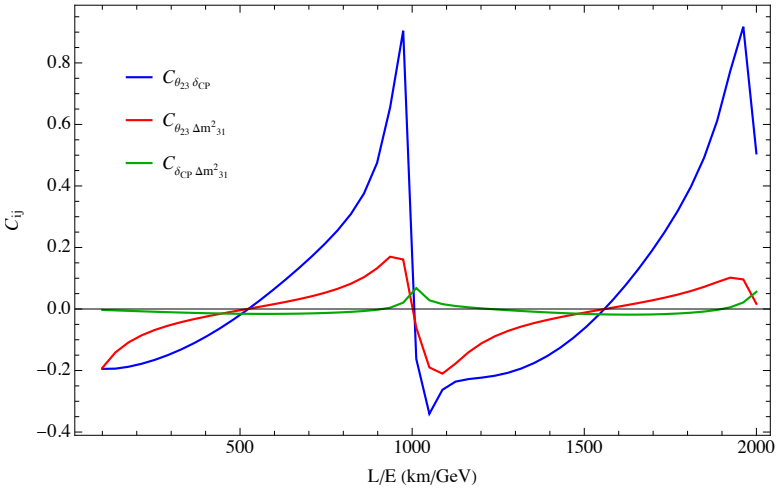
The QFIM is

$$\mathcal{F}_{ij} = 4\,\mathrm{Re}\left[\langle \partial_i \psi | \partial_j \psi \rangle - \langle \partial_i \psi | \psi \rangle \langle \psi | \partial_j \psi \rangle\right].$$

$$\mathcal{F} = \begin{pmatrix} F_{\theta_{23}\theta_{23}} & F_{\theta_{23}\delta_{\mathrm{CP}}} & F_{\theta_{23}\Delta m^2_{31}} \\ F_{\delta_{\mathrm{CP}}\theta_{23}} & F_{\delta_{\mathrm{CP}}\delta_{\mathrm{CP}}} & F_{\delta_{\mathrm{CP}}\Delta m^2_{31}} \\ F_{\Delta m^2_{31}\theta_{23}} & F_{\Delta m^2_{31}\delta_{\mathrm{CP}}} & F_{\Delta m^2_{31}\Delta m^2_{31}} \end{pmatrix}.$$

- Diagonal elements: individual sensitivities.
- Off-diagonal elements: coupled response of parameters.
- The inverse QFIM governs simultaneous precision.

Intrinsic parameter correlations



The normalized correlation is

$$C_{ij} = \frac{\mathcal{F}_{ij}}{\sqrt{\mathcal{F}_{ii}\mathcal{F}_{jj}}}.$$

- θ_{23}–$\delta_{\rm CP}$: strongest correlation.
- θ_{23}–Δm^2_{31}: moderate and sign-changing.
- $\delta_{\rm CP}$–Δm^2_{31}: comparatively weak.

Near the DUNE region,

$$L/E \simeq 520\ \mathrm{km/GeV},$$

the correlations are small.

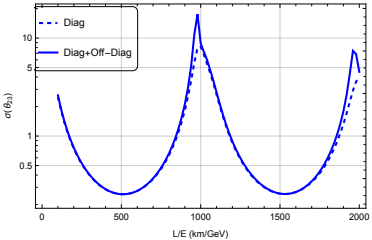
Do correlations change the precision bound?

The full QFIM gives the multiparameter QCRB

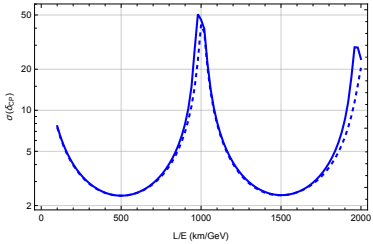
$$\sigma(\theta_\alpha) \geq \sqrt{(\mathcal{F}^{-1})_{\alpha\alpha}}.$$

Ignoring correlations gives

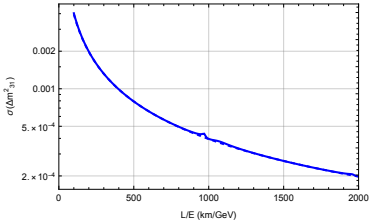
$$\sigma_{\text{diag}}(\theta_\alpha) = \frac{1}{\sqrt{\mathcal{F}_{\alpha\alpha}}}.$$



θ_{23}



δ_{CP}



Δm^2_{31}

Result: correlations produce moderate corrections for θ_{23} and δ_{CP} in selected regions, while the Δm^2_{31} precision is almost unchanged.

A deeper question: probability degeneracy

Different parameter sets can produce the same appearance probability:

$$P_{\mu e}(\boldsymbol{\Theta}_1) = P_{\mu e}(\boldsymbol{\Theta}_2).$$

Consider

$$(\theta_{23}, \delta_{\mathrm{CP}})_1 = (42^\circ, 190^\circ),$$
$$(\theta_{23}, \delta_{\mathrm{CP}})_2 = (48^\circ, 255.5^\circ),$$

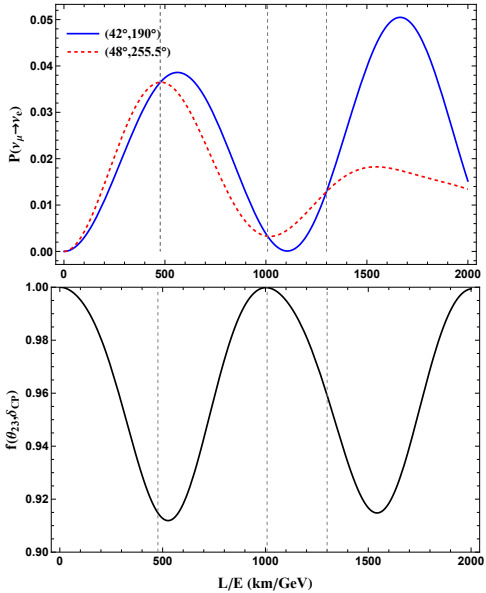
with Δm^2_{31} fixed.
At $E = 1$ GeV, the two probabilities coincide at

$$L \simeq 477, \quad 1008, \quad 1300 \ \mathrm{km}.$$

Question

Does identical appearance probability imply an identical underlying neutrino quantum state?

Probability degeneracy does not imply identical quantum states



The quantum-state overlap is quantified by the fidelity:

$$f_{12} = |\langle\psi(\Theta_1)|\psi(\Theta_2)\rangle|^2 .$$

At all three probability-degenerate points,

$$\boxed{f_{12} \simeq 0.959}.$$

$$P_{\mu e}^{(1)} = P_{\mu e}^{(2)} \quad \not\Rightarrow \quad |\psi_1\rangle = |\psi_2\rangle.$$

At the probability-degenerate points, all six independent QFIM elements take different values for the two solutions.

What have we learned?

1. **Intrinsic sensitivity:** QFI reveals distinct L/E-dependent sensitivity structures for θ_{23}, δ_{CP} and Δm^2_{31}.
2. **Measurement accessibility:** The flavor CFI does not always saturate the QFI, showing that part of the quantum information can remain inaccessible to the chosen measurement.
3. **Matter effects:** Matter strongly modifies the δ_{CP} sensitivity at a DUNE-like baseline, while its impact on θ_{23} and Δm^2_{31} is much smaller.
4. **Multiparameter estimation:** QFIM correlations modify the multiparameter precision bound for θ_{23} and δ_{CP} in selected regions, while the Δm^2_{31} bound is nearly unchanged.
5. **Quantum-state distinguishability:** Probability-degenerate solutions can correspond to distinct underlying quantum states, with

$$f_{12} \simeq 0.959.$$

Final perspective

The goal is not to replace standard oscillation analyses.

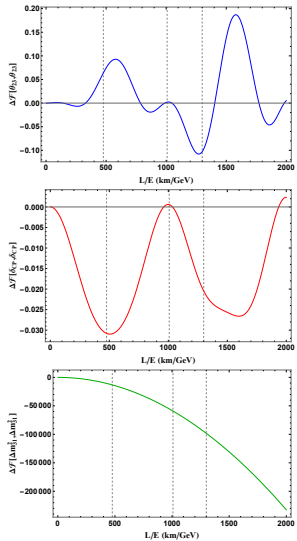
Probability + Quantum-state information

QFI and QFIM provide a complementary view of neutrino parameter estimation:

where information is encoded,
how parameters are correlated,
and how much is accessible to a given measurement.

Thank you!

QFIM difference: diagonal elements



Define

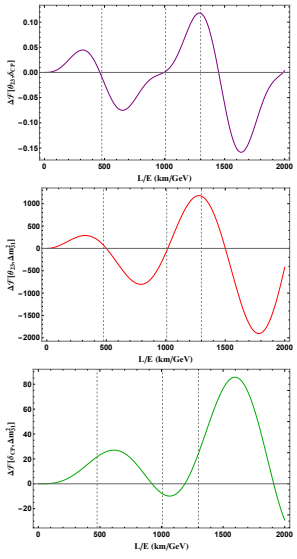
$$\Delta\mathcal{F}_{ij} = \mathcal{F}_{ij}^{(1)} - \mathcal{F}_{ij}^{(2)}.$$

At the probability-degenerate points, all three diagonal differences are nonzero.

$$|\Delta F_{\Delta m^2_{31}\Delta m^2_{31}}| \sim 2 \times 10^5$$

near $L/E = 2000$ km/GeV.
This shows that the two parameter sets have different local sensitivity structures even where their appearance probabilities coincide.

QFIM difference: off-diagonal elements



The off-diagonal elements encode cross-parameter response.
At

$$L/E \simeq 477,\ 1008,\ 1300\ \mathrm{km/GeV},$$

all three off-diagonal differences are also nonzero.

Key result

The two probability-degenerate solutions have different local QFIM geometries.