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Bogoliubov analysis of particles created by bubble collisions

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In preparation
with Jinno, Nakai, & Wang

1. Introduction

First-order phase transition is ubiquitous in BSM physics.

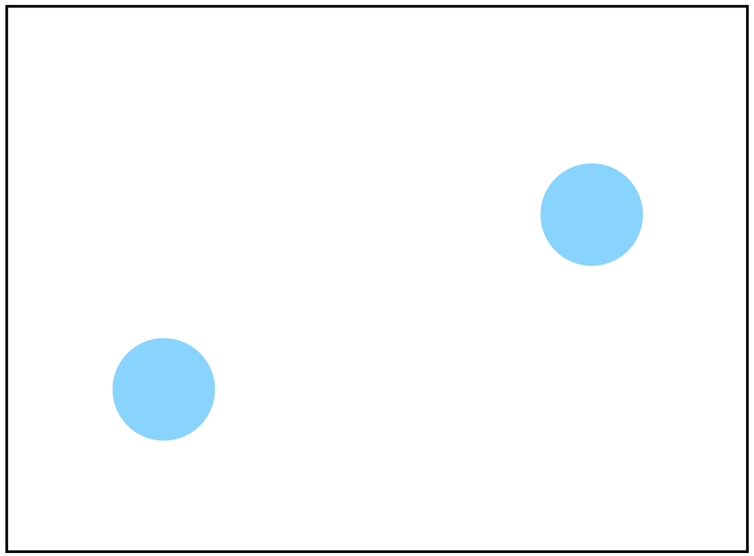
Non-equilibrium + Inhomogeneous

- Gravitational waves e.g. Athron, Balazs, Fowlie, Morris, Wu (2024) for recent review
- Particle production: dark matter, baryon, magnetic field
- Topological defects: monopole, cosmic string

Kibble (1976), Vilenkin (1981)

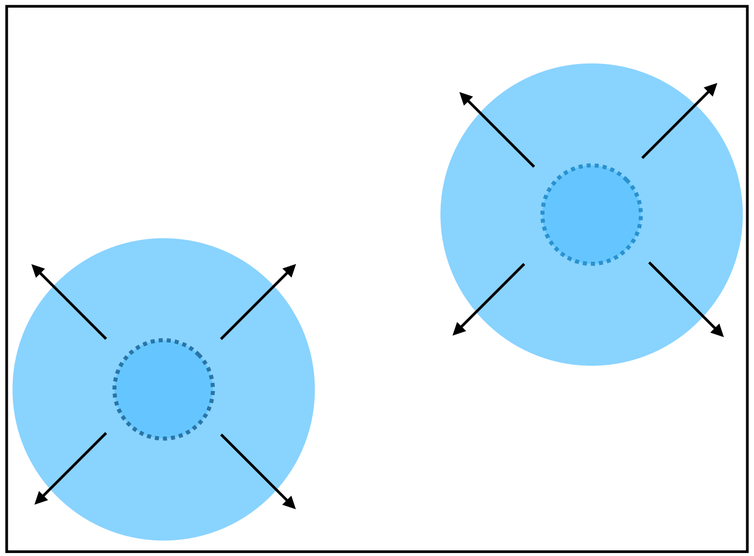
Particle production through bubble dynamics

The production stage can be classified into three epochs.



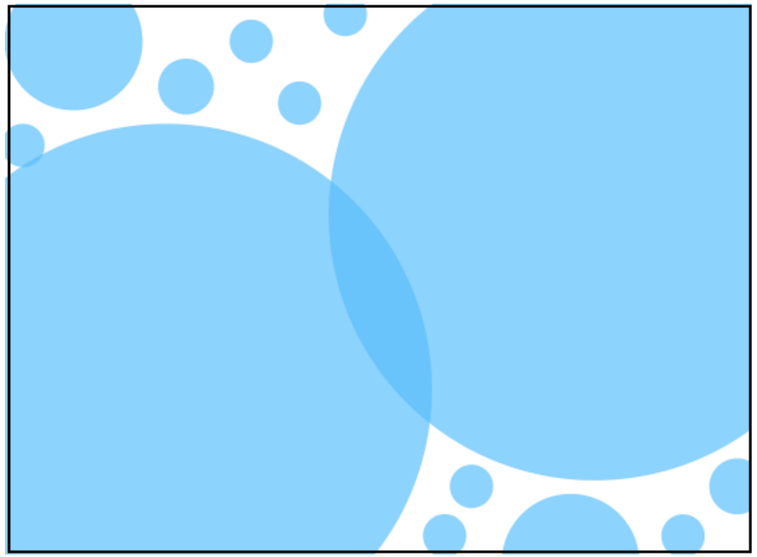
Nucleation

Rubakov (1984), Yamamoto, Tanaka, Sasaki (1995), Hashiba, Yamada, Yokoyama (2021), Shakya (2025), etc.



Expansion

Bodeker, Moore (2017), Baker, Kopp, Long (2020), Azatov, Vanvlasselaer (2021), Jinno, **SN**, Nakai, Wang (2026), Shakya (2025), etc.

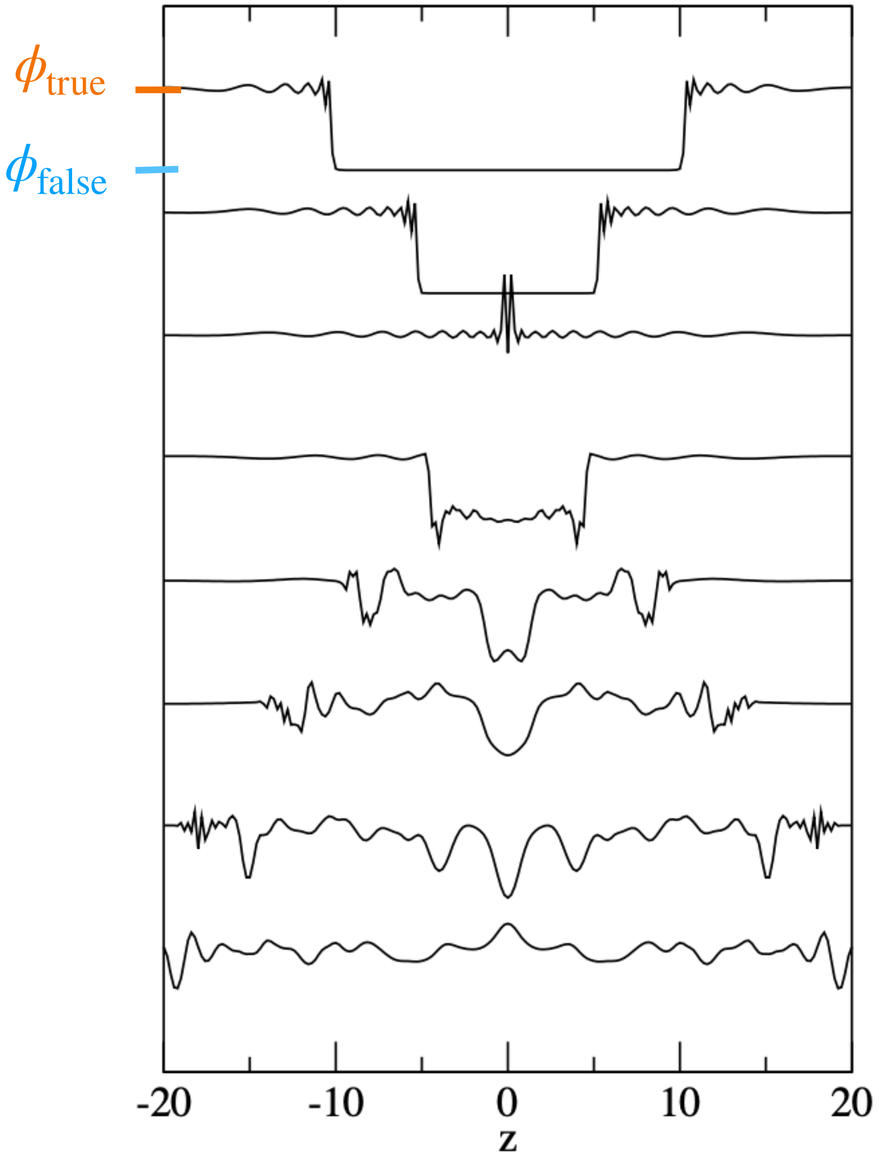


Collision

Watkins, Widrow (1992), Konstandin, Servant (2011), Falkowski, No (2013), Shakya (2025), Mansour, Shakya (2025), An, Jiang (2026), Ghoshal, Pal, Strumia (2026), etc.

→ Listen to Yaoduo Wang's Talk!!

Collision-induced production



Bubble collisions generate inhomogeneous scalar waves.

Coupling to the background field ϕ induces an effective mass.

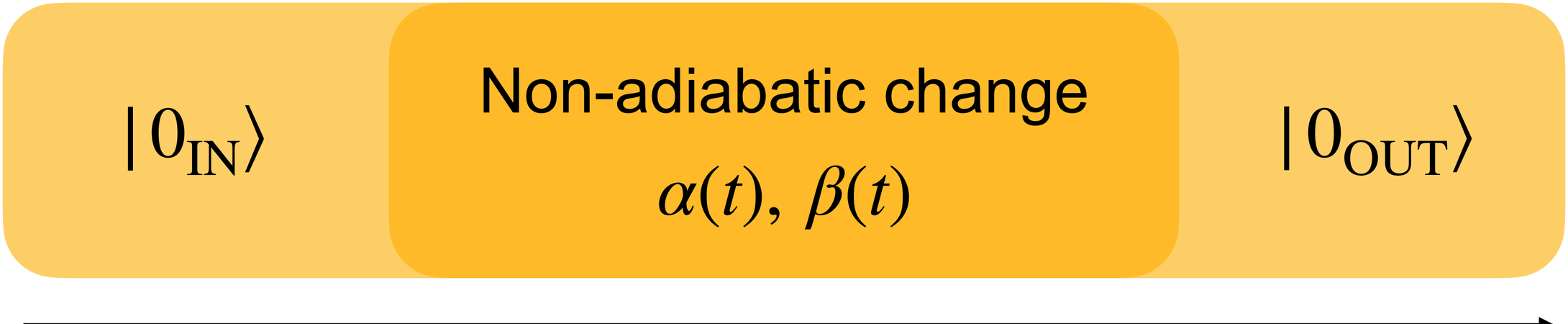
e.g. $\phi^2 |H|^2$, $\phi\bar{\psi}\psi \rightarrow m_{\text{eff}}(\phi)$

The non-adiabatic change creates particles from vacuum.

Konstandin, Servant (2011)

Bogoliubov transformation formalism

e.g. Kolb, Long (2025) for recent review



$$\hat{a}(t) = \alpha(t)a + \beta(t)b^{\dagger}$$
$$\hat{b}(t) = -\beta^{*}(t)a + \alpha^{*}(t)b^{\dagger}$$

$$n_k \sim \langle 0_{\mathrm{IN}}|\hat{a}^{\dagger}\hat{a}|0_{\mathrm{IN}}\rangle = |\beta|^2$$

However, applying to the bubble collision is not easy, because inhomogeneity requires calculation on lattice.

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2. Dynamics of background field

3. Evolution in Milne spacetime

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2. Dynamics of background field

Let us assume the following two assumptions:

① Bubble walls are planar

② SO(1,1) symmetry

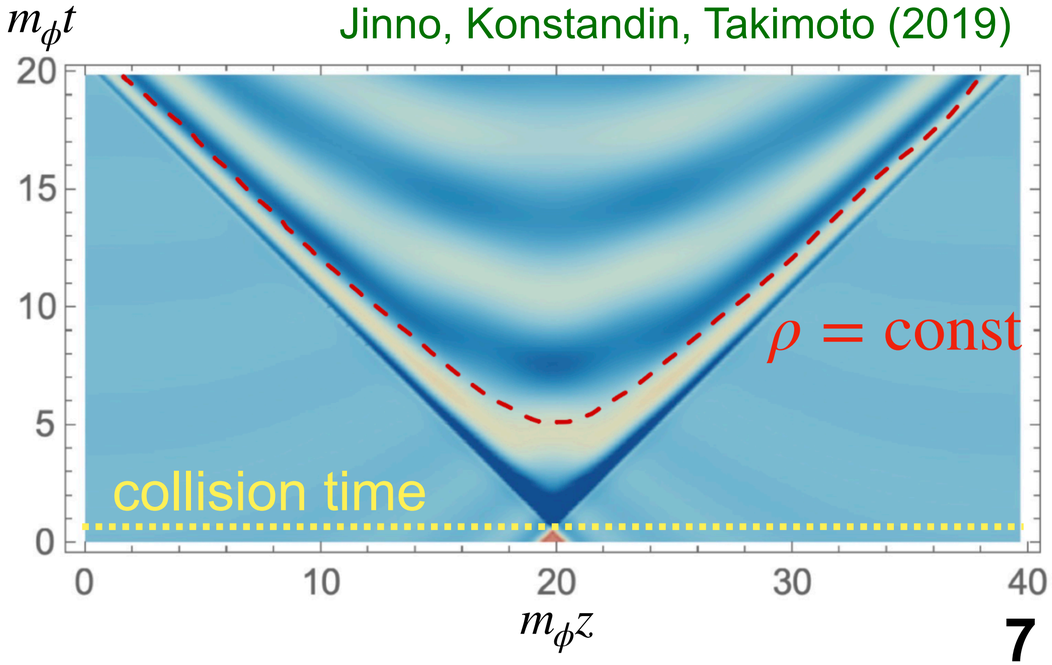
← Justified by large Lorentz factor γ_w

Under the assumptions, the scalar field obeys the trapping equation.

$$\frac{d^2\phi}{d\rho^2} + \frac{1}{\rho}\frac{d\phi}{d\rho} + \frac{dV}{d\phi} = 0$$

$$\rho = \sqrt{t^2 - z^2}$$

Jinno, Konstandin, Takimoto (2019)



A model

We consider a toy model with a Dirac fermion coupled to the order parameter field ϕ.

$$\mathcal{L}_\psi = \bar{\psi} i\gamma^\mu D_\mu \psi - m_\psi \bar{\psi}\psi + y\phi\bar{\psi}\psi \qquad m_{\rm eff}(t,z) = m_\psi - y\phi(t,z)$$

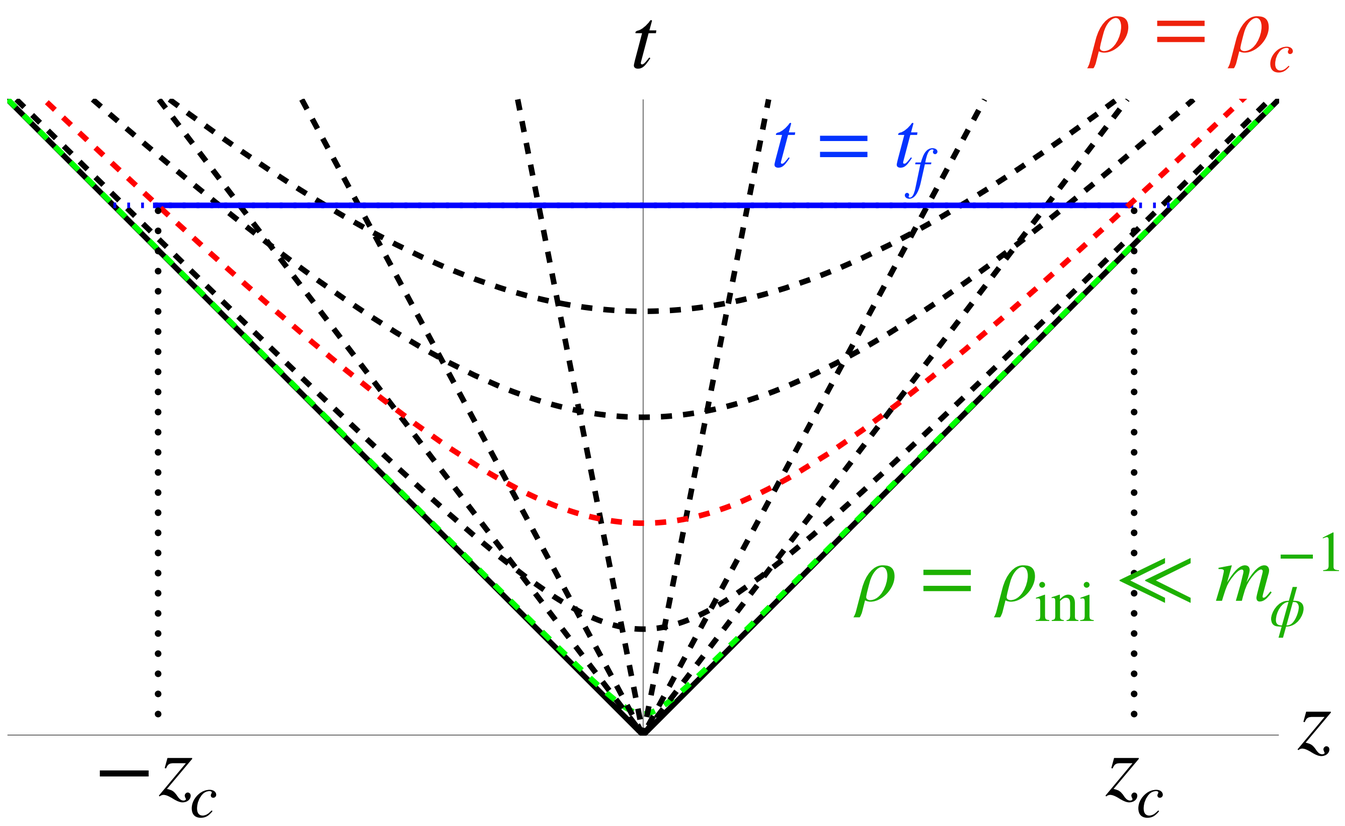
$$(i\gamma^\mu \partial_\mu - m_{\rm eff}(t,z))\psi(t,z) = 0$$

$$\to m_{\rm eff}(\rho)$$

We can mimic the problem to a homogeneous one in terms of Milne coordinates (ρ, η) : Cf) Yamamoto, Tanaka, Sasaki (1995)

$$t = \rho\cosh\eta, \quad z = \rho\sinh\eta$$

Procedure of analysis



Evolution in Milne spacetime starts at $\rho = \rho_{\rm ini}$.

↓

Particle production ends at $\rho = \rho_c$.

↓

Counting the particle number on a Minkowski time slice, $t = t_f$.

Note: the system is apparently non-adiabatic at the initial stage due to coordinate artifact.

3. Evolution in Milne spacetime

Dirac equation in Milne spacetime

$$\left[i\gamma^0\left(\partial_\rho+\frac{1}{2\rho}\right)+\frac{i}{\rho}\gamma^3\partial_\eta+i\vec{\gamma}_T\cdot\vec{\partial}_T-m_{\text{eff}}(\rho)\right]\psi(\rho,\eta,\vec{x}_T)=0$$

Translation symmetry in x, y

SO(1,1) symmetry
↔ Translation in η

Quantization

$$\psi(\rho,\eta,\vec{x}_T)=\frac{1}{\sqrt{\rho}}\int\frac{d^2k_T d\kappa}{(2\pi)^{3/2}}\frac{1}{\sqrt{2}}e^{i\vec{k}_T\cdot\vec{x}_T+i\kappa\eta}\sum_{r=\pm}\left[U_r(\rho,\vec{k}_T,\kappa)A_r(\vec{k}_T,\kappa)+V_r(\rho,\vec{k}_T,\kappa)B_r^\dagger(-\vec{k}_T,-\kappa)\right]$$

κ: boost momentum

$\vec{k}_T$: transverse momentum

$$\left\{A_r(\vec{k}_T,\kappa),A_s^\dagger(\vec{k}'_T,\kappa')\right\}=\left\{B_r(\vec{k}_T,\kappa),B_s^\dagger(\vec{k}'_T,\kappa')\right\}$$
$$=\delta_{rs}\delta(\kappa-\kappa')\delta^{(2)}(\vec{k}_T-\vec{k}'_T).$$

Hamiltonian

$$U_r = \begin{pmatrix} U_+^{(r)}(\rho, \vec{k}_T, \kappa)\xi_r \\ U_-^{(r)}(\rho, \vec{k}_T, \kappa)\xi_r \end{pmatrix} \longrightarrow (i\partial_\rho \mp m_{\text{eff}}(\rho))U_\pm^{(r)} - \left(\frac{\kappa}{\rho} \pm irk_T\right) U_\mp^{(r)} = 0.$$

$$\hat{h}\xi_r(\vec{k}_T) = r\xi_r(\vec{k}_T) \qquad \hat{h} \equiv \frac{\vec{k}_T \cdot \vec{\sigma}_T}{k_T} \quad \text{: Transverse helicity}$$

The detail is skipped.

$$H(\rho) = \int d^2x_T d\eta\ \chi^\dagger \left[-i\gamma^0\vec{\gamma}_T \cdot \vec{\partial}_T - \frac{i}{\rho}\gamma^0\gamma^3\partial_\eta + \gamma^0 m_{\text{eff}}(\rho)\right] \chi \qquad \psi(\rho, \eta, \vec{x}_T) \equiv \frac{1}{\sqrt{\rho}}\chi(\rho, \eta, \vec{x}_T)$$

$$= \int d^2k_T d\kappa\ \vec{A}^\dagger \mathcal{H}_{\vec{k}_T,\kappa}(\rho)\vec{A}$$

positive helicity

$$\mathcal{H}_{\vec{k}_T,\kappa}(\rho) = \begin{pmatrix} E_\kappa^{(+)} - E_T^{(+)} & F^{(+)*} & 0 & 0 \\ F^{(+)} & -E_\kappa^{(+)} + E_T^{(+)} & 0 & 0 \\ 0 & 0 & E_\kappa^{(-)} + E_T^{(-)} & F^{(-)*} \\ 0 & 0 & F^{(-)} & -E_\kappa^{(-)} - E_T^{(-)} \end{pmatrix}$$

negative helicity

Bogoliubov transformation

$$\hat{A}_r(\vec{k}_T,\kappa,\rho) = \alpha^{(r)}(\vec{k}_T,\kappa,\rho)A_r(\vec{k}_T,\kappa) + \beta^{(r)}(\vec{k}_T,\kappa,\rho)B_r^\dagger(-\vec{k}_T,-\kappa),$$
$$\hat{B}_r^\dagger(\vec{k}_T,\kappa,\rho) = -\beta^{(r)*}(\vec{k}_T,\kappa,\rho)A_r(\vec{k}_T,\kappa) + \alpha^{(r)*}(\vec{k}_T,\kappa,\rho)B_r^\dagger(-\vec{k}_T,-\kappa).$$

Diagonalizing the Hamiltonian, we obtain

$$|\beta^{(r)}|^2 = \frac{|F^{(r)}|^2}{2\omega(\omega + E_\kappa^{(r)} - rE_T^{(r)})} = \frac{\omega - E_\kappa^{(r)} + rE_T^{(r)}}{2\omega}$$

$$\omega^2 = k_T^2 + \left(\frac{\kappa}{\rho}\right)^2 + m_{\text{eff}}^2(\rho)$$

$\sim n_k$

however…

$$\to \frac{1 - \mathrm{Re}[U_+^{(r)*}U_-^{(r)}]^2}{2} \neq 0 \quad (\rho \to 0)$$ depending on κ

What is this?

Violation of adiabaticity

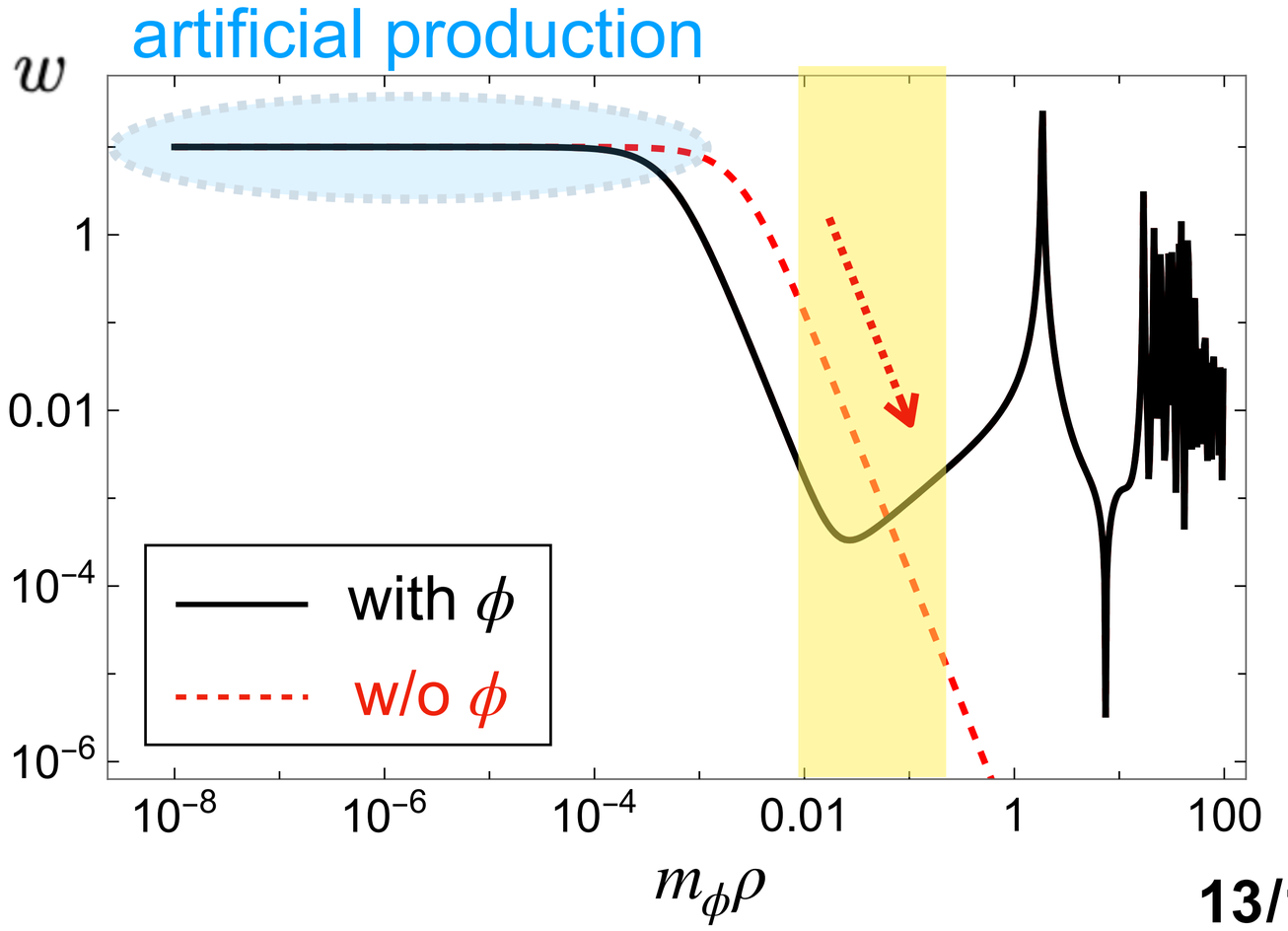
At the initial stage, the system evolves non-adiabatically due to Milne geometry.

Define the violation of adiabaticity as

$$w \equiv \left|\frac{\omega'}{\omega^2}\right| = \omega^{-3}\left|-\frac{\kappa^2}{\rho^3} + y m_{\rm eff}\phi'\right|$$

$w \ll 1$ adiabatic

$w \sim 1$ non-adiabatic



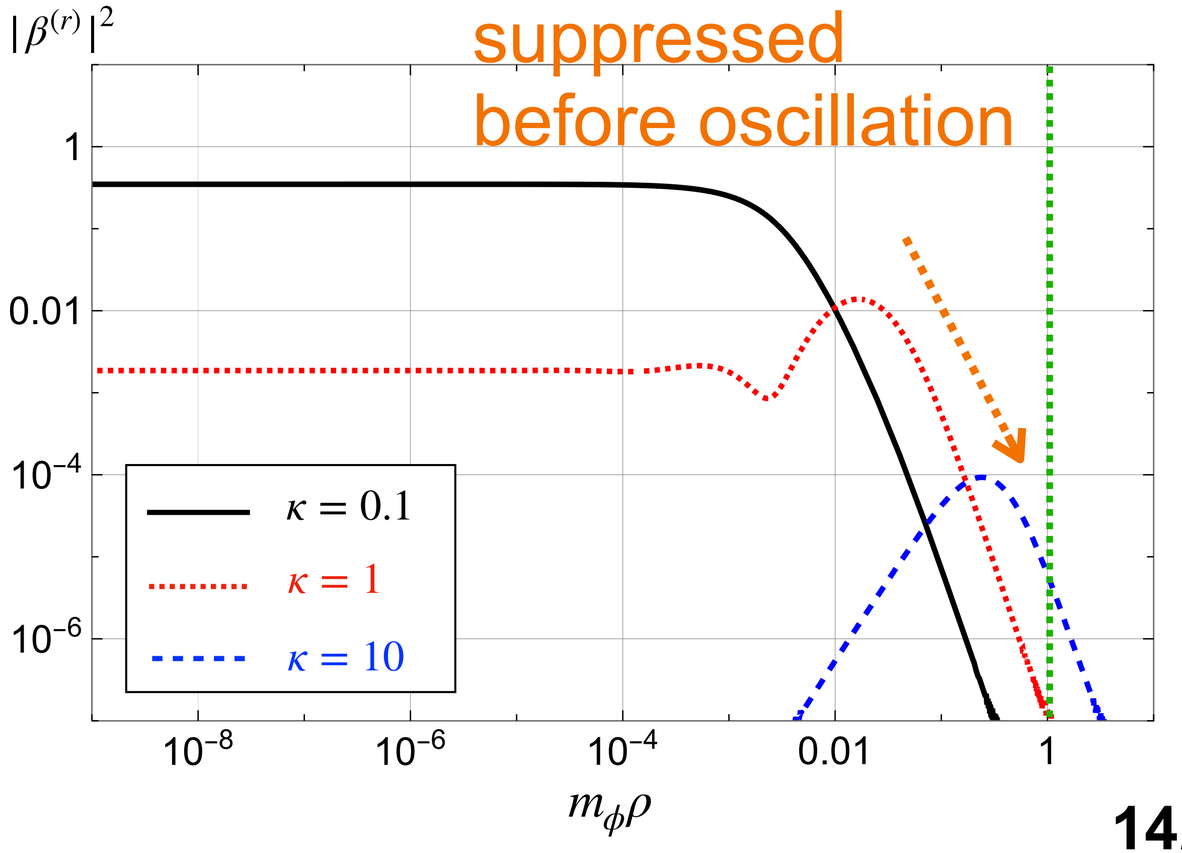
Adiabatic vacuum

We want an initial adiabatic state with no particle before the bubble collision.

$$w \ll 1 \quad \text{and} \quad |\beta^{(r)}|^2 \ll 1$$
$$(\rho \lesssim m_\phi^{-1})$$

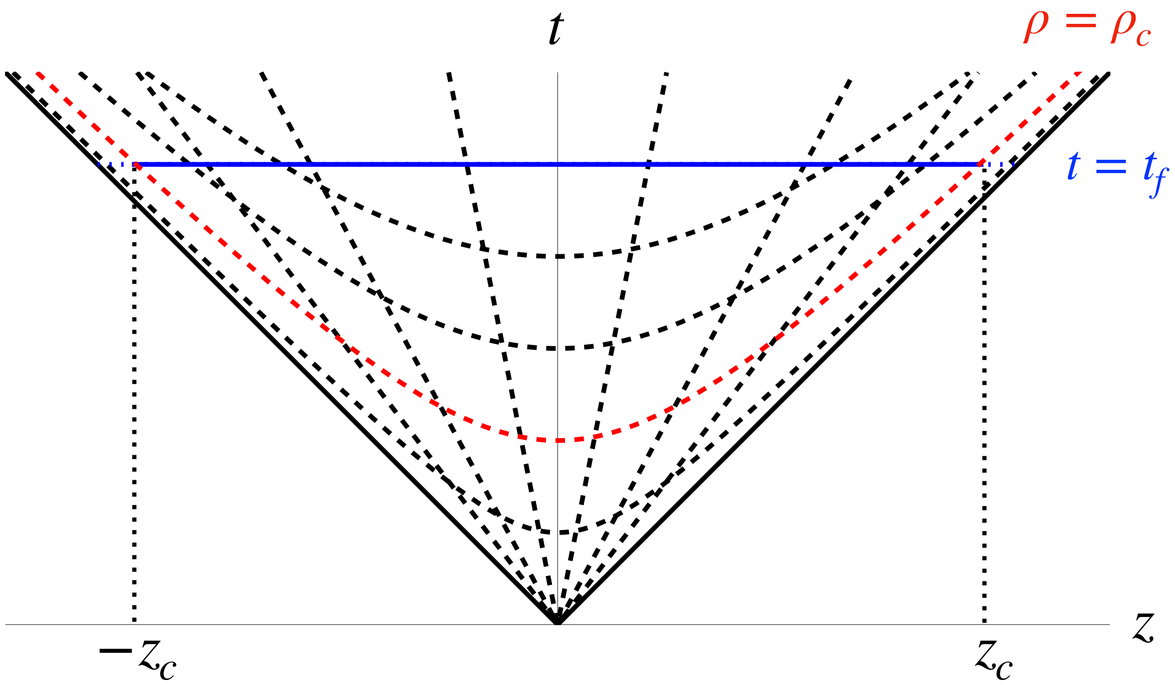
We find the initial condition so that $A_r \propto a_r$, $B_r^\dagger \propto b_r^\dagger$.

$$U_\pm^{(r)}(k_T,\kappa) = -i\sqrt{\frac{\pi\rho_i}{2}}\frac{(m_{\rm eff} - irk_T)\widetilde{H}^{(2)}_{-w^*}(m_T\rho_i) \pm m_T\widetilde{H}^{(2)}_{w}(m_T\rho_i)}{\sqrt{2m_T}},$$
$$V_\pm^{(r)}(k_T,\kappa) = i\sqrt{\frac{\pi\rho_i}{2}}\frac{(m_{\rm eff} - irk_T)\widetilde{H}^{(1)}_{-w}(m_T\rho_i) \mp m_T\widetilde{H}^{(1)}_{w^*}(m_T\rho_i)}{\sqrt{2m_T}},$$



Palermo, Roselli (2026)

4. Counting the number of particles



$$n = \frac{1}{V}\int d^2x_T \int_{-z_c}^{z_c} dz\ \langle 0|\psi_+^\dagger \psi_+|0\rangle$$

$$\frac{N}{A} \simeq \frac{1}{8\pi^2}\sum_r \int d^2k_T d\kappa\ |\beta^{(r)}|^2$$

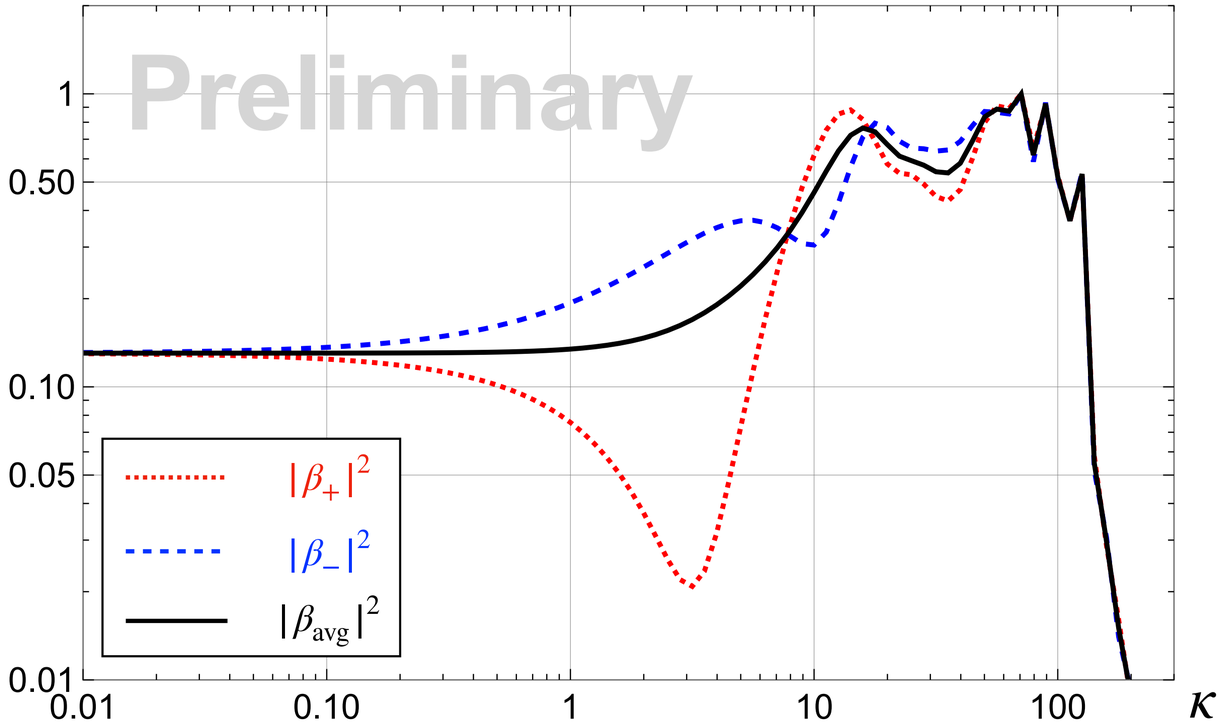
spectrum

Approximating the collision area by β, we obtain the number density of the produced fermions,

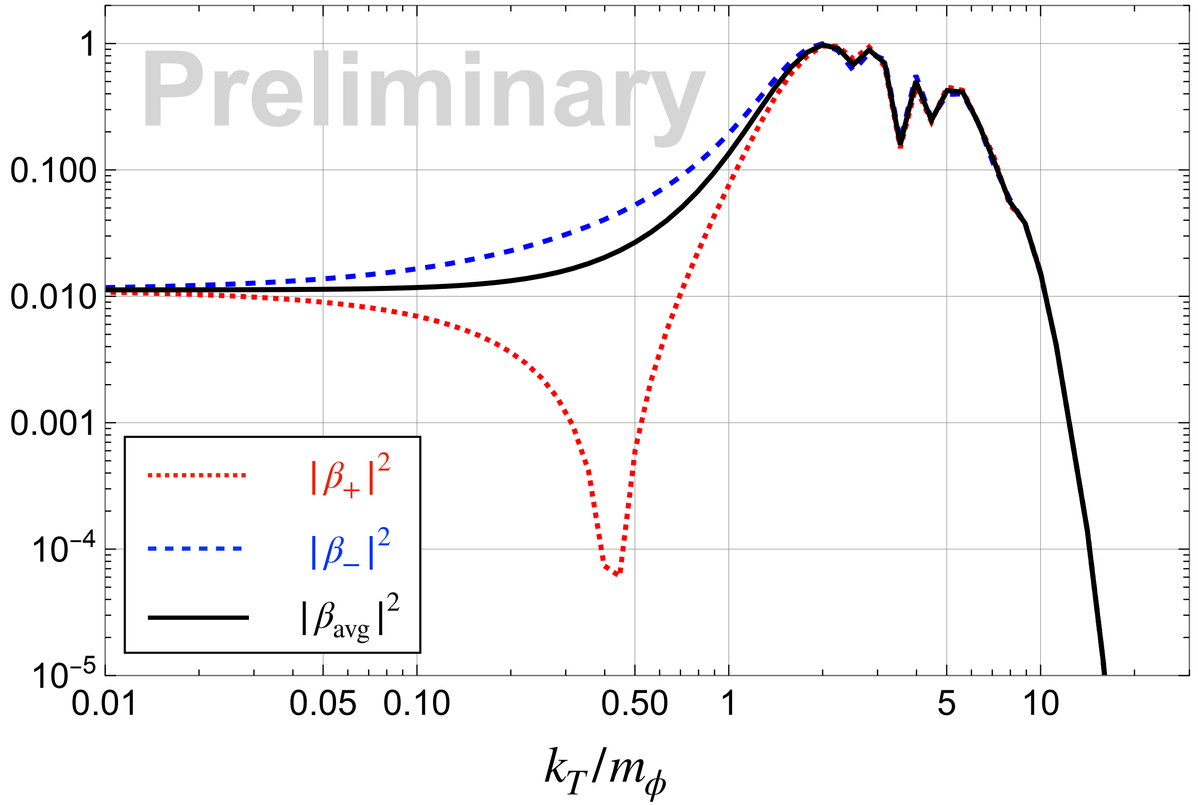
$$n_{\rm eff} \simeq \frac{3}{\beta^{-1}}\frac{N}{A}$$

β : inverse duration of phase transition

Particle spectrum



$k_T = m_\phi$



$\kappa = 1$

Here we set $m_\psi = 100 m_\phi \quad q \equiv y v_\phi^2 / 4 m_\phi^2 = 5000$

Summary

• We studied fermion production during bubble collisions by the Bogoliubov approach in Milne spacetime.

• We set a proper initial condition that reduces the Milne-coordinate artifact to establish the Bogoliubov formalism in Milne spacetime.

• The number density is estimated on a Minkowski time slice, and we obtained the particle spectrum numerically.

Thanks!

Back up

Methods of analysis

1. Off-shell approximation

Watkins, Widrow (1992), Konstandin, Servant (2011), Falkowski, No (2013), Shakya (2025), Mansour, Shakya (2025)

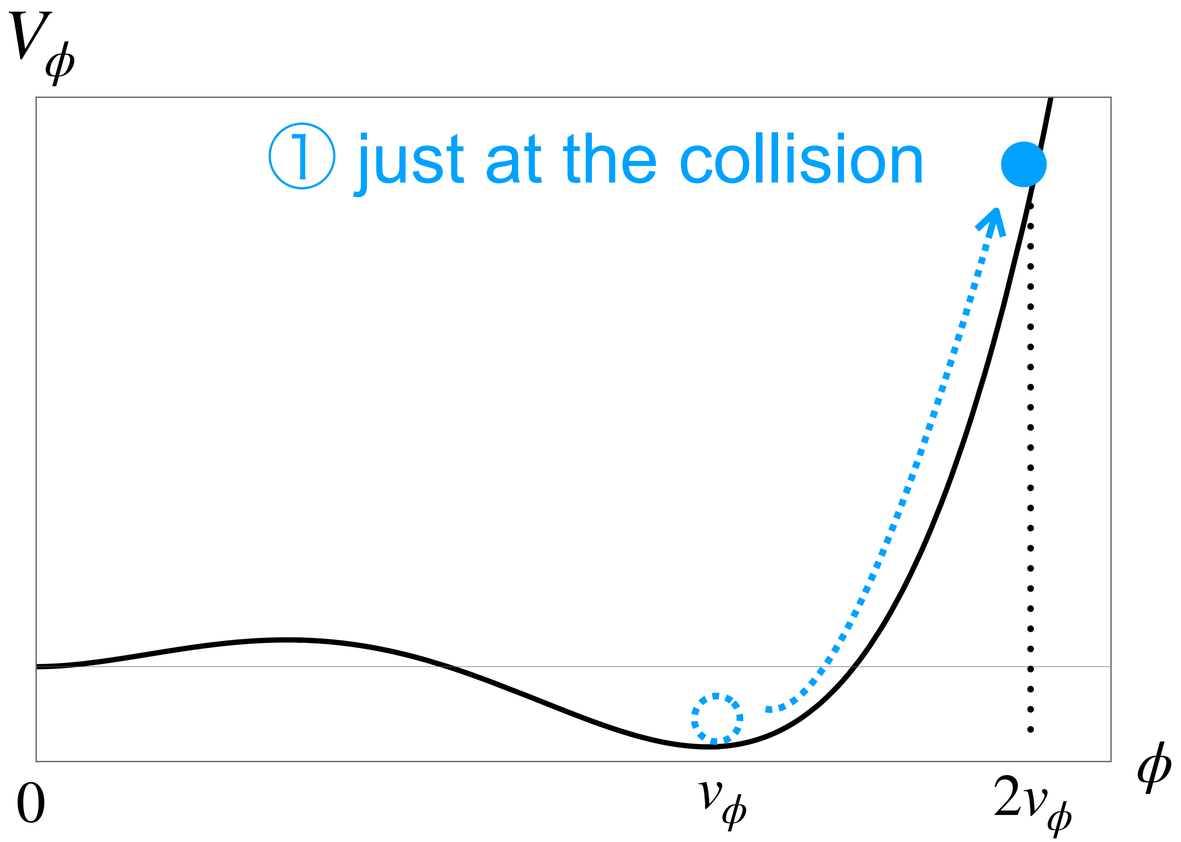
$$|\langle \text{out} | \text{in} \rangle|^2 = e^{-2\text{Im}\Gamma}$$
$$\simeq 1 - 2\text{Im}\Gamma$$

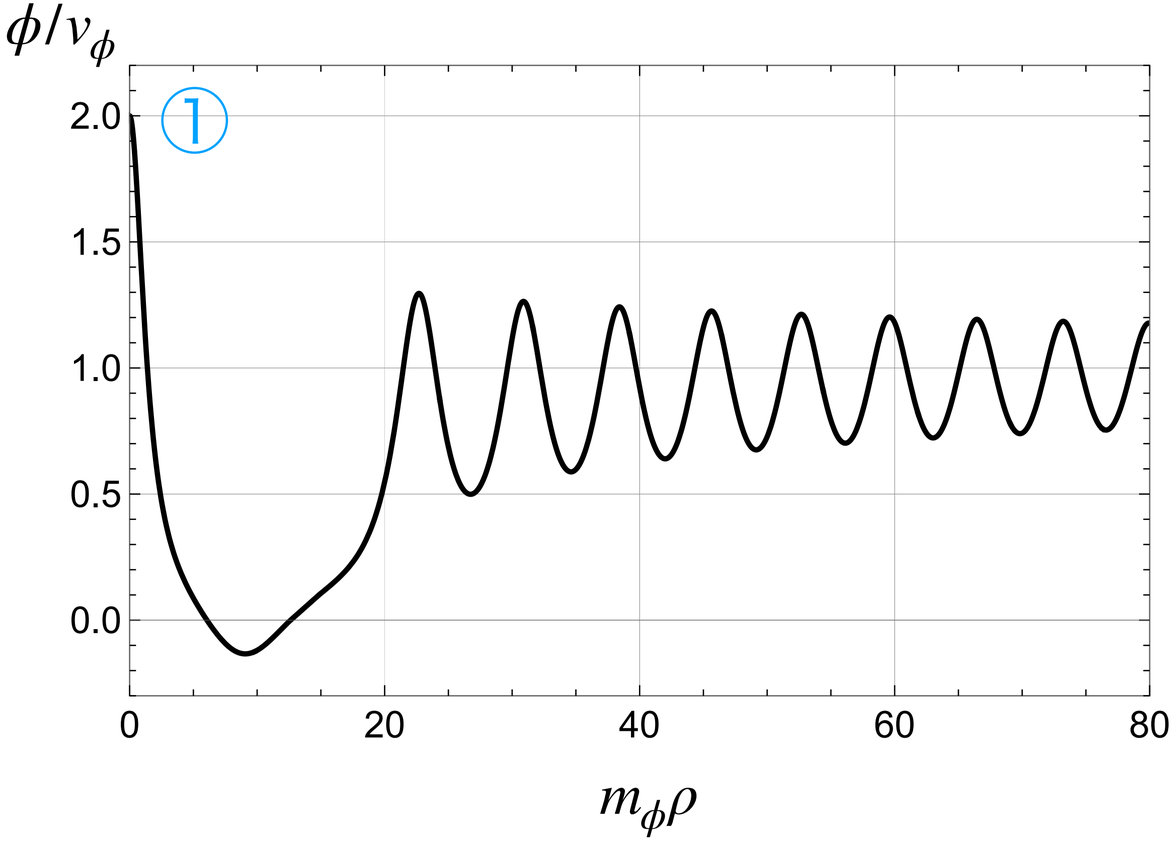
Note: there are several theoretical uncertainties: Spectral leakage An, Jiang (2026)
Gauge dependence
Numerical resolution

2. On-shell approximation Ghoshal, Pal, Strumia (2026)

3. Bogoliubov transformation **Our work**

We consider an example potential: $V(\phi) = c\left[\frac{1}{2}bv_\phi^2\phi^2 - \frac{1}{3}(b+2)v_\phi\phi^3 + \frac{1}{2}\phi^4 + V_0\right]$





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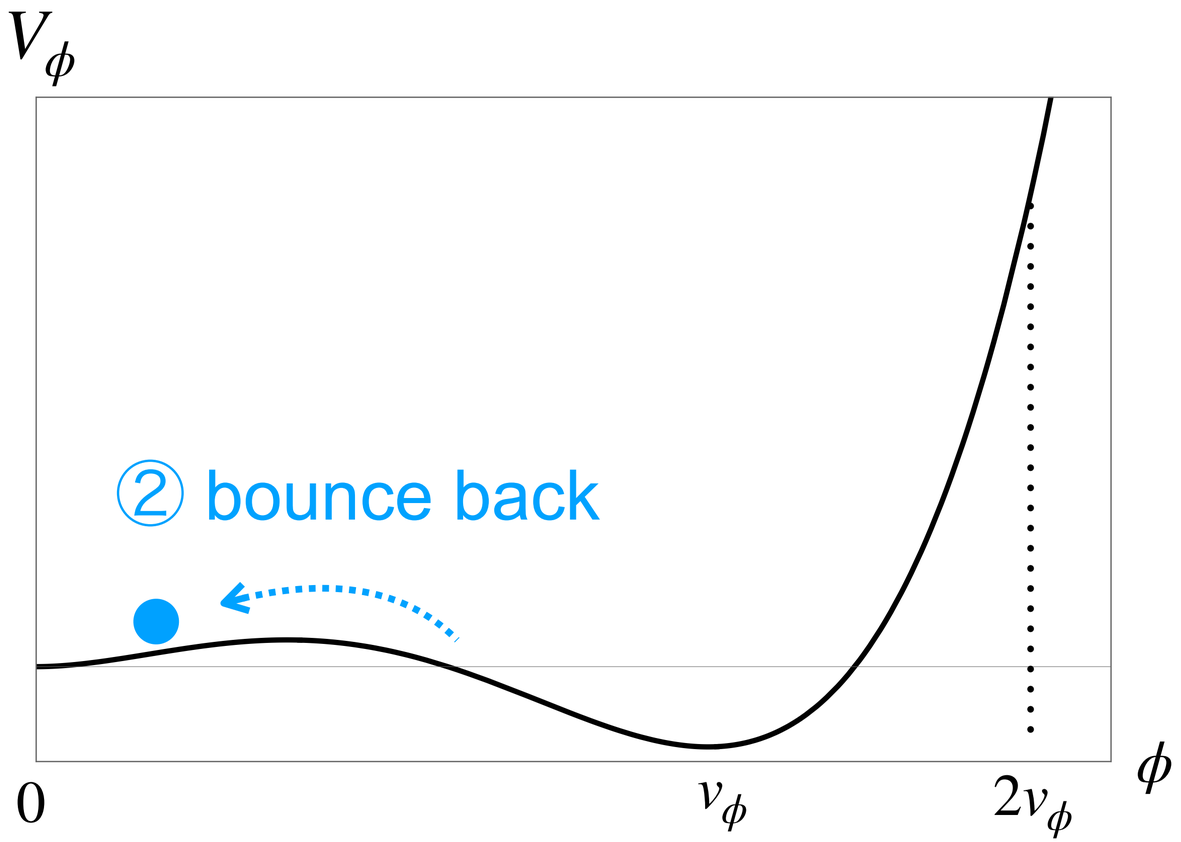

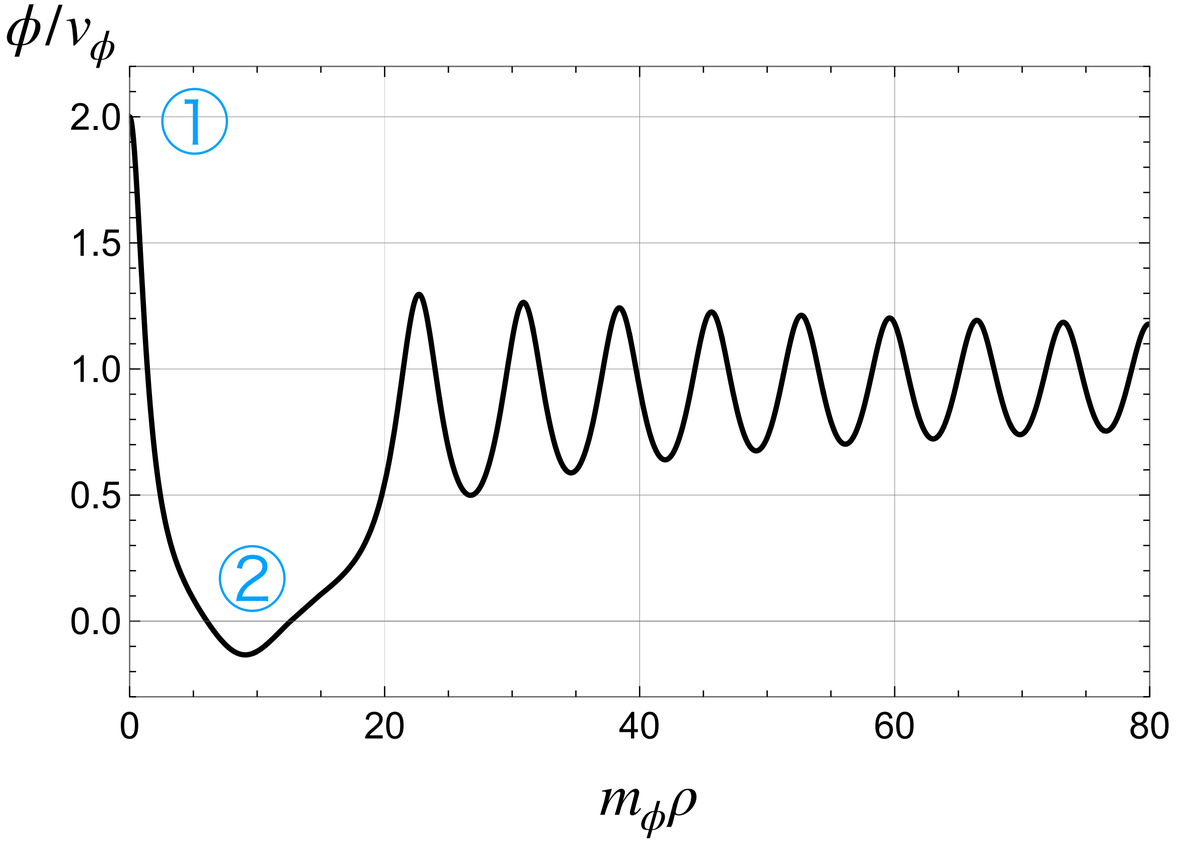

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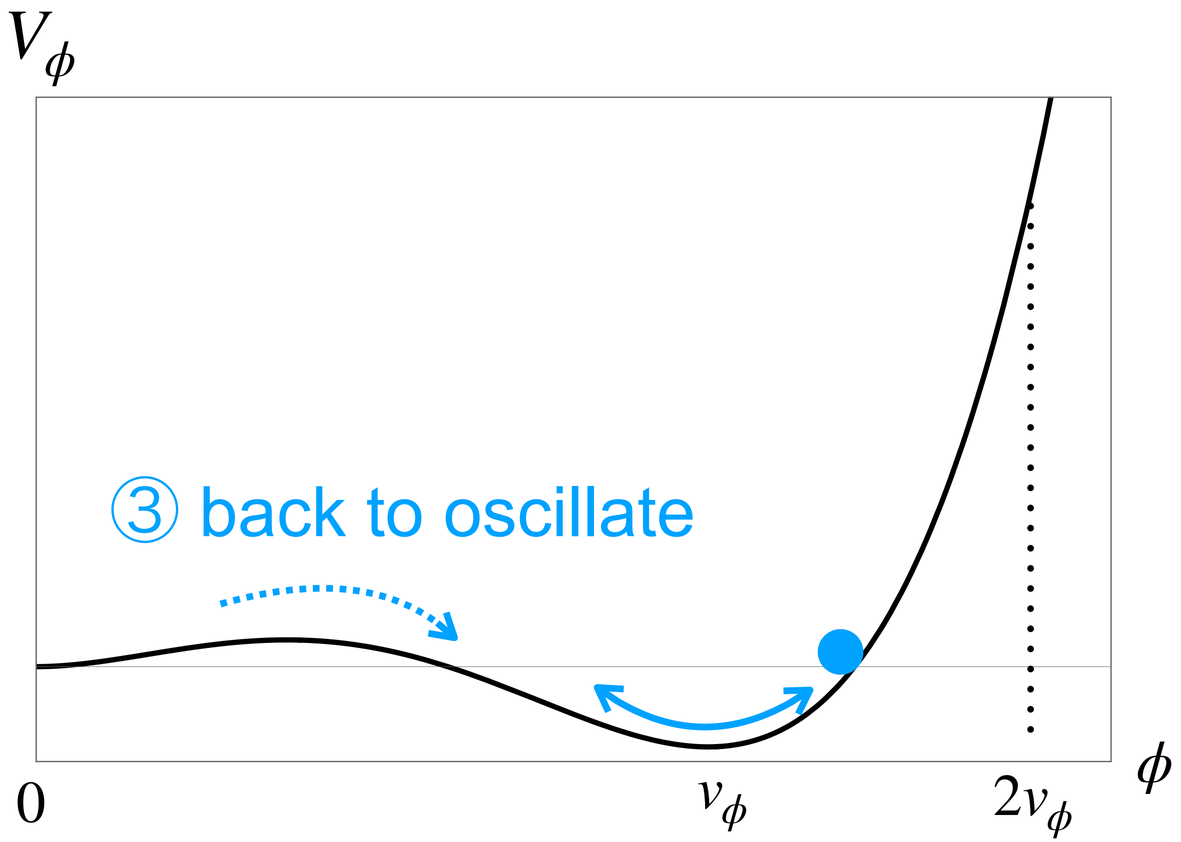
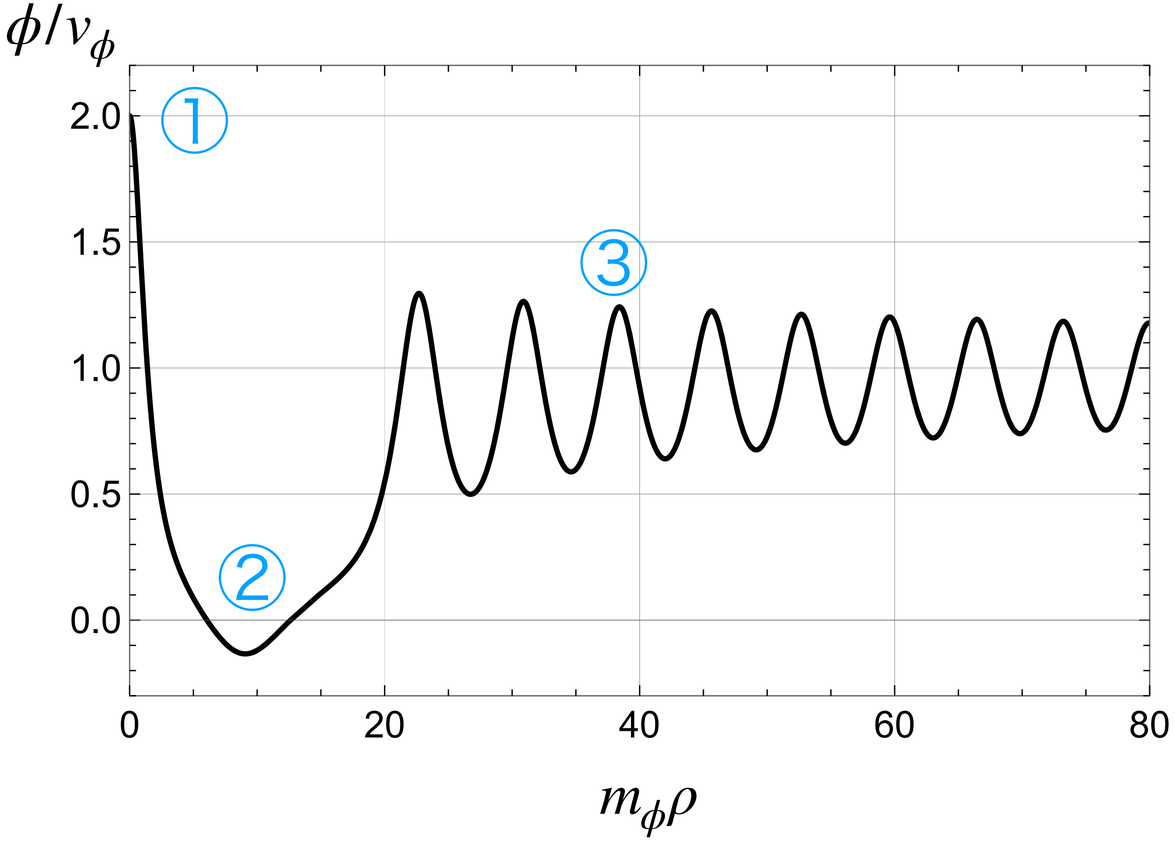




The scalar wave propagates outward from the collision surface.

Milne artifact

Uncoupled second-order equation

Non-adiabatic source

$$\left[\partial_\rho^2 + \frac{\kappa/\rho}{\kappa/\rho \pm irk_T}\frac{1}{\rho}\partial_\rho + \omega^2 \pm i\left(\partial_\rho m_{\mathrm{eff}} + \frac{\kappa/\rho}{\kappa/\rho \pm irk_T}\frac{m_{\mathrm{eff}}(\rho)}{\rho}\right)\right] U_\pm^{(r)}(\rho) = 0,$$

Even if $m_{\mathrm{eff}} = \mathrm{const}$,
it is nonzero.

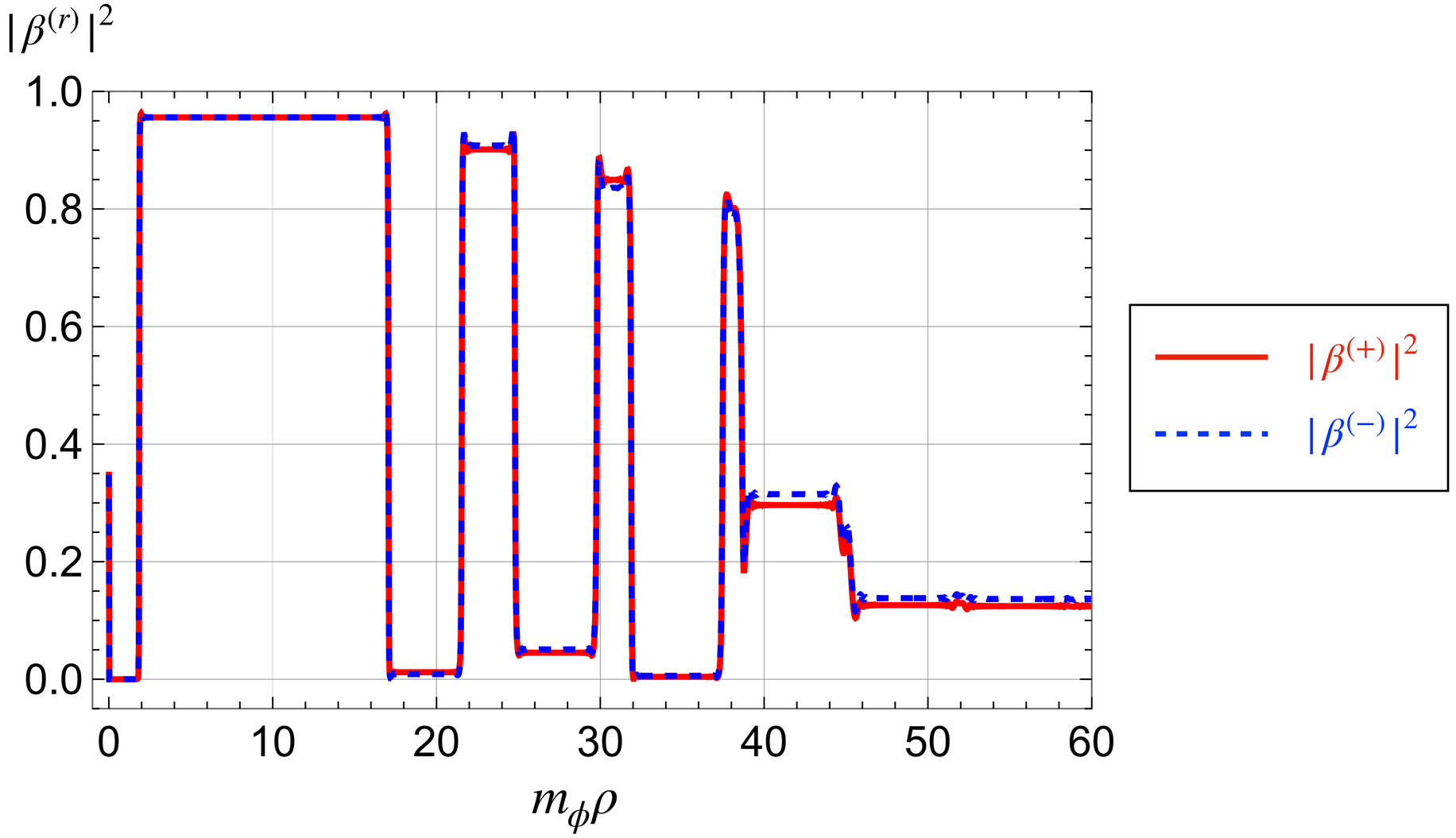
At $\rho \ll m_\phi^{-1}$, this artificial effect is significant.

Bogoliubov coefficients

$$|\beta^{(r)}|^2 = \frac{|F^{(r)}|^2}{2\omega(\omega + E_\kappa^{(r)} - rE_T^{(r)})} = \frac{\omega - E_\kappa^{(r)} + rE_T^{(r)}}{2\omega}$$

$$\frac{\alpha^{(r)}}{\beta^{(r)}} = \frac{E_\kappa^{(r)} - rE_T^{(r)} + \omega}{F^{(r)*}} \qquad \omega^2 = k_T^2 + \left(\frac{\kappa}{\rho}\right)^2 + m_{\text{eff}}^2(\rho)$$

$$E_\kappa^{(r)} = \frac{\kappa}{\rho}\text{Re}[U_+^{(r)*}U_-^{(r)}] - m_{\text{eff}}(1 - |U_+^{(r)}|^2),$$
$$E_T^{(r)} = k_T\text{Im}[U_+^{(r)*}U_-^{(r)}].$$

$$e^{-i\varphi_r}F^{(r)} = r\frac{1}{2}\frac{\kappa}{\rho}(U_+^{(r)2} - U_-^{(r)2}) - rm_{\text{eff}}U_+^{(r)}U_-^{(r)} - \frac{ik_T}{2}(U_+^{(r)2} + U_-^{(r)2}).$$

Time evolution of Bogoliubov coefficients