Primordial Black Holes as Gravitational-Wave Probes of Reheating: From PTAs to LISA and ET

(Based on JCAP 01 (2025) 118; JCAP 08 (2025) 074; and JHEP 10 (2024) 142.)

NEXUS 2026 -New directions EXploring the Unknown beyond the Standard Model.

Sep 9-13, 2026

Md Riajul Haque
Siyuan Postdoctoral Fellow
Tsung Dao Lee Institute
Shanghai Jiao Tong University

李政道研究所
TSUNG-DAO LEE INSTITUTE

Outline of the talk

Motivation

Observational challenges in probing the early Universe: why consider PBHs?

Goals

Induced Gravitational-wave sources associated with PBHs

Stable PBHs as dark matter

- Induced GWs associated with PBH formation

Evaporating PBHs ($M_{PBH} < 10^9$ g)

- Induced GWs associated with isocurvature perturbations
- Induced GWs associated with adiabatic inflationary perturbations

Constraining the reheating history with scalar-induced GWs Using the NANOGrav 15-year data set

Probing the post-inflationary era through induced GWs from PBH reheating using the future GW detectors LISA and ET.

Conclusions

Observational challenges in probing the early universe

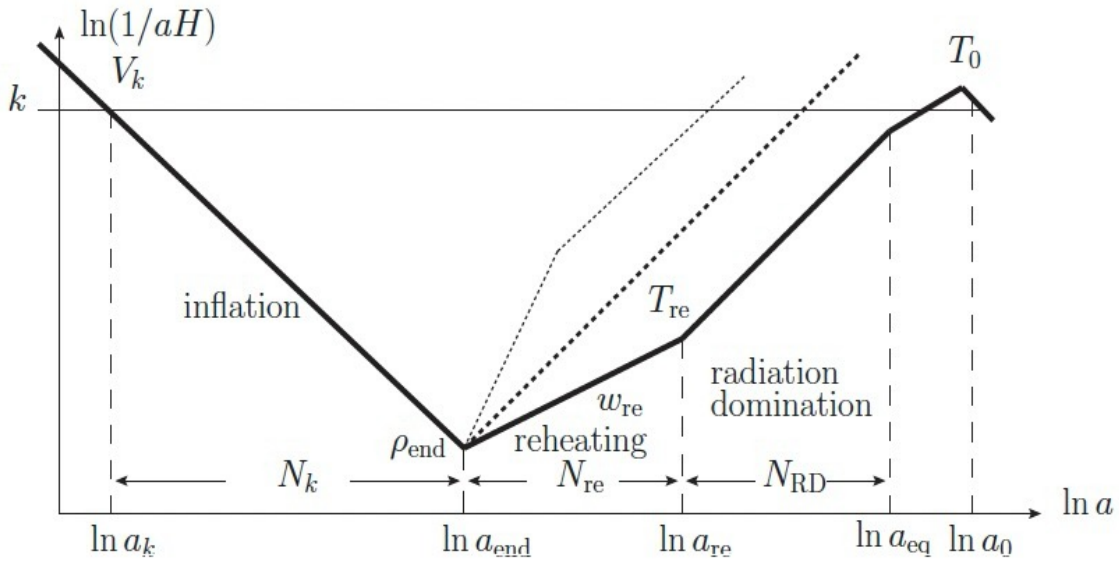
Our knowledge about the cosmic history of the Universe

Cosmic microwave background (CMB)

Provides evidence for an early inflationary phase with
- Energy scale $\sim 10^{16}\,\mathrm{GeV}$
- Duration $\Delta t_{\mathrm{inf}} \geq 10^{-36}$ Sec

Big-Bang Nucleosynthesis (BBN)

Predicts quantities such as light-element abundances
- Energy scale $E_{\mathrm{BBN}} \sim 1$ MeV
- Time scale $t_{\mathrm{BBN}} \sim 1$ Sec

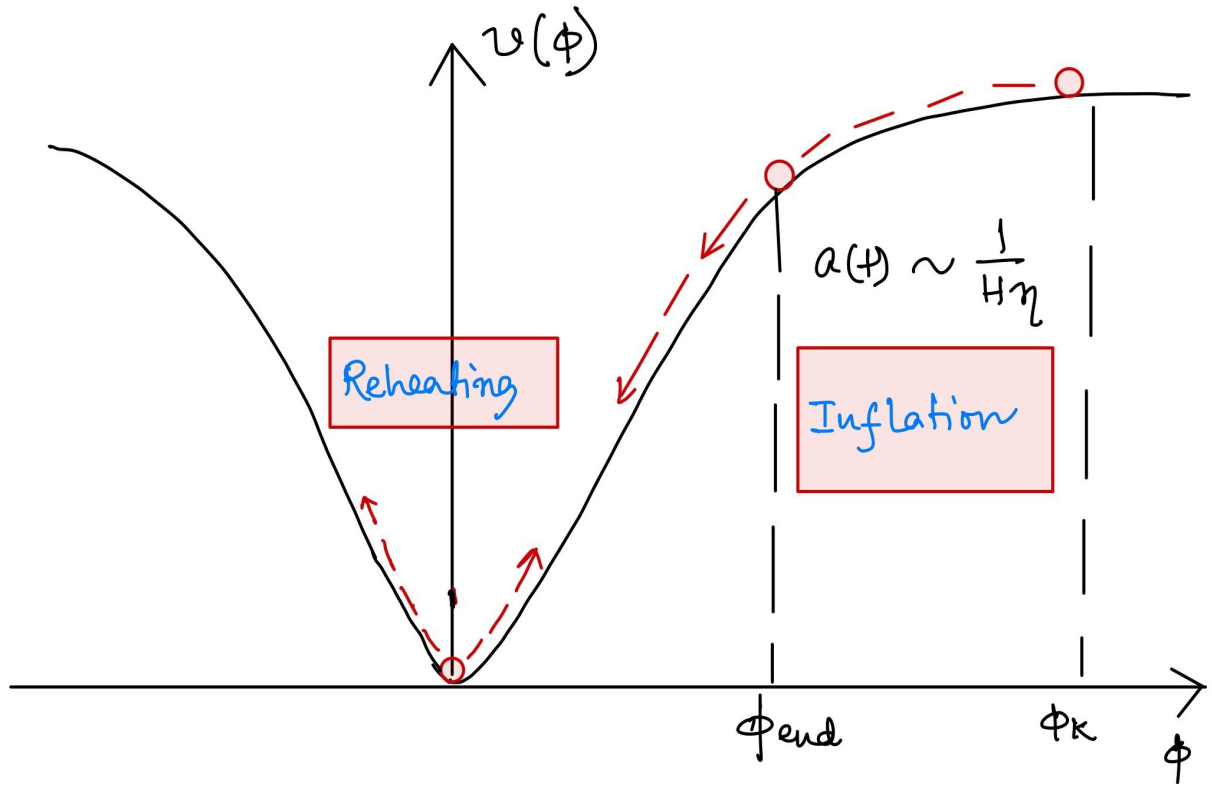


- There is a massive gap in terms of energy (and time) scale between the periods of inflation and BBN, which is poorly understood from both theory and observation

L. Dai, M. Kamionkowski and J. Wang, Phys. Rev. Lett. **113**, 041302 (2014)

Importance of Reheating phase

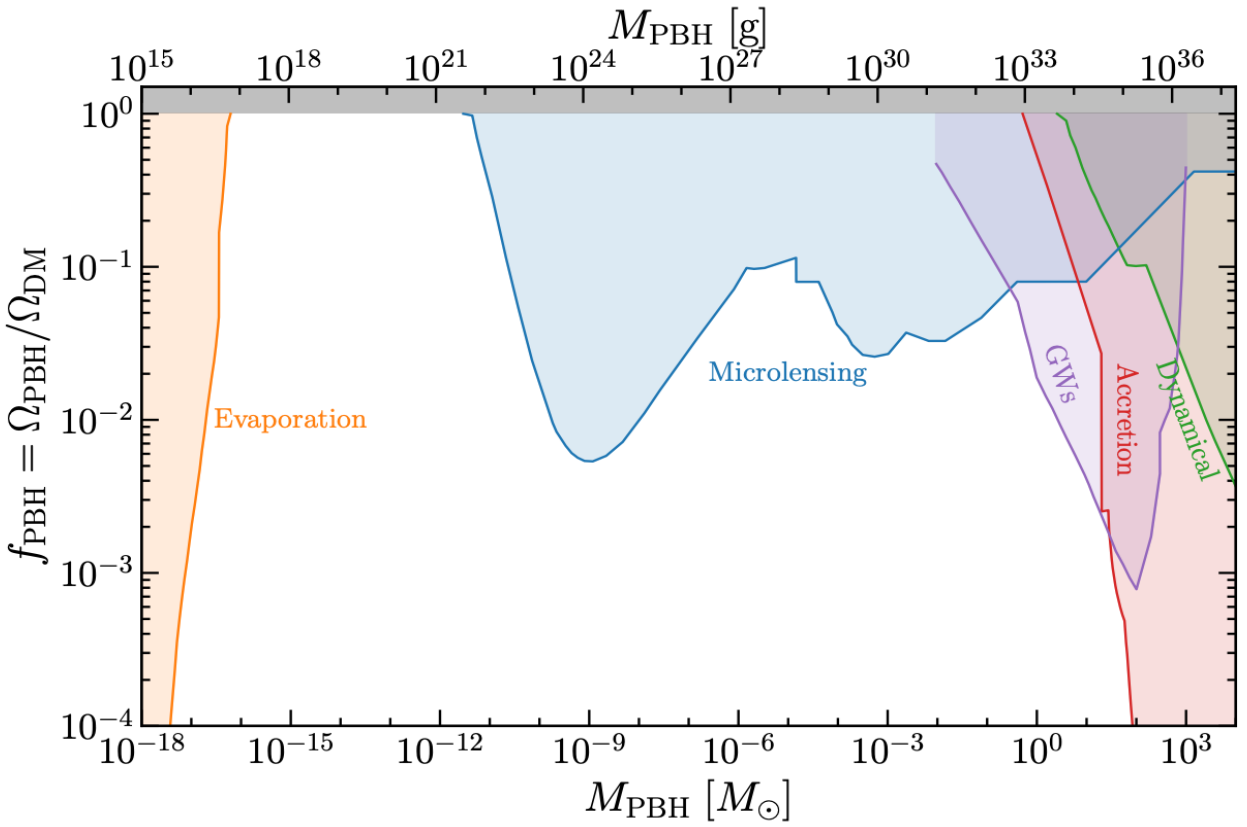
- The end point of inflation
- The universe is cold, dark, and dominated by the homogeneous inflaton field.
- How does the Universe transition to a hot, thermalized, radiation-dominated state after inflation, which is required for nucleosynthesis.
- Reheating!



A. Albrecht, P. J. Steinhardt, M. S. Turner and F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982).
A. D. Dolgov and A. D. Linde, Phys. Lett. B **116**, 329 (1982)

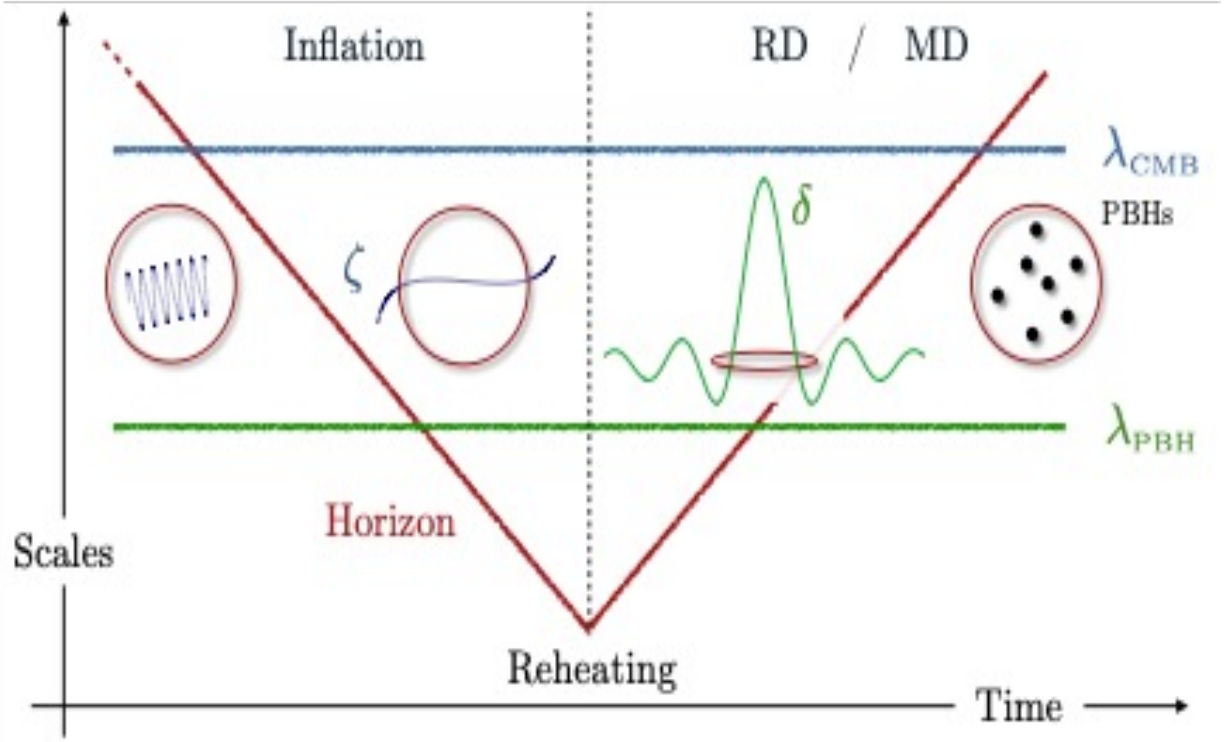
Why Primordial Black Holes?

- A novel and promising candidate for cold dark matter
- There are several mechanisms, including inflation, that can generate PBH abundantly.
- Non-baryonic, non-relativistic and nearly collisionless.
- Ultralight PBH can be responsible for the production of DM as well as SM particles.
- Several gravitational wave sources associated with PBHs give rise to the detection possibilities of the Early Universe.
- LIGO–Virgo–KAGRA observations motivate the possibility that a fraction of the observed binary black holes could have a primordial origin.

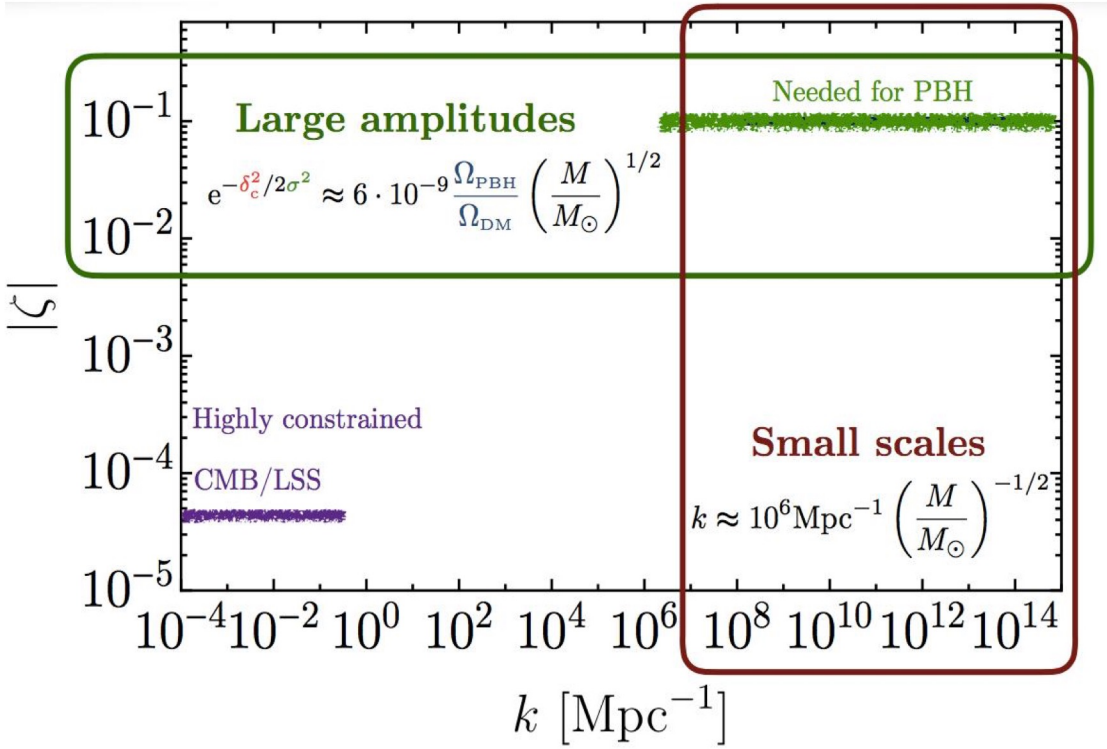


A. M. Green and B. J. Kavanagh, J. Phys. G **48**, 043001 (2021)
B. B. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Rep. Prog. Phys. **84**, 116902 (2021).

Formation of PBHs during reheating/radiation domination



❖ A schematic representation of the standard PBH formation scenario.



G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

Formation of the PBHs during reheating

❖ We assume that the inflationary scalar power spectrum with a broken power law is given by

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1} + A_0 \begin{cases} \left(\dfrac{k}{k_{\rm peak}}\right)^4 & k \le k_{\rm peak} \\ \left(\dfrac{k}{k_{\rm peak}}\right)^{n_0} & k \ge k_{\rm peak} \end{cases}$$

where $A_{\rm S}$ and $n_{\rm S}$ are the amplitude and spectral index of the power spectrum at the CMB pivot scale of $k_* = 0.05\,{\rm Mpc}^{-1}$.

❖ We shall assume that the threshold value of the density contrast for the formation of PBHs is given by following analytical expression

$$\delta_{\rm c}^{\rm an} = \frac{3\,(1+w_{\rm re})}{5+3\,w_{\rm re}}\,\sin^2\left(\frac{\pi\,\sqrt{w_{\rm re}}}{1+3\,w_{\rm re}}\right)$$

❖ Fraction of the dark matter contributed from PBH today

$$f_{\rm PBH}(M) = \beta(M)\,\frac{\Omega_{\rm m}\,h^2}{\Omega_{\rm c}\,h^2}\left(\frac{g_{\rm s,eq}}{g_{\rm s,re}}\right)\left(\frac{g_{\rm re}}{g_{\rm eq}}\right)^{\frac{1}{1+w_{\rm re}}}\left(\frac{T_{\rm re}}{T_{\rm eq}}\right)^{\frac{1-3\,w_{\rm re}}{1+w_{\rm re}}}\left(\frac{M}{\gamma\,M_{\rm eq}}\right)^{-\frac{2\,w_{\rm re}}{1+w_{\rm re}}}$$

T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev. D **88**, 084051 (2013).

GWs

After inflation:

1st order: Free wave propagating $\Box h_{ij} = 0$

2nd order: Massless field with source $\Box h_{ij} \sim \widehat{TT}^{ab}_{ij}(\partial_a \Phi \partial_b \Phi)$

$$\Omega_{\rm GW}(k) = \frac{d\rho_{GW}}{d\ln k} = \frac{k^2}{12\mathscr{H}^2}\mathscr{P}_h(k,\tau)$$

$$\Omega_{\rm GW}^{\rm induced} \sim \frac{1}{12}\Omega_{r,0}\mathscr{P}_{\mathscr{R}}^2 \sim 10^{-6}\mathscr{P}_{\mathscr{R}}^2(k \gg k_{\rm CMB})$$

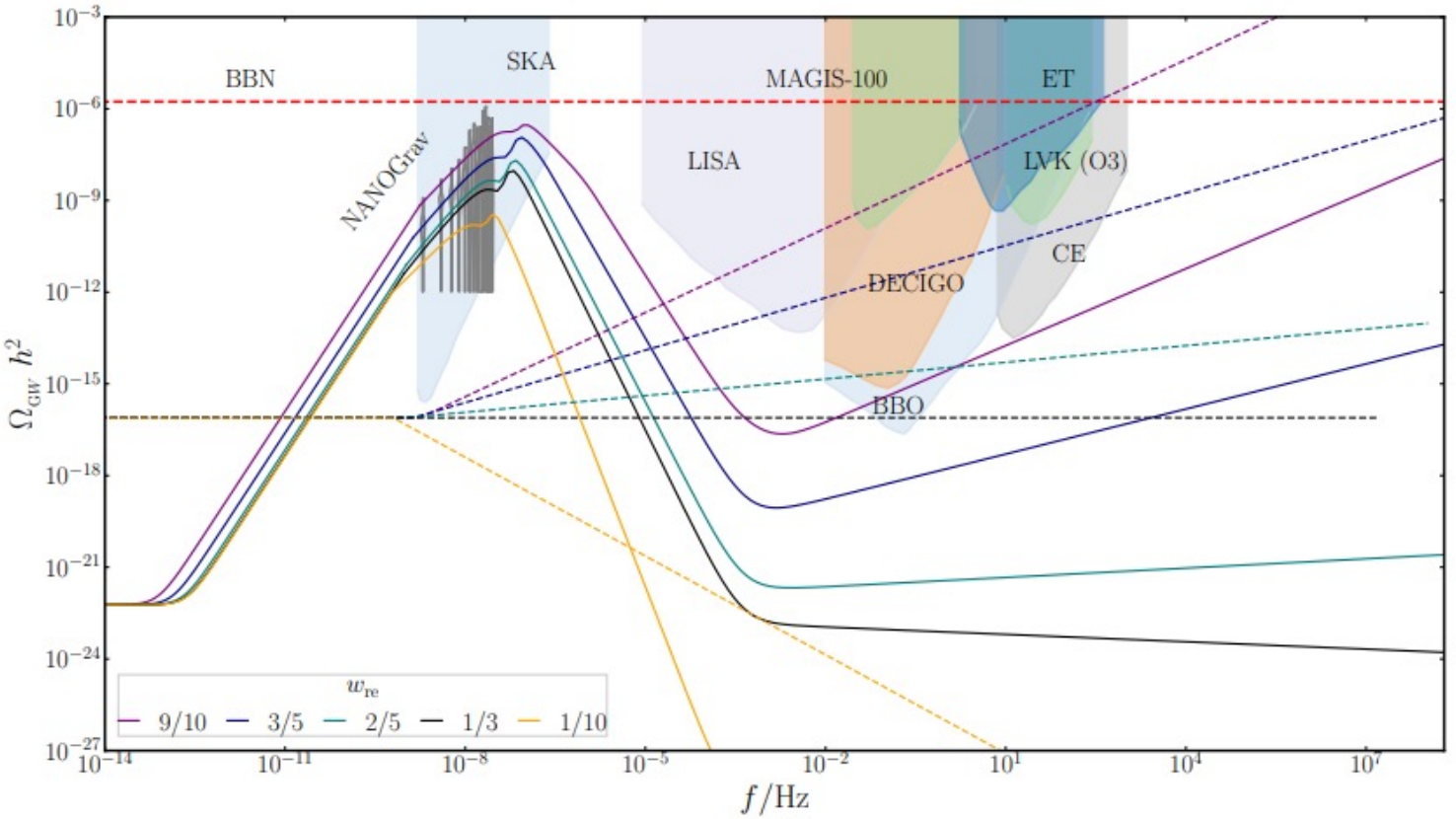
Density ratio of radiation today $\Omega_{r,0}$~4x10^{-5}

Amplitude of Primordial Fluctuations => Amplitude of induced GWs

Primordial spectrum => Content of universe (w, c_s) => induced GW spectrum

Inomata & Nakama: 1812.00674.
Byrnes et al. 2008.03289

Generation of scalar-induced secondary GWs during the epoch of reheating



The dimensionless spectral energy density of primary and secondary GWs

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, JCAP **01** (2025) 118.

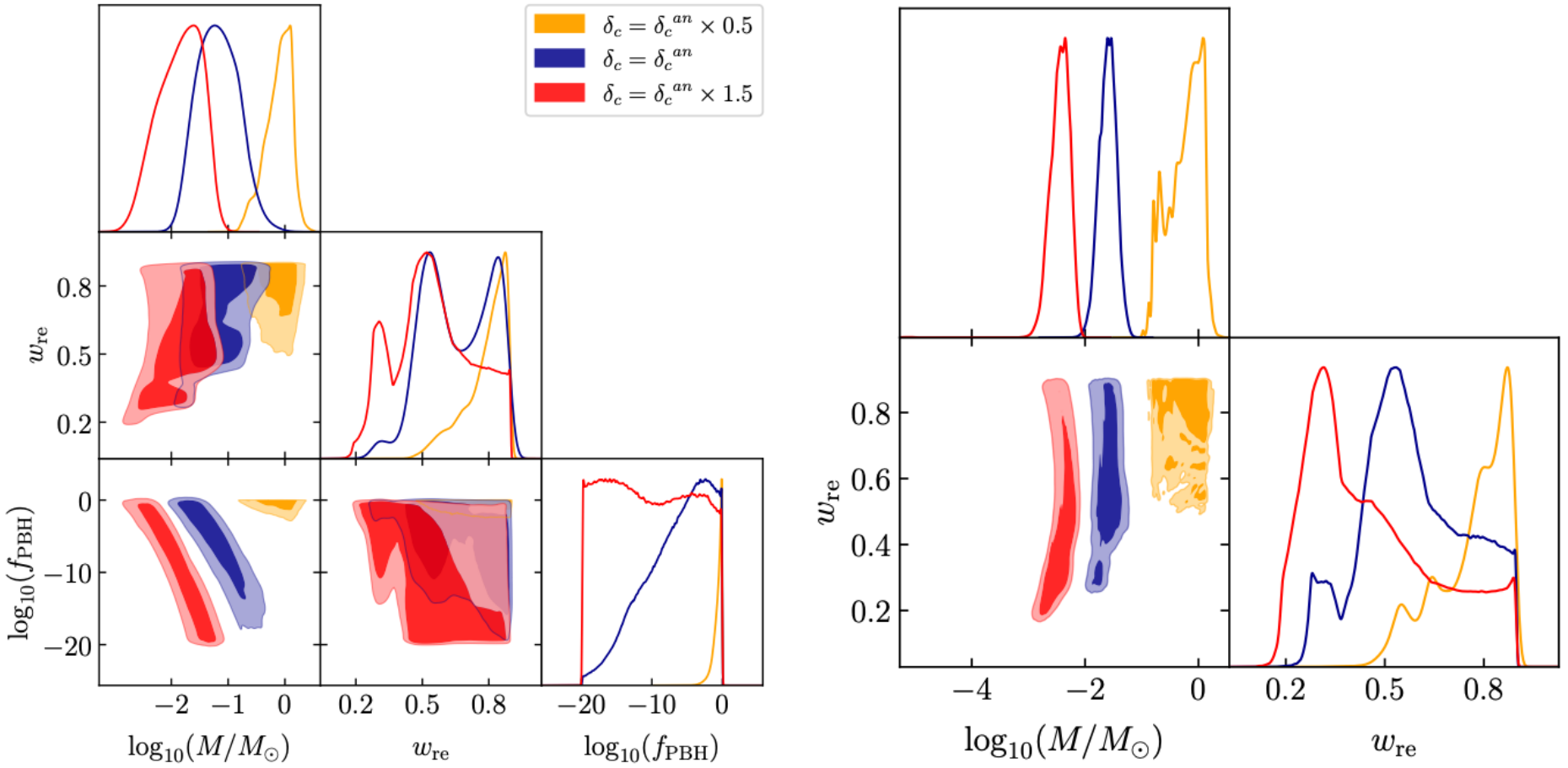
Best-fit values

Model	Parameter	Prior	Mean value		
R4pF	$\log_{10}\left(\frac{k_{\rm peak}}{\rm Mpc^{-1}}\right)$	$[6, 9]$	$7.62^{+0.35}_{-0.41}$		
	$\log_{10}(A_0)$	$[-3, 0]$	$-1.23^{+0.38}_{-0.66}$		
	$w_{\rm re}$	$[0.1, 0.9]$	0.52 ± 0.23		
	n_0	$[-3.0, -1.5]$	-2.26 ± 0.43		
R3pF	$\log_{10}\left(\frac{k_{\rm peak}}{\rm Mpc^{-1}}\right)$	$[6, 9]$	$7.54^{+0.36}_{-0.44}$		
	$\log_{10}(A_0)$	$[-3, 0]$	$-1.26^{+0.26}_{-0.64}$		
	$w_{\rm re}$	$[0.1, 0.9]$	$0.55^{+0.39}_{-0.14}$		
			$\mathbf{0.5\,\delta_c^{an}}$	$\boldsymbol{\delta_c^{an}}$	$\mathbf{1.5\,\delta_c^{an}}$
R3pB	$\log_{10}\left(\frac{M}{M_\odot}\right)$	$[-6, 3.5]$	$-0.12^{+0.28}_{-0.15}$	$-1.18^{+0.35}_{-0.39}$	$-1.85^{+0.49}_{-0.30}$
	$\log_{10}(f_{\rm PBH})$	$[-20, 0]$	$-0.67^{+0.68}_{-0.16}$	$-6.6^{+6.5}_{-1.9}$	$-10.2^{+8.2}_{-9.6}$
	$w_{\rm re}$	$[0.1, 0.9]$	$0.78^{+0.11}_{-0.030}$	$0.66^{+0.23}_{-0.19}$	0.55 ± 0.17
R2pB	$\log_{10}\left(\frac{M}{M_\odot}\right)$	$[-6, 3.5]$	$-0.24^{+0.38}_{-0.45}$	$-1.60^{+0.16}_{-0.14}$	$-2.45^{+0.20}_{-0.13}$
	$w_{\rm re}$	$[0.1, 0.9]$	$0.77^{+0.13}_{-0.038}$	0.59 ± 0.16	$0.464^{+0.095}_{-0.25}$

The best-fit values arrived upon comparison with the NANOGrav 15-year data.

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, JCAP **01** (2025) 118.

Constraints on the epoch of reheating through scalar induced GWs



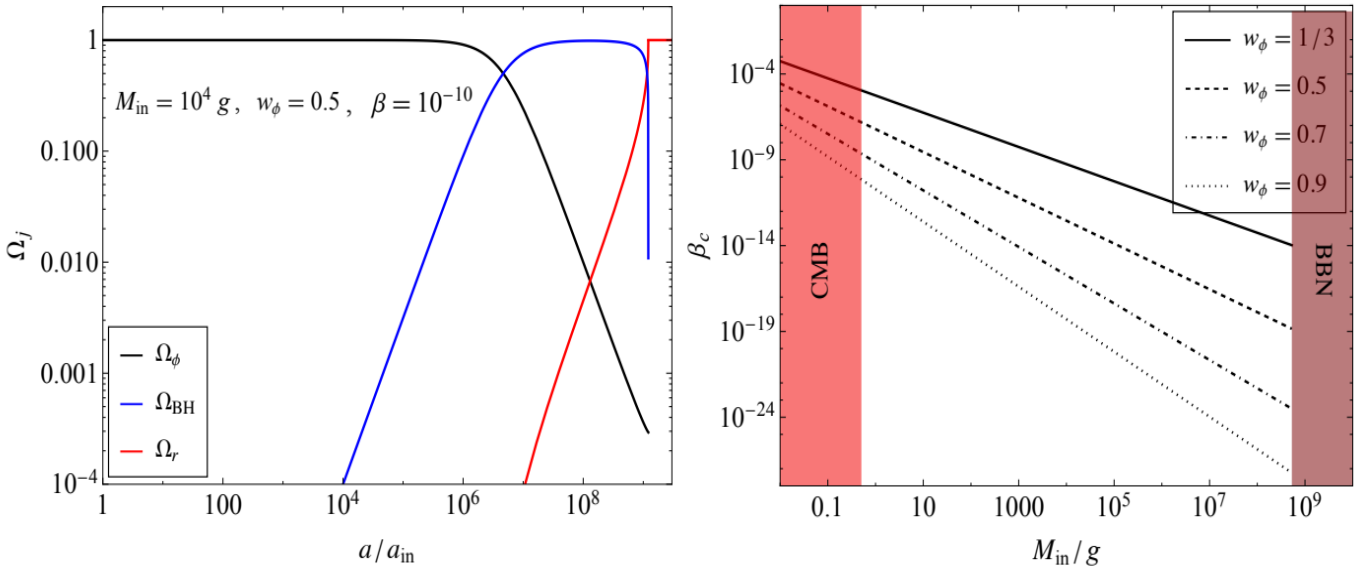
- we have plotted the marginalized posterior distributions of the parameters that have been arrived at upon comparing the models with the NANOGrav 15-year data.

S. Maity, N. Bhaumik, **M. R. Haque**, D. Maity and L. Sriramkumar, JCAP **01** (2025) 118.

PBH Reheating

Main idea

- PBH for a sufficiently large initial abundance, may temporarily dominate the total energy density.
- Light PBHs evaporate through Hawking radiation and transfer their energy into ordinary radiation, thereby restoring the standard RD before BBN.
- Two induced-GW sources are associated with PBH reheating: adiabatic perturbations and PBH isocurvature perturbations.

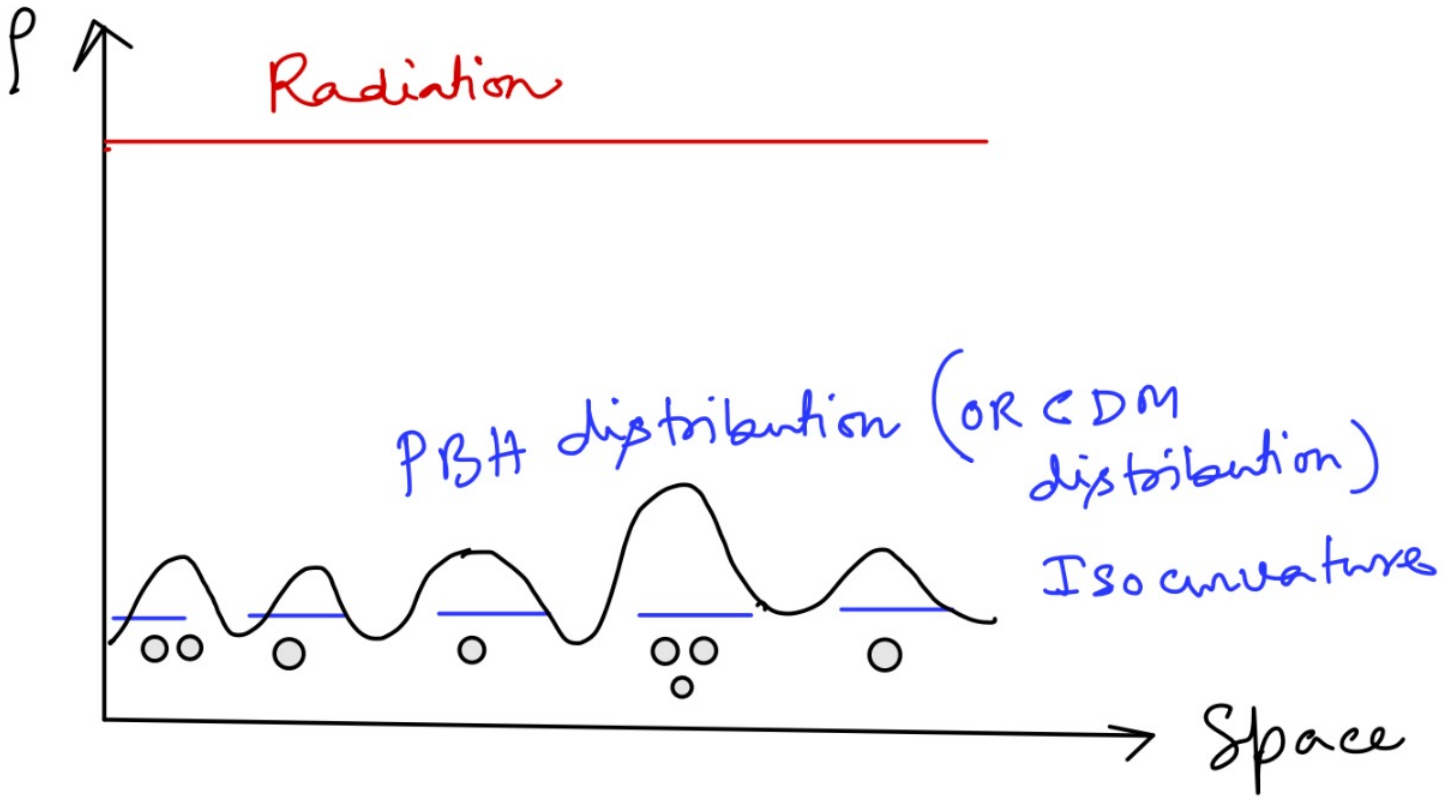


Example dynamics of PBH reheating (left) and the critical initial PBH abundance βc as a function of Min (right).

PBH reheating provides a non-standard reheating mechanism and a natural setting for two distinct induced-GW sources.

M. R. Haque, E. Kpatcha, D. Maity and Y. Mambrini, Rev. D 108, 063523 (2023)

Isocurvature perturbation evolution



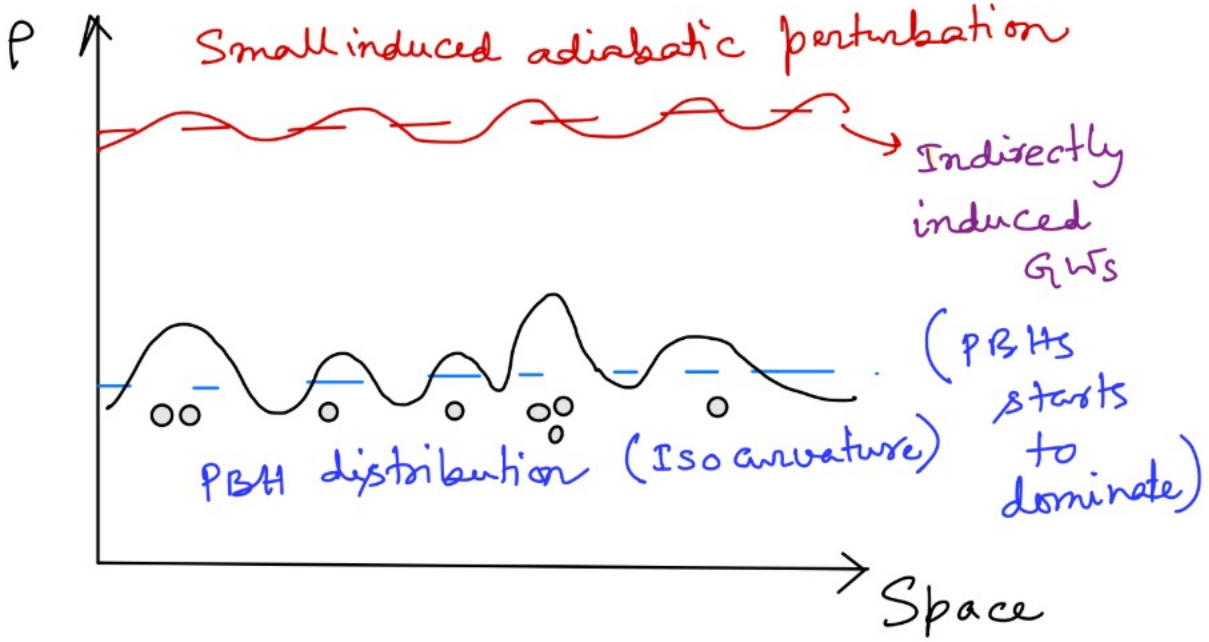
Explanations:

The inhomogeneous PBH distribution give rise to the density fluctuations

These fluctuations must be isocurvature as they form by the collapse of the homogeneous radiation

G. Domènech, C. Lin and M. Sasaki, JCAP **04** (2021) 062

Isocurvature perturbation evolution



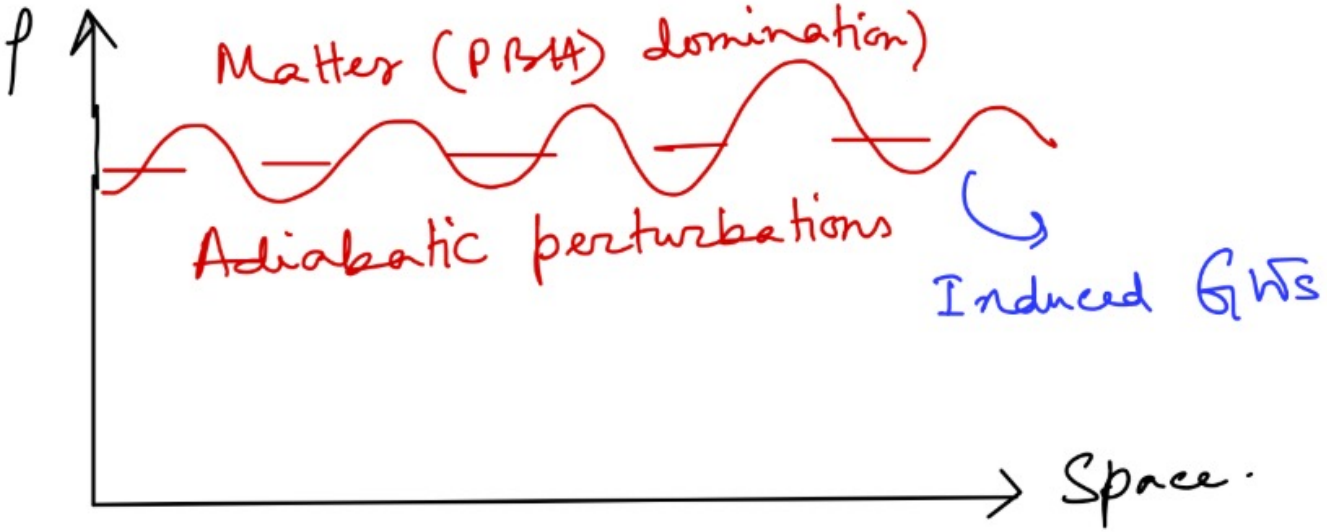
Explanations:

PBH energy density gets relative weight

Isocurvature induces curvature fluctuations

Curvature fluctuations induces GWs

Induced GWs from PBH number density fluctuations



Explanations:

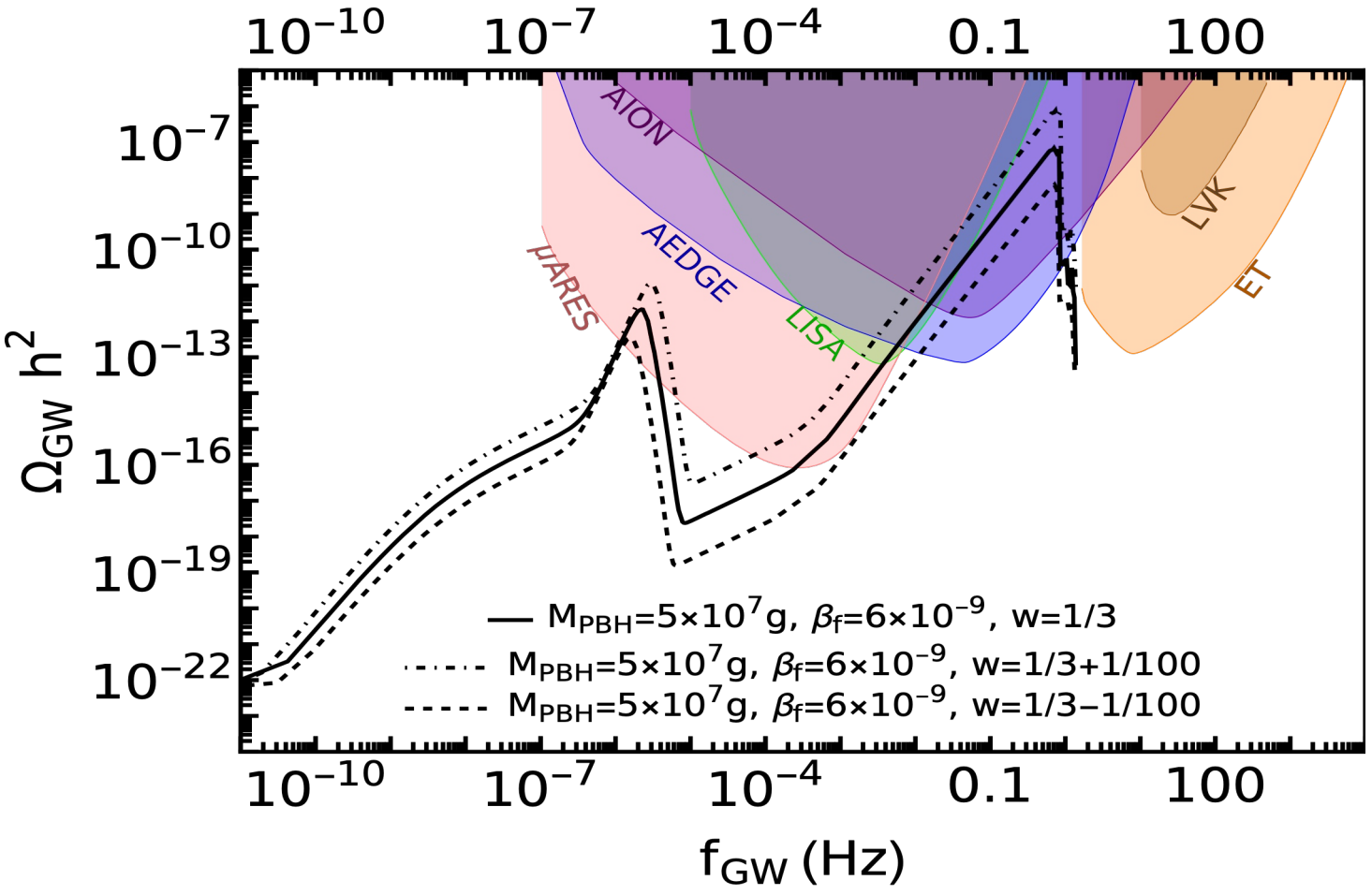
PBH dominates the energy density

Curvature fluctuations induced GWs

PBH number fluctuations→isocurvature→curvature perturbation→induced GWs

G. Domènech, C. Lin and M. Sasaki, JCAP **04** (2021) 062

Doubly peaked GWs for the PBH reheating scenario



N. Bhaumik, M. R. Haque, R.K. Jain, and M. Lewicki, JHEP **10** (2024) 142.

Signal-to-Noise Ratio (SNR): Combined GW signal

$$\mathrm{SNR} = \sqrt{\tau_{\mathrm{obs}} \int_{f_{\mathrm{min}}}^{f_{\mathrm{max}}} df \left(\frac{\Omega_{\mathrm{GW}}(f,\theta)}{\Omega_{\mathrm{noise}}(f)} \right)^2}$$

Combined GW source used:

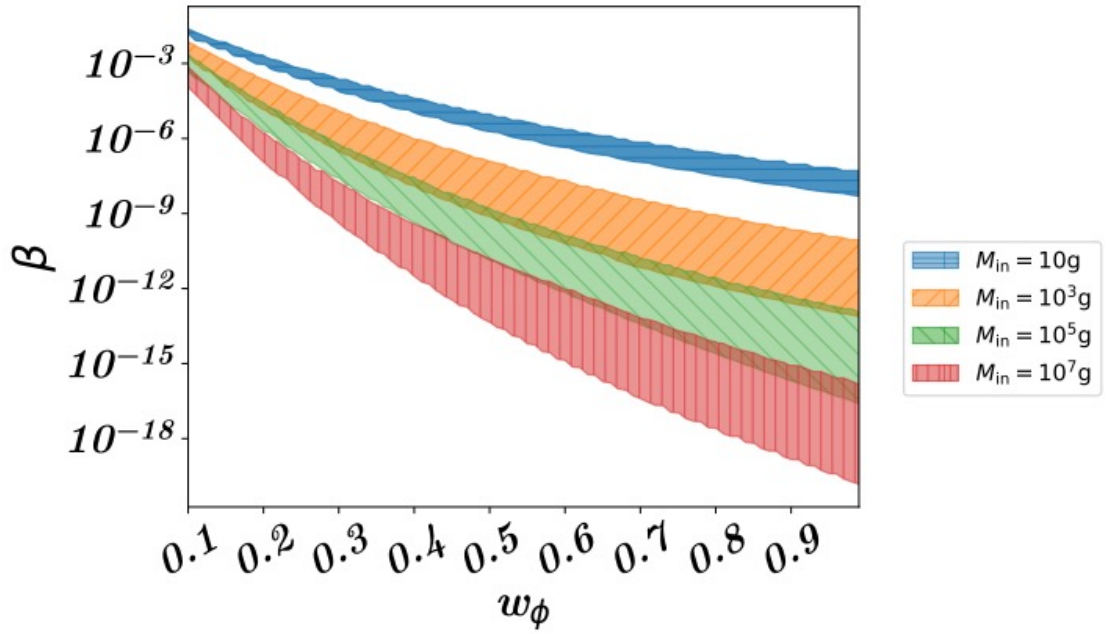
$$\Omega_{\mathrm{GW}}^{\mathrm{com}}(f) = \Omega_{\mathrm{GW}}^{\mathrm{iso}}(f) + \Omega_{\mathrm{GW}}^{\mathrm{ad}}(f)$$

ET
1–10^4 Hz | **5 yr**

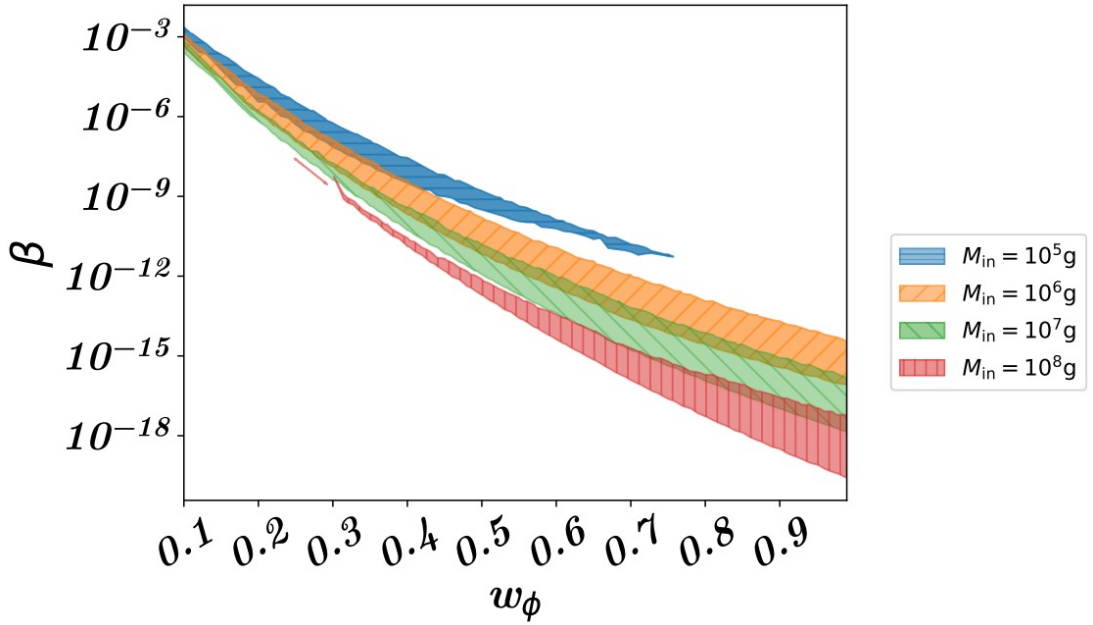
LISA
10^{-4}–1 Hz | **4 yr**

Detection criterion: SNR ≥ 1

Upper boundary: GW overproduction constrained by ΔN_{eff} at BBN



(a) ET



(b) LISA

ET — combined signal
$M_{in} \simeq 0.5 – 3.97 \times 10^7$ g

LISA — combined signal
$M_{in} \simeq 2.3 \times 10^4 – 4.8 \times 10^8$ g

Combined adiabatic + isocurvature signal: ET and LISA give complementary coverage of PBH reheating.

D. Paul, M. R. Haque and S. Pal, JCAP **08** (2025) 074.

Fisher Matrix Formalism: Uncertainty in PBH Parameters with ET

Assuming a Gaussian likelihood for the binned GW signal

$$L(\theta) = \prod_{b=1}^{N_b} \sqrt{\frac{n_b}{2\pi\Omega_n^2(f_b)}} \exp\left[-\frac{n_b\left(\Omega_{\rm sig}(f_b,\theta) - \Omega_{\rm fid}(f_b)\right)^2}{\Omega_n^2(f_b)}\right]$$

The Fisher matrix is defined as: $F_{ij} = <-\frac{\partial^2 \ln L}{\partial\theta_i \partial\theta_j}>$.

The number of independent modes in the b-th frequency bin :

$$n_b \equiv [(f_b - f_{b-1})\,\tau_{\rm obs}]$$

Under the Gaussian approximation:

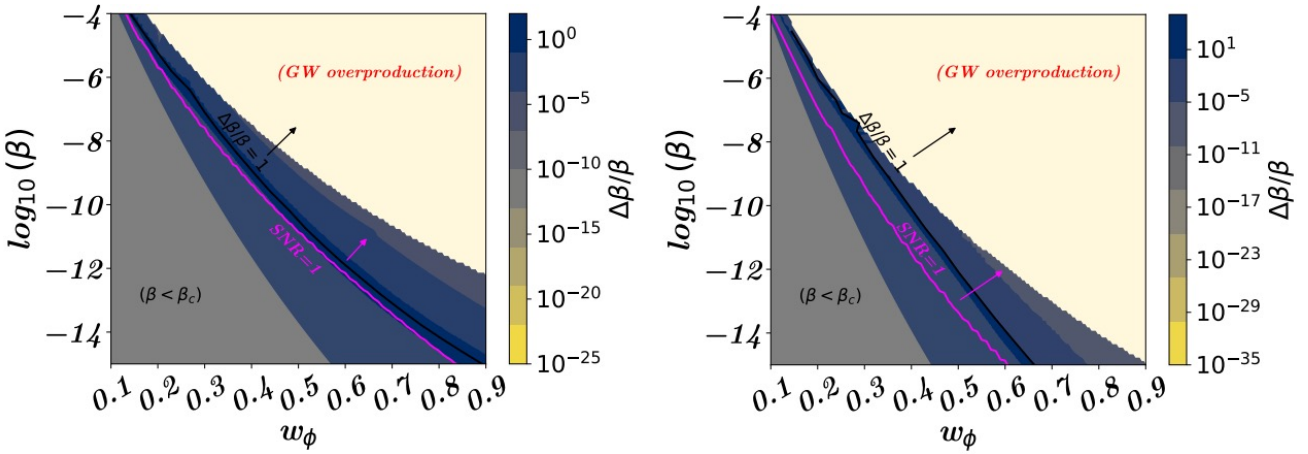
$$F_{ij} = \tau_{\rm obs} \sum_{b=1}^{N_b} \frac{2\Delta f_b}{\Omega_n^2(f_b)} \frac{\partial\Omega_{\rm GW}(f_b)}{\partial\theta_i} \frac{\partial\Omega_{\rm GW}(f_b)}{\partial\theta_j},$$

Δf_b: frequency-bin width, $\theta = \{w_\varphi, \beta\}$

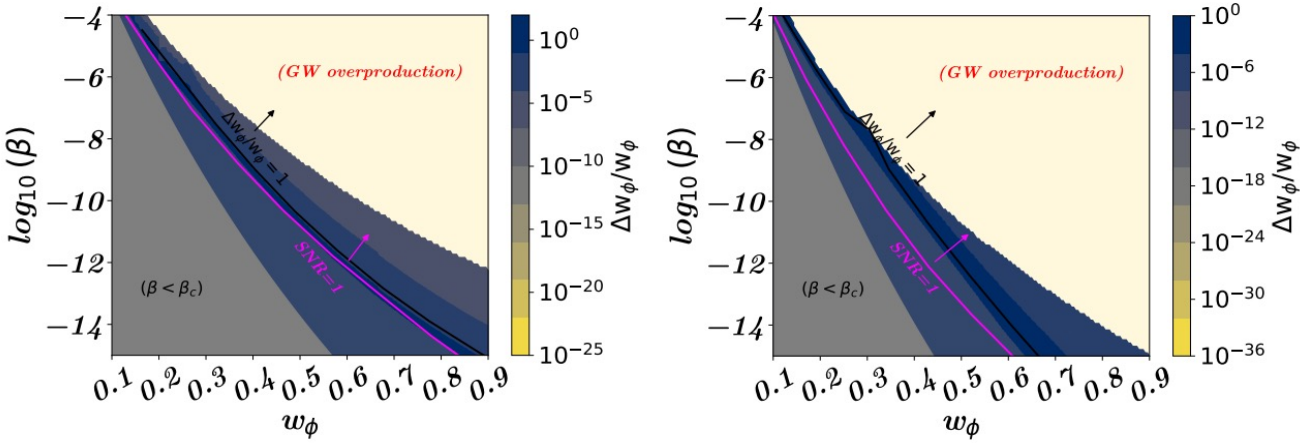
The covariance matrix $C_{ij} = (F^{-1})_{ij}$

Projected 1σ uncertainty is: $\delta\theta_i = \sqrt{C_{ii}}$.

We impose the condition: $\frac{\delta\theta_i}{\theta_i} \leq 1$



(a) $M_{\rm in} = 10^5\ g$ (b) $M_{\rm in} = 10^7\ g$



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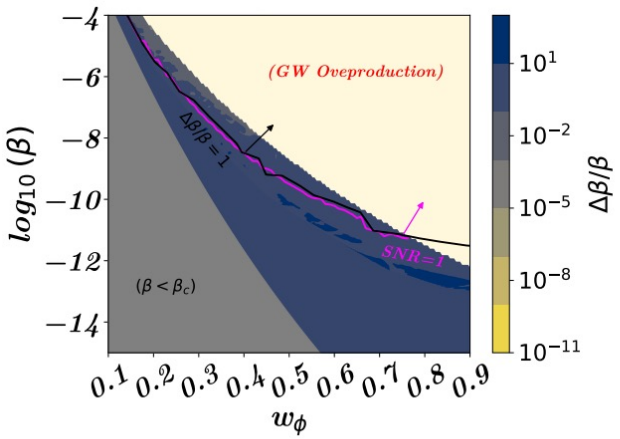
Measurable region = **SNR > 1** ∩ **Δθ/θ < 1**

D. Paul, M. R. Haque and S. Pal, JCAP **08** (2025) 074.

Fisher Matrix Formalism: Uncertainty in PBH Parameters with LISA



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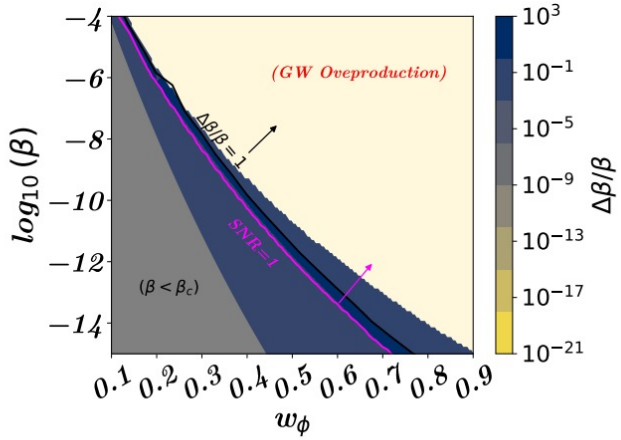
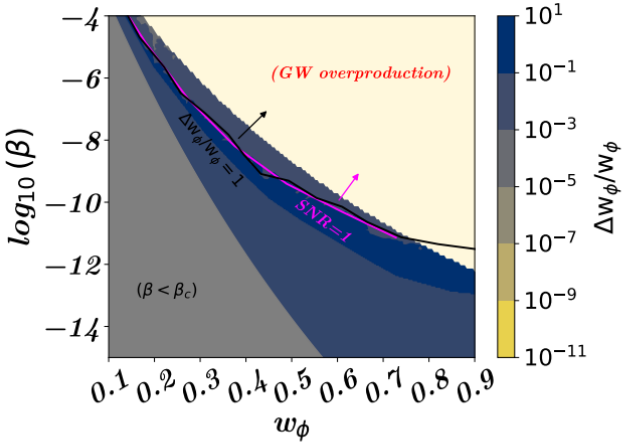


(b) $M_{\rm in} = 10^7\ g$



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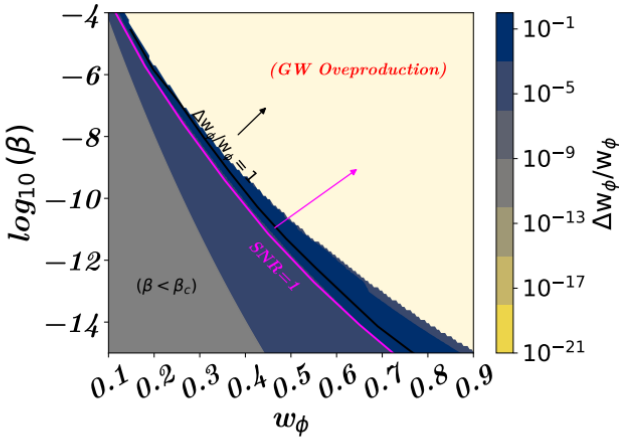

(b) $M_{\rm in} = 10^7\ g$

Main Fisher results

- Compared with ET: Fisher uncertainties are larger for LISA, but it still provides complementary constraints on PBH reheating.
- The combined Fisher constraints are mainly driven by the adiabatic GW contribution, while the GW-overproduction bound reduces the allowed parameter space.

Measurable region = **SNR > 1** ∩ **Δθ/θ < 1**

D. Paul, M. R. Haque and S. Pal, JCAP **08** (2025) 074.

Conclusions

Main takeaways

- PBHs provide a powerful probe of the poorly constrained post-inflationary and reheating epoch through their associated stochastic GW signals.
- Scalar-induced GWs can constrain the reheating history, and the NANOGrav 15-year data can be consistently explained in scenarios involving PBH formation during reheating.
- PBH reheating naturally generates two induced-GW components, adiabatic and isocurvature, which can produce a characteristic double-peaked GW spectrum.
- LISA and ET offer complementary probes of PBH reheating: ET is sensitive to $M_{in} \simeq 0.5$–3.97×10^7 g, while LISA extends the reach to $M_{in} \simeq 2.3 \times 10^4$–$4.8 \times 10^8$ g; Fisher forecasts put further constraints on the reheating parameters.

Thank You