Probing positivity bounds at colliders

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NEXUS 2026, TDLI
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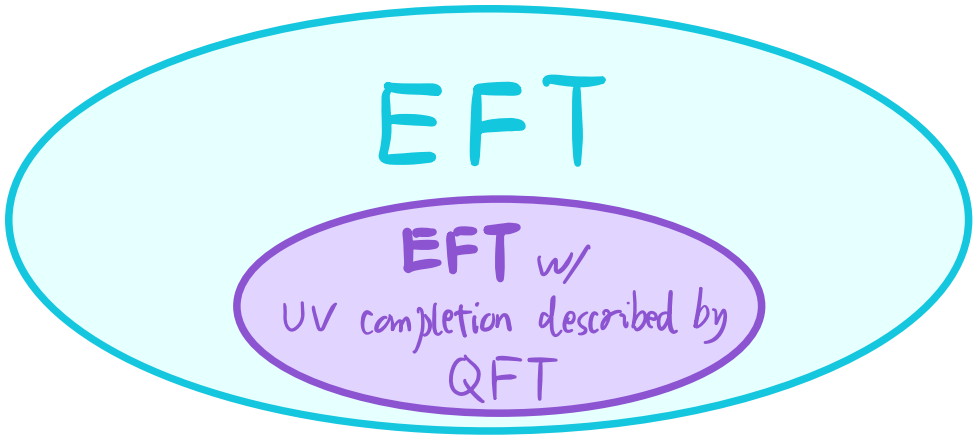
[2011.03055] JG, Lian-Tao Wang, Cen Zhang
[2311.07663] JG, Chi Shu
current work with Xiao Cao & Yunxiao Ye

Introduction

- **Can all EFTs be UV completed?**
- Assuming the UV physics is consistent with the fundamental principles of QFT (analyticity, locality, unitarity, Lorentz invariance), the Wilson coefficients of some dim-8 (or even higher dimensional) operators are bounded to one side.
  - which are usually called positivity bounds (rather than negativity bounds) ...
  - [hep-th/0602178] Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi + many papers...



- Two important implications:
  - Assuming UV physics is described by QFT $\Rightarrow$ we can rule out a large region in the EFT parameter space.
  - **Or** we can test the fundamental principles of QFT if we measure these dim-8 coefficients well enough.

Can we really do it?

- Can QFT really break down at the TeV scale?
  - Parity was once regarded as a sacred principle...
  - It is important to test the predictions of QFT even if we expect QFT to be correct. (just like entanglement...)

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- Can we really probe dim-8 operators?
  - If dim-8 effects are sizable, how do we know if the EFT is still valid?
- Could it be something else?
  - Light new particles?
  - Wrong EFT?
- Loops? [2408.10318] Yunxiao Ye, Bin He, JG, [2507.06302] Yunxiao Ye, Xiao Cao, Yu-Hang Wu, JG

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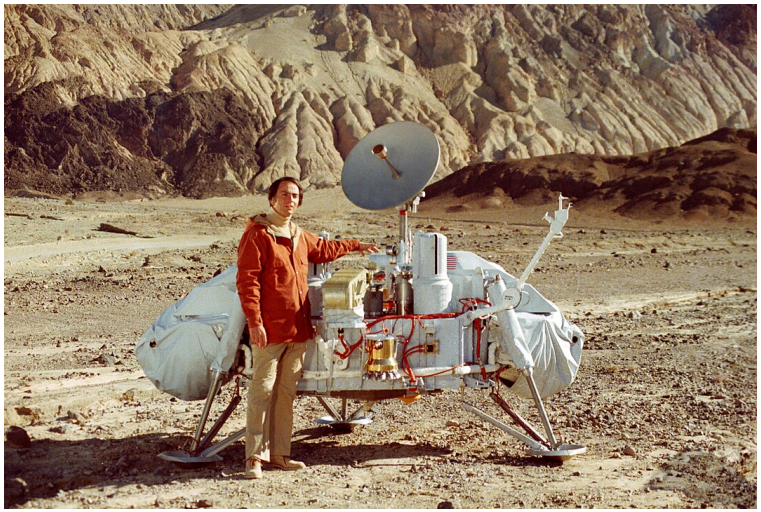
- "Extraordinary claims require extraordinary evidence." – Carl Sagan

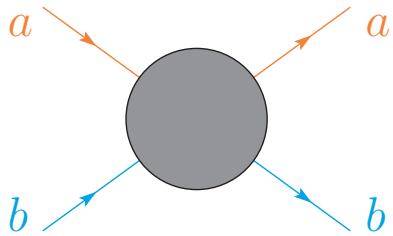


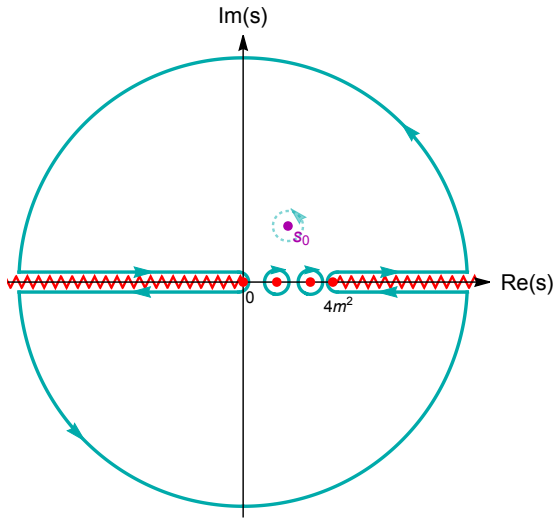
Dispersion relations

- Consider a forward ($t \to 0$, $u = 4m^2 - s$) elastic amplitude, assume it's analytic at s_0:

$$\tilde{\mathcal{A}}_{ab}(s) \equiv \mathcal{A}_{ab}(s,t,u)|_{t=0} = \sum_{n\geq 0} \kappa_n (s-s_0)^n ,$$

$$\kappa_n = \frac{1}{2\pi i} \oint_{s=s_0} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s-s_0)^{n+1}} ,$$





- Applying the fundamental principles of QFT
  - Analyticity (Cauchy's theorem applies)
  - Locality (simple poles from tree-level factorization, branch cuts from loops)

    (Note: Not the same locality as in "Bell non-locality".)
  - Unitarity (Optical theorem, $\mathrm{Im}\mathcal{A} \sim \sigma_{\mathrm{tot}}$)
  - Lorentz invariance (Crossing symmetry)

- Dispersion relation tells us that

$$\kappa_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s\sqrt{1-\frac{4m^2}{s}} \left(\frac{\sigma_{\mathrm{tot}}^{ab}}{(s-s_0)^{n+1}} + (-1)^n \frac{\sigma_{\mathrm{tot}}^{a\bar{b}}}{(s-4m^2+s_0)^{n+1}} \right) + \kappa_n^{\infty} ,$$

Sum rules and positivity bounds

- Assuming s_0 is real and $0 < s_0 < 4m^2$,

$$\kappa_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s \sqrt{1 - \frac{4m^2}{s}} \left(\frac{\sigma_{\rm tot}^{ab}}{(s - s_0)^{n+1}} + (-1)^n \frac{\sigma_{\rm tot}^{a\bar{b}}}{(s - 4m^2 + s_0)^{n+1}} \right) + \kappa_n^{\infty} \,,$$

  - Froissart-Martin bound: $\kappa_n^{\infty} = 0$ for $n > 1$.
  - $\kappa_n > 0$ for even n.

- At tree level, consider the limit $m^2,\ s_0 \ll \Lambda^2$ (massless EFT).

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \cdots$$

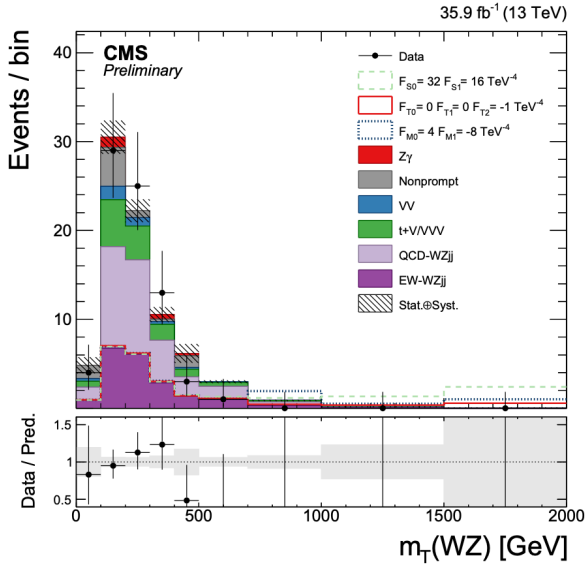
$$\tilde{\mathcal{A}}(s) = \kappa_0 + \kappa_1 s + \kappa_2 s^2 + \cdots$$

- $\kappa_1 \sim c^{(6)}$ no positivity bounds,
  $\kappa_2 \sim c^{(8)}$ YES positivity bounds!
  ...
- Many generalizations and modified versions...
  - Improved positivity bounds [1710.09611] de Rham *et al.* (see also [1710.02539] Bellazzini *et al.*), Arc variable [2011.00037] Bellazzini *et al.*, EFT-Hedron [2012.15849] Arkani-Hamed, Huang, Huang, ...

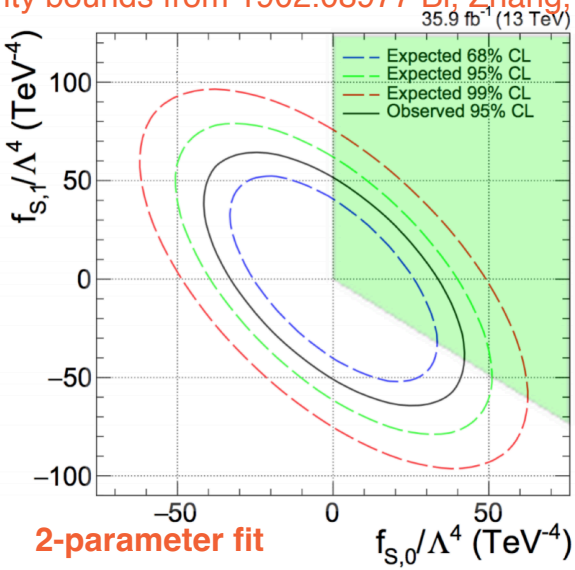
Probing positivity bounds on dimension-8 operators

- The dimension-8 contribution has a large energy enhancement ($\sim E^4/\Lambda^4$)!
- It is difficult for LHC to probe these bounds.
  - Low statistics in the high energy bins.
  - Example: Vector boson scattering.
  - $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!
- Can we separate the dim-8 and dim-6 effects?
  - Precision measurements at several different $\sqrt{s}$?
  - Also make use of angular distributions?
    See *e.g.* studies of dim-8 4f interactions [2003.11615], Alioli *et al.*, [2009.02212] Fuks et al., [2204.13121] Li et al.
  - Or find some special process where dim-8 gives the leading new physics contribution?

CMS-PAS-SMP-18-001



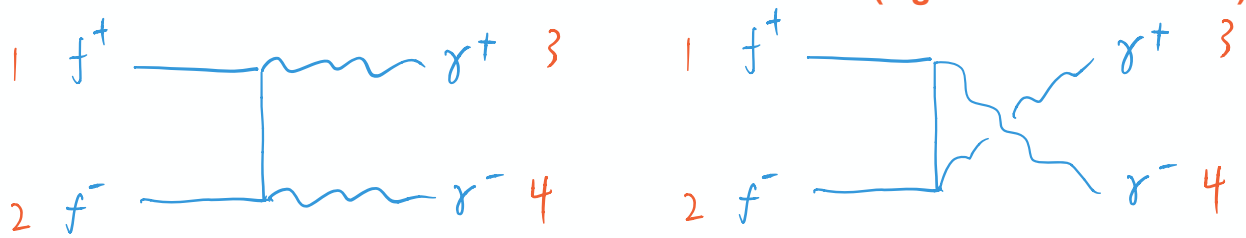
positivity bounds from 1902.08977 Bi, Zhang, Zhou



2-parameter fit

The diphoton channel [arXiv:2011.03055] Phys.Rev.Lett. 129, 011805, JG, Lian-Tao Wang, Cen Zhang

(signs denote helicities)



- $e^+e^- \to \gamma\gamma$ (or $\mu^+\mu^- \to \gamma\gamma$), SM, non-resonant.
  - Tree level SM: the only helicity configuration is $\mathcal{A}(f^+f^-\gamma^+\gamma^-)$.
- Leading order contribution: **dimension-8 contact interaction**. ($f^+f^- \to \bar{e}_L e_L$ or $e_R\bar{e}_R$)

$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\rm SM+d8} = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle} + \frac{a}{v^4}[13][23]\langle 24\rangle^2 .$$

- Operators: Also contributes to $ZZ/Z\gamma$ final states with opposite helicities.
  [1806.09640] Bellazzini, Riva ($q\bar{q} \to ZZ/Z\gamma$),
  see the d8 basis in [2005.00008] Li et al., [2005.00059] Murphy

$$a_L = -2\frac{v^4}{\Lambda^4}\left(c_W^2 c_{l^2B^2D} - 2s_W c_W c^{(2)}_{l^2WBD} + s_W^2 c^{(1)}_{l^2W^2D}\right),$$
$$a_R = -2\frac{v^4}{\Lambda^4}\left(c_W^2 c_{e^2B^2D} + s_W^2 c_{e^2W^2D}\right),$$

$$Q_{l^2B^2D} = i(\bar{l}\gamma^\mu \overleftrightarrow{D}^\nu l)B_{\mu\rho}B_\nu{}^\rho ,$$
$$Q^{(2)}_{l^2WBD} = i(\bar{l}\gamma^\mu\tau^I \overleftrightarrow{D}^\nu l)(B_{\mu\rho}W^{I\rho}_\nu + B_{\nu\rho}W^{I\rho}_\mu),$$
$$Q^{(1)}_{l^2W^2D} = i(\bar{l}\gamma^\mu \overleftrightarrow{D}^\nu l)W^I_{\mu\rho}W^{I\rho}_\nu ,$$
$$Q_{e^2B^2D} = i(\bar{e}\gamma^\mu \overleftrightarrow{D}^\nu e)B_{\mu\rho}B_\nu{}^\rho ,$$
$$Q_{e^2W^2D} = i(\bar{e}\gamma^\mu \overleftrightarrow{D}^\nu e)W^I_{\mu\rho}W^{I\rho}_\nu .$$

Positivity bounds

- The $e^+e^- \to \gamma\gamma$ amplitude:

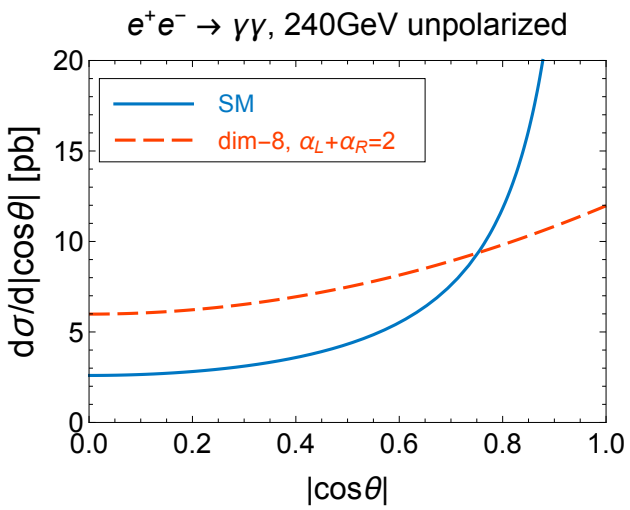
$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\mathrm{SM+d8}} = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle} + \frac{a}{v^4}[13][23]\langle 24\rangle^2 = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle}\left(1+\frac{a}{2e^2v^4}tu\right)$$

- The $e\gamma \to e\gamma$ amplitude:

$$\mathcal{A}(f^+\gamma^+f^-\gamma^-)_{\mathrm{SM+d8}} = 2e^2\frac{\langle 34\rangle^2}{\langle 12\rangle\langle 32\rangle}\left(1+\frac{a}{2e^2v^4}su\right) \underset{t\to 0}{=} -2e^2\left(1-\frac{a}{2e^2v^4}s^2\right)$$

- Positivity bound implies $a \geq 0$.
- The interference term is bounded to be destructive in $e\gamma \to e\gamma$, and **constructive in $e^+e^- \to \gamma\gamma$!**
- All other contributions are either vanishing or suppressed!
  - SM$\times$d6 at tree level: Dipole operators, interference suppressed by m_e.
  - d6^2: Dipole operators are very well constrained by $g-2$ and EDM measurements.
  - SM$\times$d6 at 1-loop: Contributions are either very-well constrained by other measurements with tree-level contributions, or forbidden by selection rules.

The diphoton cross section



- **Differential cross section** (The production polar angle θ is "folded" since the photon polarizations are not measured.)

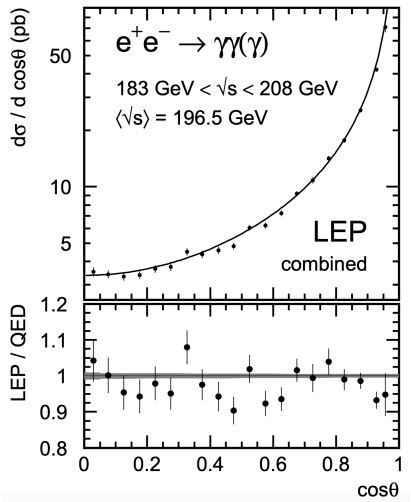
$$\frac{d\sigma(e^+e^- \to \gamma\gamma)}{d|\cos\theta|} = \frac{(1-P_{e^-})(1+P_{e^+})}{4}\frac{e^4}{4\pi s}\left(\frac{1+c_\theta^2}{1-c_\theta^2} + a_L\frac{s^2(1+c_\theta^2)}{4e^2v^4}\right) + \frac{(1+P_{e^-})(1-P_{e^+})}{4}\frac{e^4}{4\pi s}\left(\frac{1+c_\theta^2}{1-c_\theta^2} + a_R\frac{s^2(1+c_\theta^2)}{4e^2v^4}\right),$$

- Positivity bounds: $a_L \geq 0$, $a_R \geq 0$.
- **Positivity bound directly on the cross section!**

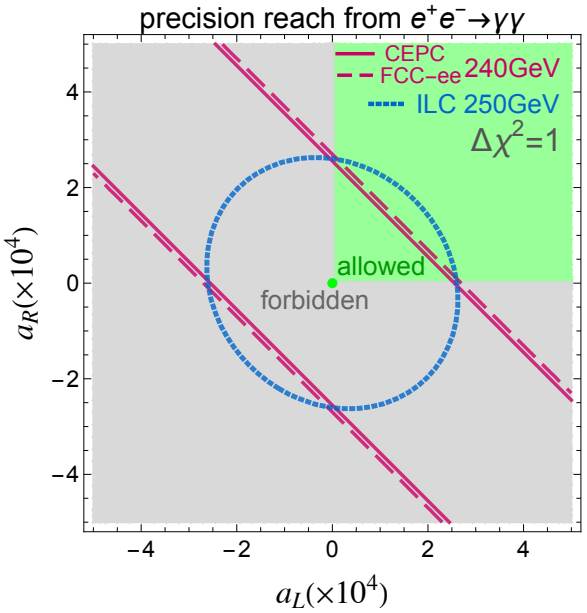
$$\sigma(e^+e^- \to \gamma\gamma) \geq \sigma_{\rm SM}(e^+e^- \to \gamma\gamma)\,.$$

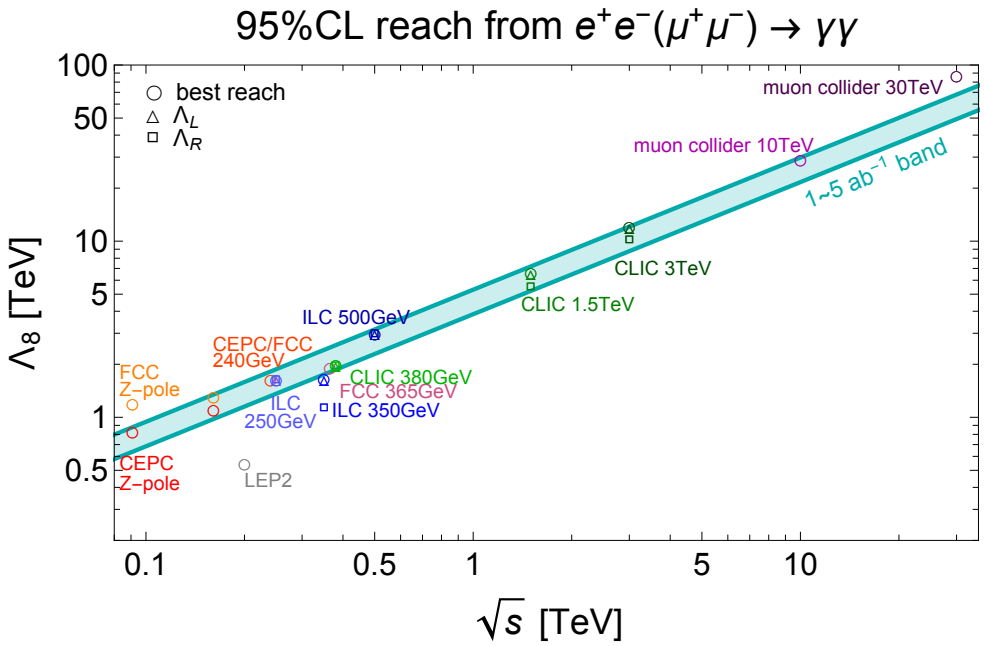
- The LEP measurement was $\sim 1.5\sigma$ below the SM prediction.

1302.3415



Future projections

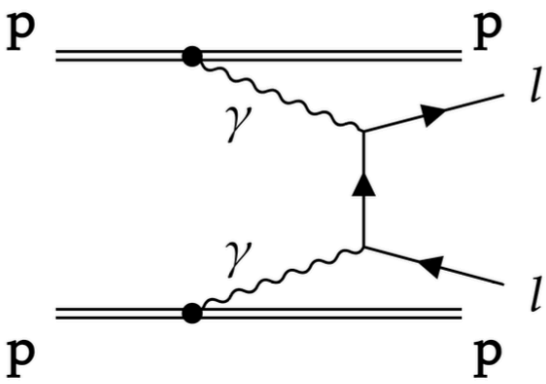




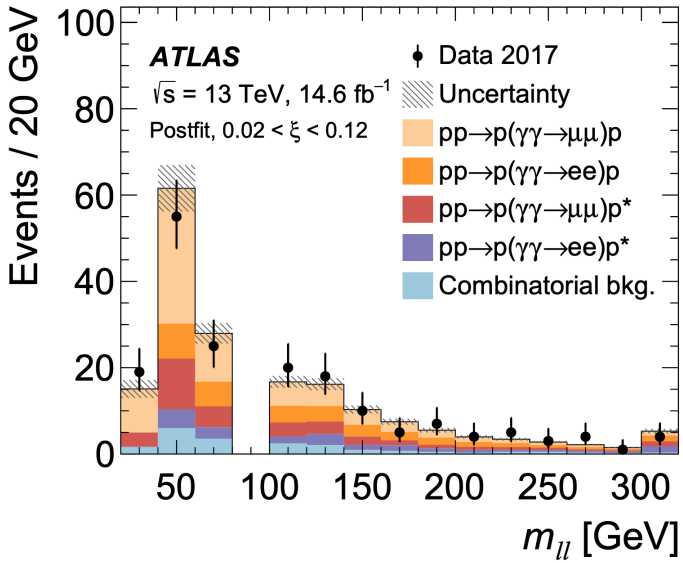
- χ^2 fit to the binned distribution
  - Statistics only, 19 bins in $\cos\theta \subset [0, 0.95]$.
  - Agrees reasonably well with LEP result ($\lesssim 10\%$ in the reach on Λ).
- Is beam polarization useful? **Yes and no!**
  - One could measure σ_L and σ_R simultaneously.
- High energy still wins!

$$\frac{\Lambda_2}{\Lambda_1} = \left(\frac{E_2}{E_1}\right)^{\frac{3}{4}} \left(\frac{L_2}{L_1}\right)^{\frac{1}{8}} .$$

The $\gamma\gamma \to \ell^+\ell^-$ channel [2311.07663] JG, Chi Shu

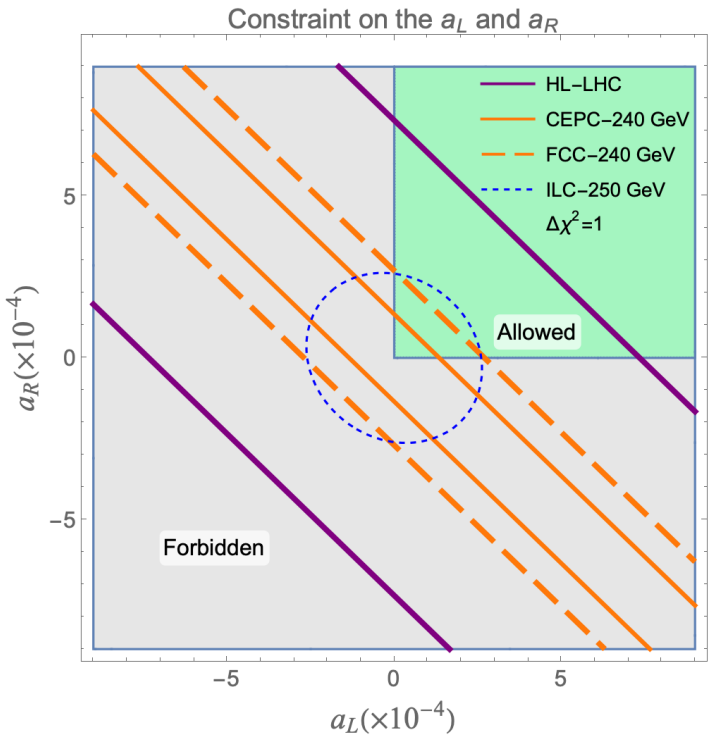


ATLAS 2009.14537



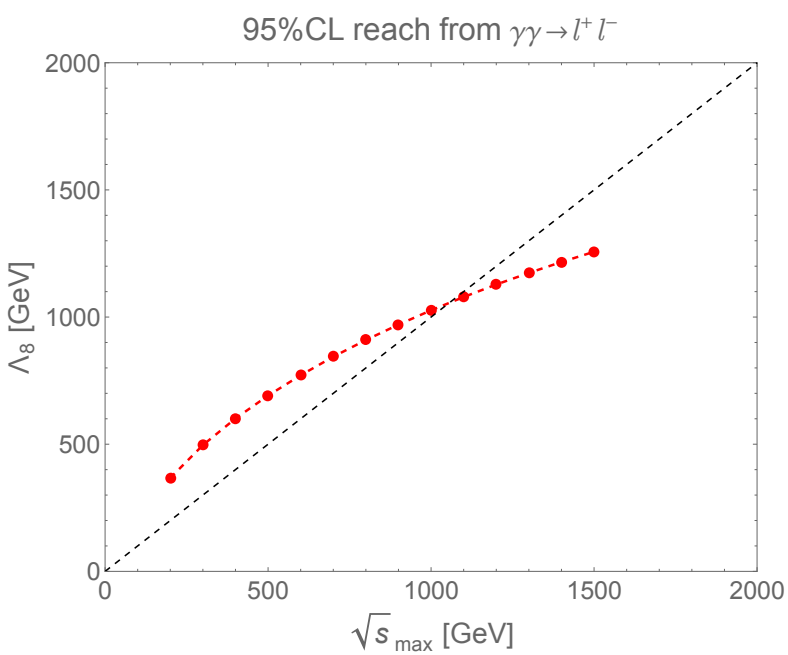
- The "reverse" amplitude, $\gamma\gamma \to \ell^+\ell^-$, probes the same physics as $e^+e^- \to \gamma\gamma$ does.
- The proton can radiate a photon without breaking apart. The **forward proton tagger** can select $\gamma\gamma$ fusion process. (Low background!)
- By comparing MC simulation (with the effective photon "PDF" from [2207.03012] H.-S. Shao, D. d'Enterria) with the ATLAS analysis we get a naive estimation of 11% selection efficiency. We then scale the luminosity to HL-LHC (2 detectors, $3\,\mathrm{ab}^{-1}$ each).

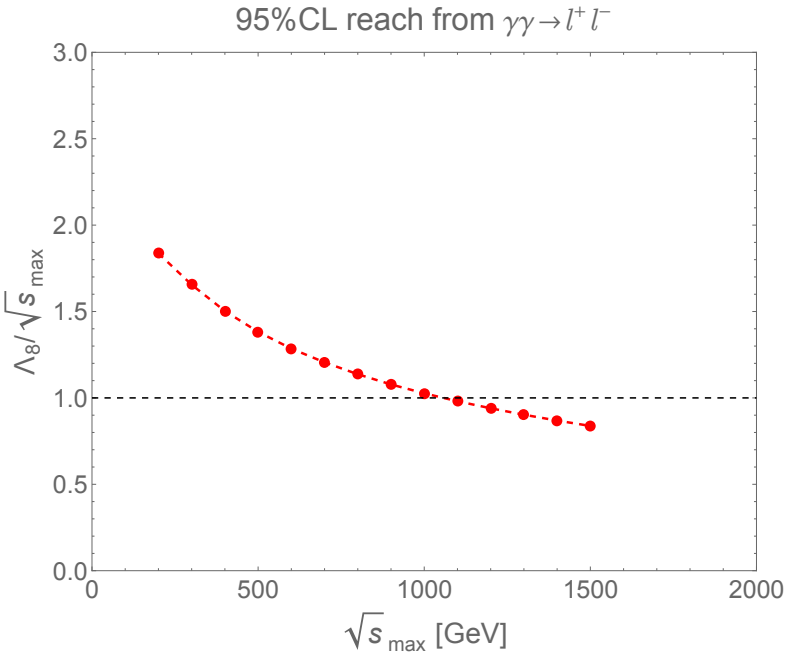
$e^+e^- \to \gamma\gamma$ vs. $\gamma\gamma \to \ell^+\ell^-$



- HL-LHC vs. future lepton colliders (CEPC 240GeV updated to $20\,\mathrm{ab}^{-1}$).
- HL-LHC: A maximum cut on $m_{\ell\ell}$ of 1.5 TeV is imposed.
- The process can also be probed with heavy ion collisions (UPC) but we found the future reach to be around ~ 0.1, mainly due to smaller luminosity and lower E_{cm} of the $\gamma\gamma$ pair.

Is the EFT valid?





- If $E \sim \Lambda$, EFT breaks down!
- Λ depends on coupling (in the UV theory). We define $\Lambda_8 = v/a^{1/4}$, assuming couplings are of order one .
- A more stringent maximum cut on $m_{\ell\ell}$ (denoted as $\sqrt{s}_{\max}$) improves the EFT validity with a price on the reach on Λ_8.

$e^+e^- \to e^+e^-$ [2009.02212] Fuks, Liu, Zhang, Zhou

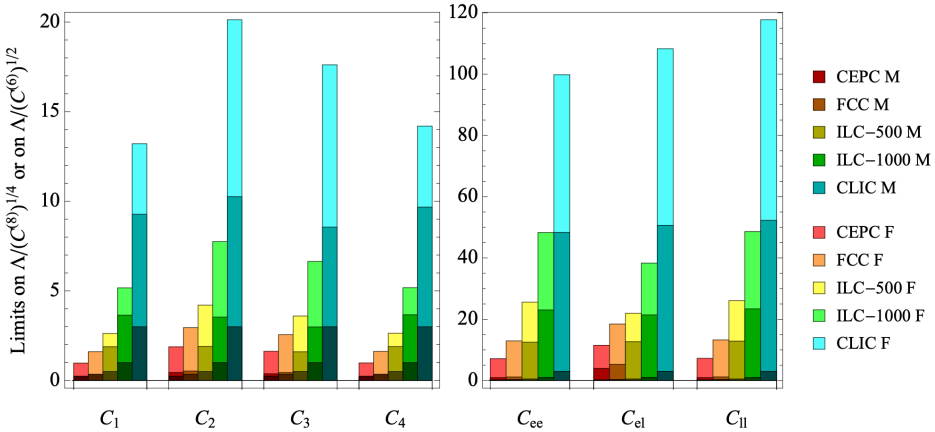
- Many operators, many positivity bounds...

$$O_{ee} = (\bar{e}\gamma^\mu e)\ (\bar{e}\gamma_\mu e)\ ,$$
$$O_{el} = (\bar{e}\gamma^\mu e)\ (\bar{l}\gamma_\mu l)\ ,$$
$$O_{ll} = (\bar{l}\gamma^\mu l)\ (\bar{l}\gamma_\mu l)\ ,$$

$$O_1 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{e}\gamma_\mu e)\ ,$$
$$O_2 = \partial^\alpha(\bar{e}\gamma^\mu e)\partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_3 = D^\alpha(\bar{e}l)\ D_\alpha(\bar{l}e),$$
$$O_4 = \partial^\alpha(\bar{l}\gamma^\mu l)\ \partial_\alpha(\bar{l}\gamma_\mu l)\ ,$$
$$O_5 = D^\alpha(\bar{l}\gamma^\mu\tau^I l)\ D_\alpha(\bar{l}\gamma_\mu\tau^I l)\ ,$$

$$C_1 \le 0,$$
$$C_4 + C_5 \le 0,$$
$$C_5 \le 0,$$
$$C_3 \ge 0,$$
$$2\sqrt{C_1(C_4+C_5)} \ge C_2,$$
$$2\sqrt{C_1(C_4+C_5)} \ge -(C_2+C_3).$$

- Multiple runs with different energies & beam polarizations are very useful! Angular distributions also help.

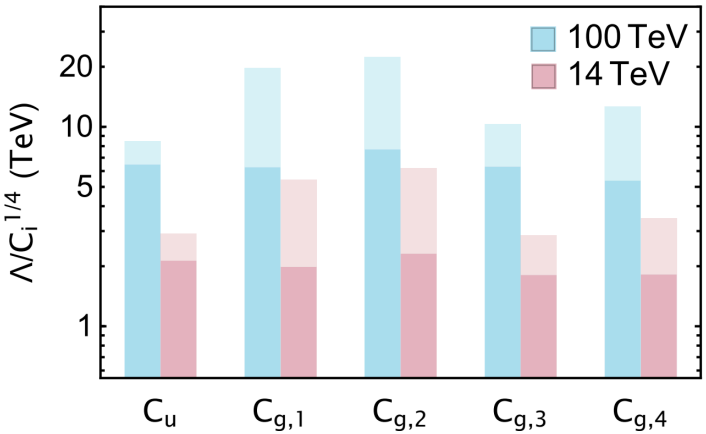


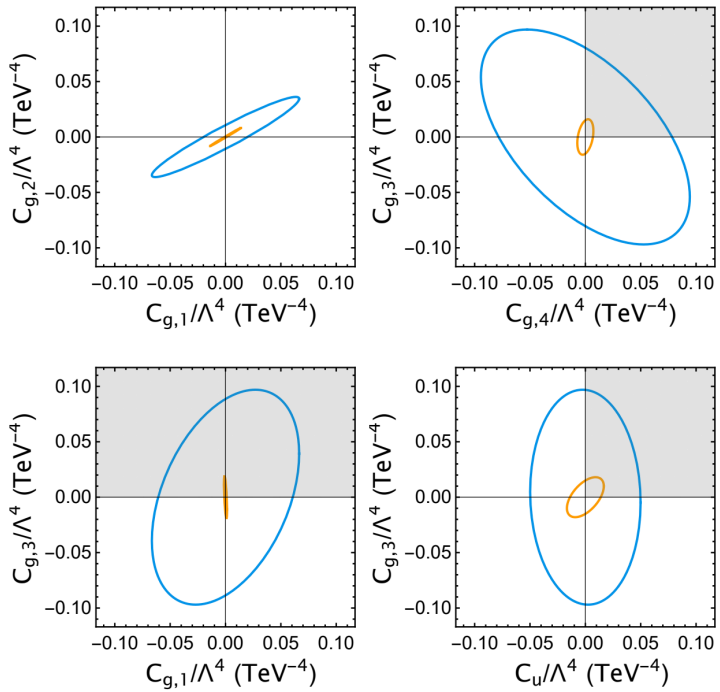
$pp \to \gamma\gamma$ [2512.04336] Liu, Lyu, Wu

- The leading BSM contribution to $q\bar{q} \to \gamma\gamma$ and $gg \to \gamma\gamma$ both come from d8 operators. They have different helicity structures.

$$\mathcal{M}(q_L\bar{q}_L\gamma^+\gamma^-)_{\mathrm{d8}} = \frac{C_{q_L}}{\Lambda^4}[13][23]\langle 24\rangle^2,$$
$$\mathcal{M}(q_R\bar{q}_R\gamma^+\gamma^-)_{\mathrm{d8}} = \frac{C_{q_R}}{\Lambda^4}\langle 14\rangle\langle 24\rangle[23]^2,$$

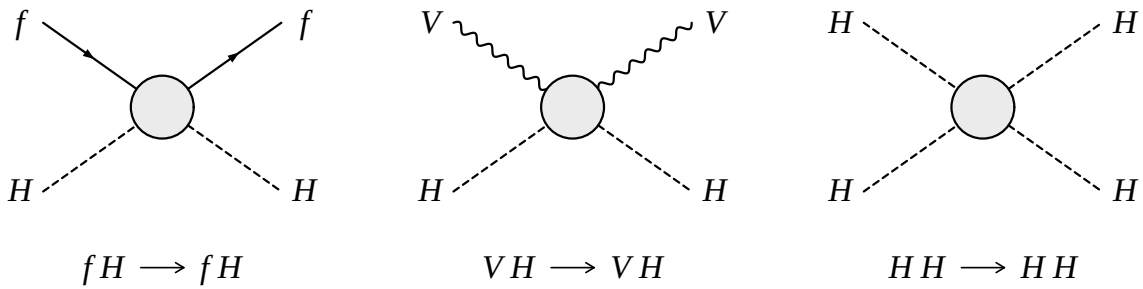
$$\mathcal{M}(g^+g^+\gamma^+\gamma^+)_{\mathrm{d8}} = \frac{1}{\Lambda^4}\Big(4[12]^2[34]^2(C_{g,1}-C_{g,2}) + 2([13]^2[24]^2+[14]^2[23]^2)(C_{g,3}-C_{g,4})\Big)\delta^{a_1a_2},$$
$$\mathcal{M}(g^+g^+\gamma^-\gamma^-)_{\mathrm{d8}} = \frac{1}{\Lambda^4}4[12]^2\langle 34\rangle^2(C_{g,1}+C_{g,2})\delta^{a_1a_2},$$
$$\mathcal{M}(g^+g^-\gamma^+\gamma^-)_{\mathrm{d8}} = \frac{1}{\Lambda^4}2[13]^2\langle 24\rangle^2(C_{g,3}+C_{g,4})\delta^{a_1a_2},$$
$$\mathcal{M}(g^+g^-\gamma^-\gamma^+)_{\mathrm{d8}} = \frac{1}{\Lambda^4}2[14]^2\langle 23\rangle^2(C_{g,3}+C_{g,4})\delta^{a_1a_2},$$





Can we probe the positivity bounds involving the Higgs boson?

(current work with Xiao Cao & Yunxiao Ye)



- dim-8 *HHHH* contact interaction $\sim (DH)^4$
  - Almost impossible to probe? (VBS?)
- dim-8 *VVHH* contact interaction can be probed with $gg \to hh$, $W^+W^- \to hh$, VBS ...
  - Small overall rates, problem with high energy tails...
- dim-8 *ffHH* can be probed with $\ell^+\ell^- \to hZ$, $\ell^+\ell^- \to WW$, $\ell^+\ell^- \to ZZ$ ...
  - No tree-level SM $f\bar{f}hh$ amplitude if we ignore the Yukawa coupling.
  - $\ell^+\ell^- \to hZ/WW/ZZ$: Lepton colliders with fixed c.o.m. energy, no high energy tail problem.

2-lepton-2-higgs operators and their positivity bounds

- Consider all the dim-8 operators that generate contact 2-lepton-2-higgs interactions:

$$
\begin{aligned}
\mathcal{O}^{(1)}_{l^2H^2D^3} &= i(\bar{l}_p\gamma^\mu D^\nu l_r)(D_{(\mu}D_{\nu)}H^\dagger H) + h.c. \\
\mathcal{O}^{(2)}_{l^2H^2D^3} &= i(\bar{l}_p\gamma^\mu D^\nu l_r)(H^\dagger D_{(\mu}D_{\nu)}H) + h.c. \\
\mathcal{O}^{(3)}_{l^2H^2D^3} &= i(\bar{l}_p\gamma^\mu \tau^I D^\nu l_r)(D_{(\mu}D_{\nu)}H^\dagger \tau^I H) + h.c. \\
\mathcal{O}^{(4)}_{l^2H^2D^3} &= i(\bar{l}_p\gamma^\mu \tau^I D^\nu l_r)(H^\dagger D_{(\mu}D_{\nu)}\tau^I H) + h.c. \\
\mathcal{O}^{(1)}_{e^2H^2D^3} &= i(\bar{e}_p\gamma^\mu D^\nu e_r)(D_{(\mu}D_{\nu)}H^\dagger H) + h.c. \\
\mathcal{O}^{(2)}_{e^2H^2D^3} &= i(\bar{e}_p\gamma^\mu D^\nu e_r)(H^\dagger D_{(\mu}D_{\nu)}H) + h.c. \,.
\end{aligned}
\tag{1}
$$

- The bounds are

$$
\begin{aligned}
a_L \equiv\ & c^{(1)}_{l^2H^2D^3} + c^{(2)}_{l^2H^2D^3} + c^{(3)}_{l^2H^2D^3} + c^{(4)}_{l^2H^2D^3} < 0\,, \\
a_R \equiv\ & c^{(1)}_{e^2H^2D^3} + c^{(2)}_{e^2H^2D^3} < 0\,, \\
& c^{(1)}_{l^2H^2D^3} + c^{(2)}_{l^2H^2D^3} - 2c^{(3)}_{l^2H^2D^3} - 2c^{(4)}_{l^2H^2D^3} < 0\,.
\end{aligned}
\tag{2}
$$

- Also define combinations not subject to bounds:

$$
\begin{aligned}
b_L &\equiv c^{(1)}_{l^2H^2D^3} - c^{(2)}_{l^2H^2D^3} + c^{(3)}_{l^2H^2D^3} - c^{(4)}_{l^2H^2D^3}\,, \\
b_R &\equiv c^{(1)}_{e^2H^2D^3} - c^{(2)}_{e^2H^2D^3}\,.
\end{aligned}
\tag{3}
$$

PRELIMINARY

$\ell^-\ell^+ \to hZ$ (in the high energy limit)

$$
\begin{aligned}
\frac{d\sigma_{d8}}{d\cos\theta}(l_L^- l_R^+ \to hZ_R) &= \frac{e^2(2s_w^2-1)\cos^4\left(\frac{\theta}{2}\right)m_Z^2}{32\pi c_w^2\Lambda^4 s_w^2}\left(-2s_w^2 a_L + a_L + b_L\right)\\
\frac{d\sigma_{d8}}{d\cos\theta}(l_L^- l_R^+ \to hZ_L) &= \frac{e^2(2s_w^2-1)\sin^4\left(\frac{\theta}{2}\right)m_Z^2\left(-2s_w^2 a_L + a_L + b_L\right)}{32\pi c_w^2\Lambda^4 s_w^2}\\
\frac{d\sigma_{d8}}{d\cos\theta}(l_L^- l_R^+ \to hZ_0) &= \frac{e^2(2s_w^2-1)\sin^2(\theta)\left(-3b_L m_H^2 + m_Z^2\left((-4s_w^2 a_L) + 2a_L + 3b_L\right) + sb_L\right)}{256\pi c_w^2\Lambda^4 s_w^2}\\
\frac{d\sigma_{d8}}{d\cos\theta}(l_R^- l_L^+ \to hZ_R) &= \frac{e^2\sin^4\left(\frac{\theta}{2}\right)m_Z^2\left(b_R - 2s_w^2 a_R\right)}{16\pi c_w^2\Lambda^4}\\
\frac{d\sigma_{d8}}{d\cos\theta}(l_R^- l_L^+ \to hZ_L) &= \frac{e^2\cos^4\left(\frac{\theta}{2}\right)m_Z^2\left(b_R - 2s_w^2 a_R\right)}{16\pi c_w^2\Lambda^4}\\
\frac{d\sigma_{d8}}{d\cos\theta}(l_R^- l_L^+ \to hZ_0) &= \frac{e^2\sin^2(\theta)\left(-3b_R m_H^2 + m_Z^2(3b_R - 4s_w^2 a_R) + sb_R\right)}{128\pi c_w^2\Lambda^4}
\end{aligned}
\tag{4}
$$

- $a < 0$, b is not bounded.
- The leading energy contributions are not subject to positivity bounds.
- It comes from $\ell^+\ell^- h\phi_0$ contact interaction which cancels in the $\ell H \to \ell H$ elastic amplitude.

PRELIMINARY

$\ell^-\ell^+ \to hZ$

- Other dim-8 contributions also contribute at subleading $(\sim m^2/s)$ level.
  - *e.g.*, $(\bar{\ell}\gamma^\nu \ell)\, \partial^\mu (H^\dagger H)\, B_{\mu\nu}$
- For unpolarized beams there is a flat direction consistent with positivity.
- The left- and right-handed SM *Zee* couplings have opposite signs.

PRELIMINARY

$\ell^-\ell^+ \to ZZ$ (in the high energy limit)

$$\frac{d\sigma}{d\cos\theta}\left(\ell_L^-\ell_R^+ \to Z_0Z_0\right) \simeq \frac{e^4 m_Z^4 (1-2s_W^2)^4}{16\pi s^3 c_W^4 s_W^4}\cot^2\theta - \frac{e^2 a_L m_Z^2 (1-2s_W^2)^2}{64\pi\Lambda^4 c_W^2 s_W^2}\cos^2\theta,$$

$$\frac{d\sigma}{d\cos\theta}\left(\ell_R^-\ell_L^+ \to Z_0Z_0\right) \simeq \frac{e^4 m_Z^4 s_W^4}{\pi s^3 c_W^4}\cot^2\theta - \frac{e^2 a_R m_Z^2 s_W^2}{16\pi\Lambda^4 c_W^2}\cos^2\theta. \quad (5)$$

- Leading contribution to Z_0Z_0, but the SM Z_0Z_0 is suppressed (m_Z^2/s) compared with the (opposite helicity) transverse modes.
  ([1806.09640] Bellazzini, Riva)
- Many other contributions ...
  (See also work on NTGCs by Ellis, He, Xiao *et al.*)
- Do we have enough observables for a global fit with all dim-8 contributions?
- Next step: Look at *WW*.

PRELIMINARY

Conclusion

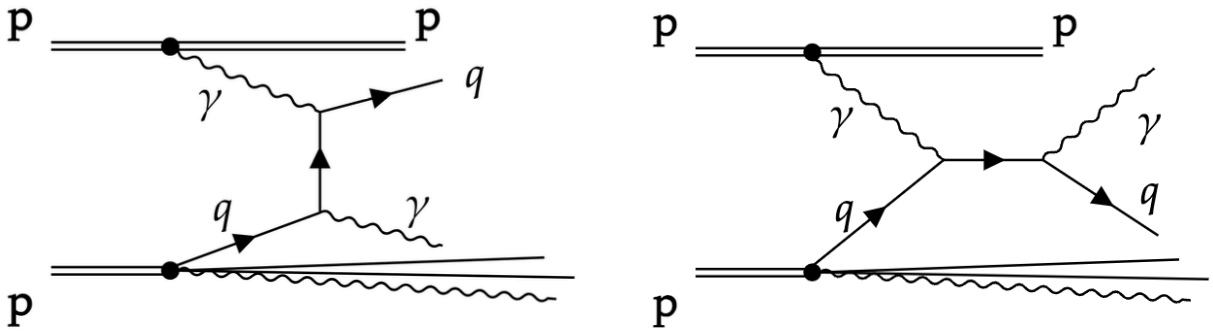
- **It's important to test the fundamental principles of QFT!**
  - Positivity bounds are concrete predictions of the fundamental principles of QFT that can be tested.

- **It's possible to test some of the positivity bounds!**
  - Measurements of $e^+e^- \to \gamma\gamma$ ($\mu^+\mu^- \to \gamma\gamma$) at future e^+e^- ($\mu^+\mu^-$) colliders offer a unique opportunity to directly probe dimension-8 operators and their positivity bounds.
  - The measurement of $\gamma\gamma \to \ell^+\ell^-$ at the HL-LHC is also promising, but the reach is slightly worse and the EFT validity could be an issue.

- **It's very difficult (but maybe still possible) to test the Higgs positivity bounds!**

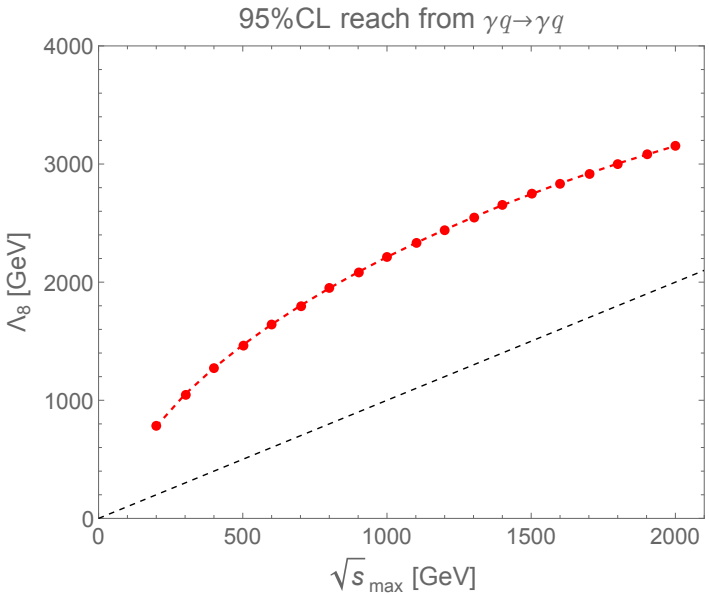
backup slides

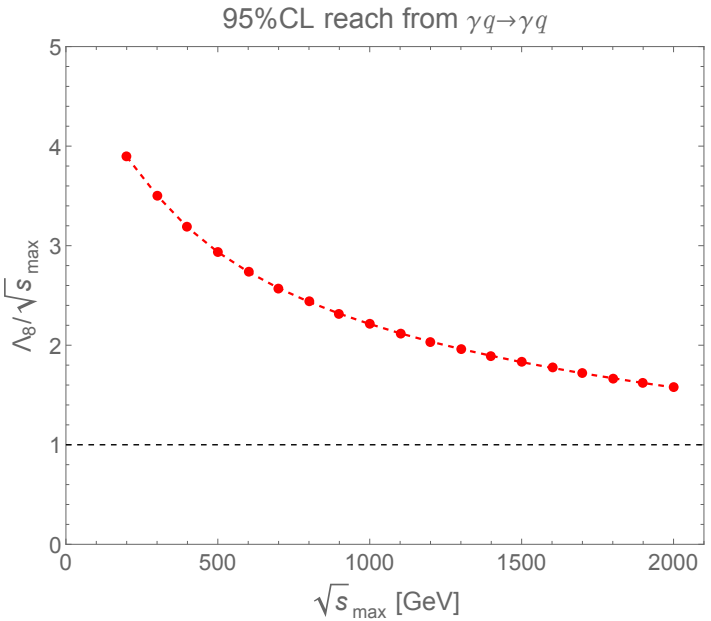
The $\gamma q \to \gamma q$ channel?



- One proton breaks, one proton doesn't....
- Potentially has a larger rate and controllable background.
- We don't really know the signal and background rates...
- Positivity gives us a good motivation to measure and study this process.

The $\gamma q \to \gamma q$ channel

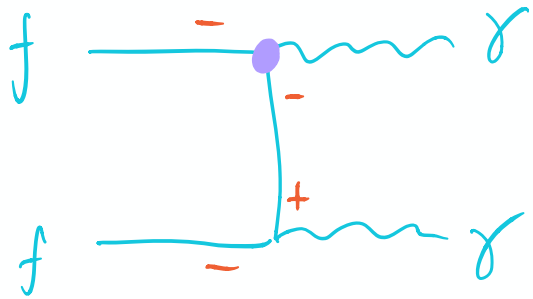




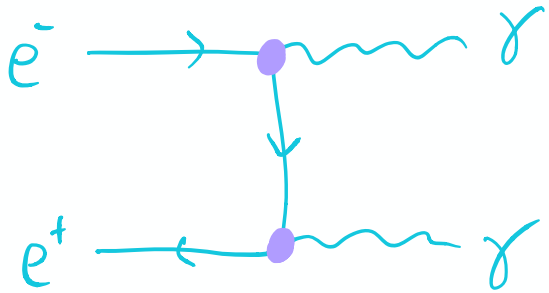
- A **very** ideal analysis suggests the reach on $a = a_L + a_R$ can get to $\sim 10^{-5}$, assuming it is universal for all quarks.
- More studies on the signal and background rates are needed.

All other contributions are either vanishing or suppressed!

- **The only tree-level d6 contribution are from dipole operators and have different fermion helicities.**

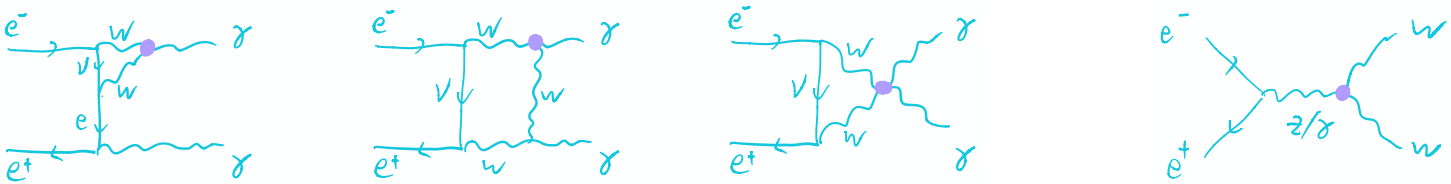


- **SM$\times$d6 at tree level:** no interference (suppressed by m_e).
- **d6^2:** Dipole operators are very well constrained by $g-2$ and EDM measurements.

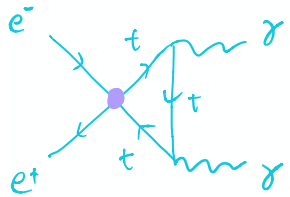


All other contributions are either vanishing or suppressed!

- **SM×d6 at 1-loop:** are either very-well constrained by other measurements with tree-level contributions, or forbidden by selection rules.
  - O_{3W} is very well constrained by $e^+e^- \to WW$ measurements.



  - Other contributions are constrained by Z-pole measurements or suppressed by the small y_e.
  - Contribution from the $eett$ 4f operator is forbidden by angular momentum selection rules. ([2001.04481] Jiang, Shu, Xiao, Zheng)



$= 0$ (for e^+e^- with opposite helicities)

- **other d8:** They have different helicities and do not interfere with SM.

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy

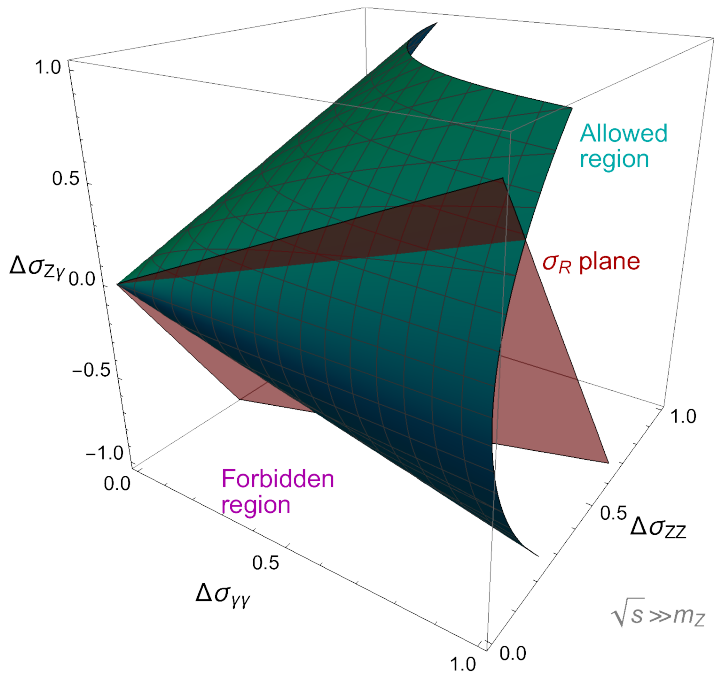
- $Z\gamma$, ZZ processes are more complicated due to the massive Z.
  - Other helicity states contribute in both SM and BSM (*e.g.* nTGCs).
  - In the high energy limit, $\mathcal{A}(f^+f^-V^+V^-)$ dominates in SM.

- In the $\sqrt{s} \gg m_Z$ limit,

$$\sigma(e^+e^- \to ZZ) \geq \sigma_{\rm SM}(e^+e^- \to ZZ)\,.$$

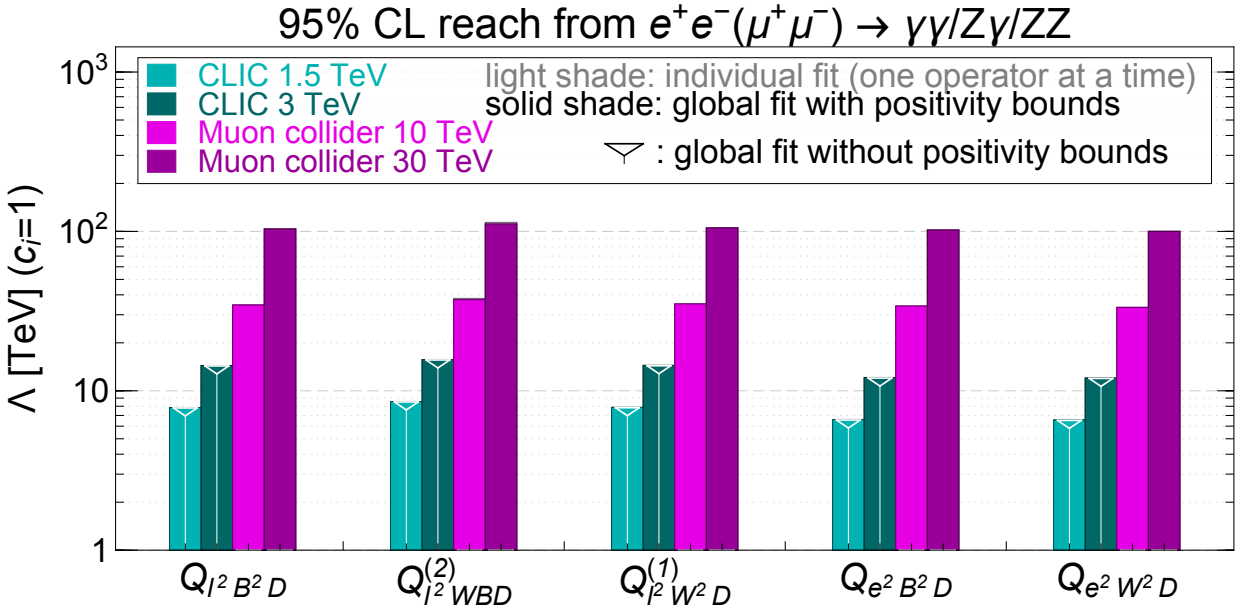
- Consider the elastic amplitude of $eV \to eV$,
  - V is an arbitrary mixing state of γ and Z,
  - scan over the mixing angle to obtain the strongest bound ($\Delta\sigma \equiv \sigma - \sigma_{\rm SM}$),

$$(\Delta\sigma_{Z\gamma})^2 \leq 4\Delta\sigma_{\gamma\gamma}\Delta\sigma_{ZZ}\,.$$



- σ_Rs only occupy a plane in the 3d parameter space.
  - 3 operators with ℓ_L, 2 operators with e_R.

Combined $\gamma\gamma/Z\gamma/ZZ$ analysis at high energy

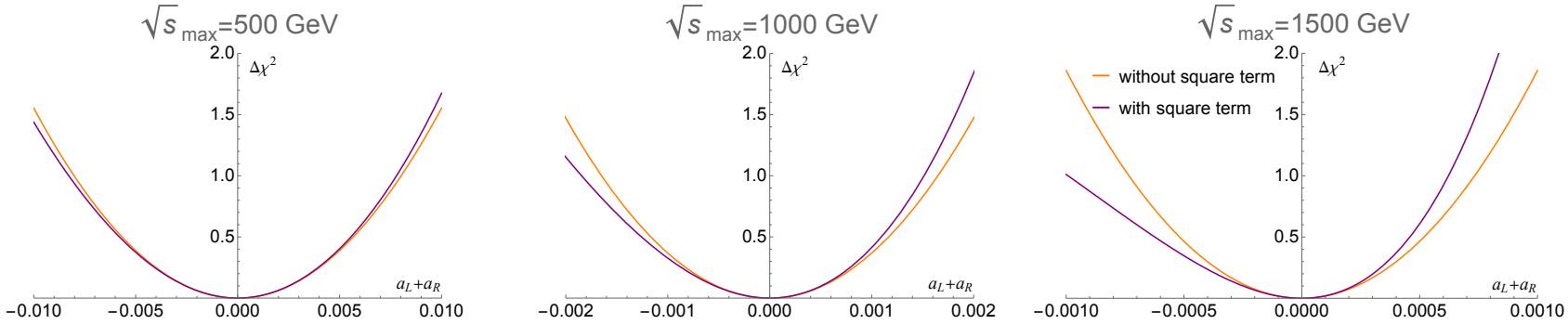


- Global fit of the $\gamma\gamma/Z\gamma/ZZ$ processes in the high energy limit.
- No beam polarizations $\Rightarrow$ flat directions.
- Flat directions are lifted once the positivity bounds are imposed!

Collider scenarios for $e^+e^- \to \gamma\gamma$ (already outdated)

$\int \mathcal{L}dt\ [\mathrm{ab}^{-1}]$				
unpolarized	91 GeV	161 GeV	240 GeV	365 GeV
CEPC	8	2.6	5.6	
FCC-ee	150	10	5	1.5
ILC	250 GeV	350 GeV	500 GeV	
$(-0.8, +0.3)$	0.9	0.135	1.6	
$(+0.8, -0.3)$	0.9	0.045	1.6	
$(\pm 0.8, \pm 0.3)$	0.1	0.01	0.4	
CLIC	380 GeV	1.5 TeV	3 TeV	
$(-0.8, 0)$	0.5	2	4	
$(+0.8, 0)$	0.5	0.5	1	
muon collider	10 TeV	30 TeV		
unpolarized	10	90		

With and without the dim-8 squared term ($\gamma\gamma \to \ell^+\ell^-$)



- χ^2 as a function of $a \equiv a_L + a_R$, for different $\sqrt{s}_{\max}$.
- (dim-8)^2 $\sim$ dim-12 !
- Even for $\sqrt{s}_{\max} = 1500\,\text{GeV}$, the 1-sigma bound on a changes from $\pm 7.3 \times 10^{-4}$ to $[-9.9\,,\,6.2] \times 10^{-4}$.

Positivity bounds at one-loop level

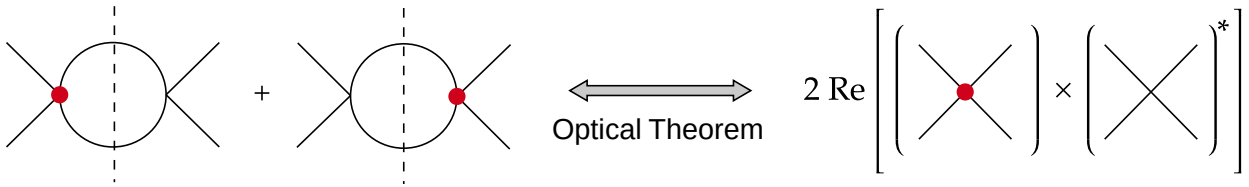
- So far we've assumed that tree-level effects dominates.
- What's the implication of positivity bounds at loop level?
  - The kinematic dependence becomes more complicated due to the log terms.
  - The definitions of renormalized parameters are ambiguous and depend on the renormalization scheme.
  - Couplings can **run**!
- Some interesting observations have been found in previous studies.
  - IR effects can be important. [2011.00037, 2112.12561] Bellazzini *et al.*
  - The naive tree-level positivity bound could appear to be violated. [2110.01624] Chala, Santiago
  - Some of the RG mixing of dim-8 coefficients are subject to positivity bounds. [2301.09995] Chala, [2309.16611] Chala, Li, [2505.02910] Liao, Nepveu, Shen

The main messages

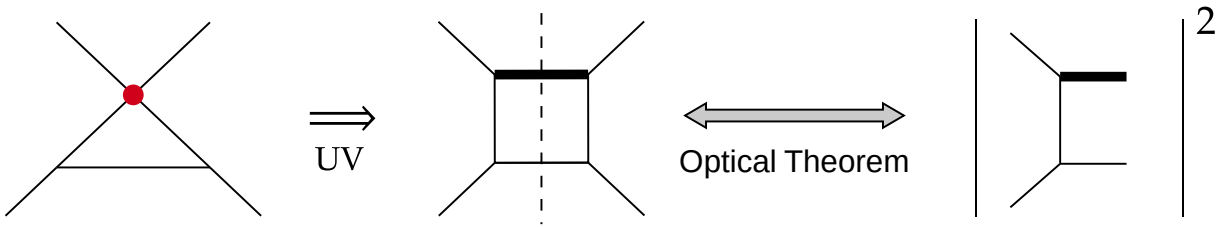
- **"Something" is positive,** but that something can have a lot of contributions and each contribution is not necessarily positive!
- It is important to include all contributions up to a fixed loop order in the UV theory for the dispersion relation (positivity bound) to be valid.
  - A loop generated dim-8 coefficient may appear to violate its tree-level bound simply because there are other contributions at the same loop level to the dispersion relation.
- Running does not violate any bound or give us any new bound. It only changes the definitions of couplings.
- In some cases the one-loop RG mixing of dim-8 coefficients are subject to positivity bounds, depending on the one-loop structure.

Two different scenarios

- An “asymmetric” one-loop diagram of the forward elastic amplitude corresponds to the interference term of a cross section and can take either sign!



- A “symmetric” one-loop diagram corresponds to a squared contribution to the cross section and could be subject to a positivity bound (which can have several contributions in the EFT).



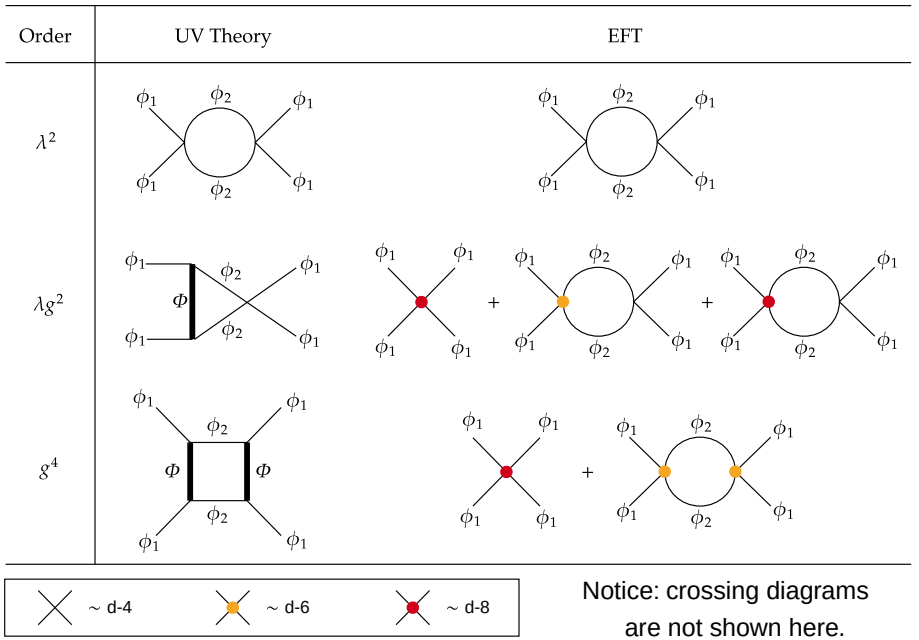
A (counter) example [2408.10318] Yunxiao Ye, Bin He, JG

- Consider a model with 2 real light scalars $\phi_{1,2}$ and a real heavy scalar Φ,

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu \Phi \partial_\mu \Phi - M^2 \Phi^2\right) + \frac{1}{2}\partial^\mu \phi_1 \partial_\mu \phi_1 + \frac{1}{2}\partial^\mu \phi_2 \partial_\mu \phi_2 - gM\Phi\phi_1\phi_2 - \frac{1}{4}\lambda\phi_1^2\phi_2^2 \,,$$

- At low energy we can integrate out Φ and match it to the EFT with $\phi_{1,2}$.

- A positivity bound can be obtained from the forward elastic amplitude $\phi_1\phi_1 \to \phi_1\phi_1$ (or $1 \leftrightarrow 2$).
- For $\lambda \gg g^2$, the new physics contributions are proportional to λg^2, which can obviously take either sign.
- **What's wrong?**



Notice: crossing diagrams are not shown here.

Back to the dispersion relation

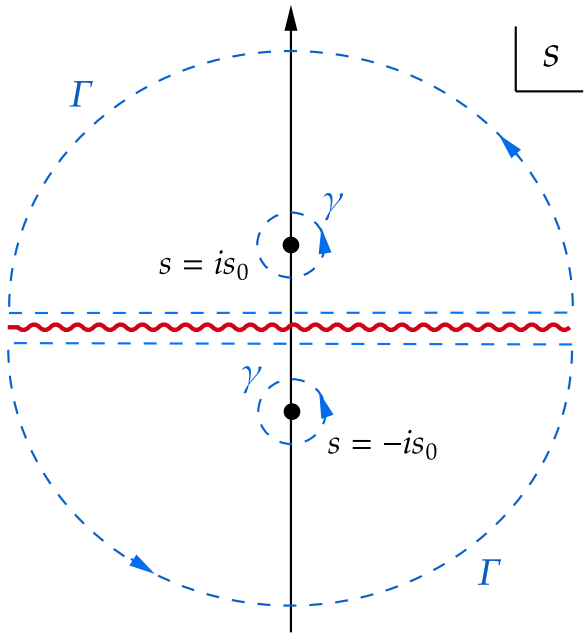
- Two different scales:
  - s_0 labels the position of the pole in the comples s plane. (often denoted as μ^2, which we will avoid doing...)
  - μ is the renormalization scale (in $\overline{MS}$).

- Due to the cross symmetry of $\phi_1\phi_1 \to \phi_1\phi_1$, we can consider the massless case (which makes life much easier) and use a modified version of the dispersion relation.

  [2011.11652] Herrero-Valea, Santos-Garcia, Tokareva

$$\Sigma \equiv \left(\oint_{s=is_0} + \oint_{s=-is_0} \right) \frac{ds}{2\pi i} \frac{s^3 \tilde{\mathcal{A}}(s)}{(s^2+s_0^2)^3}$$

$$= \frac{2}{\pi} \int_0^\infty ds \, \frac{s^4 \sigma(s)}{(s^2+s_0^2)^3} \geq 0 \,.$$



- This holds for any real s_0, but $s_0 \ll \Lambda^2$ is required for the EFT validity.
  - Dim-10 contributions are further suppressed by a factor of s_0/Λ^2.
  - With loops, we don't want to let $s_0 \to 0$ in which case Σ may diverge.

Logs are important!

$$\oint_\gamma \frac{ds}{2\pi i}\frac{s^3}{(s^2+s_0^2)^3} = 0\,,$$

$$\oint_\gamma \frac{ds}{2\pi i}\frac{s^3}{(s^2+s_0^2)^3}\, s = 0\,,$$

$$\oint_\gamma \frac{ds}{2\pi i}\frac{s^3}{(s^2+s_0^2)^3}\, s^2 = 1\,,$$

$$\oint_\gamma \frac{ds}{2\pi i}\frac{s^3}{(s^2+s_0^2)^3}\log(s) = -\frac{1}{4}\frac{1}{s_0^2}\,,$$

$$\oint_\gamma \frac{ds}{2\pi i}\frac{s^3}{(s^2+s_0^2)^3}\, s\log(s) = \frac{3\pi}{16 s_0}\,,$$

$$\oint_\gamma \frac{ds}{2\pi i}\frac{s^3}{(s^2+s_0^2)^3}\, s^2\log(s) = \frac{3}{4} + \log(s_0)\,.$$

- At tree level, only dim-8 ($\sim s^2$) or above contributes to the dispersion relation.
- At loop level, dim-4 ($\sim \log(s)$) and dim-6 ($\sim s\log(s)$) also contributes!

The 2-scalar EFT

	$\mathcal{A}_1 \equiv \mathcal{A}(\phi_1\phi_1\phi_1\phi_1)$	$\mathcal{A}_2 \equiv \mathcal{A}(\phi_2\phi_2\phi_2\phi_2)$	$\mathcal{A}_{12} \equiv \mathcal{A}(\phi_1\phi_2\phi_1\phi_2)$
$D4$	$\mathcal{A}_1^{[4]} = c_1^{[4]}$	$\mathcal{A}_2^{[4]} = c_2^{[4]}$	$\mathcal{A}_{12}^{[4]} = c_{12}^{[4]}$
$D6$	$\mathcal{A}_1^{[6]} = 0$	$\mathcal{A}_2^{[6]} = 0$	$\mathcal{A}_{12}^{[6]} = c_{12}^{[6]}\, t$
$D8$	$\mathcal{A}_1^{[8]} = c_1^{[8]}(s^2+t^2+u^2)$	$\mathcal{A}_2^{[8]} = c_2^{[8]}(s^2+t^2+u^2)$	$\mathcal{A}_{12}^{[8]} = c_{12,su}^{[8]}(s^2+u^2) + c_{12,t}^{[8]}t^2$

$$\begin{aligned}\mathcal{L} =& \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + \frac{c_1^{[4]}}{4!}\phi_1^4 + \frac{c_2^{[4]}}{4!}\phi_2^4 + \frac{c_{12}^{[4]}}{4}\phi_1^2\phi_2^2 + c_{12}^{[6]}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_2\right)\phi_1\phi_2 \\ &+ \frac{c_1^{[8]}}{2}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_1\right)\left(\partial_\nu\phi_1\right)\left(\partial^\nu\phi_1\right) + \frac{c_2^{[8]}}{2}\left(\partial_\mu\phi_2\right)\left(\partial^\mu\phi_2\right)\left(\partial_\nu\phi_2\right)\left(\partial^\nu\phi_2\right) \\ &+ 2c_{12,su}^{[8]}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_2\right)\left(\partial_\nu\phi_1\right)\left(\partial^\nu\phi_2\right) + c_{12,t}^{[8]}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_1\right)\left(\partial_\nu\phi_2\right)\left(\partial^\nu\phi_2\right),\end{aligned}$$

- The Wilson coefficients in the massless scalar EFT can be conveniently parameterized in terms of tree level on-shell amplitudes.
- (Note: We've absorbed Λ in the definition of couplings.)

$\mathcal{A}(\phi_1\phi_1\phi_1\phi_1)$ at the 1 loop level

$$\mathcal{A}_1^{[4]} = c_1^{[4]} + \frac{1}{32\pi^2}\left(\left(c_1^{[4]}\right)^2 + \left(c_{12}^{[4]}\right)^2\right)\left(-\log\frac{-s}{\mu^2} - \log\frac{-t}{\mu^2} - \log\frac{-u}{\mu^2} + 6\right),$$

$$\mathcal{A}_1^{[6]} = \frac{1}{16\pi^2}\left(c_{12}^{[4]}c_{12}^{[6]}\right)\left(-s\log\frac{-s}{\mu^2} - t\log\frac{-t}{\mu^2} - u\log\frac{-u}{\mu^2}\right).$$

$$\mathcal{A}_1^{[8],\text{tree}} = c_1^{[8]}\left(s^2+t^2+u^2\right),$$

$$\begin{aligned}\mathcal{A}_1^{[8],\text{1-loop}} = \frac{1}{16\pi^2}s^2\Bigg[&-\log\frac{-s}{\mu^2}\left(\frac{1}{2}\left(c_{12}^{[6]}\right)^2 + \frac{2}{3}c_{12}^{[4]}c_{12,su}^{[8]} + c_{12}^{[4]}c_{12,t}^{[8]} + \frac{5}{3}c_1^{[4]}c_1^{[8]}\right)\\ &+\left(c_{12}^{[6]}\right)^2 + \frac{13}{9}c_{12}^{[4]}c_{12,su}^{[8]} + 2\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{31}{9}c_1^{[4]}c_1^{[8]}\Bigg]\\ &+ \quad (s\longleftrightarrow t) \quad + \quad (s\longleftrightarrow u).\end{aligned}$$

- $\overline{MS}$ scheme. μ is the renormalization scale at which the couplings are defined.
- Amplitudes are independent of μ (which gives RG equations).

Positivity bound at the 1-loop level

The Σ (≥ 0) for $\phi_1\phi_1 \to \phi_1\phi_1$ in the EFT up to 1-loop and dim-8 is

$$\Sigma = 2c_1^{[8]} + \frac{1}{64\pi^2}\frac{1}{s_0^2}\left(\left(c_1^{[4]}\right)^2 + \left(c_{12}^{[4]}\right)^2\right) + \frac{1}{16\pi^2}\frac{1}{s_0}\frac{3\pi}{8}c_{12}^{[4]}c_{12}^{[6]} + \left(\frac{3}{4} + \log\frac{s_0}{\mu^2}\right)\beta_1^{[8]} + \frac{1}{16\pi^2}\left(2\left(c_{12}^{[6]}\right)^2 + \frac{26}{9}c_{12}^{[4]}c_{12,su}^{[8]} + 4\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{62}{9}c_1^{[4]}c_1^{[8]}\right),$$

where

$$\beta_1^{[8]} \equiv \mu\frac{dc_1^{[8]}}{d\mu} = -\frac{1}{16\pi^2}\left(\frac{4}{3}c_{12}^{[4]}c_{12,su}^{[8]} + 2\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{10}{3}c_1^{[4]}c_1^{[8]} + \left(c_{12}^{[6]}\right)^2\right).$$

- Σ is independent of μ just like the amplitude. Requiring $\mu\frac{d}{d\mu}\Sigma = 0$ gives us the β function!
  - By changing μ we don't really change the positivity bound! We just change the definition of $c_1^{[8]}$.
- Σ does depend on s_0. $\Sigma \geq 0$ is valid as long as $0 < s_0 \ll \Lambda^2$.

Positivity bound at the 1-loop level

$$\Sigma = 2c_1^{[8]} + \frac{1}{64\pi^2}\frac{1}{s_0^2}\left(\left(c_1^{[4]}\right)^2 + \left(c_{12}^{[4]}\right)^2\right) + \frac{1}{16\pi^2}\frac{1}{s_0}\frac{3\pi}{8}c_{12}^{[4]}c_{12}^{[6]} + \left(\frac{3}{4} + \log\frac{s_0}{\mu^2}\right)\beta_1^{[8]}$$
$$+ \frac{1}{16\pi^2}\left(2\left(c_{12}^{[6]}\right)^2 + \frac{26}{9}c_{12}^{[4]}c_{12,su}^{[8]} + 4\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{62}{9}c_1^{[4]}c_1^{[8]}\right).$$

- If $c_1^{[8]}$ is generated at one loop, while the other coefficients are generated at the tree level, then $c_1^{[8]}$ and the 1-loop contribution of tree-level coefficients are at the same loop order.
  - The tree level bound $c_1^{[8]} \geq 0$ does not necessarily hold at loop level!
- Dim-4 and dim-6 contributions are important at the one-loop level (because of the log s terms)!
- In general $\beta_1^{[8]}$ can take either sign.

Back to the $\Phi\phi_1\phi_2$ model

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu\Phi\partial_\mu\Phi - M^2\Phi^2\right) + \frac{1}{2}\partial^\mu\phi_1\partial_\mu\phi_1 + \frac{1}{2}\partial^\mu\phi_2\partial_\mu\phi_2 - gM\Phi\phi_1\phi_2 - \frac{1}{4}\lambda\phi_1^2\phi_2^2\,,$$

- $\frac{c_1^{[8]}}{2}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_1\right)\left(\partial_\nu\phi_1\right)\left(\partial^\nu\phi_1\right)$ is only generated at one-loop level.
- $c_1^{[8]}$ needs to be matched to the one-loop level, which is (at matching scale M):

$$c_1^{[8]}(M) = \frac{1}{16\pi^2}\frac{g^2}{M^4}\frac{1}{45}\left(55\lambda - 166g^2\right)\,,$$

- Other coefficients are matched to the tree level

$$c_{12}^{[4]}(M) = 2g^2 - \lambda\,, \qquad c_{12}^{[6]}(M) = -\frac{g^2}{M^2}\,, \qquad c_{12,su}^{[8]}(M) = \frac{g^2}{M^4}\,,$$

- Plug them in Σ, run $c_1^{[8]}$ down to μ ...

Back to the $\Phi\phi_1\phi_2$ model

- This gives (the dependence on μ cancels as intended)

$$\Sigma = \frac{\lambda^2}{64\pi^2}\frac{1}{s_0^2} + \frac{\lambda g^2}{16\pi^2}\left(-\frac{1}{s_0^2} + \frac{3\pi}{8}\frac{1}{M^2 s_0} + \frac{5}{9}\frac{1}{M^4} + \frac{4}{3}\frac{1}{M^4}\log\frac{s_0}{M^2}\right)$$
$$+ \frac{g^4}{16\pi^2}\left(\frac{1}{s_0^2} - \frac{3\pi}{4}\frac{1}{M^2 s_0} - \frac{47}{20}\frac{1}{M^4} - \frac{11}{3}\frac{1}{M^4}\log\frac{s_0}{M^2}\right),$$

- It can be verified by calculating $\frac{2}{\pi}\int_0^\infty ds\,\frac{s^4[\sigma(\phi_1\phi_1\to\phi_2\phi_2)+\sigma(\phi_1\phi_1\to\Phi\Phi)]}{\left(s^2+s_0^2\right)^3}$ at tree level and expand to $\mathcal{O}(M^{-4})$.
- The beta function is now

$$\beta_1^{[8]} = \frac{1}{16\pi^2}\left(\frac{4}{3}\frac{\lambda g^2}{M^4} - \frac{11}{3}\frac{g^4}{M^4}\right).$$

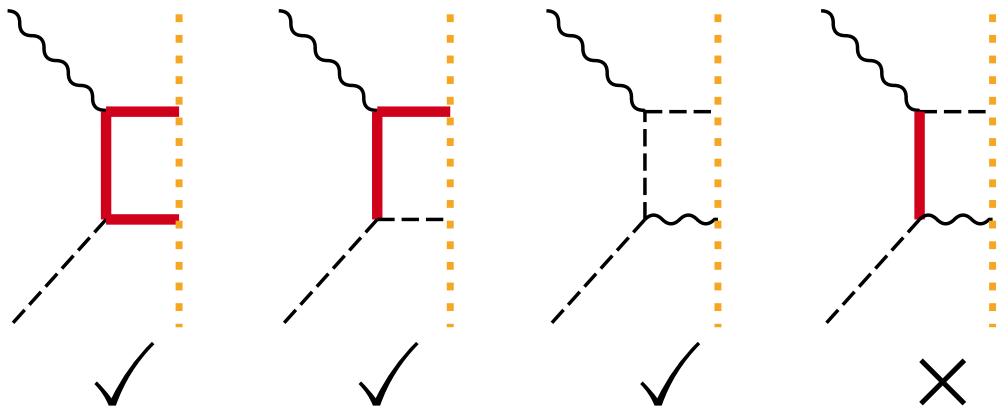
- Consider the limit $\lambda \gg g^2$, obviously $\beta_1^{[8]}$ can take either sign in this case!
- In the opposite limit $\lambda \to 0$, it seems that the positivity bound is violated when $s_0 \approx M$, but when $s_0 \approx M$ the EFT expansion breaks down and we cannot truncate Σ at $\mathcal{O}(M^{-4})$...

Scalar QED EFT [2507.06302] Yunxiao Ye, Xiao Cao, Yu-Hang Wu, JG

- The situation is a bit different if external photons (gauge bosons) are involved!
  - Gauge invariance imposes strong constraints on the form of the UV completion.

- For simplicity, we focus on scalar QED EFT here, where the only light d.o.f. are a complex (charge 1) scalar and a photon.

- We assume heavy particles have spin ≤ 1.
  - No tree-level BSM contribution to $\phi\gamma \to \phi\gamma$ and $\gamma\gamma \to \gamma\gamma$.
  - At one loop, SM and BSM contributions are separately positive. Only SM contributes to $\sigma(SM \to SM)$. (By SM I mean scalar QED...)



Scalar QED EFT

$$
\begin{aligned}
\mathcal{L}_{[\mathcal{O}]\leq 4} =& -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) - \frac{1}{4}\lambda_1(\phi^\dagger\phi)^2\,,\\
\mathcal{L}_6\cdot\Lambda^2 =& c_{\phi^6}(\phi^\dagger\phi)^3 + c_{D^2\phi^4}\phi^\dagger\phi(D_\mu\phi)^\dagger D^\mu\phi + c_{F^2\phi^2}\phi^\dagger\phi F^{\mu\nu}F_{\mu\nu}\,,\\
\mathcal{L}_8\cdot\Lambda^4 =& c^{(1)}_{F^4}(F_{\mu\nu}F^{\mu\nu})^2 + c^{(2)}_{F^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2\\
&+ c^{(1)}_{F^2D^2\phi^2}(D_\mu\phi)^\dagger(D_\nu\phi)F^{\mu\rho}F^{\nu}_{\rho} + c^{(2)}_{F^2D^2\phi^2}(D^\mu\phi)^\dagger(D_\mu\phi)F_{\nu\rho}F^{\nu\rho}\\
&+ c^{(1)}_{D^4\phi^4}((D_\mu\phi)^\dagger)^2(D_\nu\phi)^2 + c^{(2)}_{D^4\phi^4}\phi^\dagger\phi(D_\mu D_\nu\phi)^\dagger(D_\mu D_\nu\phi)\\
&+ c_{F^2\phi^4}(\phi^\dagger\phi)^2F_{\mu\nu}F^{\mu\nu} + c_{D^2\phi^6}(\phi^\dagger\phi)^2(D_\mu\phi)^\dagger D^\mu\phi + c_{\phi^8}(\phi^\dagger\phi)^4\,.
\end{aligned}
$$

- Tree-level positivity bounds:

$$
2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4} \geq 0\,,\quad c^{(1)}_{F^2D^2\phi^2} < 0\,,\quad c^{(1)}_{F^4} > 0\,,\quad c^{(2)}_{F^4} > 0\,.
$$

- $c^{(1)}_{F^2D^2\phi^2}$, $c^{(1)}_{F^4}$ and $c^{(2)}_{F^4}$ are generated at the 1-loop level in the UV theory under our assumptions.

$\phi\gamma \to \phi\gamma$

- Consider γ in the helicity basis, now ϕ is a complex scalar. However, the charge conjugation symmetry in (scalar) QED ensures the amplitude still has a $s \leftrightarrow u$ symmetry.
- Repeating what we did in the scalar case, we find

$$
\begin{aligned}
\Sigma =& \frac{e^4}{8\pi^2}\frac{1}{s_0^2} - \frac{1}{\Lambda^4}\frac{1}{2}c^{(1)}_{F^2D^2\phi^2} \\
&+ \frac{1}{\Lambda^4}\frac{1}{96\pi^2}\left[6e^2 c^{(1)}_{F^2D^2\phi^2}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\log\left(\frac{-t}{\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{-t}{\mu^2}\right) - \frac{1}{2}\zeta_2\right)\right. \\
&\quad + e^2(2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4} + 18c^{(1)}_{F^2D^2\phi^2} + 96c^{(1)}_{F^4} + 96c^{(2)}_{F^4})\left(-\log\left(\frac{-t}{\mu^2}\right) + 2\right) \\
&\quad \left. + (6e^2 c^{(1)}_{F^2D^2\phi^2} + 8c^2_{F^2\phi^2})\left(-\frac{3}{2} - 2\log\left(\frac{s_0}{\mu^2}\right) + 4\right)\right] \\
&+ \frac{1}{576\pi^2\Lambda^4}\left[14e^2 c^{(1)}_{D^4\phi^4} + 7e^2 c^{(2)}_{D^4\phi^4} - 1152e^2 c^{(2)}_{F^4} + 16c^2_{F^2\phi^2} + 12e^2 c^{(1)}_{F^2D^2\phi^2} - 96e^2 c^{(2)}_{F^2D^2\phi^2}\right],
\end{aligned}
$$

- No t-channel simple pole in this case.
- IR divergence may exist for fixed order amplitudes (but here we only need to worry about the leading order contribution of $c^{(1)}_{F^2D^2\phi^2}$) ....
- $\log t$ can be regulated by giving ϕ a small mass m.

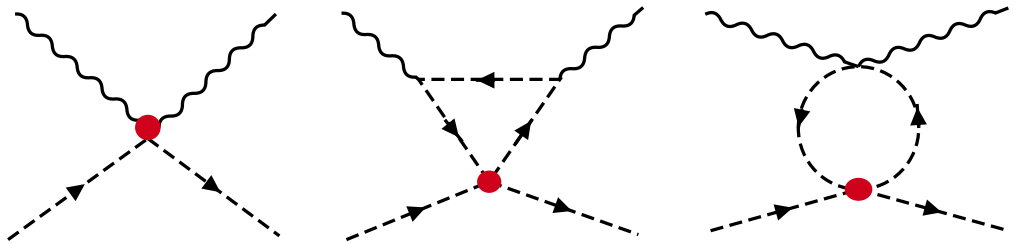
$\phi\gamma \to \phi\gamma$

- We can subtract the SM contribution by requiring at least one final state heavy particle in $\sigma_{\rm tot}(\phi\gamma)$.

$$\Sigma' = \Sigma - \Sigma_{\rm SM} \geq 0\,.$$

- Only $c^{(1)}_{D^4\phi^4}$ and $c^{(2)}_{D^4\phi^4}$ can be generated at the tree level (under our assumptions).
- Up to one loop in the UV, we have (omitting $\mathcal{O}(m^2)$ corrections)

$$\Sigma' \cdot 2\Lambda^4 \approx -c^{(1)}_{F^2D^2\phi^2} + \frac{19e^2}{288\pi^2}(2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4}) - \frac{e^2}{48\pi^2}(2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4})\log\frac{m^2}{\mu^2} \geq 0\,.$$



$c^{(1)}_{F^2D^2\phi^2}$ (generated at one loop)

$2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4}$

$\phi\gamma \to \phi\gamma$

- Up to one loop in the UV, we have

$$\Sigma' \cdot 2\Lambda^4 \approx -c^{(1)}_{F^2D^2\phi^2} + \frac{19e^2}{288\pi^2}(2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4}) - \frac{e^2}{48\pi^2}(2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4}) \log\frac{m^2}{\mu^2} \geq 0\,.$$

- Here Σ' has no s_0 dependence (because only t enters these loops) ...
- The leading order β function for $c^{(1)}_{F^2D^2\phi^2}$ is (which can also be obtained by requiring $\mu\frac{d}{d\mu}\Sigma' = 0$)

$$\beta(c^{(1)}_{F^2D^2\phi^2}) = \frac{e^2}{24\pi^2}\left(2c^{(1)}_{D^4\phi^4} + c^{(2)}_{D^4\phi^4}\right)\,.$$

- $c^{(1)}_{F^2D^2\phi^2}$ does not necessarily obey the tree-level bound ($c^{(1)}_{F^2D^2\phi^2} < 0$).
  - For $m \to 0$, Σ' is dominated by the log term, which implies a bound on β (< 0) that tends to restore the tree-level bound of $c^{(1)}_{F^2D^2\phi^2}$ in the IR.
  - In other words, for small μ we are absorbing a large positive log term in the definition of $c^{(1)}_{F^2D^2\phi^2}$...

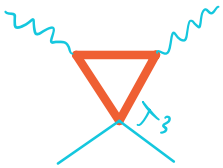
UV models

- A heavy scalar Φ with charge 2

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) + (D^\mu\Phi)^\dagger(D_\mu\Phi) - M^2\Phi^\dagger\Phi$$
$$-\frac{1}{4}\lambda_1(\phi^\dagger\phi)^2 - \frac{1}{4}\lambda_2(\Phi^\dagger\Phi)^2 - \lambda_3\Phi^\dagger\Phi\phi^\dagger\phi - (\frac{1}{2}gM\Phi^\dagger\phi\phi + h.c.)\,.$$

- Tree level matching: $c^{(1)}_{D^4\phi^4} = g^2\,,\ c^{(2)}_{D^4\phi^4} = 0.$
- One loop matching (and running):

$$c^{(1)}_{F^2D^2\phi^2} = \frac{1}{16\pi^2}\frac{1}{6}e^2g^2\left(9 + 4\log\left(\frac{\mu^2}{M^2}\right)\right)\,.$$



- Tree-level positivity bound is "violated" for $\mu \gtrsim 0.3M$.
- **Note:** λ_3 does not contribute to $c^{(1)}_{F^2D^2\phi^2}$! (Otherwise we can get a violation of positivity bound.)
- Σ' is given by (with g, e defined at the matching scale M)

$$\Sigma' = \frac{e^2g^2}{576M^4\pi^2}\left(11 - 12\log\left(\frac{m^2}{M^2}\right)\right) > 0.$$

$\gamma\gamma \to \gamma\gamma$, 1 loop

- Same linear polarization ($\gamma_y\gamma_y \to \gamma_y\gamma_y$)

$$\begin{aligned}\Sigma'_{yy}\cdot\Lambda^4 =& 16c_{F^4}^{(1)} + \frac{1}{4\pi^2}c_{F^2\phi^2}^2\left(-\frac{3}{2}-2\log\left(\frac{s_0}{\mu^2}\right)+4\right)\\ &-\frac{1}{96\pi^2}e^2c_{F^2D^2\phi^2}^{(1)}\left(-\frac{3}{2}-2\log\left(\frac{s_0}{\mu^2}\right)-2\log\left(\frac{m_\gamma^2}{\mu^2}\right)+8\right)\\ &-\frac{1}{144\pi^2}e^2(25c_{F^2D^2\phi^2}^{(1)}+72c_{F^2D^2\phi^2}^{(2)})\,.\end{aligned}$$

- Orthogonal linear polarization ($\gamma_y\gamma_x \to \gamma_y\gamma_x$)

$$\Sigma'_{yx}\cdot\Lambda^4 = 16c_{F^4}^{(2)} + \frac{1}{96\pi^2}e^2c_{F^2D^2\phi^2}^{(1)}\left(\frac{3}{2}+2\log\left(\frac{s_0}{\mu^2}\right)+2\log\left(\frac{m_\gamma^2}{\mu^2}\right)-8\right)-\frac{7}{144\pi^2}e^2c_{F^2D^2\phi^2}^{(1)}\,.$$

- If all operators above are generated at one-loop level, the contributions from $c_{F^4}^{(1)}$, $c_{F^4}^{(2)}$ and $c_{F^2\phi^2}^2$ are two-loop effects from the UV model.
- If $c_{D^4\phi^4}^{(1)}$ or $c_{D^4\phi^4}^{(2)}$ are generated at tree level, their contribution to the above equations are also at the two-loop level from the UV model.