Mechanisms of Kinetic Equilibrium between SIMP and radiation

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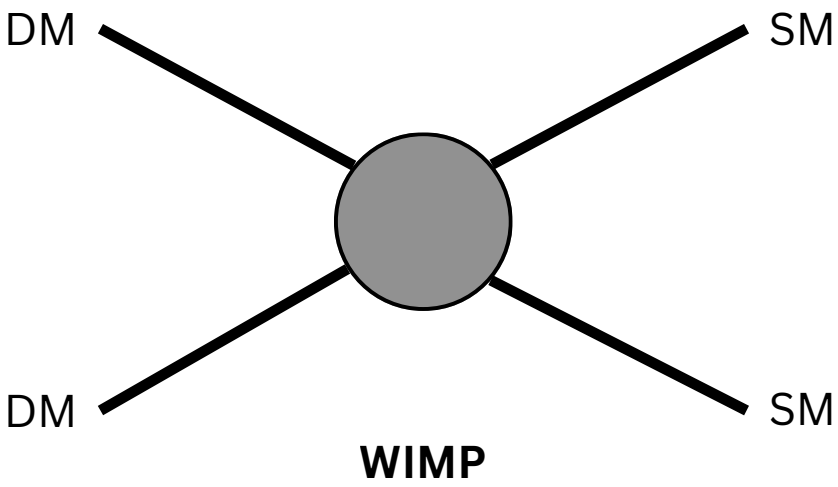
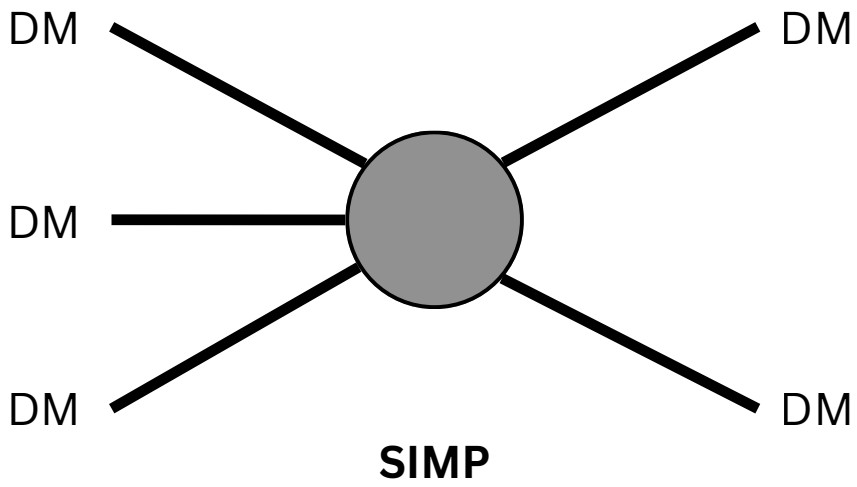
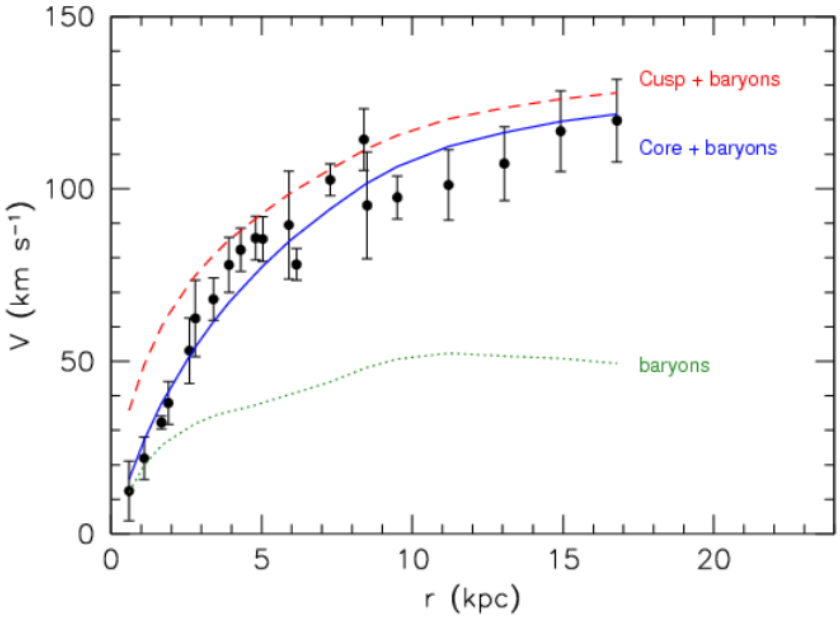
What is SIMP dark matter?





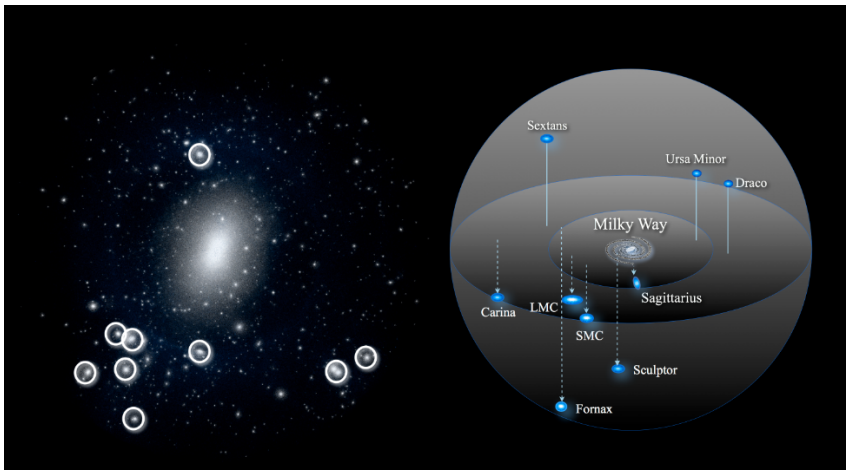
- **WIMP:** Freeze-out driven by $2 \to 2$ annihilation into Standard Model particles.
- **SIMP:** Freeze-out driven by number-changing processes within the dark sector, e.g. $3 \to 2$.

Why SIMP dark matter?



Core-Cusp Problem (Moore 1994)

Rotation curves of low-mass galaxies indicate a flat DM density at the center, while CDM simulations predict a steep density cusp.



Too Big to Fail (Boylan-Kolchin et al. 2011)

Simulations of the Milky Way's sub-halos predict many massive dwarf satellites that are not observed.

The SIMP Overheating Problem

- The $3 \to 2$ annihilations deposit energy into the dark matter sector, slowing its cooling rate.
- If this heat is not dissipated, the resulting warm/hot dark matter would suppress small-scale galaxy formation.
- **Standard solution:** Couple the SIMP via a portal to the Standard Model bath.
- **Our novel mechanism:** Introduce an ultra light dark axion as a heat sink; because it is relativistic and weakly interacting, it efficiently carries away energy and keeps the dark pions cold.

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QCD-like models and the WZW term

- QCD-like theories, after chiral symmetry breaking, can generate dark pions with the desired $3 \to 2$ process via the Wess-Zumino-Witten (WZW) term.
- Existence of such term depends on the flavor group and the symmetry-breaking pattern.
- A commonly used example in the literature is: $\mathrm{SU}(2N_f) \to \mathrm{Sp}(2N_f)$, though many other symmetry-breaking patters are possible.
- In this case, the WZW term is

$$\mathcal{L}_{\mathrm{WZW}} = \frac{2N_c}{15\pi^2 f_\pi^5} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr}\left(\pi \partial_\mu \pi \partial_\nu \pi \partial_\rho \pi \partial_\sigma \pi\right).$$

Hochberg et al. (2015)

SIMP Lagrangian: Dark Pion Sector

- Consider two quarks ($N_f = 2$) charged under the gauge group Sp(4).
- The global flavor symmetry SU(4) is broken by the chiral condensate

$$\langle q_i q_j \rangle = \mu^3 J_{ij}, \qquad J = \begin{pmatrix} 0 & I_2 \\ -I_2 & 0 \end{pmatrix}.$$

- This gives the breaking pattern SU(4) $\to$ Sp(4), producing $15 - 10 = 5$ Goldstone bosons (dark pions).

Pion matrix:

$$\Sigma = e^{i\pi/f_\pi} J e^{i\pi^T/f_\pi} = e^{2i\pi/f_\pi} J,$$

$$\pi = \sum_{a=1}^{5} \pi_a T_a = \frac{1}{2\sqrt{2}} \begin{pmatrix} \pi_3 & \pi_1 - i\pi_2 & 0 & \pi_5 - i\pi_4 \\ \pi_1 + i\pi_2 & -\pi_3 & -\pi_5 + i\pi_4 & 0 \\ 0 & -\pi_5 - i\pi_4 & \pi_3 & \pi_1 + i\pi_2 \\ \pi_5 + i\pi_4 & 0 & \pi_1 - i\pi_2 & -\pi_3 \end{pmatrix}.$$

SIMP Lagrangian

The SIMP Lagrangian is given by:

$$\mathcal{L} = \underbrace{\frac{f_\pi^2}{4}\,\mathrm{Tr}\left[(D^\mu\Sigma)(D_\mu\Sigma)^\dagger\right]}_{\text{Kinetic term}} - \underbrace{\frac{f_\pi^2}{2}B_0\,\mathrm{Tr}\left[M_q\Sigma + \mathrm{h.c.}\right]}_{\text{Mass term}} + \underbrace{\frac{8}{15\pi^2 f_\pi^5}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\sigma\pi\right)}_{\text{WZW term}}.$$

With:

$$B_0 \approx \frac{m_\pi^2}{m_u + m_d} \quad , \quad M_q = \begin{pmatrix} 0 & m_q \\ -m_q & 0 \end{pmatrix} \quad , \quad m_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

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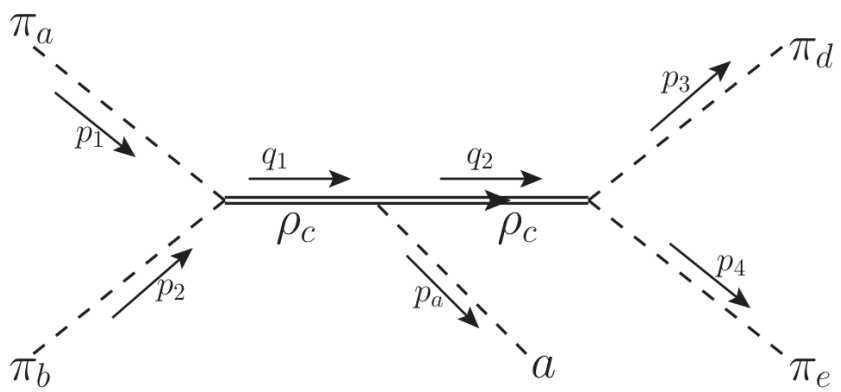
Extending the Effective Theory

- To couple the axion to the pions, we must go beyond the pure pion Lagrangian and incorporate the dark rho mesons.
- With the rho mesons acting as off-shell propagators, the following process becomes possible:

$$\pi + \pi \longleftrightarrow \pi + \pi + a.$$

- This process provides the necessary interaction between the axion and the pion sector.



New Lagrangian

- The new Lagrangian after chiral symmetry breaking is:

$$\mathcal{L} = \overbrace{\boxed{\frac{f_\pi^2}{4}\mathrm{Tr}\left[(D^\mu\Sigma)(D_\mu\Sigma)^\dagger\right]}}^{\pi\pi\rho \text{ vertex}} - \frac{f_\pi^2}{2}B_0\mathrm{Tr}\left[M_q(a)\Sigma + \mathrm{h.c.}\right]$$

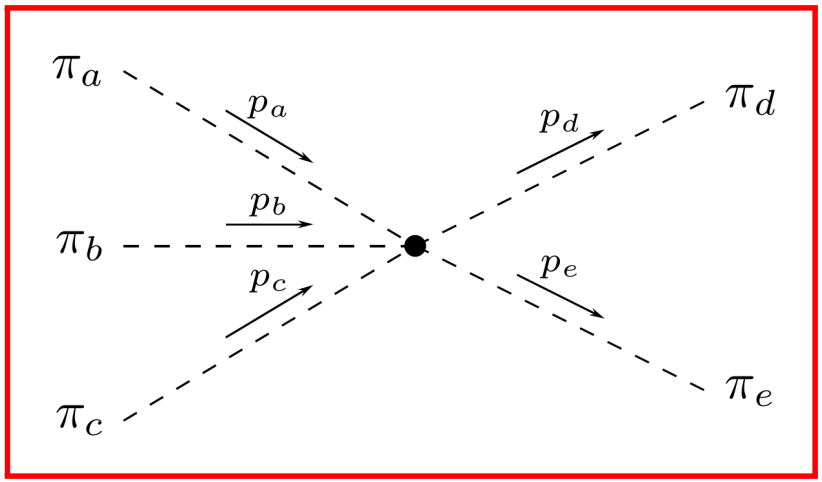
$$+ \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) - \frac{m_a^2}{2}a^2 + \overbrace{\boxed{\frac{a}{f}\sum_{\mathscr{A}_1,\mathscr{A}_2} c_{\mathscr{A}_1\mathscr{A}_2}F_{\mathscr{A}_1\mu\nu}\tilde{F}^{\mu\nu}_{\mathscr{A}_2} + \mathcal{L}_{WZW}(a)}}^{\rho\rho a \text{ vertex}}$$

With:

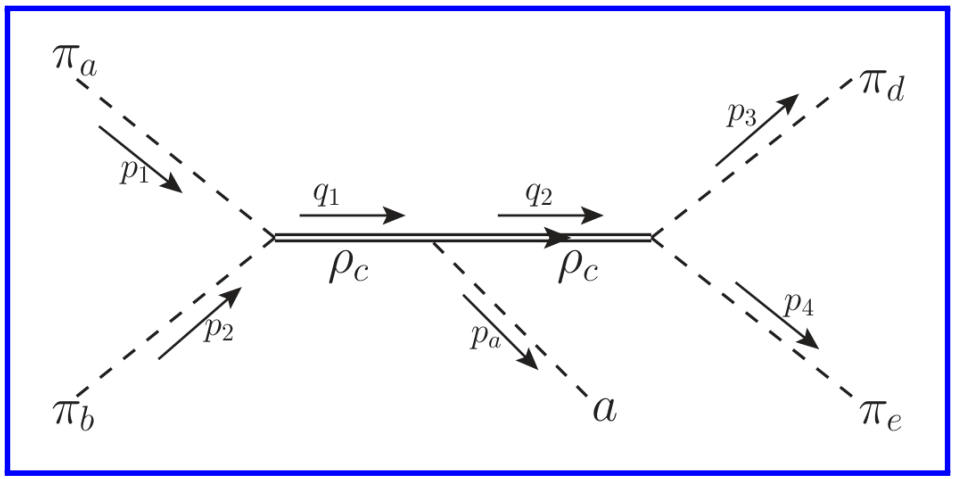
$$D_\mu\Sigma \supset \partial_\mu\Sigma + ig_\rho\left(\rho_\mu\Sigma + \Sigma\rho_\mu^T\right) + i\frac{\partial_\mu a}{f_a}\left(\mathbb{K}\Sigma + \Sigma\mathbb{K}^T\right)$$

$$\mathbb{K} = \begin{pmatrix} k_L(a) & 0 \\ 0 & -k_R(a) \end{pmatrix}$$

Boltzmann equation



$$\langle \sigma v^2 \rangle_{3\pi \to 2\pi} \propto m_\pi^3 f_\pi^{-10} T_\pi^2$$



$$\mathcal{T}(T_\pi, T_a) \propto h(m_\pi, T_\pi, T_a) f_a^{-2} (T_\pi - T_a)$$

$$\frac{dY_\pi(x)}{dx} = -\frac{1}{Hx}\langle \sigma v^2 \rangle_{3\pi \to 2\pi} s^2 Y_\pi^2 \left(Y_\pi - Y_{\pi,eq}\right),$$

$$\frac{dT_\pi(x)}{dx} = -\frac{2T_\pi}{x} + \frac{2sY_\pi}{3Hx}\left[(m_\pi + \frac{3}{2}T_\pi)\langle \sigma v^2 \rangle_{3\pi \to 2\pi} s \left(Y_\pi - Y_{\pi,eq}\right) + \mathcal{T}(T_\pi, T_a)\right]$$

$$\frac{dT_a(x)}{dx} = -\frac{T_a}{x} + \frac{15}{2\pi^2 H x T_a^3}\mathcal{T}(T_\pi, T_a) s^2 Y_\pi^2,$$

with: $x = m_\pi/T_\pi$, $\quad Y = n_\pi/s$, $\quad s =$ entropy density.

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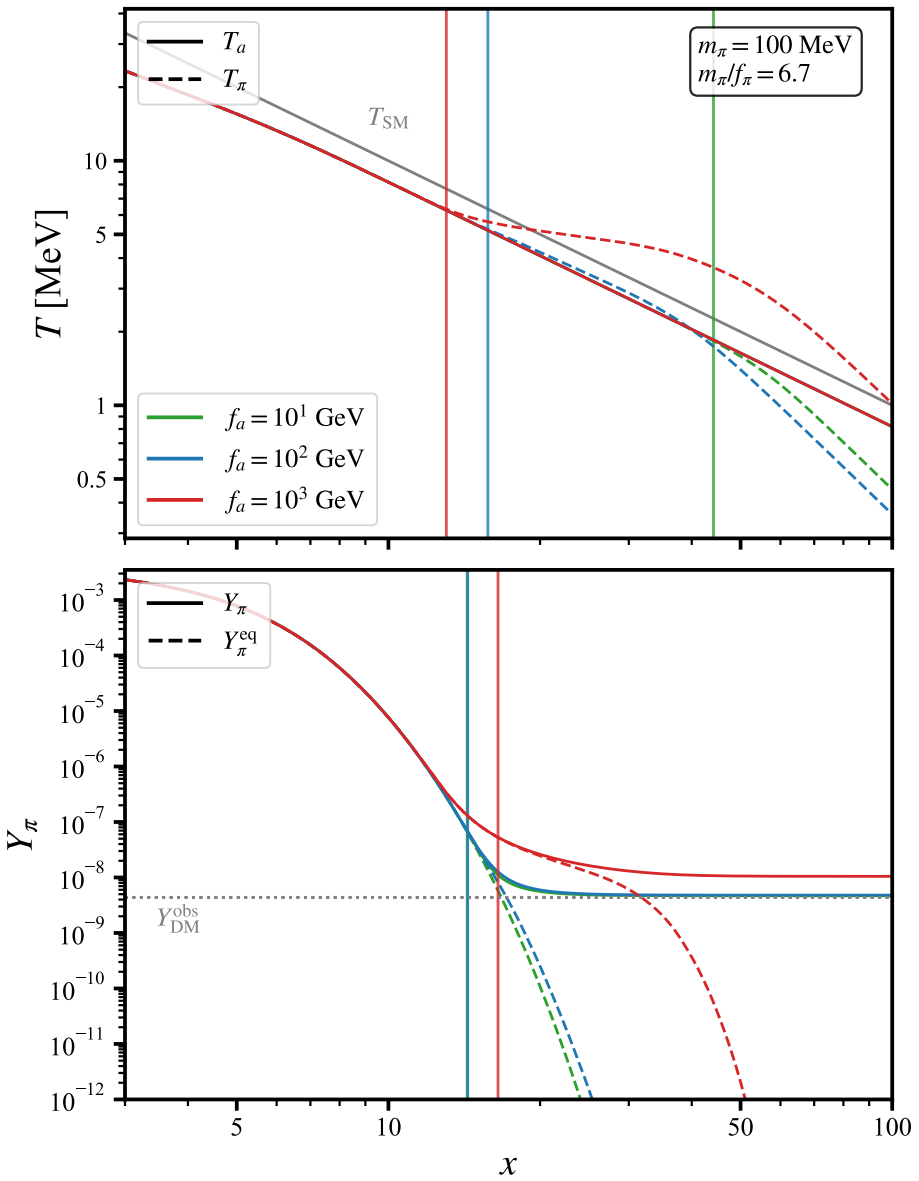
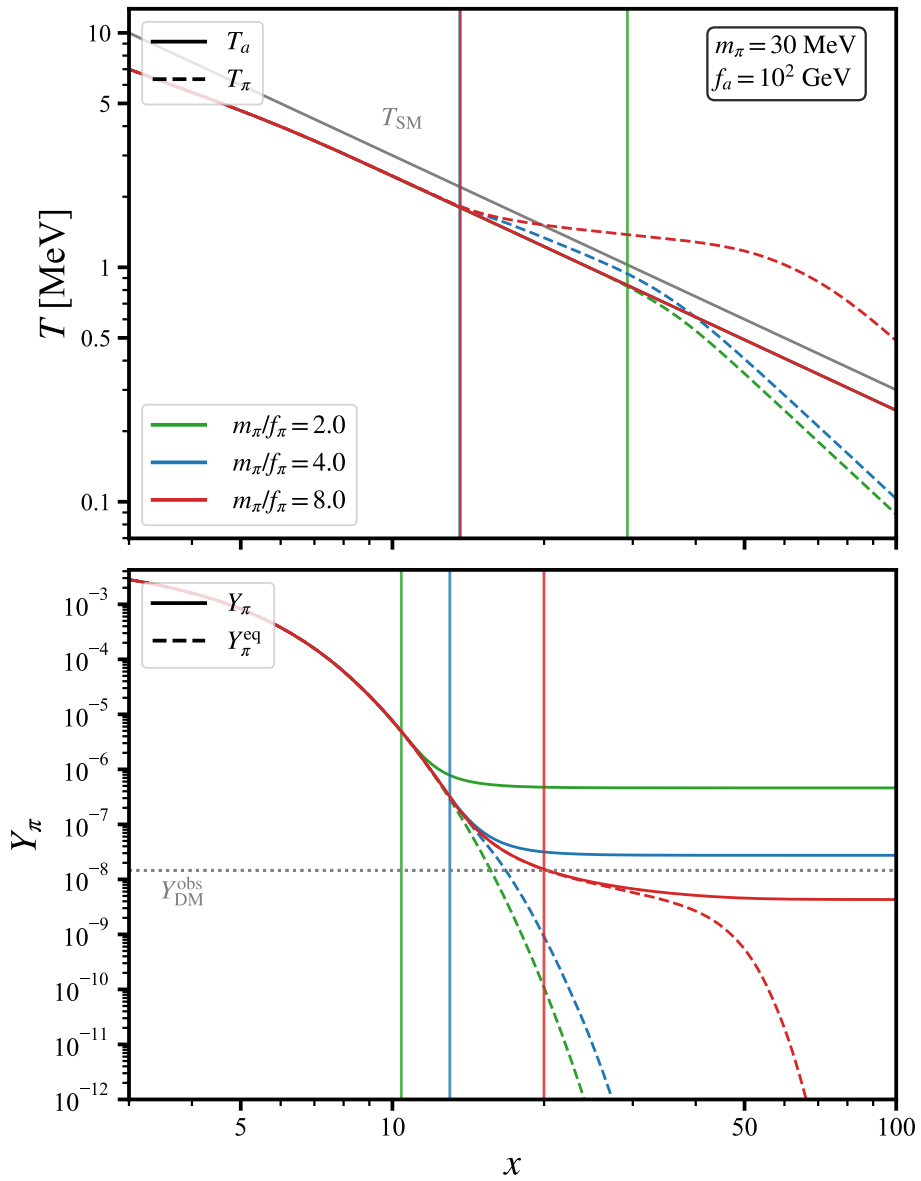
The effect of f_a on DM relic density

- Larger f_a weakens $2\pi \leftrightarrow 2\pi + a$, leading to an earlier kinetic decoupling (smaller x_k).
- $x_k > x_f$: pions and axions remain in kinetic equilibrium through freeze-out. **The relic density is then set solely by the $3 \to 2$ annihilation rate $\propto \mathbf{m_\pi^3/f_\pi^{10}}$.**
- $x_k < x_f$: decoupling before freeze-out leads to a cannibal phase with slow (logarithmic) cooling, delaying freeze-out thus enhancing the relic density, now depending on f_a too.

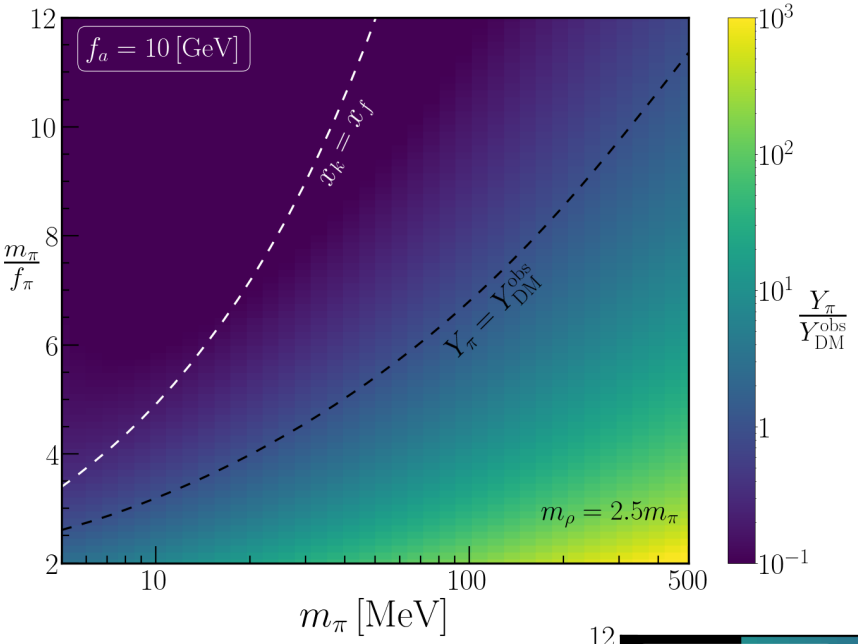


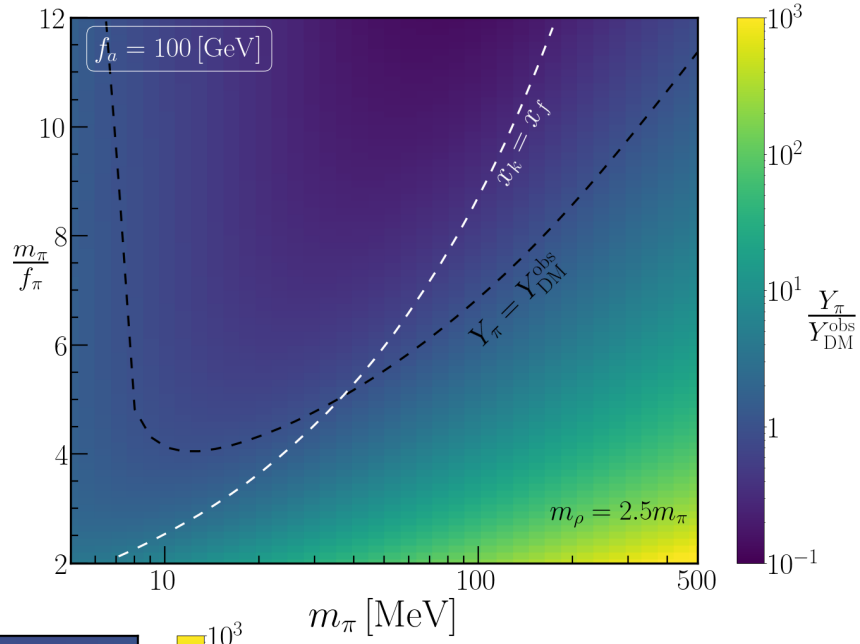
The effect of f_π on DM relic density

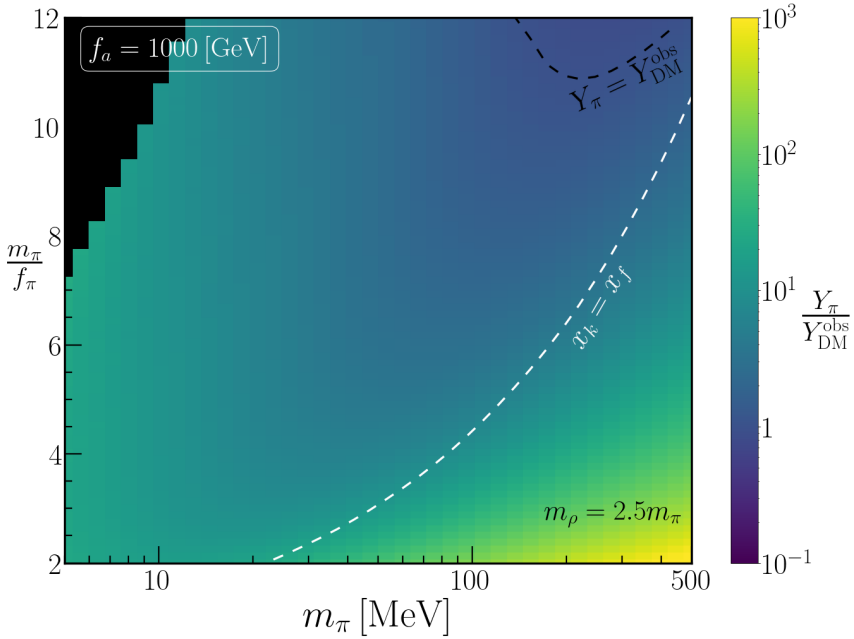
- f_π controls $\langle \sigma v^2 \rangle_{3\pi \to 2\pi}$, which affects both x_f and x_k.
- Decreasing f_π strengthens $3 \to 2$ annihilation $\Rightarrow$ later freeze-out (x_f increases) but earlier kinetic decoupling (x_k decreases).
- $x_k < x_f$:the cannibal phase delays freeze-out, so the final relic density is larger than the naive $3 \to 2$ scaling would predict.



Parameter scan







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Conclusion

- We showed that SIMP dark matter can account for the observed relic abundance while remaining cold, without requiring a portal to the Standard Model.
- This is achieved by introducing an ultra light dark axion that acts as a heat sink, efficiently draining the excess energy from $3 \to 2$ annihilations.
- For dark pion masses $m_\pi \sim 100$–500 MeV, the correct relic density can be obtained for f_a as large as 1 TeV.
- For $f_a = 100$ GeV, the viable mass range extend down to $m_\pi \sim 10$ MeV.
- This significantly broadens the allowed parameter space for SM couplings, since the portal is no longer needed to cool the dark sector.