Thermal $\mathbb{Z}_3$ domain wall of Standard Model in the universe

Yiming Yang
University of Wisconsin-Madison

2026.09.10

Work with Yang Bai,

Content

1. Background
2. SM 1-form symmetry
3. PNJL model
4. $\mathbb{Z}_3$ DW property
5. Cosmological evolution
6. Summary

1-form symmetry

- Ordinary symmetry is also called 0-form symmetry $G^{(0)}$, acting on **local operator** $\mathcal{O}_R(x)$. E.g. When $\Sigma_3(t)$ is spacial 3d surface, the unitary operator $U_g(\Sigma_3(t))$ of $g \in G^{(0)}$ is defined.
- 1-form symmetry $G^{(1)}$, by contrast, is generated by $d = 4 - 2 = 2$ operators $U_g(\Sigma_2)$ and acts on **line operators**.
- Compare to the unitary transformation

$$U_g(\Sigma_3(t))[\mathcal{O}_R(x)] = U_g(\Sigma_3(t))\mathcal{O}_R(x)U_g^{-1}(\Sigma_3(t)) = R(g)\mathcal{O}_R(x) \tag{1}$$

the 1-form symmetry transformation generated by closed 2d surface

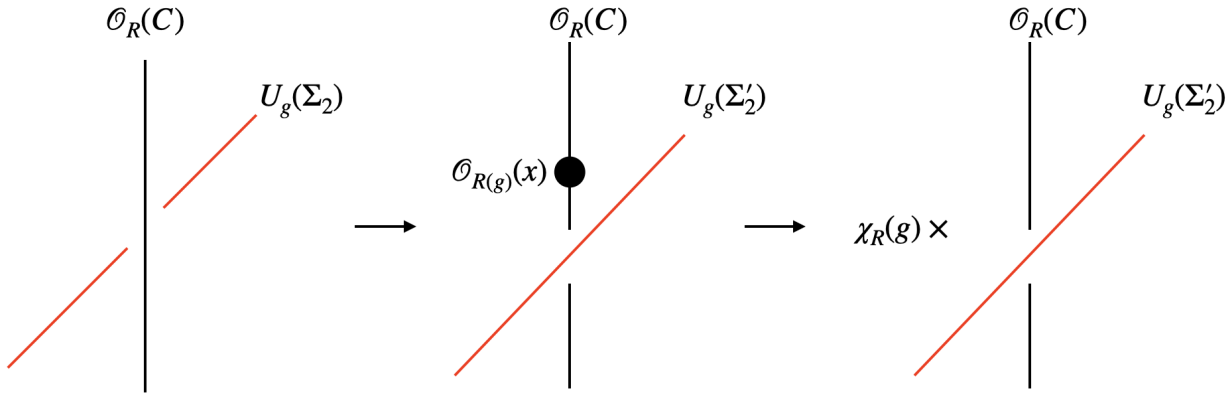
$$\left\langle U_g(S^2)\,\mathcal{O}_R(C\,)\right\rangle = \chi_R(g)^{\mathrm{Link}(S^2,C)}\left\langle \mathcal{O}_R(C\,)\right\rangle, \tag{2}$$

under the path integral sense.

1-form symmetry

- With open surface Σ_2 Σ_2'

$$\langle U_g(\Sigma_2)\,\mathcal{O}_R(C\,)\rangle = \chi_R(g)^{\mathrm{Link}(\Sigma_2,C)}\,\langle \mathcal{O}_R(C\,)U_g(\Sigma_2')\rangle\,, \tag{3}$$



so is the the closed surface.

- The topology invariance implies the higher form symmetry commutes with all element in G, so it is always in the center and $G^{(1)}$ is **abelian**.
- $\chi_R : G^{(1)} \to U(1)$ is the character specifying the 1-form charge of $\mathcal{O}_R$.

1-form $\mathbb{Z}_N$ symmetry of $SU(N)$

- If consider $SU(N)$ gauge theory, center is $\mathbb{Z}_N$. The rep R has N-ality $\mathbf{n}_R$

$$\chi_R(g_k)^{\text{Link}(\Sigma_2,C)} = \exp\left[\frac{2\pi i}{N}\, k\, \mathbf{n}_R\, \text{Link}(\Sigma_2, C)\right]. \tag{4}$$

- The line operator is **Wilson line**

$$W_R(C) = \mathcal{P}\exp\left(i \oint_C A\right), \tag{5}$$

- When crossing the surface Σ_2 along infinite small path γ, the **gauge holomony has shift** generated by the Poincaré duality 1-form C_1 of Σ_2,[1]

$$\exp\left(i \oint_\gamma A\right) \to \exp\left(i \oint_\gamma C_1 + A\right) = \exp\left(i\frac{2\pi k}{N}\right)\exp\left(i \oint_\gamma A\right), \tag{6}$$

and the 2-form $\mathrm{d}C_1$ defines transformation $U_k(\Sigma_2)$,

$$\langle U_k(\Sigma_2) W_r(C)\rangle = \chi_R \langle W_r(C)\rangle, \tag{7}$$

so the Wilson lines change the $\mathbb{Z}_N$ phase, and are the operators form the representation of 1-form symmetry center of $SU(N)$.

[1] S. Gukov and E. Witten, hep-th/0612073.

SM 1-form symmetry

- The universal covering group of SM gauge group $\widetilde{G}_{\rm SM} = SU(3)_c \times SU(2)_W \times U(1)_Y$, without dynamical matter fields and magnetic monopoles, has global 1-form symmetry

$$\mathbb{Z}_{3,c}^{\mathfrak{e}(1)} \times \mathbb{Z}_{2,W}^{\mathfrak{e}(1)} \times U(1)_Y^{\mathfrak{e}(1)} \times U(1)_Y^{\mathfrak{m}(1)}, \tag{8}$$

- Including the SM matter fields breaks the electric 1-form symmetry of the down to a diagonal subgroup, $\mathbb{Z}_6^{\mathfrak{e}(1)}$, $q_6(R) = 2\,\mathbf{n}_3^{\mathfrak{e}} + 3\,\mathbf{n}_2^{\mathfrak{e}} + 6\,Y \pmod 6$

particle	$SU(3)_c \times SU(2)_L \times U(1)_Y$	$(\mathbf{n}_3^{\mathfrak{e}}, \mathbf{n}_2^{\mathfrak{e}}, 6Y)$
Q_L	(**3**,**2**,1/6)	(1,1,1)
u_R	(**3**,**1**,2/3)	(1,0,4)
d_R	(**3**,**1**,-1/3)	(1,0,-2)
L_L	(**1**,**2**,-1/2)	(0,1,-3)
e_R	(**1**,**1**,-1)	(0,0,-6)
H	(**1**,**2**,1/2)	(0,1,3)

SM 1-form symmetry

- General Wilson line operator in rep $R = (R_c, R_W)_Y$ is

$$\mathcal{W}_R(C) \equiv \mathrm{Tr}_R\, \mathcal{P} \exp\left[i \oint_C dx^\mu \left(g_s G^a_\mu T^a_{R_c} + g_W W^i_\mu T^i_{R_W} + g_Y Y B_\mu \right) \right], \quad (9)$$

The full set of Wilson line is $G_W = (\mathbf{n}_3, \mathbf{n}_2, 6Y)$ is the unbroken 1-form group. The diagonal electric $\mathbb{Z}_6^{\mathfrak{e}(1)}$ charge

$$q_6(R) = 2\,\mathbf{n}_3^{\mathfrak{e}} + 3\,\mathbf{n}_2^{\mathfrak{e}} + 6\,Y \pmod 6\,, \quad (10)$$

is $q_6(R) \equiv 0 \pmod 6$ for all matters, so corresponding **6 genuine Wilson line operators** for exact global 1-form symmetry.[1]

[1] D. Tong, JHEP 07 (2017) 104, [1705.01853].

SM 1-form symmetry

- A more general way is to consider the lattice of possible Wilson lines $W_{\mathbf{n}_3,\mathbf{n}_2,6Y}$. Dynamical charged matter fields can **screen** Wilson lines. For instance, since Q_L carries the charge label $(1,1,1)$, the composite object

$$W_{\mathbf{n}_3,\mathbf{n}_2,6Y}(C)\,Q_L(x\in C) \tag{11}$$

transforms in the **same way** as the Wilson line $W_{\mathbf{n}_3+1,\mathbf{n}_2+1,6Y+1}(C)$. These two Wilson lines therefore belong to the same screening class. The group of unscreened is smaller than G_W.

$$D_1 = G_W/Q_{\text{matter}} = \frac{\mathbb{Z}_3\times\mathbb{Z}_2\times\mathbb{Z}}{\langle(\mathbf{n}_3^{\mathfrak{e}},\mathbf{n}_2^{\mathfrak{e}},6Y) : \text{matter}\in\text{SM}\rangle}\ . \tag{12}$$

- Using the **Smith normal form**, $D_1\simeq\mathbb{Z}_6$. The global electric 1-form symmetry is the **Pontryagin dual** of D_1. Since $\mathbb{Z}_6$ is self-dual, one obtains the global electric 1-form symmetry

$$G^{\mathfrak{e}(1)} = \widehat{D}_1 \simeq \mathbb{Z}_6^{\mathfrak{e}(1)}\ . \tag{13}$$

Quotient of SM group

- The local QFT only cares the algebra, so the SM gauge group can be $G_{\text{SM}} = SU(3)_c \times SU(2)_W \times U(1)_Y/\Gamma$, $\Gamma = 1, \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_6$.
- This makes the 1-form symmetry also smaller, $G^{(1)} = \mathbb{Z}_{6/p}$.
- E.g. $G_{\text{SM}} = SU(3)_c \times SU(2)_W \times U(1)_Y/\mathbb{Z}_3 \cong U(3) \times SU(2)$. The genuine Wilson line lattice is

$$D_1 = G_W/Q_{\text{matter}} = \frac{\mathbb{Z}_2 \times \mathbb{Z}}{\langle (\mathbf{n}_2^{\mathfrak{e}}, 6Y) : \text{matter} \in \text{SM} \rangle} \cong \mathbb{Z}_2. \tag{14}$$

- If one want to add new particles in this theory (BSM), the it must satisfy $2\,\mathbf{n}_3^{\mathfrak{e}} + 6\,Y \equiv 0 \pmod 3$. Some freedom to assign exotic charged matter, like $(\mathbf{3}, \mathbf{1}, 0)$ disappear compared to $\Gamma = 1$, but particle like $(\mathbf{3}, \mathbf{2}, 2/3)$ is still allowed .
- With current understanding and experiment, we cannot decide what quotient Γ is in SM at all!

Quotient of SM group

- We need to know the quotient. Some GUT are only allowed when SM gauge group is small enough. E.g. $SU(5)$ GUT requires $\Gamma = \mathbb{Z}_6$.
- Some searches are done about exotic charge.[1]
- Study of the 1-form symmetry can on the other side be used to tell the quotient. If the $\mathbb{Z}_3$ object is discovered, the possibility of quotient $\mathbb{Z}_3$ and $\mathbb{Z}_6$ is **excluded**.
- The 1-form symmetry itself is not easy for phenomenology. However, in thermal field theory, when **compactifying the time direction** to thermal circle gives an unwrapped and a wrapped descendant. i.e., a 4d 1-form symmetry gives both a 3d 1-form symmetry and a 3d 0-form symmetry. [2]

$$G^{(1)}_{d=4} \to G^{(1)}_{d=3} \times G^{(0)}_{d=3} \ . \tag{15}$$

- The breaking of 0-form induces domain wall in the thermal history of universe.

[1] S. Koren and A. Martin, SciPost Phys. 18 (2025) 004, [2406.17850].

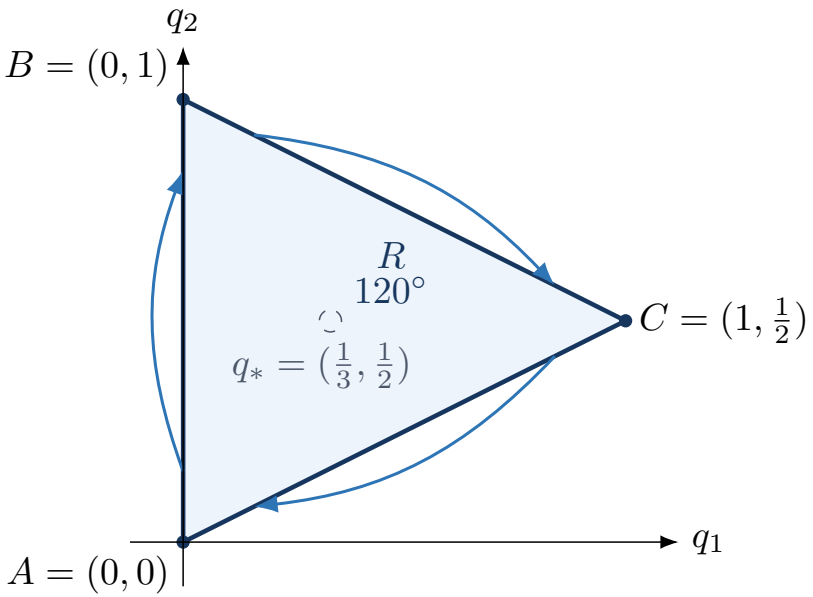
[2] D. Gaiotto, A. Kapustin, Z. Komargodski and N. Seiberg, JHEP 05 (2017) 091, [1703.00501].

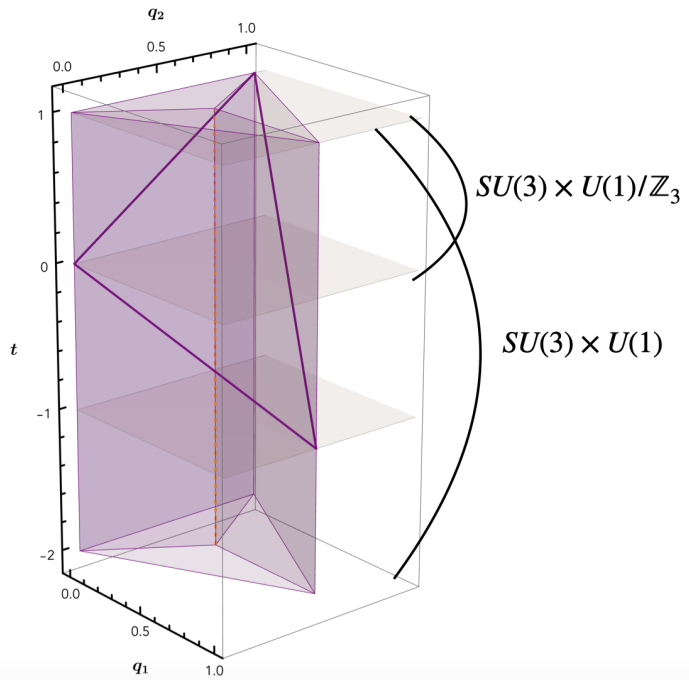
$\mathbb{Z}_3$ symmetry in $SU(3)_c \times U(1)_{\rm EM}$

- q_1, q_2, t to present the $\mathbb{Z}_3$

$$G_0(\mathbf{x}) = \frac{2\pi T}{g_s}\left[q_1(\mathbf{x})\mathrm{diag}\left(\frac{1}{2}, -\frac{1}{2}, 0\right) + q_2(\mathbf{x})\mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)\right],$$
$$A_0(\mathbf{x}) = \frac{2\pi T}{e}\, t(\mathbf{x})\ . \tag{16}$$

- The permutation of the three diagonal components, and the trivial periodicity constraint the **smallest cell**.



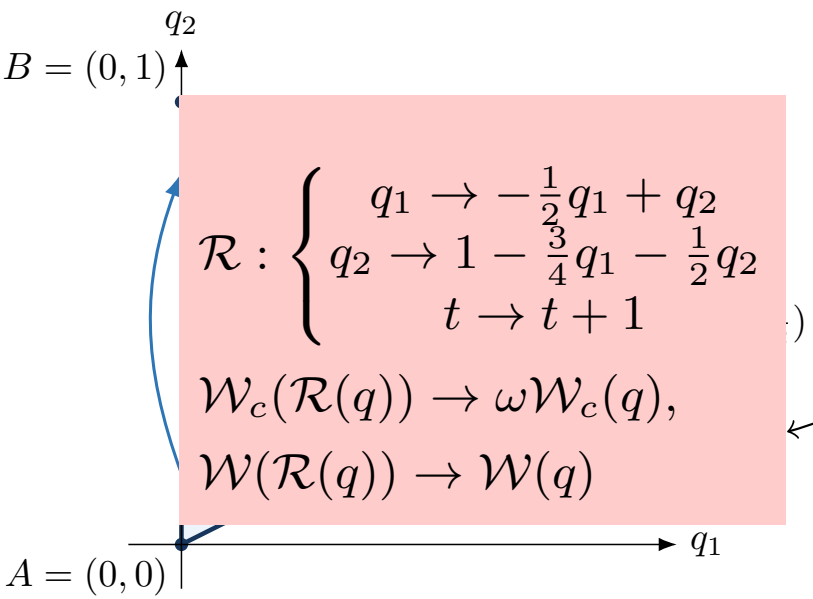


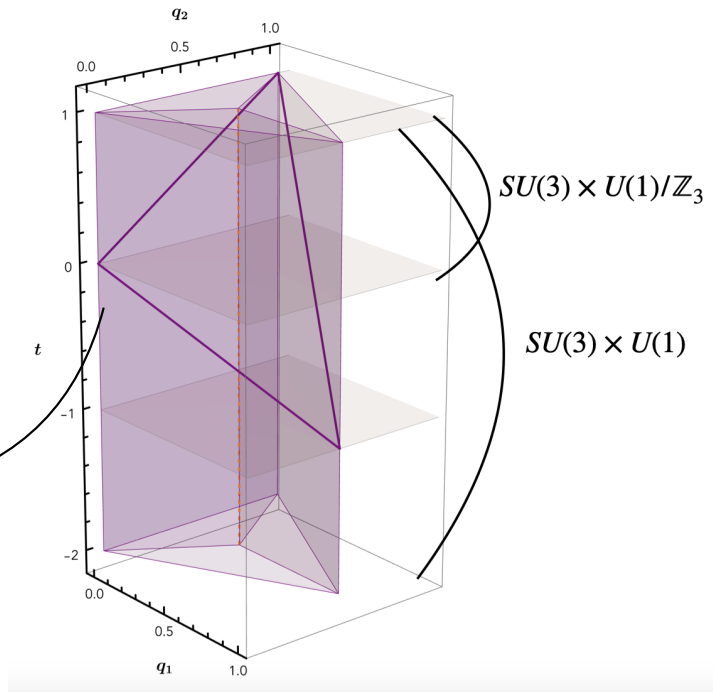
$\mathbb{Z}_3$ symmetry in $SU(3)_c \times U(1)_{\rm EM}$

- q_1, q_2, t to present the $\mathbb{Z}_3$

$$G_0(\mathbf{x}) = \frac{2\pi T}{g_s}\left[q_1(\mathbf{x})\mathrm{diag}\left(\frac{1}{2}, -\frac{1}{2}, 0\right) + q_2(\mathbf{x})\mathrm{diag}\left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)\right],$$
$$A_0(\mathbf{x}) = \frac{2\pi T}{e}\, t(\mathbf{x})\ . \tag{17}$$

- The permutation of the three diagonal components, and the trivial periodicity constraint the **smallest cell**.





PNJL-like model

- Defining the **Polyakov loop** as the trace of the thermal Wilson line

$$L_c(\vec{x}) \equiv \frac{1}{3}\,\mathrm{Tr}\left(\mathcal{W}_c(q_i(\vec{x}))\right)\,, L(\vec{x}) \equiv \frac{1}{3}\,\mathrm{Tr}\left(\mathcal{W}_c(q_i)\,\mathcal{W}_{\mathrm{EM}}^{(Q_f)}\right) = e^{2\pi i Q_f\, t}\, L_c\,, \quad (18)$$

- The effective potential, is[1]

$$\Omega(q_1, q_2, t, \Sigma; T) = U_{\mathrm{g}}\left(L_c, L_c^\dagger; T\right) + \frac{\Sigma^2}{2\,G} + \Omega_{\mathrm{m}}(q_i, t, \Sigma; T) - (0\ T \text{ quark}). \quad (19)$$

- The U_{g} is the **lattice potential**,

$$\frac{U_{\mathrm{glue}}}{T^4} = -\frac{b_2(T)}{2} L_c^\dagger L_c - \frac{b_3}{6}\left(L_c^3 + L_c^{\dagger 3}\right) + \frac{b_4}{4}\left(L_c^\dagger L_c\right)^2, \quad (20)$$
$$\text{with} \quad b_2(T) = a_0 + a_1\,(T_0/T) + a_2\,(T_0/T)^2 + a_3\,(T_0/T)^3\,.$$

- Ω_{m} is the contribution from **matter**, both quark and lepton. E.g.[2]

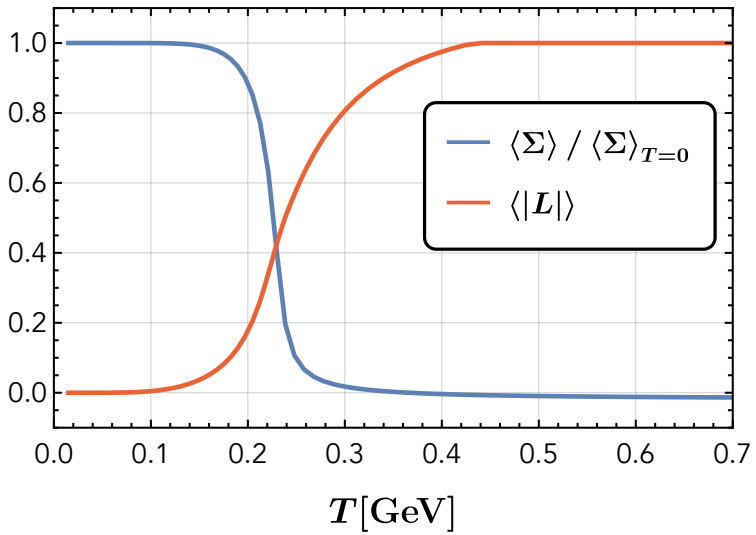
$$\Omega_{\mathrm{quark}} = -2\sum_{f=u,d} T\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\,\mathrm{Tr}\left[\log\left(1 + \mathcal{W}_c(q_i)\, e^{2\pi i Q_f t} e^{-E_{p,f}/T}\right)\right] + \text{c.c.}. \quad (21)$$

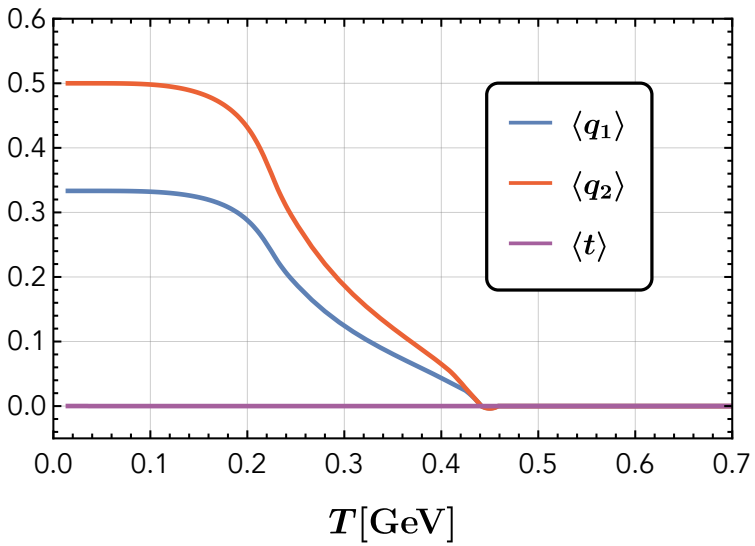
[1] C. Ratti, M. A. Thaler and W. Weise, Phys. Rev. D 73 (2006) 014019, [hep-ph/0506234].

[2] K. Fukushima and V. Skokov, Prog. Part. Nucl. Phys. 96 (2017) 154–199, [1705.00718].

PNJL-like model

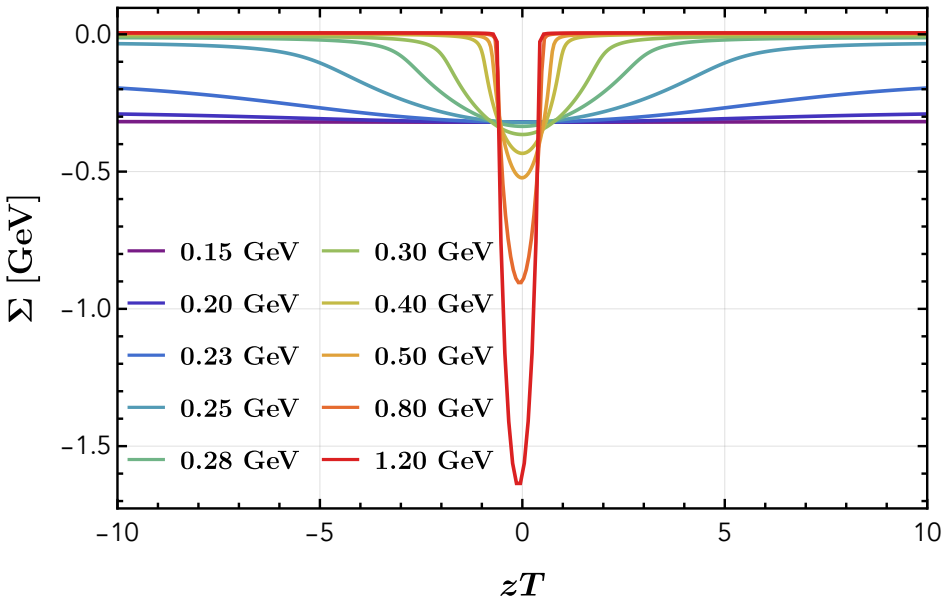
- **High** T: $\langle|L|\rangle = 1$ (deconfinement), $\langle|\Sigma|\rangle = m_0$ bare quark mass (chiral symmetric); Three vacua $(0,0,0)$, $(0,1,1)$, $(1,1/2,-1)$.
- **Low** T: $\langle|L|\rangle = 0$ (confinement), $\langle|\Sigma|\rangle = 320$ MeV (chiral symmetry breaking); Three vacua (q_1, q_2) moves exponentially close to $(1/3, 1/2)$, in three layers $t = 1, 0, -1$.
- The **exponential closing but never exactly equal** is important. This reflects again the $\mathbb{Z}_3$ is exact center always, and the domain walls last and never collapse.

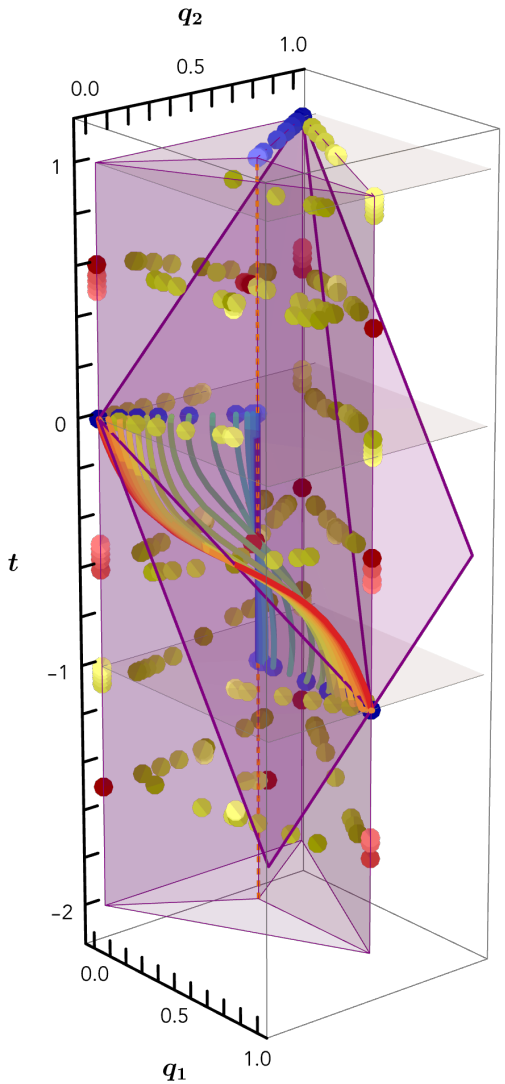




Domain wall profile

- Local extrema points: Blue minimum, Red maximum, Yellow saddle.
- deeper color for higher T
- colored lines are the domain paths with variable (q_1, q_2, t, Σ).



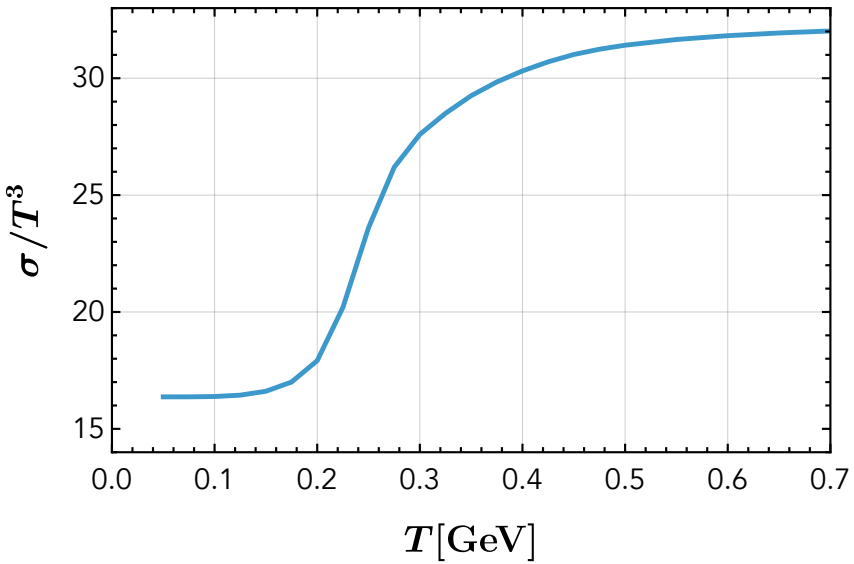


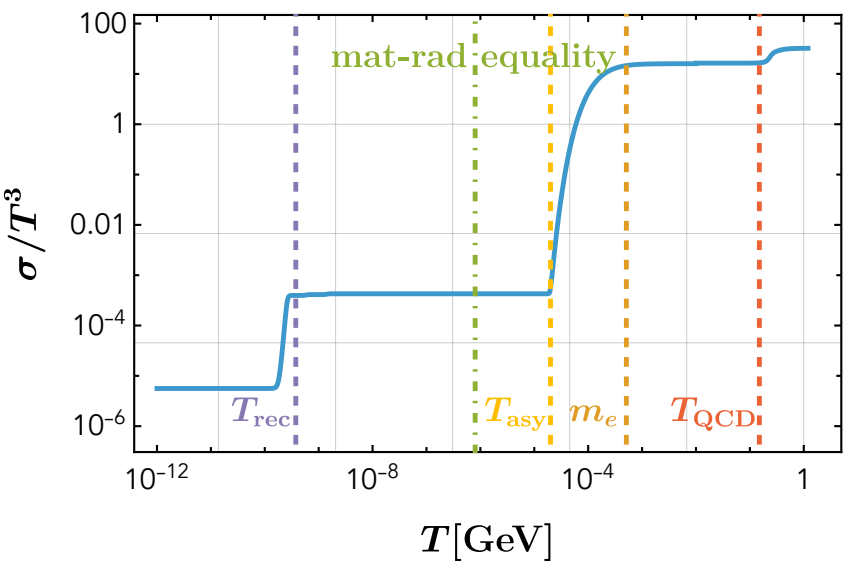
Domain wall tension

- The domain wall is thermal. Naively in high enough T, there is no other scale, so the tension can only be $\sigma \propto T^3$.
- When $m_e < T < T_{\text{confine}}$, from the profile that q_1, q_2 are almost constant, so dominated by t. Again, t-DW here is from massless electron, so $\sigma \propto T^3$.
- $T < m_e$, ignoring the chemical potential by collecting into n_e.

$$\begin{aligned}\Omega_e(t,T) &= -Tn_e\left[\mathrm{Li}_{\frac{5}{2}}(-e^{-2\pi it}e^{-\frac{m_e}{T}}) + \mathrm{Li}_{\frac{5}{2}}(-e^{2\pi it}e^{-\frac{m_e}{T}})\right]/\mathrm{Li}_{\frac{3}{2}}(-e^{-\frac{m_e}{T}}) \\ &\approx -Tn_e\left[2\cos 2\pi t - e^{-m_e/T}(\cos 4\pi t - 2\cos 2\pi t)/2\sqrt{2}\right], \qquad (22)\end{aligned}$$

so, Sine-Gordon $-\cos\pi t = \tanh\left(\sqrt{2e^2n_e/T}z\right)$.





DW interaction with plasma

- Thermal wall. The actual "wall" is the changing of plasma thermal property — gauge holomony, a boundary condition. In the grand potential, $E_p + 2\pi i Q_i \Longrightarrow$ imaginary chemical potential. [1]
- The plasma holds the wall — **No plasma, no wall**.
- Although we used $A_0 \sim 2\pi Q/g$, we should not think this gives real physical electric fields. – No tree level effects.
- The nonzero holomony is the **shift of Mastubara frequency**. Wick rotation,

$$i\omega_n + iQ \longrightarrow E \pm i0^+. \tag{23}$$

E.g., the scalar 2-point Green function in the thermal Euclidean spacetime has **pole structure**

$$G_E(i\omega_n, \vec{p}) = \frac{1}{(\omega + Q)^2 + E_{\vec{p}}^2} = \frac{1}{2E_{\vec{p}}} \left(\frac{1}{E_{\vec{p}} - (i\omega_n + iQ)} + \frac{1}{E_{\vec{p}} + (i\omega_n + iQ)} \right),$$

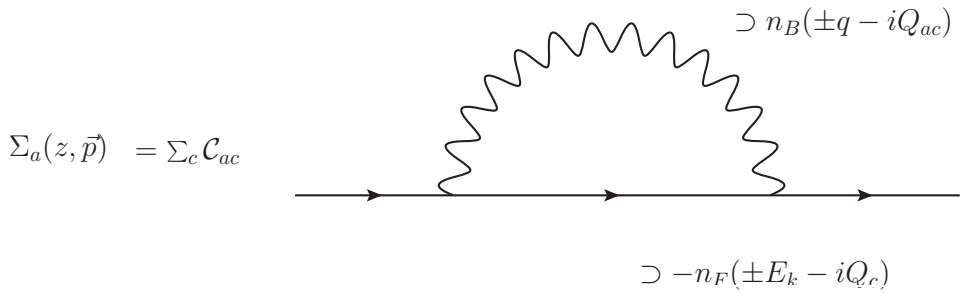
and the Feynman Green function in the real time is

$$\begin{aligned} G_F(E, \vec{p}) =& \frac{1}{p^2 - m^2 + i0^+} = \frac{1}{E_{\vec{p}}^2 - E^2 + i0^+} \\ =& \frac{1}{2E_{\vec{p}}} \left(\frac{1}{E_{\vec{p}} - (E + i0^+)} + \frac{1}{E_{\vec{p}} + (E + i0^+)} \right). \end{aligned} \tag{24}$$

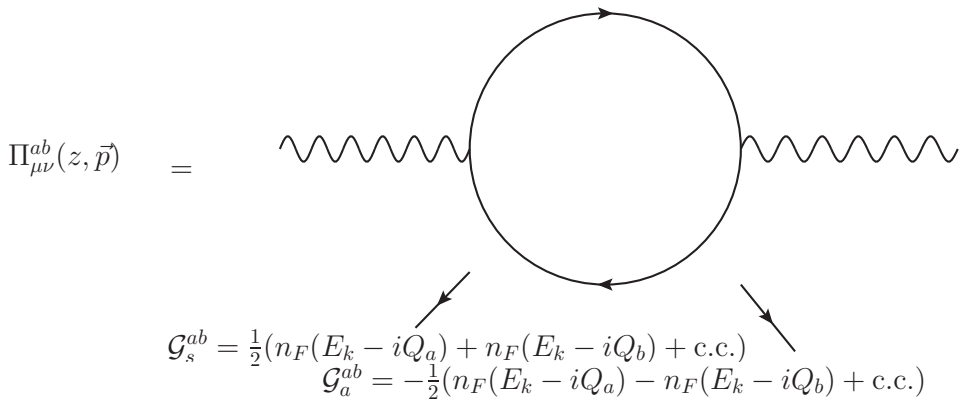
[1] A. Roberge and N. Weiss, Nucl. Phys. B 275 (1986) 734–745.

DW interaction with plasma

- The effect of gauge holomony is likely to appear in the **loop level**. When one calculate the thermal loop, the holomony appears in **distribution function**.
- fermion self energy



- gauge boson self energy



First results of cosmological evolution

- VOS model with temperature dependent tension[1] (if that can model)

$$\frac{\mathrm{d}L}{\mathrm{d}t} = HL + Dv^2L + c_w v, \quad \frac{\mathrm{d}v}{\mathrm{d}t} = (1-v^2)(\frac{k_w}{L} - Dv), \tag{25}$$

$$D = \frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}t} + 3H + \frac{1}{l_f}, \tag{26}$$

- $\dot{\sigma}/\sigma + 3H$ cancels exactly in this case $\sigma \propto T^3$. $\Longrightarrow$ Friction dominates.
- Friction is from the reflection $R(P)$.

$$\Delta P = 2\int \frac{\mathrm{d}^2 p_\parallel}{(2\pi)^3}\int_0^\infty \mathrm{d}p_z[n(-v) - n(v)]R(p)\frac{p_z^2}{E}, \tag{27}$$

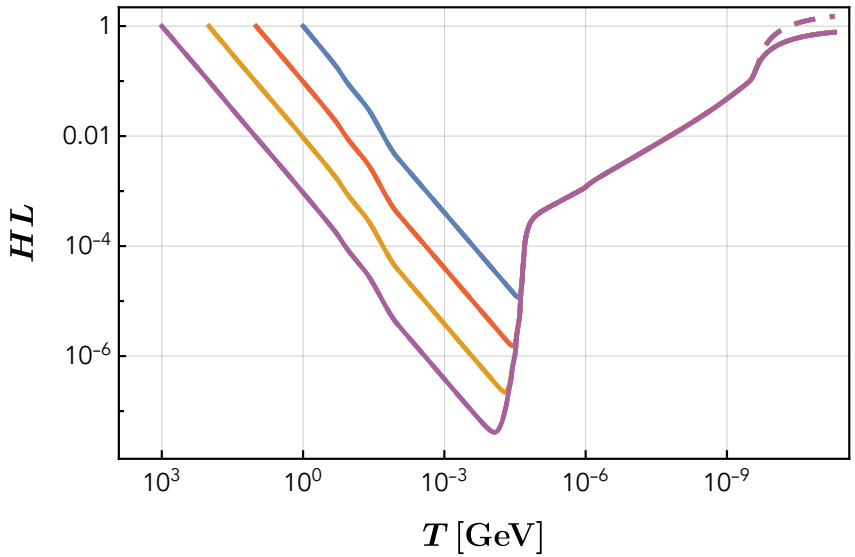
The friction of the plasma on the domain wall is

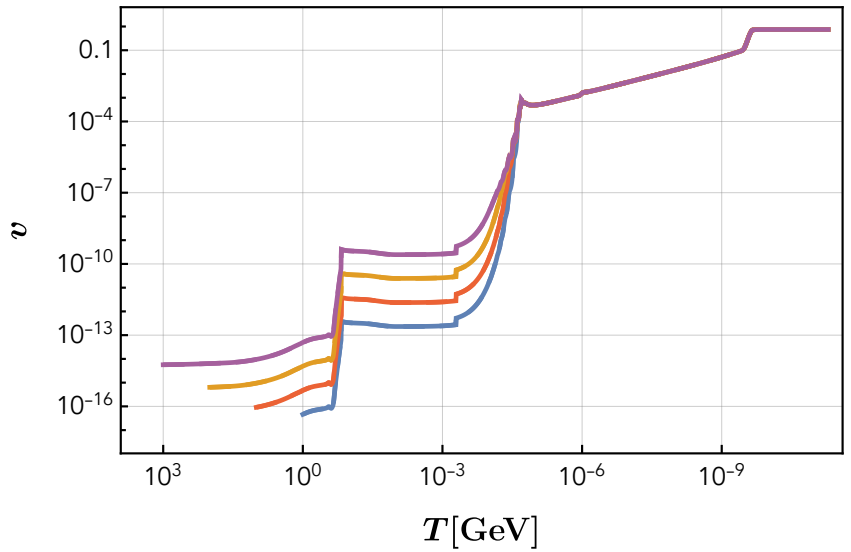
$$\frac{1}{\ell_f} = \mathcal{O}(1)\frac{\Delta P}{\sigma v \gamma}, \tag{28}$$

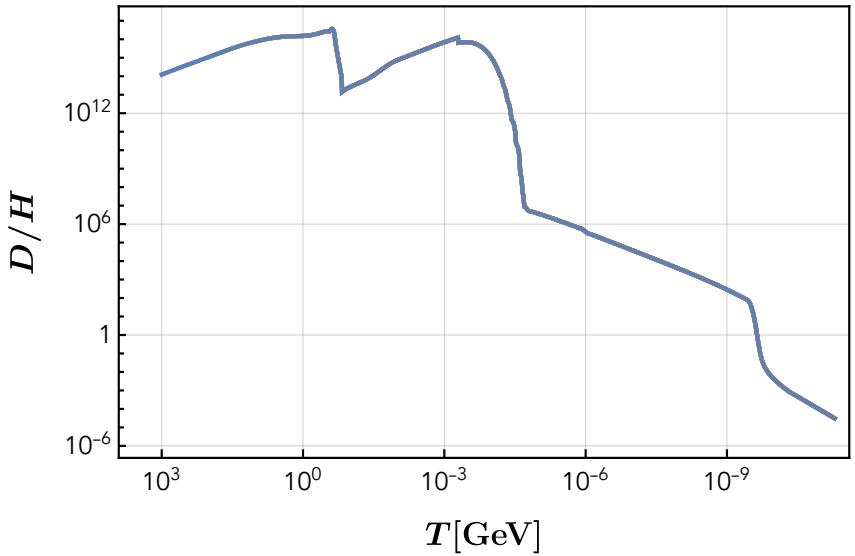
- Reflection is calculated with WKB approximation.

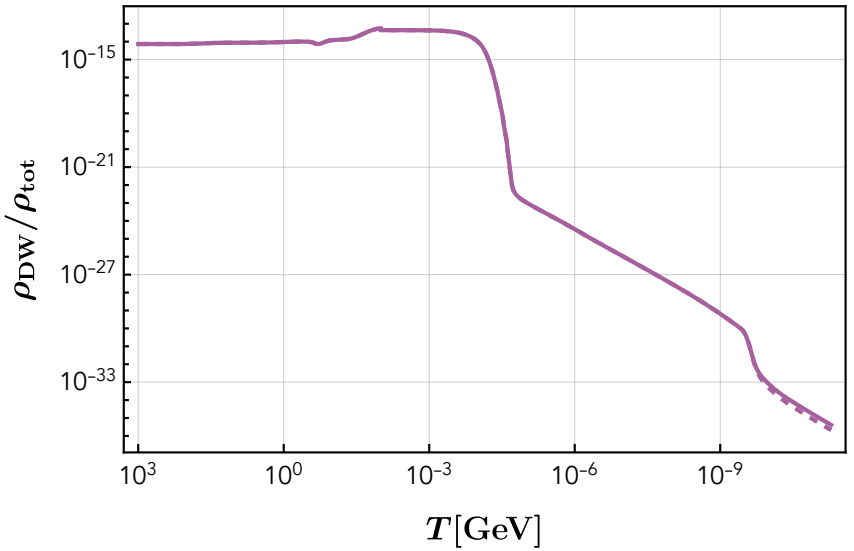
[1] P. P. Avelino, C. J. A. P. Martins and J. C. R. E. Oliveira, Phys. Rev. D 72 (2005) 083506, [hep-ph/0507272]. Y. Bai and T.-K. Chen, 2605.13958.

First results of cosmological evolution









Summary

- The 1-form symmetry (subgroup of $\mathbb{Z}_6^{\mathfrak{c}}$) is **exact** in SM, and it can be used to test the **quotient** of the SM gauge group.
- In the **thermal** field theory, this induces ordinary symmetry. Such symmetry breaking build the thermal domain wall.
- The $\mathbb{Z}_3$ holds even when QCD are confined. The domain wall will last even at current universe, although most effects are small to be truly observed.
- The plasma **supports** the wall, and wall never exists itself. The interaction between wall and plasma is intrinsic **loop level**, with the gauge holomony in the distribution function.
- To do: Fully understand the interaction, and then the evolution in the universe.
- To do: Phenomenological observation of the domain wall in the universe.

Thanks!

Backup

Fermion self energy

For quark,

$$\Sigma_a(z,\vec{p}) = g_s^2 \sum_c \mathcal{C}_{ac} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \sum_{s,t=\pm 1} \frac{t\mathcal{M}_s}{2q} \frac{1 + n_B(tq - iQ_{ac}) - n_F(sE_k - iQ_c)}{z - sE_k - tq}, \tag{29}$$

where $\vec{k}$ is the internal quark momentum, and $\vec{q}$ is the internal gluon momentum, $E_k = \sqrt{k^2 + m^2}$, and $q = |\vec{q}| = \sqrt{k^2 + p^2 - 2kp\cos\theta}$ with $p = |\vec{p}|$ and $x = \cos\theta$ the angle between $\vec{k}$ and $\vec{p}$. The spinor structure $\mathcal{M}_s$ is defined as

$$\mathcal{M}_\pm = \gamma^0 \mp \frac{k}{E_k} \vec{\gamma} \cdot \hat{\vec{k}} + 2\frac{m}{E_k}, \tag{30}$$

and the color projection operator

$$\mathcal{C}_{ac} = \frac{1}{2}\left(1 - \frac{1}{3}\delta_{ac}\right). \tag{31}$$

The $Q_{a=1,2,3}$ is the holomony for the three color and $Q_{ac} = Q_a - Q_c$ comes form the gluon contribution so it is in the adjoint way.

Fermion self energy

$$\Sigma_a(z,\vec{p}) = A_a\gamma^0 + B_a\vec{\gamma}\cdot\hat{\vec{k}} + C_a, \tag{32}$$

where

$$A_a(z,\vec{p}) = \frac{g_s^2}{8\pi^2}\sum_c \mathcal{C}_{ac}\int_0^\infty \mathrm{d}k k^2 \int_{-1}^1 \mathrm{d}x \frac{\mathcal{F}_+^{ac}+\mathcal{F}_-^{ac}}{q},$$

$$B_a(z,\vec{p}) = \frac{g_s^2}{8\pi^2}\sum_c \mathcal{C}_{ac}\int_0^\infty \mathrm{d}k k^2 \int_{-1}^1 \mathrm{d}x \frac{kx}{E_k}\frac{\mathcal{F}_-^{ac}-\mathcal{F}_+^{ac}}{q}, \tag{33}$$

$$C_a(z,\vec{p}) = \frac{g_s^2}{8\pi^2}\sum_c \mathcal{C}_{ac}\int_0^\infty \mathrm{d}k k^2 \int_{-1}^1 \mathrm{d}x \frac{2m}{E_k}\frac{\mathcal{F}_-^{ac}-\mathcal{F}_+^{ac}}{q},$$

$$\mathcal{F}_+^{ac} = \frac{\mathcal{N}_{++}^{ac}}{z-E_k-q} + \frac{\mathcal{N}_{+-}^{ac}}{z-E_k+q}, \quad \mathcal{F}_-^{ac} = \frac{\mathcal{N}_{-+}^{ac}}{z+E_k-q} + \frac{\mathcal{N}_{--}^{ac}}{z+E_k+q}, \tag{34}$$

$$\mathcal{N}_{++}^{ac} = 1 + n_B(q-iQ_{ac}) - n_F(E_k - iQ_c), \quad \mathcal{N}_{--}^{ac} = \left(\mathcal{N}_{++}^{ac}\right)^*,$$

$$\mathcal{N}_{+-}^{ac} = n_B(q+iQ_{ac}) + n_F(E_k - iQ_c), \quad \mathcal{N}_{-+}^{ac} = \left(\mathcal{N}_{+-}^{ac}\right)^*. \tag{35}$$

It is straightforward to write these formulas for electron in the EM holomony.

Gauge boson self energy

For gluons,

$$\Pi^{\mu\nu}_{ab}(z,\vec{p}) = \tag{36}$$
$$2g_s^2 \sum_f \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\mathcal{G}_s^{ab,f}}{2E_k} \frac{K\cdot P(K^\mu P^\nu + K^\nu P^\mu) - P^2 K^\mu K^\nu - \eta^{\mu\nu}(K\cdot P)^2}{(K\cdot P)^2 - (P^2)^2/4}$$
$$+ g_s^2 \sum_f \int \frac{\mathrm{d}^3k}{(2\pi)^3} \frac{\mathcal{G}_a^{ab,f}}{2E_k} \frac{4K\cdot P K^\mu K^\nu - P^2(K^\mu P^\nu + K^\nu P^\mu) + (K\cdot P)P^2\eta^{\mu\nu}}{(K\cdot P)^2 - (P^2)^2/4},$$

where we denote $P = (z, \vec{p})$ and $K = (E_k, \vec{k})$ the momentum of fermion, the antifermion has momentum $P - K$. f is the flavor of quark. The color symmetric $\mathcal{G}_s^{ab}$ and color antisymmetric $\mathcal{G}_a^{ab}$ are

$$\mathcal{G}_s^{ab,f} = \frac{1}{2}\left[n_F(E_k - iQ_a^f) + n_F(E_k - iQ_b^f) + \text{c.c.}\right],$$
$$\mathcal{G}_a^{ab,f} = -\frac{1}{2}\left[n_F(E_k - iQ_a^f) - n_F(E_k - iQ_b^f) - \text{c.c.}\right]. \tag{37}$$

Gauge boson self energy

The transverse and longitudinal mode are

$$\Pi_T^{ab}(z,\vec{p}) = \frac{g_s^2}{4\pi^2}\sum_f \int_0^\infty \mathrm{d}k \frac{k^2}{E_k}\int_{-1}^1 \mathrm{d}x \left[\mathcal{G}_s^{ab,f}\frac{\iota^2 - sk^2(1-x^2)/2}{\iota^2 - s^2/4} + \mathcal{G}_a^{ab,f}\frac{\iota[k^2(1-x^2)-s/2]}{\iota^2 - s^2/4}\right], \tag{38}$$

$$\Pi_L^{ab}(z,\vec{p}) = \frac{g_s^2}{4\pi^2}\sum_f \int_0^\infty \mathrm{d}k \frac{k^2}{E_k}\int_{-1}^1 \mathrm{d}x \left[\mathcal{G}_s^{ab,f}\frac{s(E_k^2 - k^2x^2)}{\iota^2 - s^2/4} + \mathcal{G}_a^{ab,f}\frac{\iota[4\xi^2 - s^2]}{\iota^2 - s^2/4}\right], \tag{39}$$

where $s = P^2 = z^2 - p^2$, $\iota = K\cdot P = E_k z - kpx$, $\xi = K\cdot\epsilon_L\sqrt{s} = pE_k - zkx$.

Reflection

- The quark reflection dominates when $T > T_{\text{confine}}$, because Σ has nontrivial VEV at the center. — QM-like potential barrier reflection.
- $T < T_{\text{confine}}$ has electron and photon reflection. Because photon number density is much larger than electron when $T < m_e$, one can expect the main effect is from photon. — Pöschl-Teller potential due to t profile.
- WKB for electron is trciky.

Quark reflection

- Dirac spinor $(\alpha, \beta, \gamma, \delta)^T$

$$i\alpha'(z) = E\alpha(z) - m(z)\gamma(z), \quad i\gamma'(z) = -E\gamma(z) + m(z)\alpha(z), \tag{40}$$

- Solve the scattering

$$\alpha(z) \to e^{ip_z z} + re^{-ip_z z}, \ z \to -\infty, \quad \alpha(z) \to te^{ip_z z}, \ z \to +\infty, \tag{41}$$

  and $R = |r|^2$.
- According to the numerical calculation, the $\Delta P/T^4$ can be approximately factorized into $\Delta P/T^4 \approx \widetilde{P} v\gamma$.
- One can see when quark are confined, the pressure is suppressed. On the higher temperature $T > 1.2$ GeV, we approximate the Σ profile by

$$\Sigma(z) = e^{-c_2\left(\frac{zT}{w}\right)^2 - c_4\left(\frac{zT}{w}\right)^4 - c_6\left(\frac{zT}{w}\right)^6}, \tag{42}$$

  Then the reflection are solved from the analytical barrier to arbitrary high temperature.

Photon reflection

- There we used the approximation that $n_F(E_k - 2\pi i t) + \text{c.c.} \to 2n_F(E_k)\cos 2\pi t$,

$$\Pi_T = \cos 2\pi t\, \Pi_T^{(0)}, \quad \Pi_T^{(0)} = \lim_{P^2 \to 0} \Pi_T \approx \frac{e^2 n_e}{m_e}, \tag{43}$$

- The classical momentum is

$$p_\pm = \pm\sqrt{p_{\text{as}}^2 - 2\sin^2 \pi t\, (e^2 n_e/m_e)}, \tag{44}$$

where asymptotic momentum is $p_{\text{as}}^2 = \omega^2 - p_\parallel^2 - \Pi_T^{(0)}$.

- With $-\cos \pi t = \tanh\left(\sqrt{2e^2 n_e/T}\, z\right)$, one can get the Pöschl-Teller reflection

$$\frac{d^2}{dz^2} A_T + (p_{\text{as}}^2 - 2e^2 n_e/m_e \,\text{sech}^2(\sqrt{2e^2 n_e/T}\, z)\,) A_T = 0 \tag{45}$$

- Reflection rate is known

$$R = \frac{|\sin(\pi\lambda)|^2}{\sinh^2(\pi s) + |\sin(\pi\lambda)|^2}, \tag{46}$$

where

$$a = -2d^2 e^2 n_e/m_e, \quad \lambda(\lambda+1) = -a, \quad s = d\, p_z^{\text{as}}, \quad d = \sqrt{T/2e^2 n_e}. \tag{47}$$