Quantum Forces in Standard Model and Dark Sector

Bingrong Yu (Cornell U.)

June 4, 2026

based on: **[2209.07082], [2410.19059], [2504.00104], [2507.12522], [2512.07938]**

in collaboration with Steven Ferrante, Mijo Ghosh, Yuval Grossman, Maxim Perelstein
Chinhsan Sieng, Walter Tangarife, Xun-jie Xu, and Siyu Zhou

A sketch of today's talk

- Introduction of quantum forces
- Neutrino-mediated quantum force
- Quantum forces in the dark sector
- Summary

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- **Introduction of quantum forces**
- Neutrino-mediated quantum force
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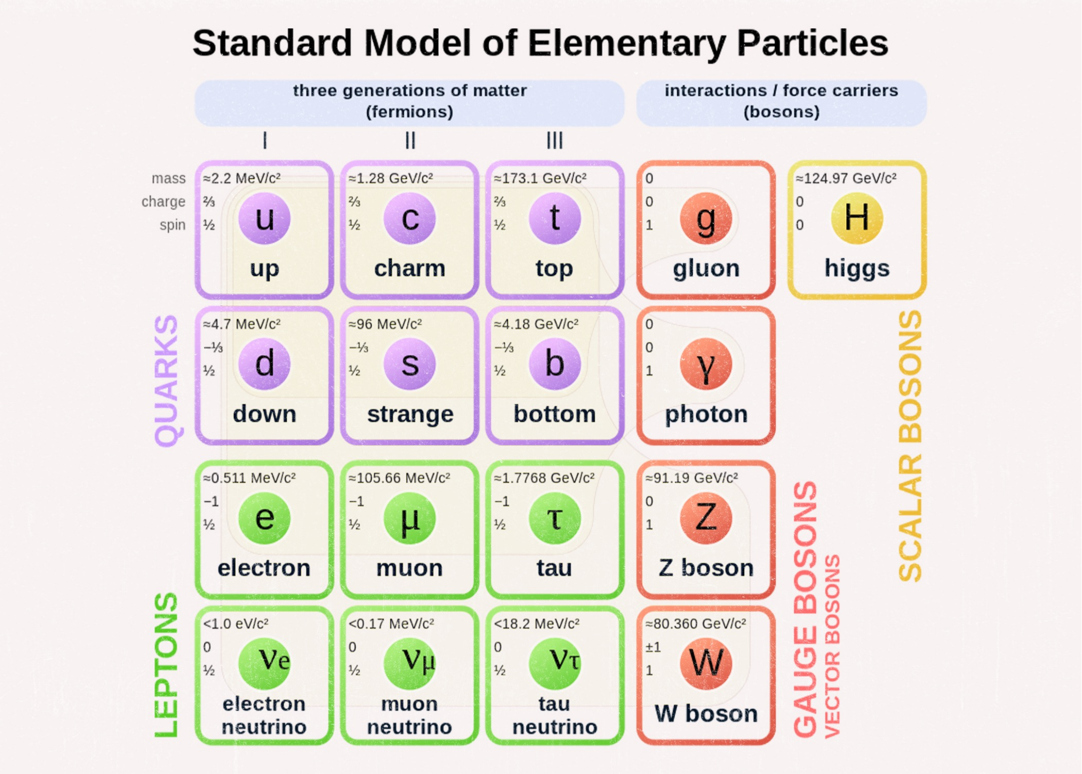
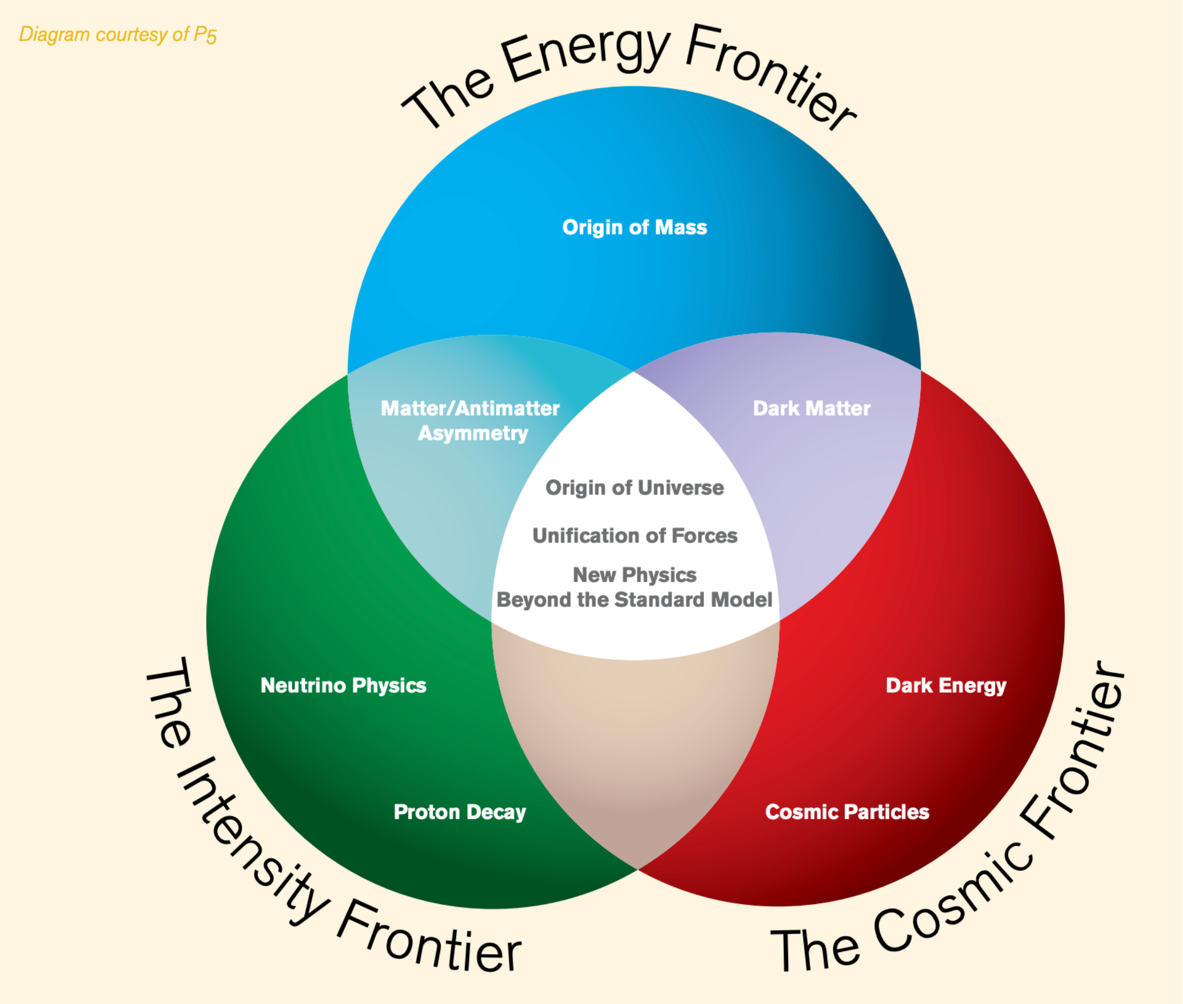
Standard Model is successful, but incomplete

- Dark matter
- Dark energy
- Neutrino mass
- Matter-antimatter asymmetry
- Hierarchy problem
- Strong CP problem
- …



Directions to probe new physics beyond the SM

- The topic I am going to talk about today (quantum forces) provides a powerful tool to probe the **precision** frontier and the **cosmic** frontier



Evolving concept of FORCE

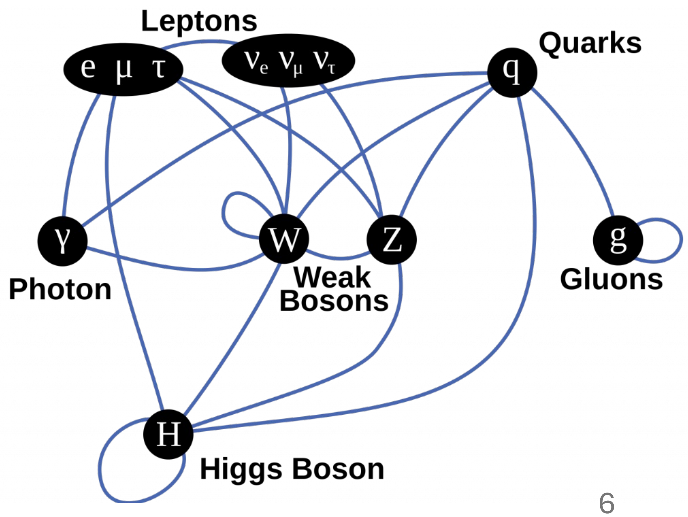
Newtonian: cause of acceleration

Maxwell: field generated by source

QFT: interaction carried by gauge boson

$$\vec{F} = m\vec{a}$$

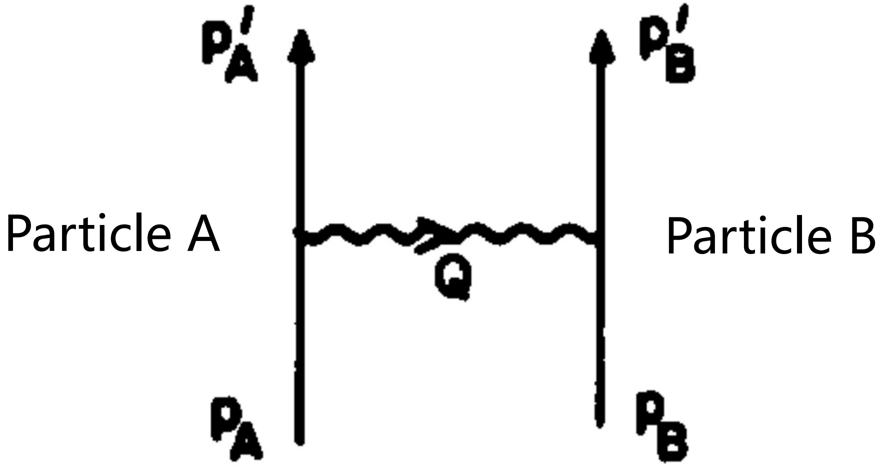
$$\partial_\mu F^{\mu\nu} = J^\nu$$



From classical force to quantum force

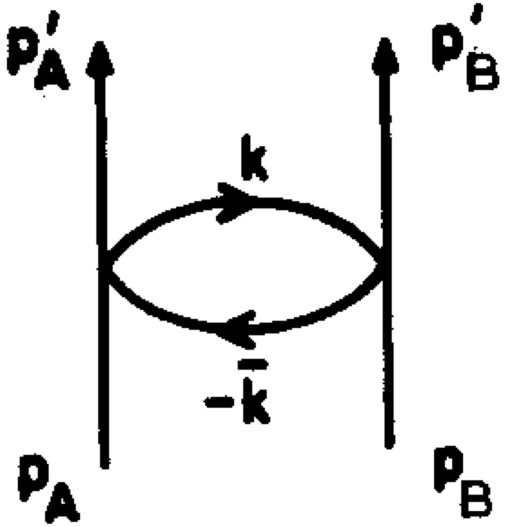
- Diagrammatic description

mass of mediator particle << mass of external particle



Classical force

only mediated by **bosons**



Quantum force

mediated by either **bosons** or **fermions**

From classical force to quantum force

- Field theory description

ϕ: light mediator particle J: source term

$\mathcal{L} \supset \phi J$

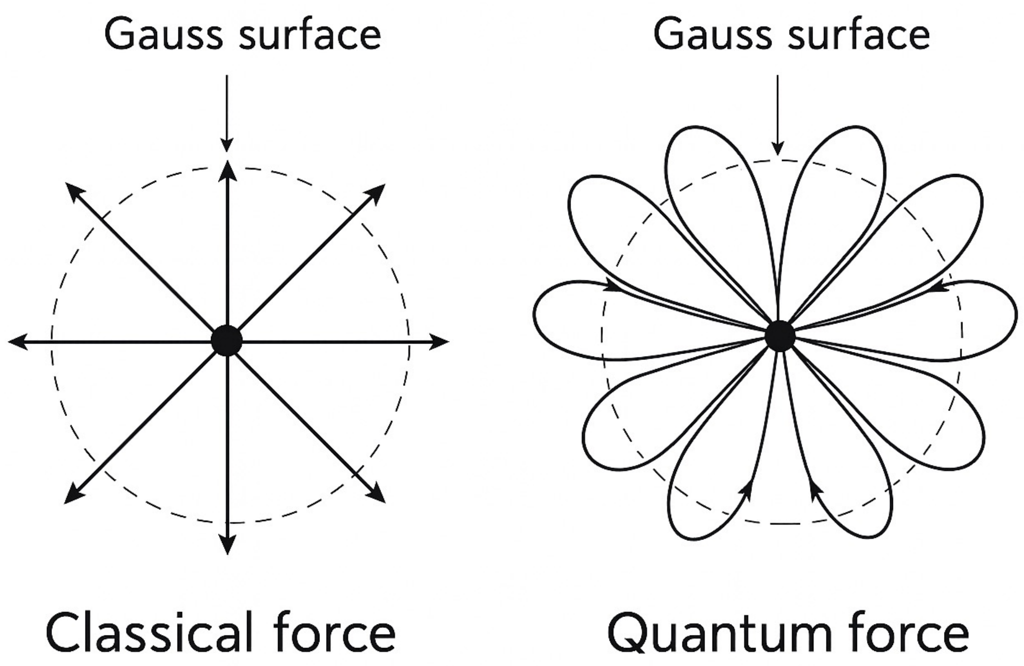
$\left(\partial^2 + m^2\right)\phi = J$ classical force $\propto \langle \phi \rangle_J$

$\mathcal{L} \supset \phi^2 J$

$\left(\partial^2 + m^2 + J\right)\phi = 0$ $\langle \phi \rangle_J = 0$

only field fluctuation does not vanish

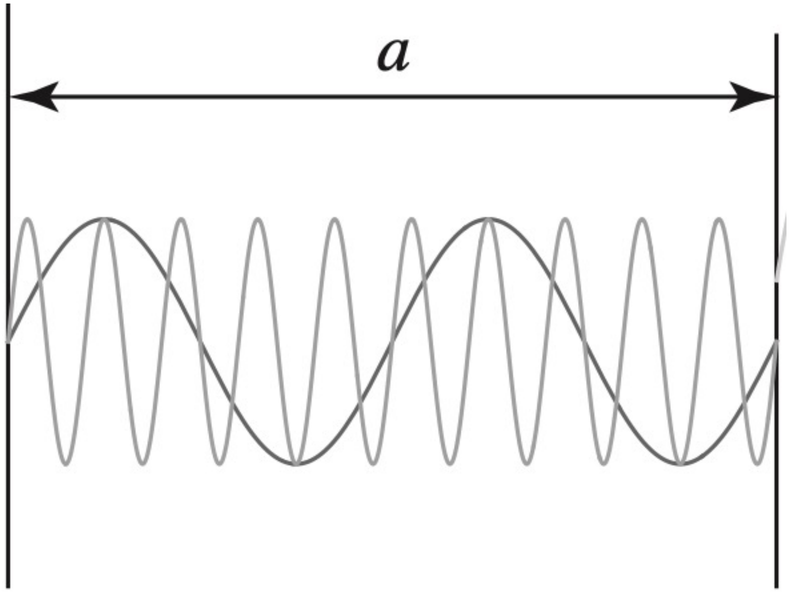
quantum force $\propto \langle \phi^2 \rangle_J$



Examples of quantum forces

Casimir effect

Casimir, 1948

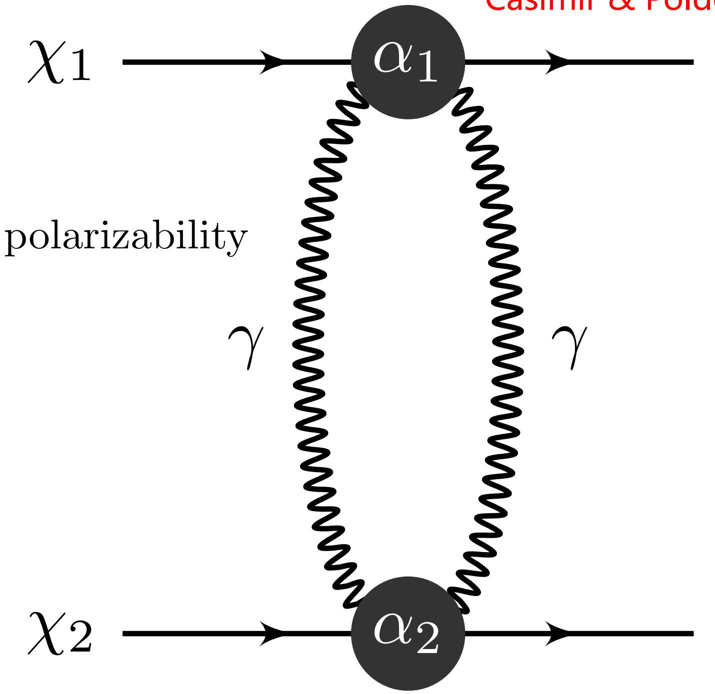


$$\mathcal{V}_{\text{Casimir}} = \pi^2 \hbar c / (720\, a^3)$$

Examples of quantum forces

Casimir-Polder force

Casimir & Polder, 1947



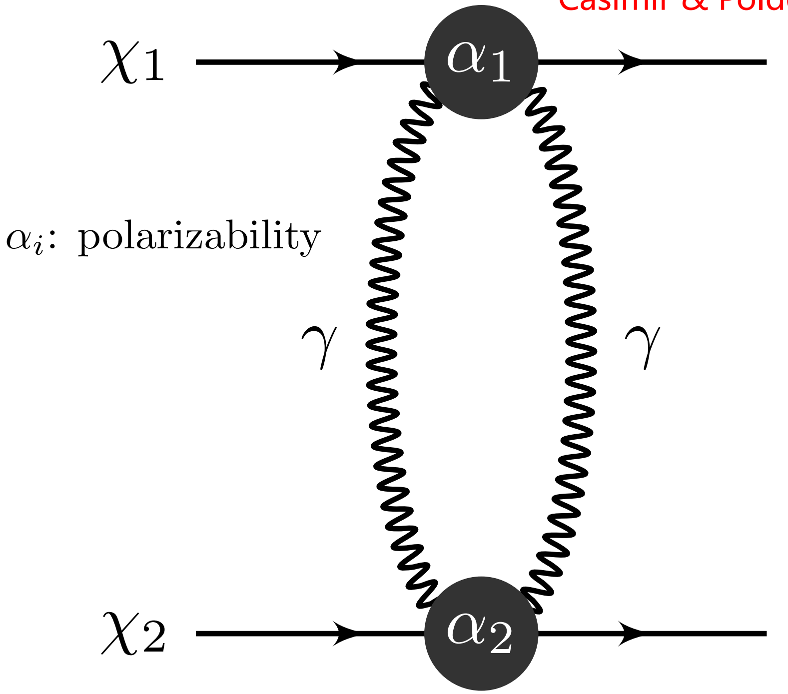
α_i: polarizability

$$V_{2\gamma}(r) \sim \frac{\alpha_1 \alpha_2}{r^7}$$

Examples of quantum forces

Casimir-Polder force

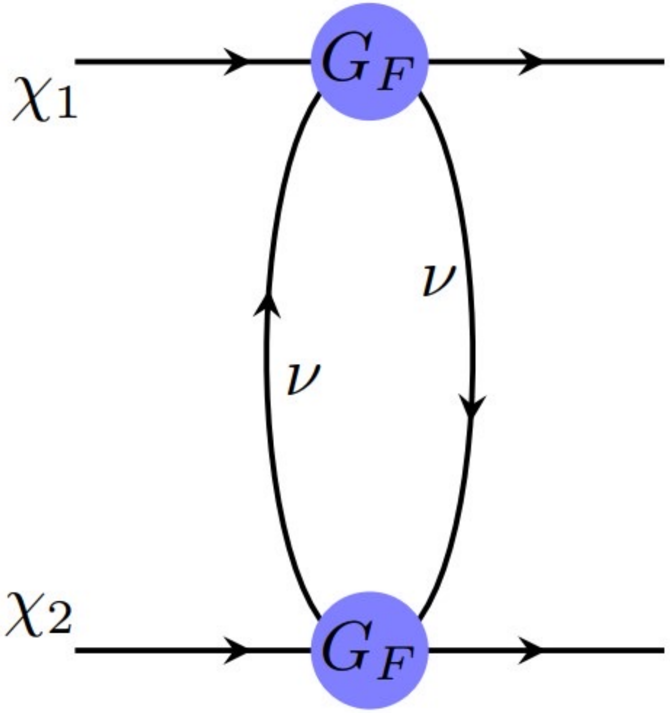
Casimir & Polder, 1947



$$V_{2\gamma}(r) \sim \frac{\alpha_1 \alpha_2}{r^7}$$

two-neutrino force

Feinberg & Sucher, 1968



$$V_{2\nu}(r) \sim \frac{G_F^2}{r^5}$$

Examples of quantum forces

Casimir-Polder force

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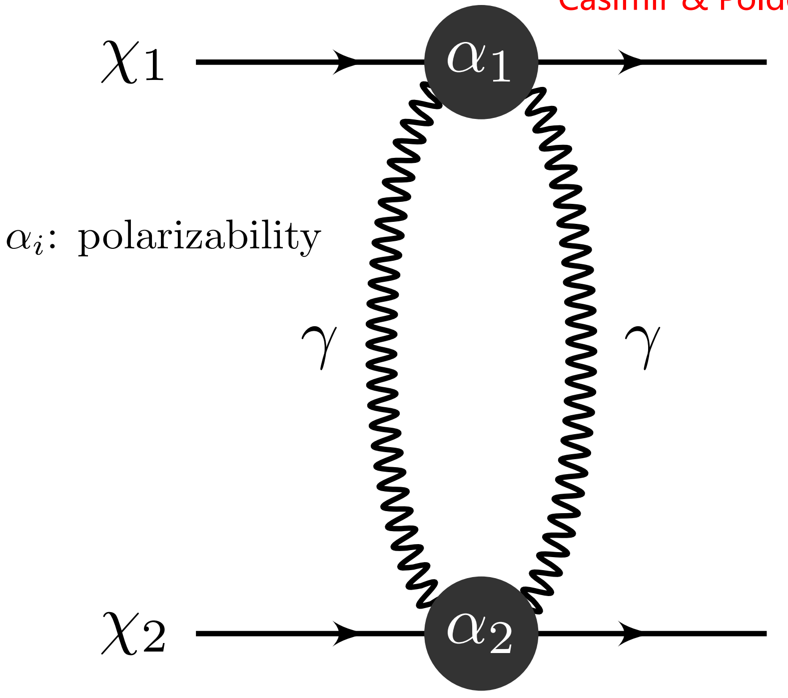

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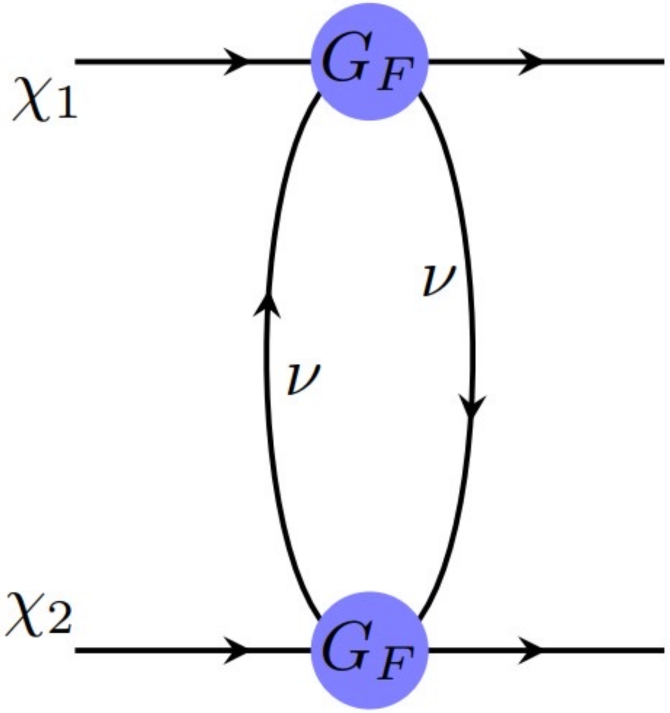
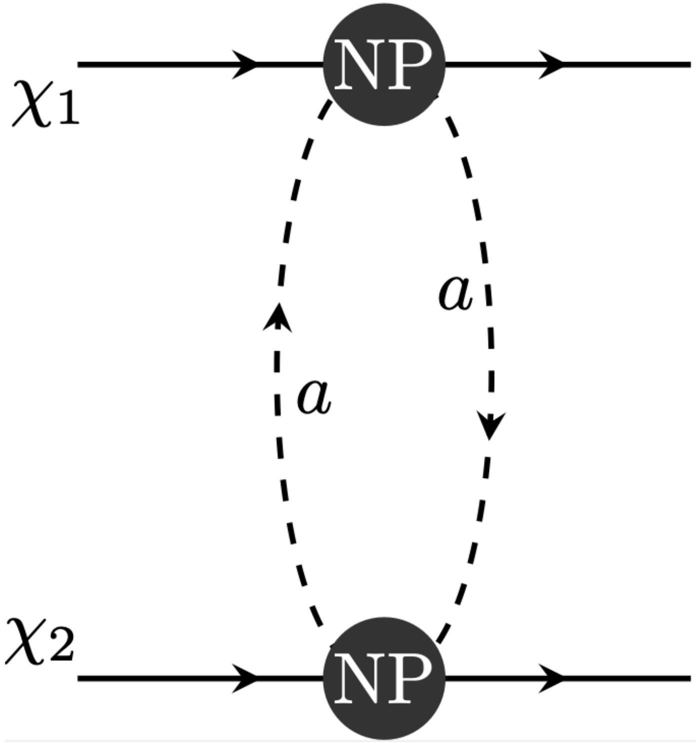


$$V_{2\nu}(r) \sim \frac{G_F^2}{r^5}$$

two-axion force

Ferrer & Nowakowski, 1998



$$V_{2a}(r) \sim \frac{(\mathrm{NP})^2}{r^3}$$

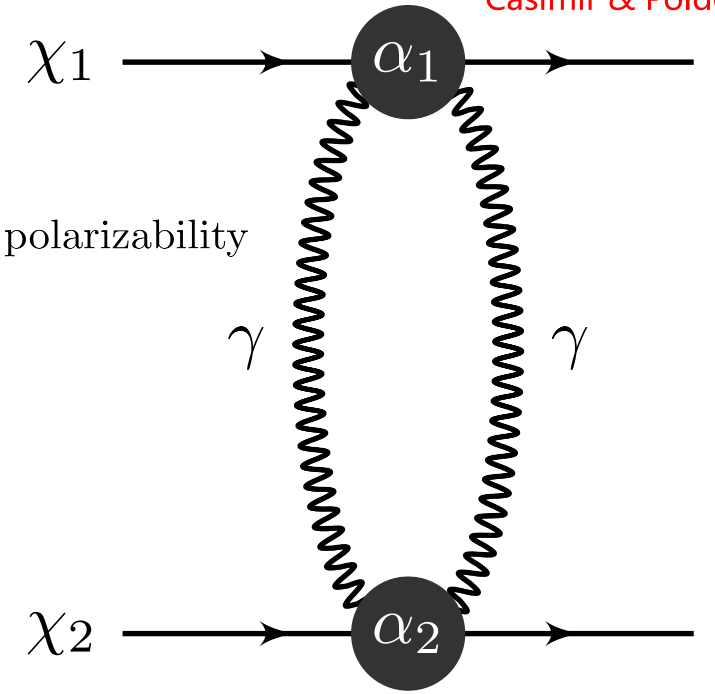
Examples of quantum forces

Quantum forces are only sensitive to IR modes

Casimir-Polder force

Casimir & Polder, 1947

α_i: polarizability



$$V_{2\gamma}(r) \sim \frac{\alpha_1 \alpha_2}{r^7}$$

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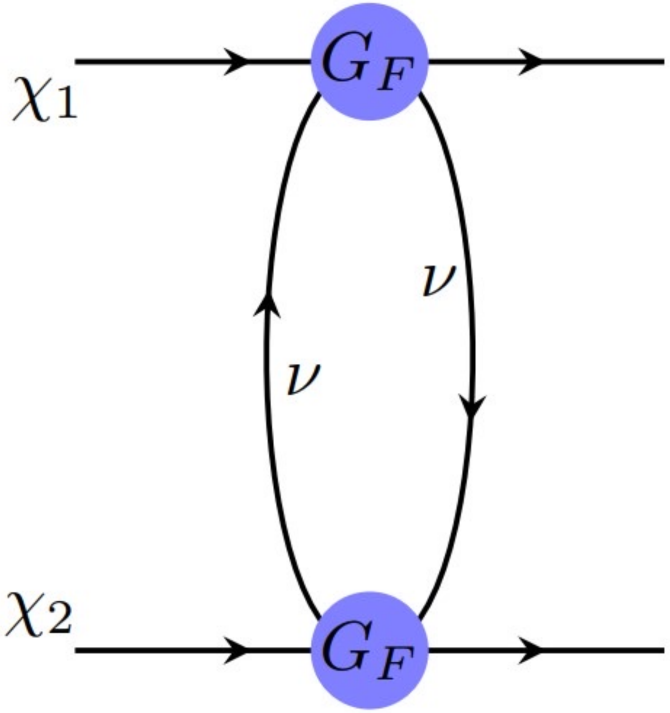

$$V_{2\nu}(r) \sim \frac{G_F^2}{r^5}$$

two-axion force

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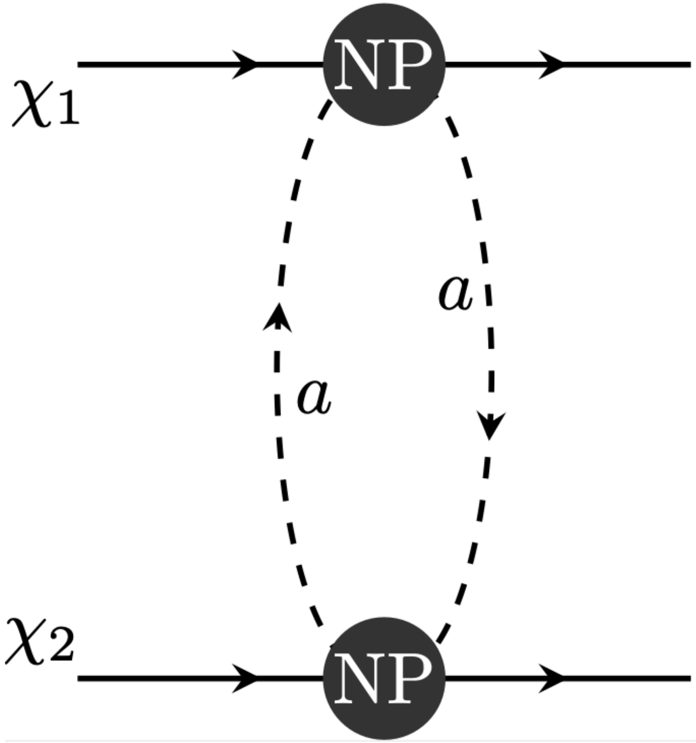

$$V_{2a}(r) \sim \frac{(\mathrm{NP})^2}{r^3}$$

Why do we care about quantum forces?

- **Theory**: interesting QFT results distinct from classical forces
  - intrinsically violate the superposition principle
  - significantly enhanced in the background: short-range force → long-range force
  - …

- **Phenomenology**:
  - **In the SM (neutrino force)**: sensitive to fundamental parameters, affect the SM precision test, astrophysical implications…
  - **In the dark sector**: fifth-force detection, DM evolution, structure formation…

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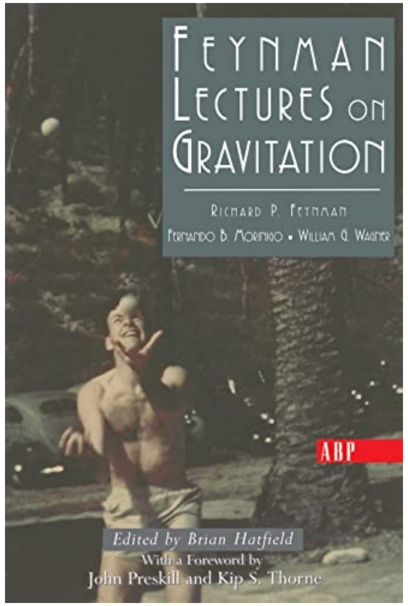
The two-neutrino force

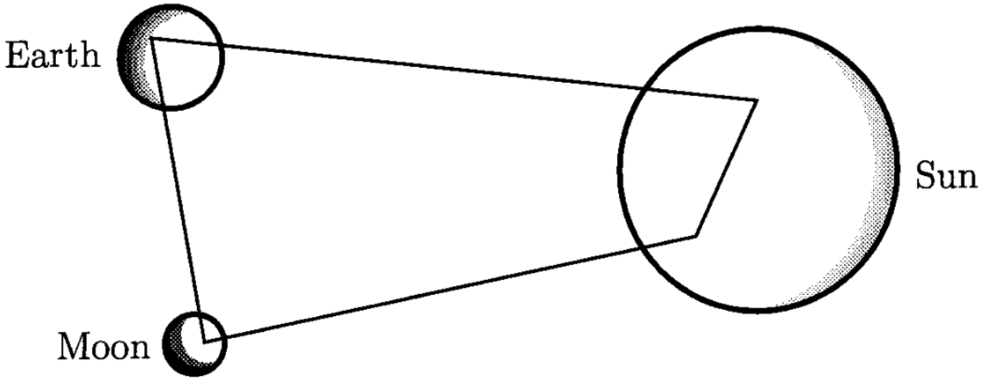
Original idea : **Feynman**



Feynman's goal:

Explain the origin of gravity from neutrino exchange

1960s, Section 2.4



$$E = -G'^3\, m_1 m_2 m_3\, \pi^2 \frac{1}{(r_{12}+r_{23}+r_{13}) r_{12} r_{23} r_{13}}. \tag{2.4.4}$$

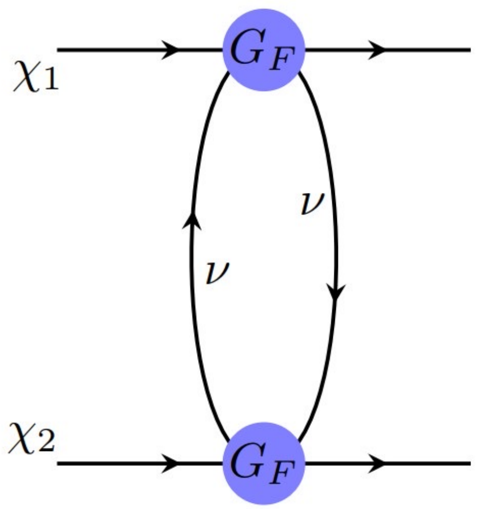
If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

$$E = -\frac{G'^3\, m_1 m_2\, \pi^2}{r_{12}} \int \frac{4\pi \rho(R) R^2 dR}{2R^3}, \tag{2.4.5}$$

Resemble gravity by two-neutrino exchange among many bodies

The two-neutrino force



- The first correct neutrino force law

Feinberg & Sucher, 1968

dispersion formalism

$$V_{2\nu}(r) = \frac{G_F^2}{4\pi^3 r^5}$$

- The neutrino mass effect

Grifols, Masso & Toldra, 1996

Dirac mass: $$V_{2\nu}^D(r) = \frac{G_F^2}{4\pi^3}\frac{m_\nu^3}{r^2}K_3(2m_\nu r) \xrightarrow{r \gg m_\nu^{-1}} \frac{G_F^2}{8}\left(\frac{m_\nu}{\pi r}\right)^{5/2} e^{-2m_\nu r}$$

Majorana mass: $$V_{2\nu}^M(r) = \frac{G_F^2}{2\pi^3}\frac{m_\nu^2}{r^3}K_2(2m_\nu r) \xrightarrow{r \gg m_\nu^{-1}} \frac{G_F^2}{4}\left(\frac{m_\nu^3}{\pi^5 r^7}\right)^{1/2} e^{-2m_\nu r}$$

force range ~ inverse of twice neutrino mass

- Neutrino force versus gravity

$$\frac{V_{2\nu}(r)}{V_{\rm grav}(r)} \sim \frac{G_F^2/r^5}{G_N m^2/r} \sim \left(\frac{m}{\rm GeV}\right)^{-2}\left(\frac{r}{\rm nm}\right)^{-4}$$

weaker than gravity when distance
is larger than a nanometer

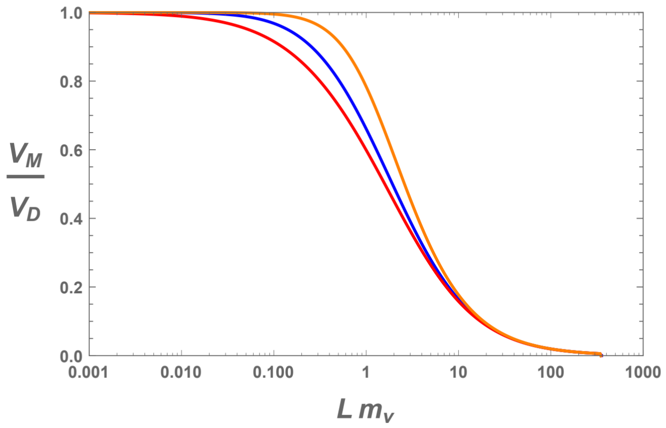
Too weak to probe by usual long-range experiments

Why probe the neutrino force?

- Prediction of QFT and SM, the unique long-range quantum force in SM

$$V_{2\nu}(r) \sim \frac{G_F^2}{r^5}$$

- Sensitive to the nature of neutrino mass (Dirac or Majorana)



- Relevant to the precision test of the SM at low energies

How to probe the neutrino force?

- **Microscopic probe:** Ghosh, Grossman, Sieng & **BY**, [2410.19059], [2512.07938]

  Fundamentally, neutrino interaction **violates parity symmetry**

  Looking at parity-violating atomic transition $\Longrightarrow$ Distinguish neutrino force effect from leading QED effect

- **Macroscopic probe:** Ghosh, Grossman, Tangarife, Xu & **BY**, [2209.07082], [2405.16801]

  Coherent enhancement from a neutrino background

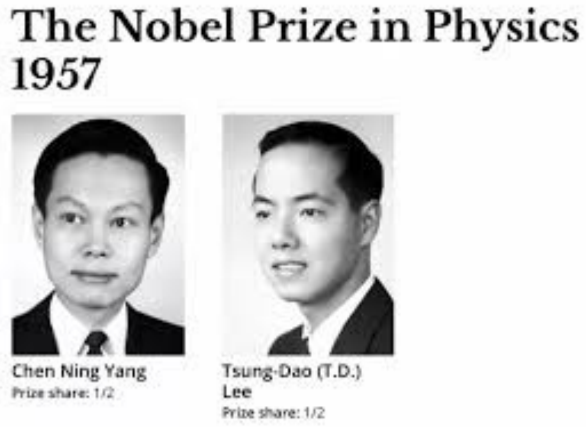
  Putting the target into a strong neutrino background $\Longrightarrow$ Change the scaling behavior from $1/r^5$ to $1/r$

1 Atomic Parity Violation

as a probe of neutrino force

Based on Ghosh, Grossman, Sieng & **BY**, [2410.19059]

The basic idea



- Look for something absent in EM: **Parity Violation**

- The physical observable: Parity-violating transitions in atom (such as the **electric-dipole** transition)

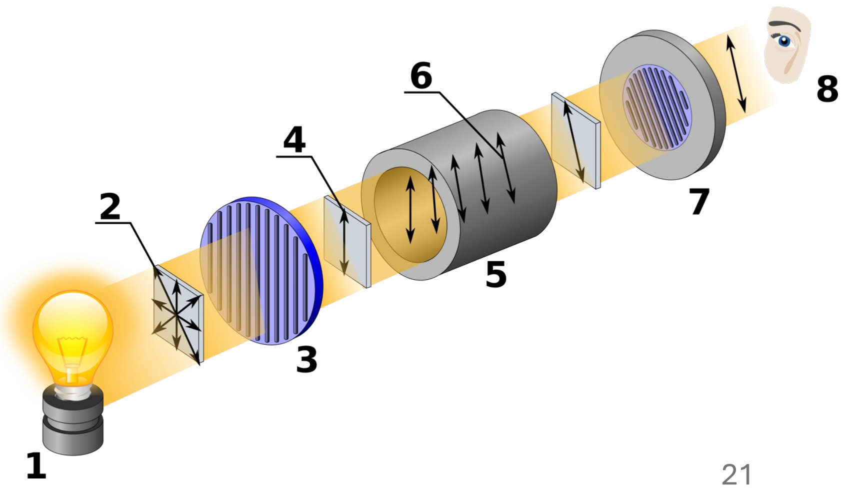
$$E_1^{\mathrm{PV}} = \langle A'|d_e|B'\rangle = \sum_{\Delta} \frac{\langle A|d_e|\Delta\rangle\langle\Delta|V_{\mathrm{PV}}|B\rangle}{E_B - E_\Delta} + (A \leftrightarrow B)$$

Neutrino force contributes to the PV matrix elements, but QED effect cannot!

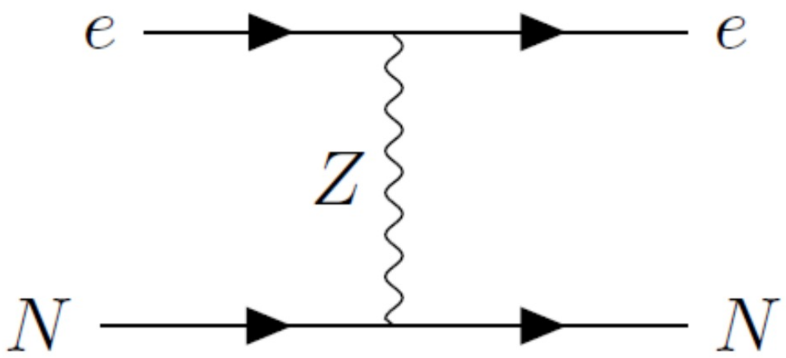
- What do we observe: **optical rotation angle** of the incident light

$$\Phi = \frac{\pi L}{\lambda}\,\mathrm{Re}\left(n_{\mathrm{R}}(\lambda) - n_{\mathrm{L}}(\lambda)\right) \propto \mathrm{Im}\, E_1^{\mathrm{PV}}$$

$n_{\mathrm{R/L}}$: refractive index of polorized photon



- The leading APV effect comes from the tree-level *Z*-exchange force



Already measured for s-wave states in Cesium
experimental error: 0.35%

Measurement of Parity Nonconservation and an Anapole Moment in Cesium

Science, 1997

C. S. Wood, S. C. Bennett, D. Cho,* B. P. Masterson,†
J. L. Roberts, C. E. Tanner,‡ C. E. Wieman§

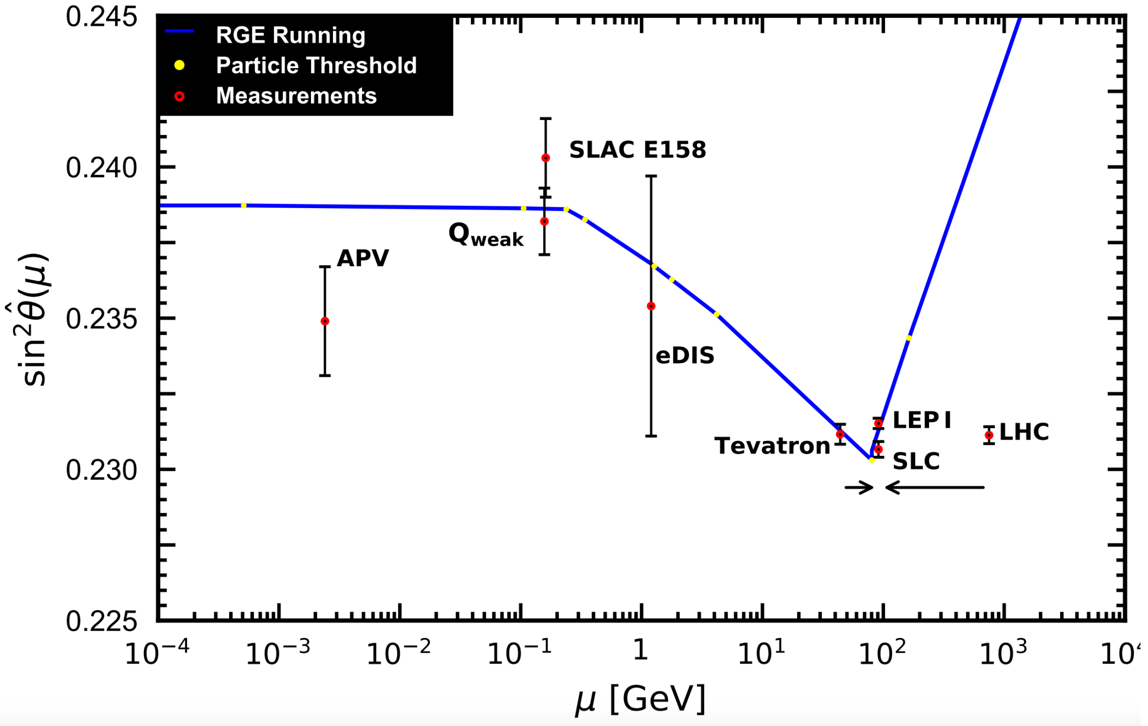
The amplitude of the parity-nonconserving transition between the 6*S* and 7*S* states of cesium was precisely measured with the use of a spin-polarized atomic beam. This measurement gives $\mathrm{Im}(E1_{pnc})/\beta = -1.5935(56)$ millivolts per centimeter and provides an improved test of the standard model at low energy, including a value for the *S* parameter of $-1.3(3)_{exp}\,(11)_{theory}$. The nuclear spin–dependent contribution was 0.077(11) millivolts per centimeter; this contribution is a manifestation of parity violation in atomic nuclei and is a measurement of the long-sought anapole moment.

- Neutrino force is the **loop** correction, but with a much longer range!

Why particle physics cares about APV

- APV measurement ⇒ weak charge Q_W ⇒ weak mixing angle $\sin^2\theta_W$ ⇒ precision test of SM at atomic scale

- Currently, there exists a 2σ slight tension between the APV measured value and the SM predicted value



Taken from PDG (2024)

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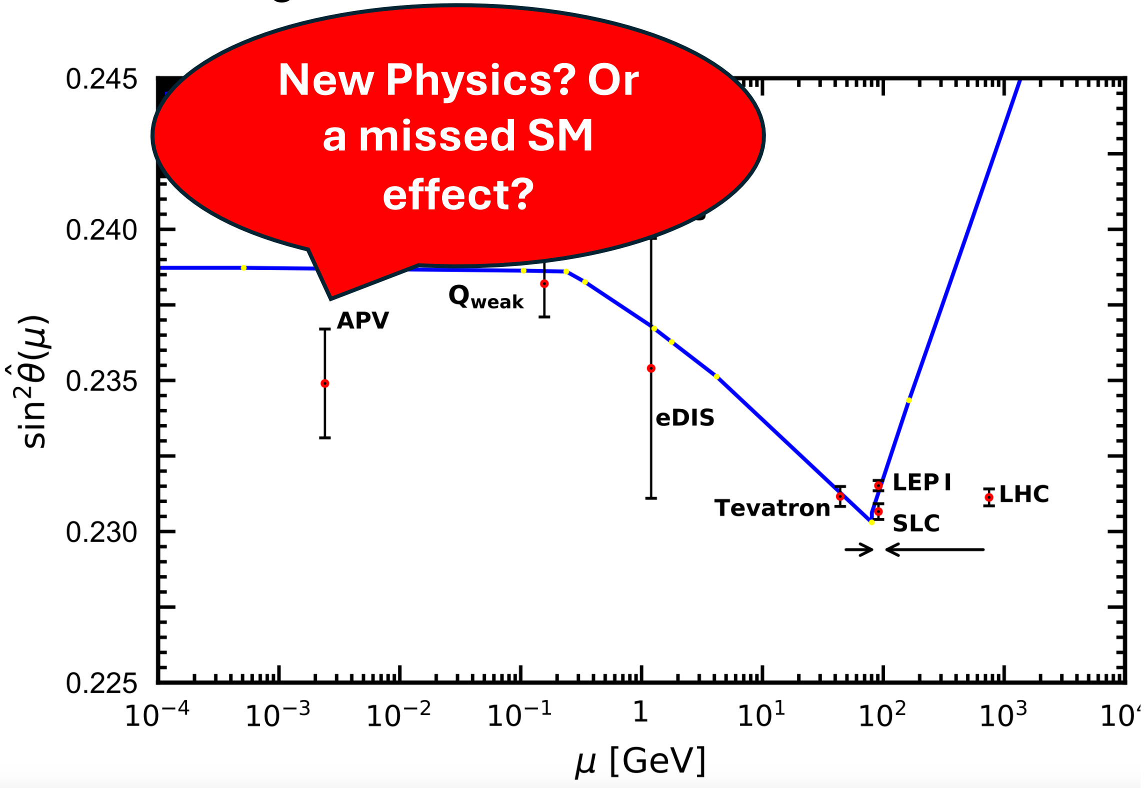
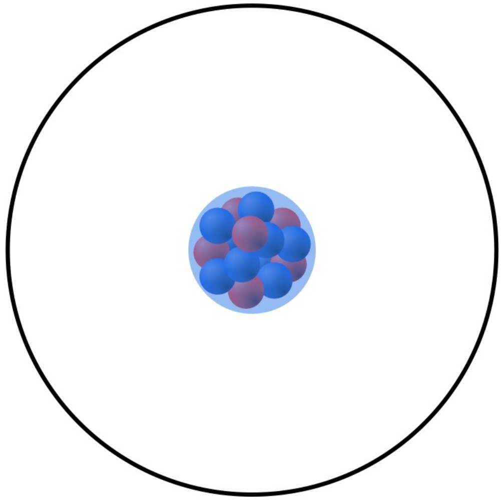


Taken from PDG (2024)

Neutrino force versus Z force

Z forces $\quad m_Z^{-1} \sim 10^{-16}$ cm

short-range

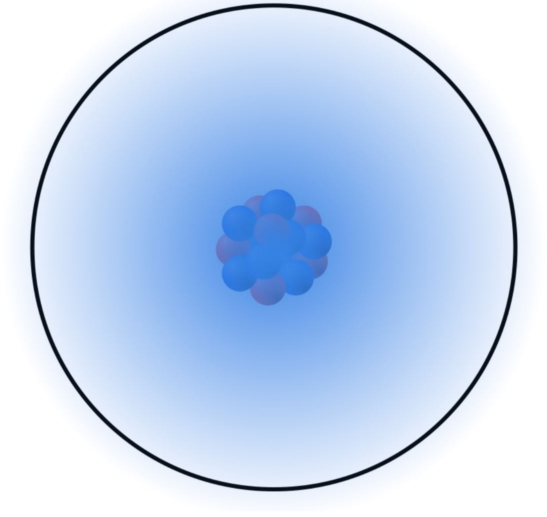


$V_Z \sim G_F \delta^3(r)$

$$\langle \Psi_f | V_Z | \Psi_i \rangle \sim G_F \Psi_f^*(0) \Psi_i(0)$$

ν forces $\quad m_\nu \sim 0.1$ eV $\quad m_\nu^{-1} \sim 10^{-4}$ cm

long-range



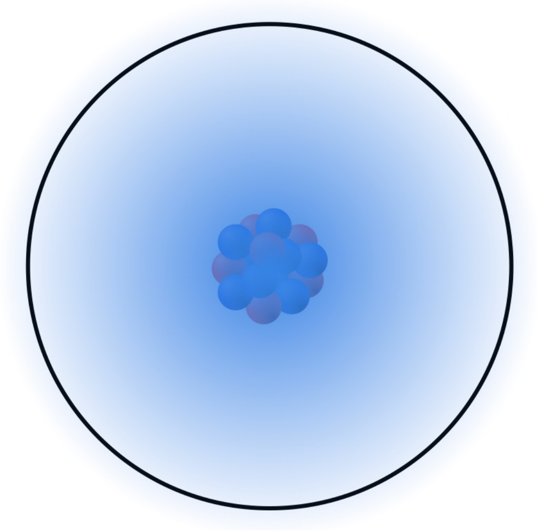
$V_{2\nu} \sim G_F^2 / r^5$

$$\langle \Psi_f | V_{2\nu} | \Psi_i \rangle \sim G_F^2 \int \mathrm{d}^3\mathbf{r}\, \Psi_f^*(\mathbf{r}) \frac{1}{r^5} \Psi_i(\mathbf{r})$$

much longer range, but *singular* at small distances

Neutrino force versus Z force

ν forces
long-range



$$\langle \Psi_f | V_{2\nu} | \Psi_i \rangle \sim G_F^2 \int \mathrm{d}^3\mathbf{r}\, \Psi_f^*(\mathbf{r}) \frac{1}{r^5} \Psi_i(\mathbf{r})$$

- However, $1/r^5$ potential is **divergent** near $r = 0$ for s-wave states

- Indicate the failure of 4-Fermi approximation at short distances

$1/r^n$ with $n > 2$ is known as singular potential

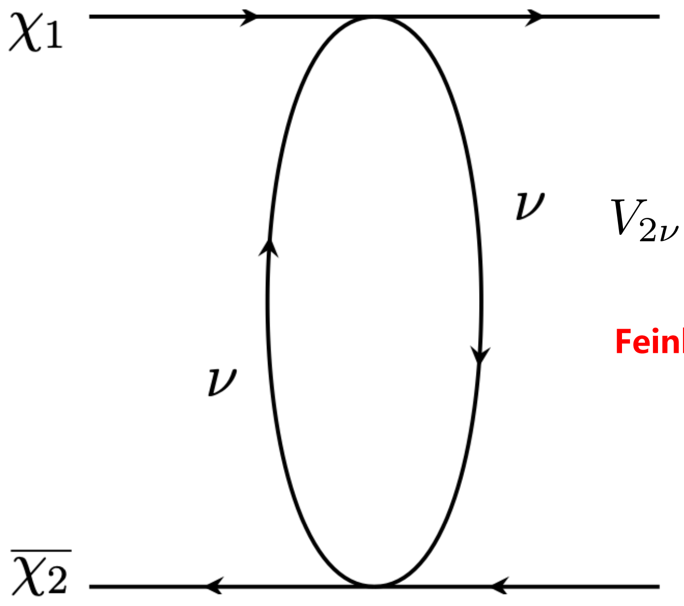
Landau and Lifshitz's argument (1960):

When the particle is approaching the center of the potential, the kinetic energy $E_k = k^2/(2m_\chi)$ with $k \sim r^{-1}$ increases as $1/r^2$ while V decreases as $-1/r^n$. Therefore, the total energy would not be bounded from below and the particle would keep falling to infinitely small r, corresponding to infinitely high energy.

- To correctly calculate the neutrino APV effects, it is **necessary** to know the **short-range behavior** of the neutrino force

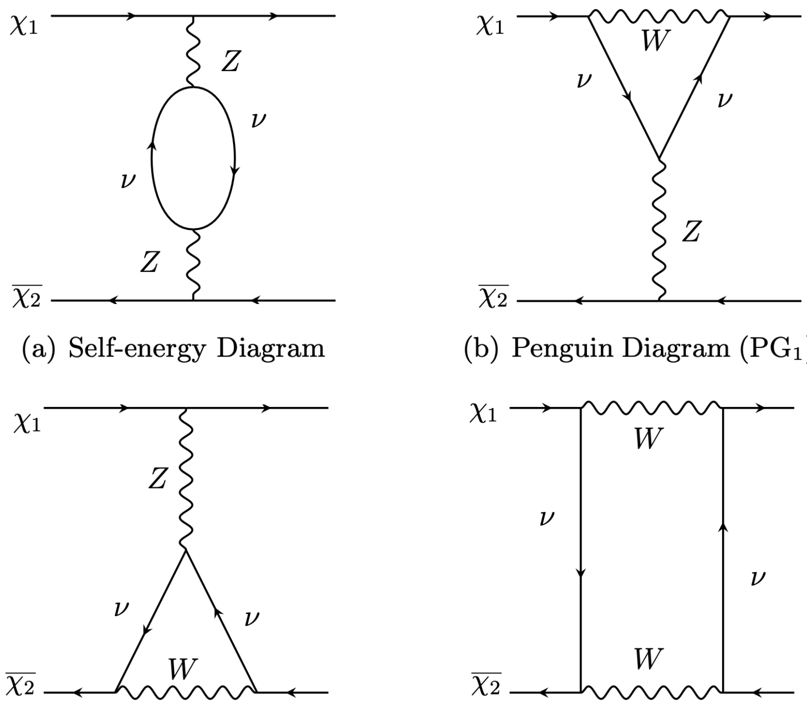
Neutrino force beyond 4-Fermi

What does the force look like at UV?



$$V_{2\nu}(r) = \frac{G_F^2}{4\pi^3 r^5}$$

Feinberg & Sucher, 1968



(a) Self-energy Diagram (b) Penguin Diagram (PG_1)

(c) Penguin Diagram (PG_2) (d) Box Diagram

4-Fermi theory: $r \gg 1/m_Z$

breaks down when $r < 1/m_Z$

full electroweak theory

valid at all distances

The full-range neutrino force

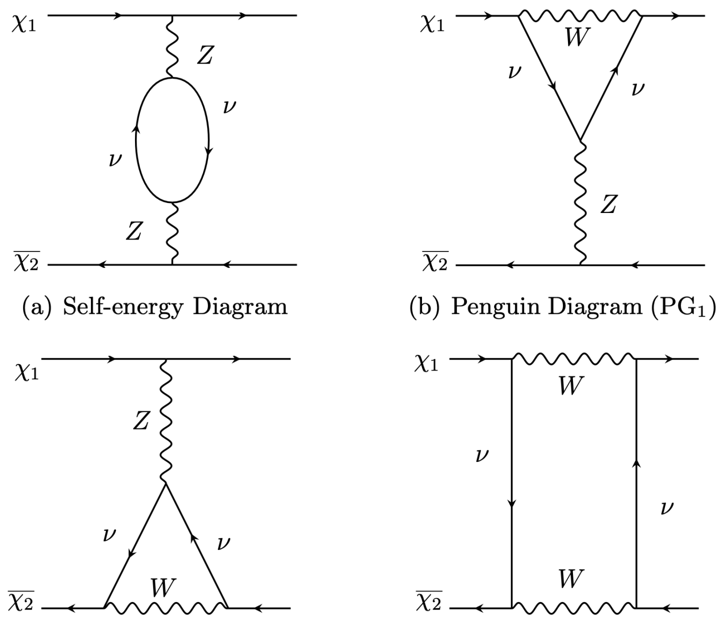
Ghosh, Grossman, Sieng & **BY**, [2410.19059], [2512.07938]

$$V_0^{\rm SE}(r) = \left(\frac{g}{4c_W}\right)^4 \frac{1}{48\pi^3 r}\left[e^x(2+x)\,{\rm Ei}(-x) + e^{-x}(2-x)\,{\rm Ei}(x) + 2\right]$$

$$x \equiv m_Z r\,, \quad {\rm Ei}(x) \equiv -\int_{-x}^{\infty} {\rm d}t\; t^{-1} e^{-t}$$

$$V_0^{\rm PG}(r) = -\frac{g^4}{2048\pi^3 c_W^2 r}\int_0^\infty {\rm d}t\, e^{-\sqrt{t}\,r}\,\frac{t\left(2m_W^2+3t\right) - 2\left(m_W^2+t\right)^2 \log\left(1+t/m_W^2\right)}{t^2\left(t-m_Z^2\right)}$$

$$V_0^{\rm box}(r) = -\frac{g^4}{512\pi^3 r}\int_0^\infty {\rm d}t\, e^{-\sqrt{t}\,r}\,\frac{m_W^2 t\left(t+2m_W^2\right) + \left(t^2-2m_W^4\right)\left(t+m_W^2\right)\log\left(1+t/m_W^2\right)}{t^2\left(t+2m_W^2\right)^2}$$



(a) Self-energy Diagram

(b) Penguin Diagram (PG_1)

(c) Penguin Diagram (PG_2)

(d) Box Diagram

This full expression is lacking in literature for more than 50 years

Renormalizable neutrino force, valid at all distances

Asymptotic behaviors

- Long-range limit $r \gg 1/m_Z$:

$$V_0^{\rm SE}(r) = -\frac{g^4}{512\pi^3 m_W^4 r^5} = -\frac{G_F^2}{16\pi^3 r^5}$$

$$V_0^{\rm PG}(r) = -\frac{g^4}{256\pi^3 c_W^2 m_Z^2 m_W^2 r^5} = -\frac{G_F^2}{8\pi^3 r^5}$$

$$V_0^{\rm box}(r) = -\frac{g^4}{128\pi^3 m_W^4 r^5} = -\frac{G_F^2}{4\pi^3 r^5}$$

All reduce to the $1/r^5$ form at long distances

- Short-range limit $r \ll 1/m_Z$:

$$V_0^{\rm SE}(r) \approx \frac{g^4}{3072\pi^3 c_W^4}\frac{\log(m_Z r)}{r}$$

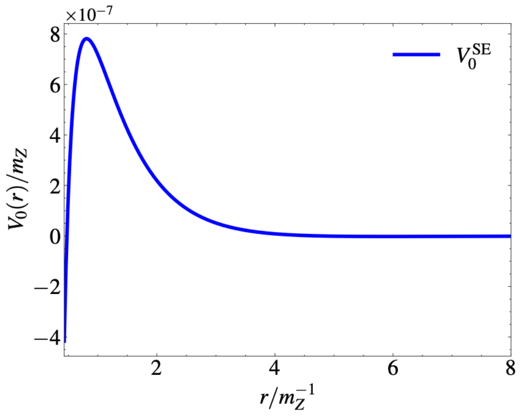
$$V_0^{\rm PG}(r) \approx \frac{g^4}{512\pi^3 c_W^2}\frac{\log^2(m_W r)}{r}$$

$$V_0^{\rm box}(r) \approx -\frac{g^4}{256\pi^3}\frac{\log^2(m_W r)}{r}$$

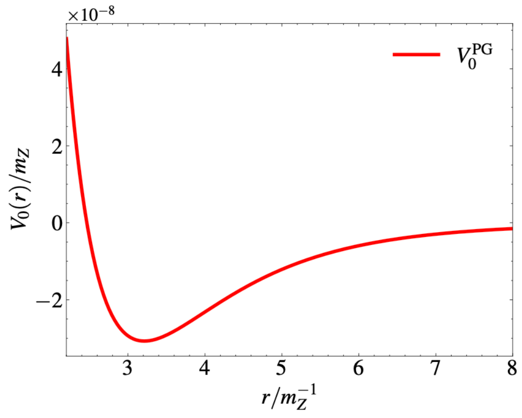
All scale as $1/r$ at short distances

But penguin and box are enhanced by $\log^2(m_W r)$

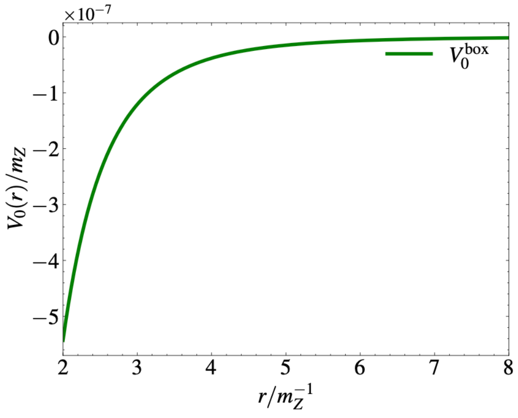
Comparison of three UV diagrams



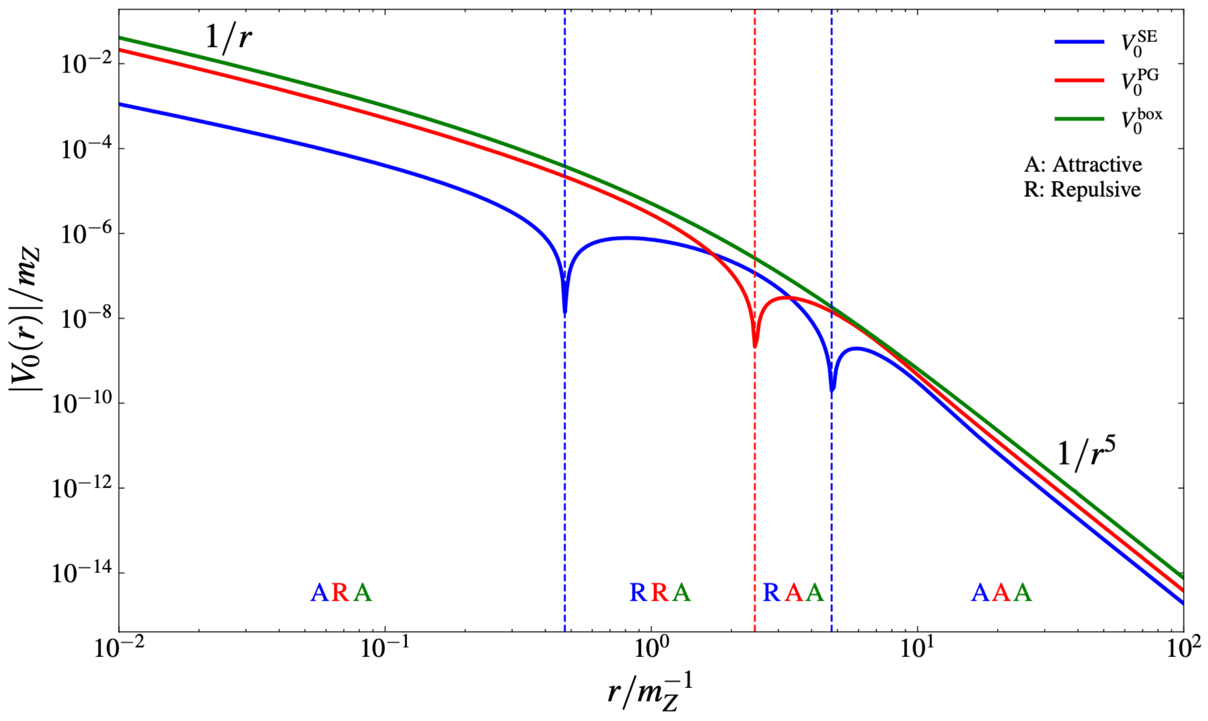
(a) Self-energy diagram



(b) Penguin diagram



(c) Box diagram



Self-energy (penguin) changes sign twice (once) around $1/m_Z$

Box does not change sign the whole range

Neutrino force contribution to APV

- Parity-violating neutrino forces in muonium and positronium:

$$V_{\rm PV}^{e\bar{\mu}}(r) = \left[\frac{2\left(\boldsymbol{\sigma}_{\bar{\mu}} - \boldsymbol{\sigma}_e\right) \cdot \mathbf{p}_e}{m_e} + \frac{\left(\boldsymbol{\sigma}_{\bar{\mu}} \times \boldsymbol{\sigma}_e\right) \cdot \nabla}{m_e}\right] \left[3\left(1 - 4s_W^2\right) V_0^{\rm SE}(r) - \left(2 - 4s_W^2\right) V_0^{\rm PG}(r)\right]$$

$$V_{\rm PV}^{e\bar{e}}(r) = \frac{2\left(\boldsymbol{\sigma}_{\bar{e}} \times \boldsymbol{\sigma}_e\right) \cdot \nabla}{m_e} \left[3\left(1 - 4s_W^2\right) V_0^{\rm SE}(r) - \left(2 - 4s_W^2\right) V_0^{\rm PG}(r) + V_0^{\rm box}(r)\right]$$

All the V_0 have been computed

- What we compute: neutrino force contribution *relative to* the *Z* force

$$\boxed{\eta_\nu \equiv \frac{C_{AB}^{\nu}}{C_{AB}^{Z}}}$$

$$C_{AB}^{j} = \frac{\langle B | V_{\rm PV}^{j} | A \rangle}{E_A - E_B}$$

The parity-violating matrix elements

- $\ell = 0$ (s-wave) state

$$\eta_\nu \equiv \frac{\text{neutrino force contribution}}{Z \text{ force contribution}}$$

$$\eta_\nu^{\rm SE} = -\frac{g^2}{32\pi^2 c_W^2}\,,$$

$$\eta_\nu^{\rm PG} = \frac{g^2\left(1-2s_W^2\right)}{96\pi^2\underline{\left(1-4s_W^2\right)}}\left[2\pi^2\left(1+c_W^2\right)^2 - 3\left(7+4c_W^2\right) + 6\left(3+2c_W^2\right)\log\left(\frac{1}{c_W^2}\right)\right.$$
$$\left. - 6\left(1+c_W^2\right)^2\log^2\left(1+\frac{1}{c_W^2}\right) - 12\left(1+c_W^2\right)^2 {\rm Li}_2\left(\frac{c_W^2}{1+c_W^2}\right)\right],$$

$$\eta_\nu^{\rm box} = \frac{g^2\left(4+3\pi^2\right)}{256\pi^2\underline{\left(1-4s_W^2\right)}}\,.$$

The loop/tree ratio is accidentally enhanced by $\left(1-4\,s_W^2\right)^{-1} \sim 20$

Numerically:

$$s_W^2 = 0.23863$$
$$\eta_\nu^{\rm SE} \approx -0.2\%\,,$$
$$\eta_\nu^{\rm PG} \approx 4\%\,,$$
$$\eta_\nu^{\rm box} \approx 12\%\,.$$

muonium: $\eta_\nu^{e\bar{\mu}} = \eta_\nu^{\rm SE} + \eta_\nu^{\rm PG} \approx 4\%$

agree with naive estimate $\dfrac{V_{\rm loop}}{V_{\rm tree}} \sim \dfrac{g^2}{16\pi^2\left(1-4s_W^2\right)} \sim \mathcal{O}(5\%)$

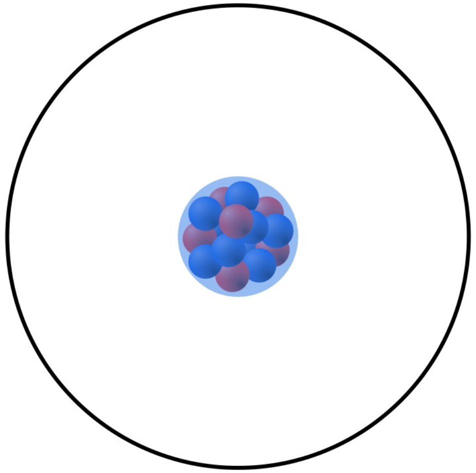
positronium: $\eta_\nu^{e\bar{e}} = \eta_\nu^{\rm SE} + \eta_\nu^{\rm PG} + \eta_\nu^{\rm box} \approx 16\%$

additionally enhanced by the box diagram

- Higher-ℓ states

$$\eta_\nu \equiv \frac{\text{neutrino force contribution}}{Z \text{ force contribution}}$$

Z forces
short-range



$$V_Z \sim G_F \delta^3(r)$$

$$\eta_\nu^{\ell=1} \sim \log\left(\frac{m_Z^2}{\alpha^2 m_e^2}\right) \eta_\nu^{\ell=0}$$

$$\eta_\nu^{\ell\geq 2} \sim \left(\frac{m_Z^2}{\alpha^2 m_e^2}\right)^{\ell-1} \eta_\nu^{\ell=0} \gg 1$$

Tree-level contribution is highly suppressed for higher-ℓ states

Neutrino force contribution is dominated

Yet, observing APV for $\ell \neq 0$ states is still unrealistic in experiments

- Heavy atom systems

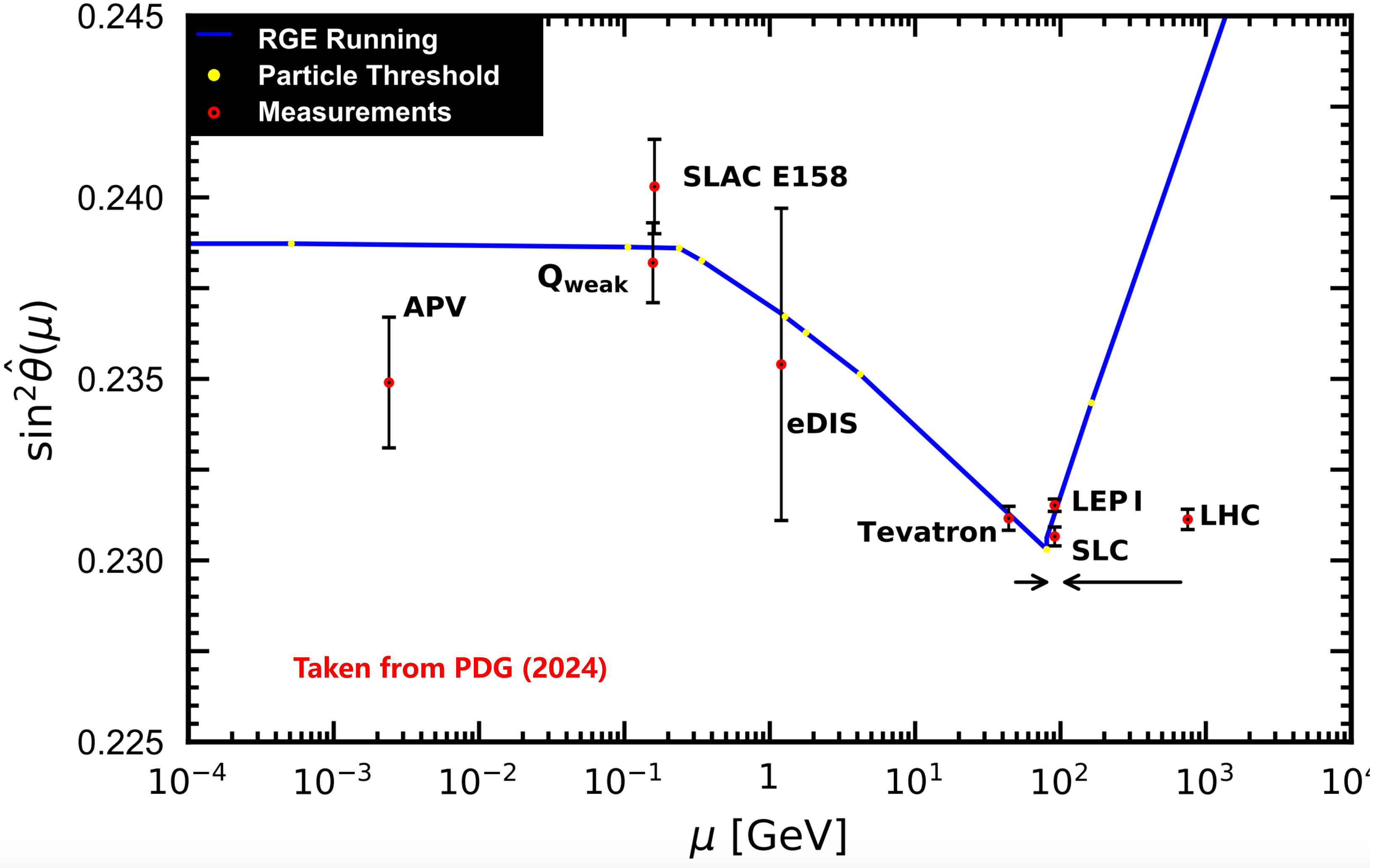
- In heavy atoms, both tree and loop are coupled to neutrons: no accidental enhancement
- We assume the many-body effects and relativistic corrections are roughly canceled in the ratio (which is valid for s-wave state)

$$\text{muonium/hydrogen}: \quad \eta_\nu \sim \frac{g^2}{16\pi^2\left(1-4s_W^2\right)} \sim 5\% \,,$$

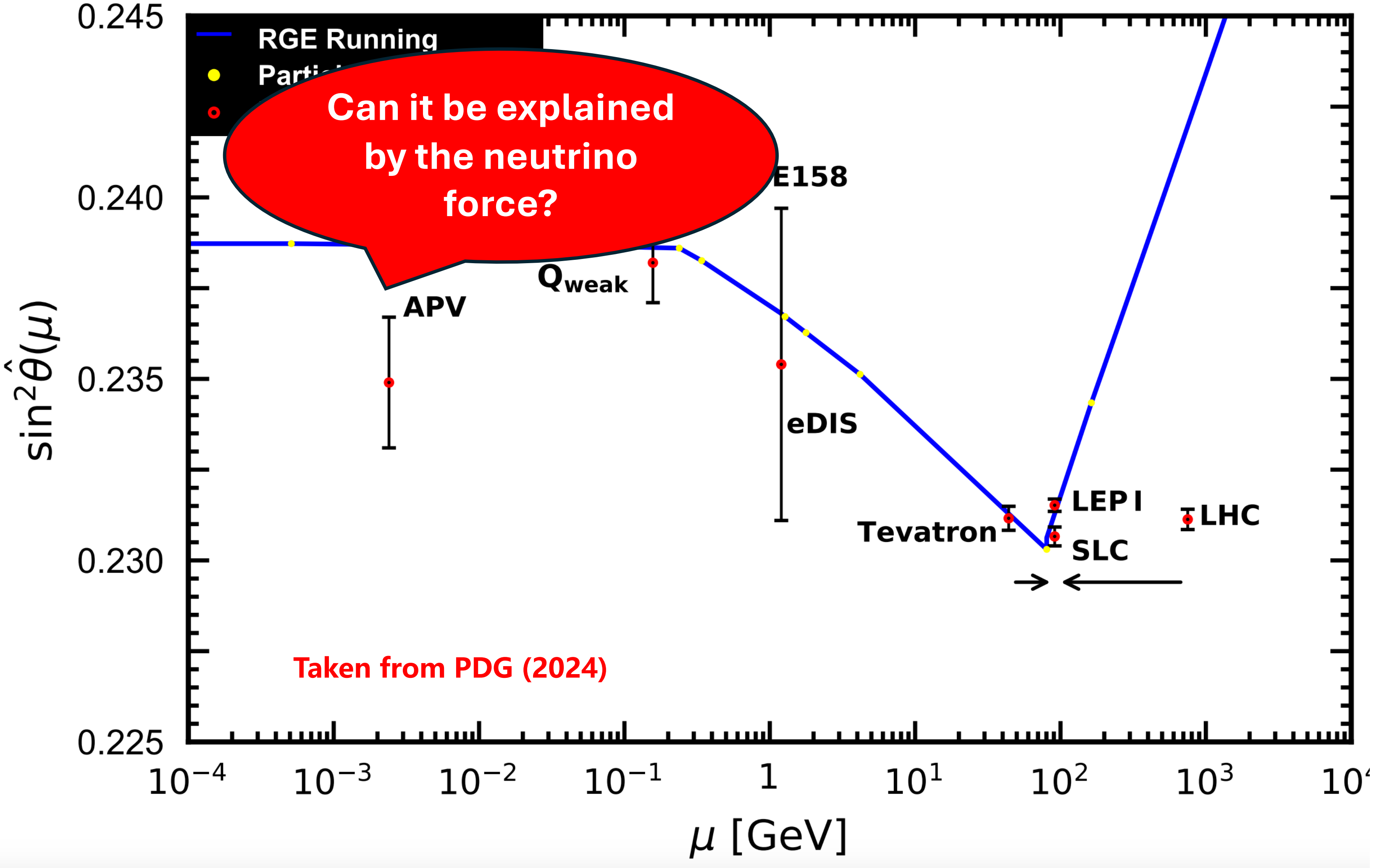
$$\text{heavy atoms}: \quad \eta_\nu \sim \frac{g^2}{16\pi^2} \sim 0.3\% \,.$$

comparable to the current APV experimental error (0.35%) in Cesium

Neutrino effects on extracting $\sin^2\theta_W$



Neutrino effects on extracting $\sin^2\theta_W$



Neutrino effects on extracting $\sin^2\theta_W$

- Neglecting the neutrino force will affect the extracted value of $\sin^2\theta_W$ from APV

$$\delta s_W^2 \equiv \left(\sin^2\theta_W' - \sin^2\theta_W\right) / \sin^2\theta_W$$
$$\approx \frac{\eta_\nu}{\mathcal{N}+3\mathcal{Z}}\left[3\mathcal{N} + (4\mathcal{N}-3\mathcal{Z})\left(1-4s_W^2\right)\right]$$

η_ν has been calculated above

θ_W: true value including neutrino force effect

θ_W': extracted value without neutrino force

$\mathcal{Z}$: number of protons

$\mathcal{N}$: number of neutrons

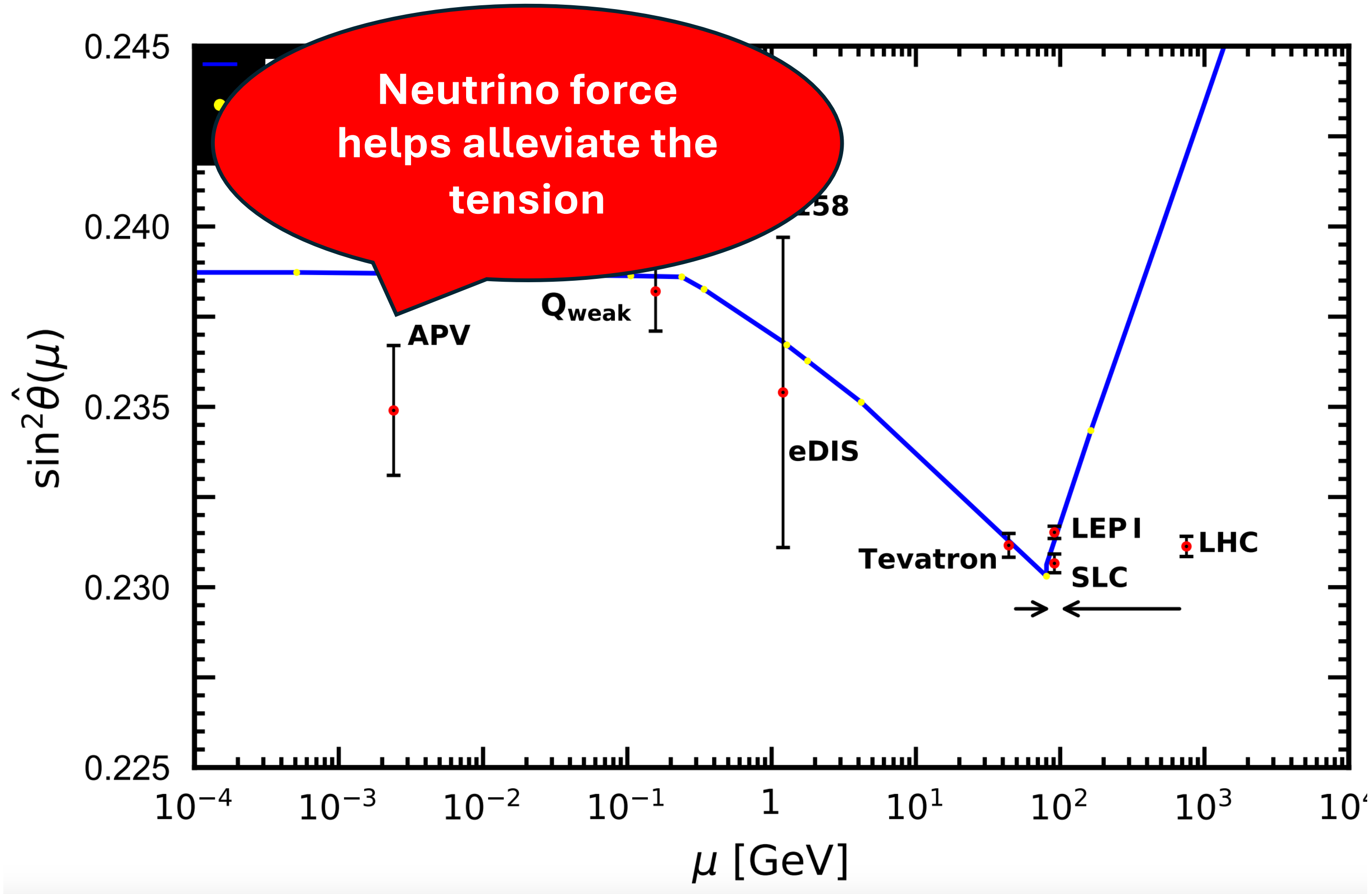
$$\delta s_W^2 \approx \begin{cases} -0.2\% & \text{muonium} \\ -0.7\% & \text{positronium} \\ -0.3\% & \text{Cesium} \end{cases}$$

Neglecting the neutrino effect

⇒ measured value < true value

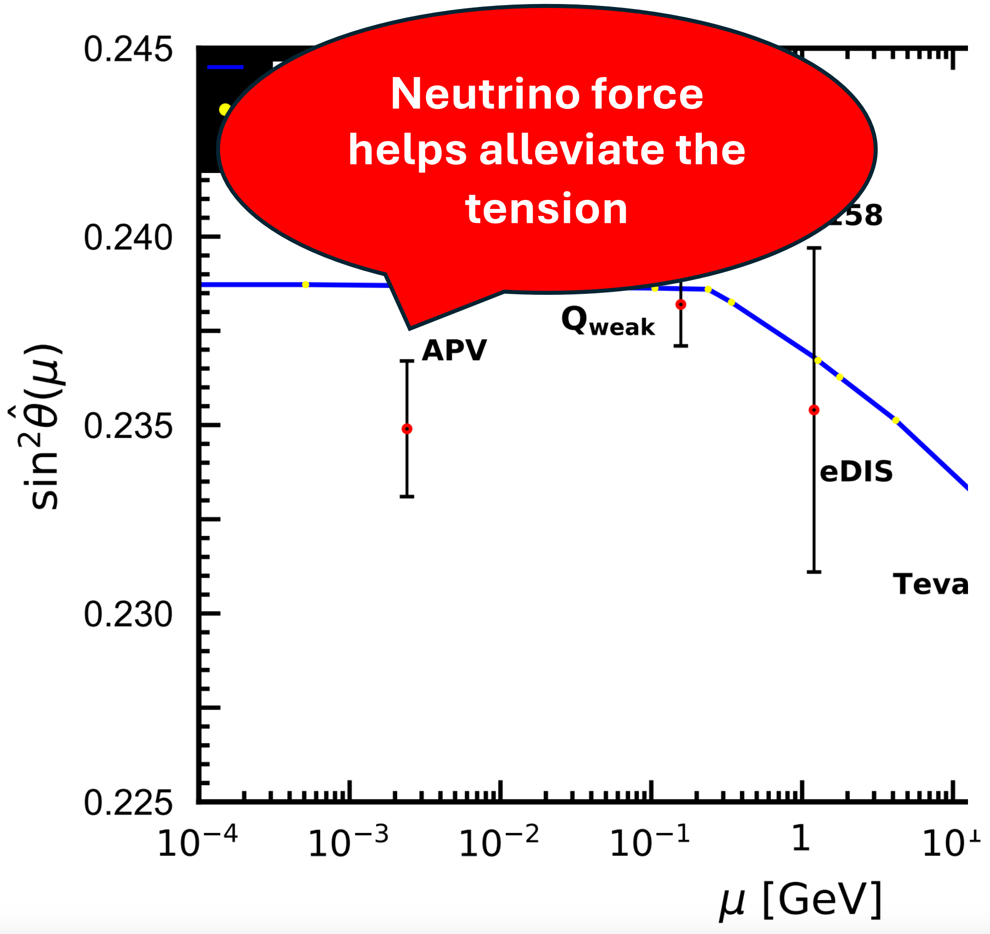
(current experimental error ~ 0.8%, current atomic theory error ~ 0.5%)

Neutrino effects on extracting $\sin^2\theta_W$



$$\delta s_W^2 \approx \begin{cases} -0.2\% & \text{muonium} \\ -0.7\% & \text{positronium} \\ -0.3\% & \text{Cesium} \end{cases}$$

Neutrino effects on extracting $\sin^2\theta_W$



ScienceNews

INDEPENDENT JOURNALISM SINCE 1921

NEWS PARTICLE PHYSICS

A strange 'neutrino force' helped heal a crack in particle physics

Science News, April 2026

Neutrinos and other particles can produce a subtle force, relevant for precise measurements

Quick Summary

- We derived an **exact** formula for the neutrino-mediated quantum force that is applicable at all length scales

- Neutrino force has sizable effects in **atoms**, with an important implication for precision measurement of the Weinberg angle at low energy scales

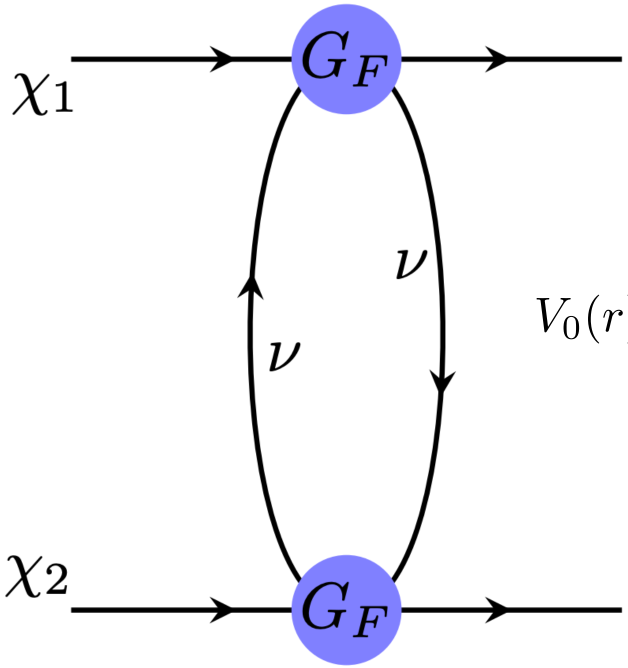
2 Neutrino forces in neutrino backgrounds

probing neutrino force via fifth force experiments

Based on Ghosh, Grossman, Tangarife, Xu & **BY**, [2209.07082]

Coherent enhancement from neutrino background

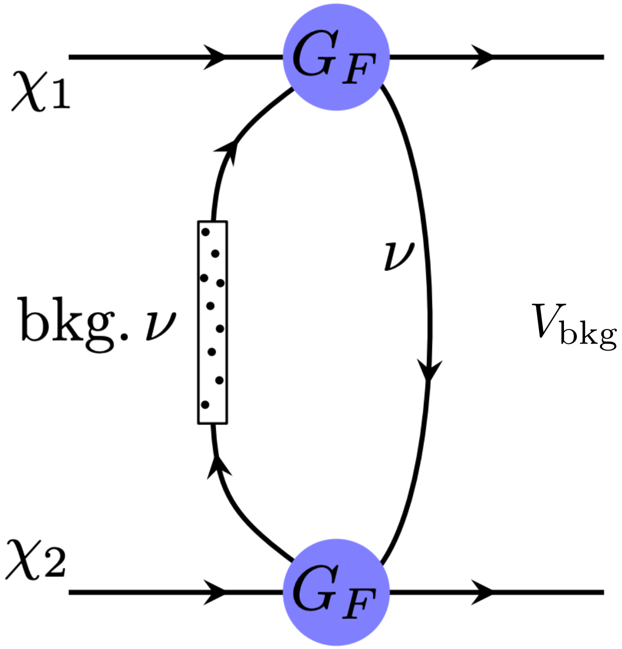
in vacuum



$V_0(r) \sim G_F^2 \times \frac{1}{r^5}$

$$V = V_0 + V_{\rm bkg}$$

in ν background



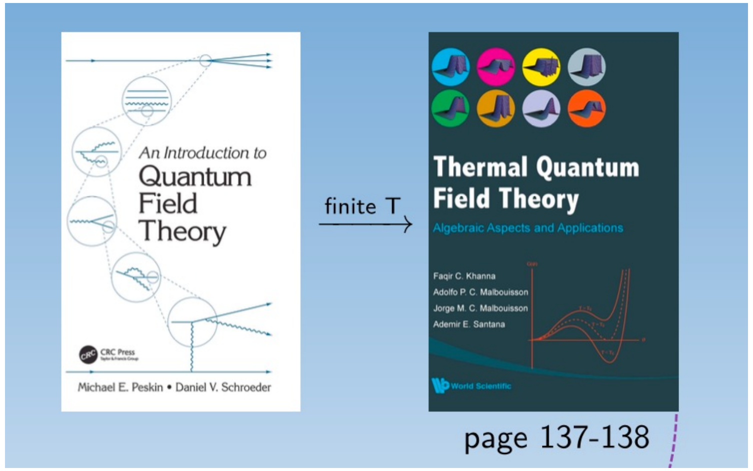
$V_{\rm bkg}(r) \sim G_F^2 \times ?$

$$S_\nu(k) = (\not{k} + m)\left\{\frac{i}{k^2 - m_\nu^2 + i\epsilon} - \underline{2\pi\delta\left(k^2 - m_\nu^2\right)\left[\Theta\left(k^0\right) n_+(\mathbf{k}) + \Theta\left(-k^0\right) n_-(\mathbf{k})\right]}\right\}$$

extra term ∝ number density

Propagator in finite temperature/density

$$S_\nu(k) = (\not{k} + m)\left\{\frac{i}{k^2 - m_\nu^2 + i\epsilon} - \underbrace{2\pi\delta\left(k^2 - m_\nu^2\right)\left[\Theta\left(k^0\right) n_+(\mathbf{k}) + \Theta\left(-k^0\right) n_-(\mathbf{k})\right]}_{\text{extra term} \propto \text{number density}}\right\}$$

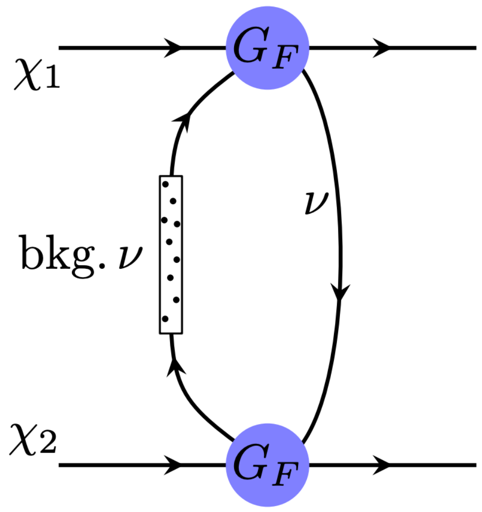


- This formula can be derived using the (real-time formalism) of finite-T QFT

- But why doe the result $\propto$ number density?

  A more intuitive derivation *without* relying on thermal equilibrium: **coherent scattering**

Cosmic neutrino background



$$V_{2\nu}(r) = V_0(r) + V_{\rm bkg}(r)$$

- relativistic limit ($m_\nu \ll T$)

$$V_0(r) = \frac{G_F^2}{4\pi^3 r^5}$$

$$V_{\rm bkg}(r) = -\frac{8G_F^2}{\pi^3}\frac{T^4}{r(1+4r^2T^2)^2} \sim \begin{cases} -1/r & \text{for } r \ll T \\ -1/r^5 & \text{for } r \gg T \end{cases}$$

- non-relativistic limit ($m_\nu \gg T$) $\quad V_0(r) \sim e^{-2m_\nu r} \quad (\text{for } r \gg m_\nu^{-1})$

$\boldsymbol{V_{\rm bkg}}$

nature of neutrino	$r \ll T^{-1}$	$r \gg T^{-1}$
Dirac	$-\frac{3\zeta(3)}{4\pi^3} G_F^2 \frac{m_\nu T^3}{r}$	$-\frac{1}{64\pi^3} G_F^2 \frac{m_\nu}{T} \frac{1}{r^5}$
Majorana	$-\frac{30\zeta(5)}{\pi^3} G_F^2 \frac{T^5}{m_\nu r}$	$-\frac{1}{8\pi^3} G_F^2 \frac{1}{m_\nu T} \frac{1}{r^7}$

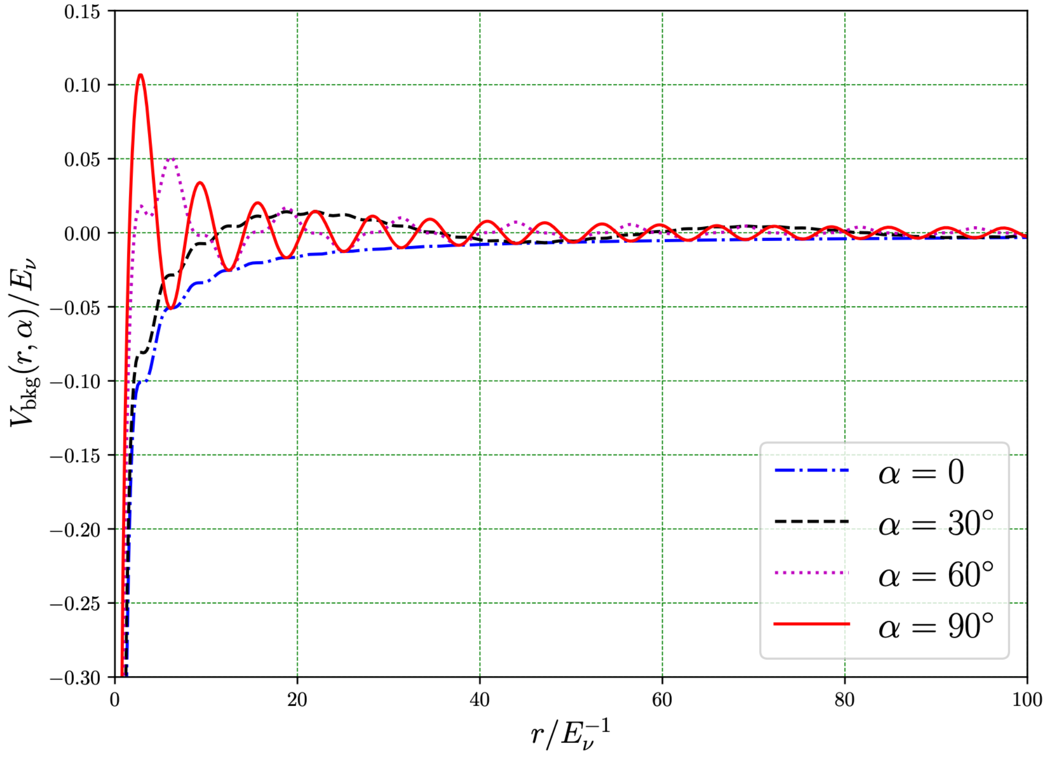
Dirac is larger than Majorana due to more mass insertions

$V_{\rm bkg}$ dominates over V_0 at long distances

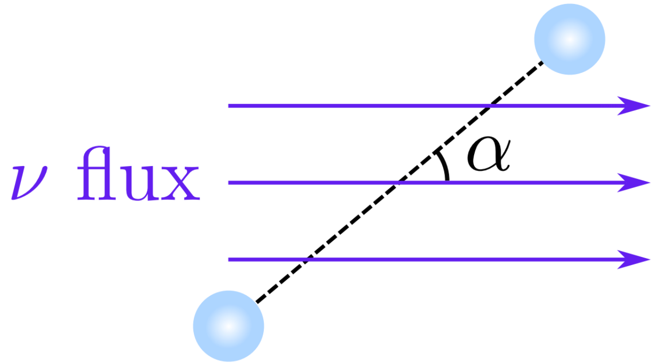
Monochromatic directional backgrounds

neutrino beam with flux Φ_0 and energy E_ν

e.g., solar, reactor, supernova neutrino flux



Ghosh, Grossman, Tangarife, Xu & **BY**, [2209.07082]



$$V_{\rm bkg}(r,\alpha) = -\frac{G_F^2}{\pi}\Phi_0 E_\nu \frac{1}{r}\left\{\cos^2\left(\frac{\alpha}{2}\right)\cos\left[(1-\cos\alpha)E_\nu r\right] + \sin^2\left(\frac{\alpha}{2}\right)\cos\left[(1+\cos\alpha)E_\nu r\right]\right\}$$

- **small angle limit:**

$$V_{\rm bkg} \approx -\frac{G_F^2}{\pi} \times \Phi_0 E_\nu \times \frac{1}{r} \times \left[1 + \mathcal{O}(\alpha^2)\right]$$

We get the 1/r potential !

Fifth-force search of background neutrino force

- Assuming small angle limit:

$$V_{\rm bkg}(r) \approx -\frac{G_F^2}{\pi} \times \Phi_0 E_\nu \times \frac{1}{r}$$ **background potential**

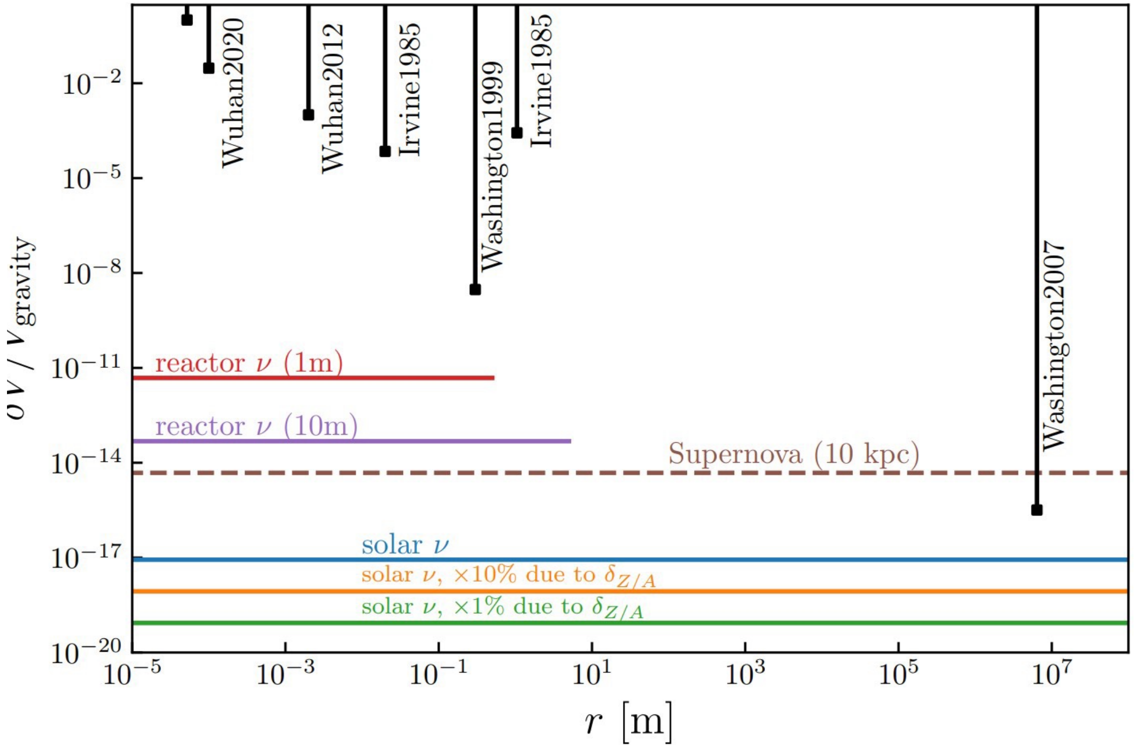
$$V_0(r) = \frac{G_F^2}{4\pi^3} \times \frac{1}{r^5}$$ **vacuum potential**

For r = 1 meter and @ 10 meters away from the reactor $\boxed{\frac{V_{\rm bkg}}{V_0} \sim 10^{22}}$

exp	$\delta V/V_{\rm gravity}$	$\langle r \rangle$	Refs
Washington2007	3.2×10^{-16}	~ 6400 km	[48]
Washington1999	3.0×10^{-9}	~ 0.3 m	[49]
Irvine1985	0.7×10^{-4}	$2-5$ cm	[45]
Irvine1985	2.7×10^{-4}	$5-105$ cm	[45]
Wuhan2012	10^{-3}	~ 2 mm	[50]
Wuhan2020	3×10^{-2}	~ 0.1 mm	[47]
Washington2020	~ 1	52 μm	[46]
Future levitated optomechanics	$\sim 10^4$	1 μm	[51]

Table 4. Sensitivities of long-range force search experiments.

Rector flux: E_ν = 2 MeV, $\Phi_0 = 5\times10^{13}$ cm^{-2}s^{-1} (10 meters away)
Solar flux: E_ν = 0.3 MeV, $\Phi_0 = 6\times10^{10}$ cm^{-2}s^{-1}
Supernova: E_ν = 10 MeV, $\Phi_0 = 1\times10^{12}$ cm^{-2}s^{-1}

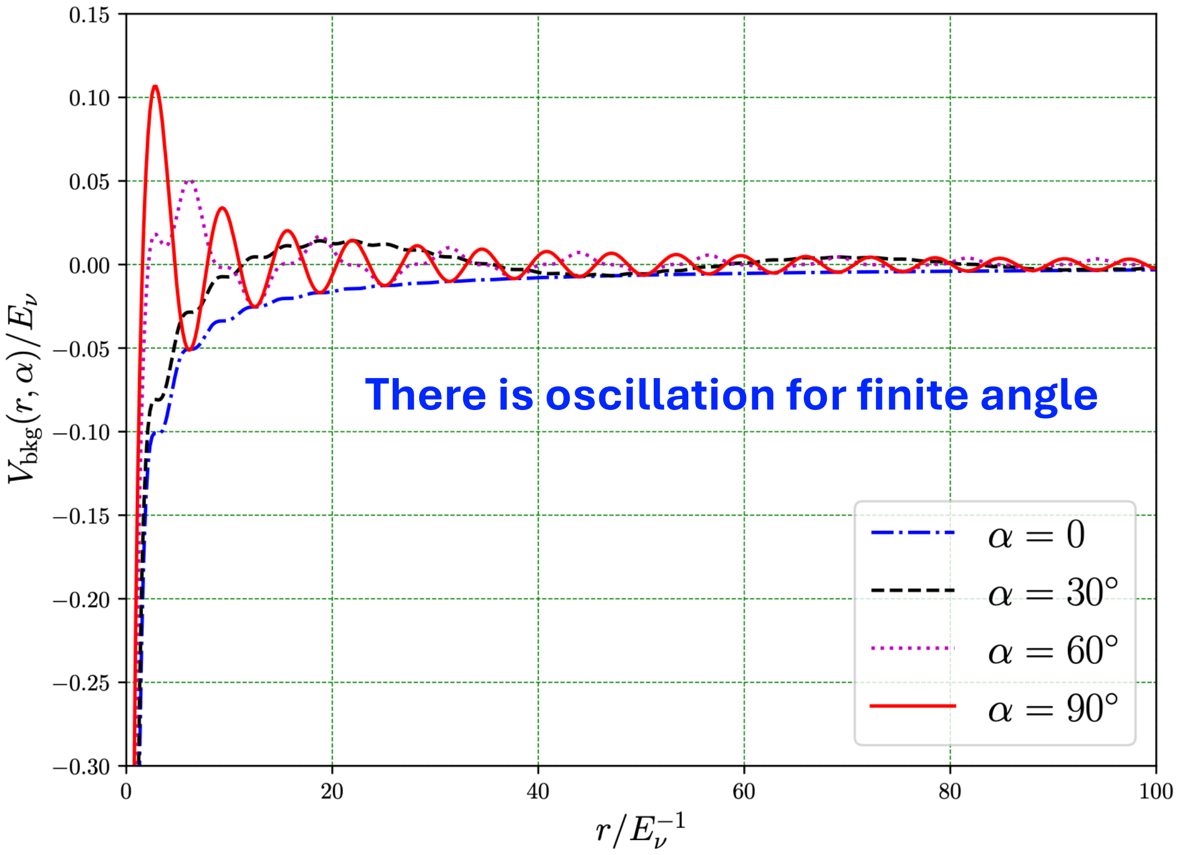


Ghosh, Grossman, Tangarife, Xu & **BY**, [2209.07082]

However…

- Decoherence effect from finite size

$$V_{\rm bkg}\left(\alpha \ll 1\right) \approx -\frac{G_F^2}{\pi} \times \Phi_0 E_\nu \times \frac{1}{r} \times \cos\left(\frac{\alpha^2 E_\nu r}{2}\right)$$



Coherent length of the neutrino beam: $L \sim 1/E_\nu$

Sum over finite size of target and energy spread of flux would smear out the leading 1/r potential

condition to avoid smearing

$$\alpha^2 \lesssim \frac{\pi}{\Delta(E_\nu r)}$$

$$\text{for } \Delta\left(E_\nu r\right) = \text{keV}\cdot\text{cm} \;\Rightarrow\; \alpha \lesssim 10^{-4}$$

This condition is difficult to satisfy in current experiments

Quick Summary

- Neutrino force is **largely enhanced** in a directional background

$$G_F^2/r^5 \quad \to \quad G_F^2/r \times \text{energy flux}$$

- Sensitivity of V_{bkg} is primarily limited by the **intensity** of existing neutrino sources and the **short coherent length** of background neutrinos

A sketch of today's talk

- Introduction of quantum forces
- Neutrino-mediated quantum force
- **Quantum forces in the dark sector**
- Summary

Quantum forces in the dark sector

- Going beyond the SM, quantum force effects could be more significant, due to possibly **larger occupation number** and **longer coherent length** of mediator particles in cosmological/astrophysical environments

- I will discuss two topics:

(i) **Perturbative** aspect: a quantum force mediated by **axions** in an axion background;

(ii) **Non-perturbative** aspect: the **Sommerfeld enhancement** induced by quantum forces and its effect on DM evolution/detection and structure formation

❶ Axion forces in axion backgrounds

Based on Grossman, **BY** & Zhou, [2504.00104]

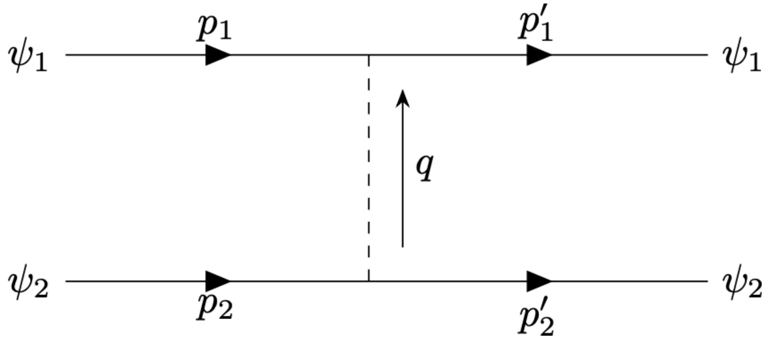
Axion as pseudo Nambu-Goldstone boson

- Well motivated in particle physics to solve strong CP problem
- Good candidate for dark matter
- Naturally light due to the protection of shift symmetry

$$V(a) = \mu_0^4 \left[1 - \cos\left(a/f_a\right)\right] \quad \Rightarrow \quad m_a = \mu_0^2/f_a$$

- A long-range fifth-force mediated by axion?

Axion force at tree level



$$\mathcal{L} \supset \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi$$

- Tree-level axion force is always spin dependent

$$V_a(\mathbf{r}) \sim \frac{1}{f_a^2} (\boldsymbol{\sigma}_1 \cdot \nabla) (\boldsymbol{\sigma}_2 \cdot \nabla) \frac{e^{-m_a r}}{4\pi r}$$

Moody & Wilczek, PRD (1984)

- Cannot lead to observable effects between unpolarized objects

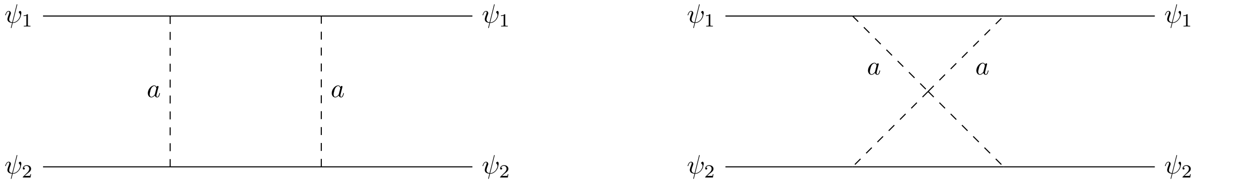
$$(\boldsymbol{\sigma}_1 \cdot \nabla) (\boldsymbol{\sigma}_2 \cdot \nabla) \overset{\text{spin average}}{=} \mathrm{Tr} \left(\sigma^i \otimes \sigma^j \right) \partial_i \partial_j = 0$$

How to get a spin-independent force? At least at one-loop level

Axion force at one-loop level

- Shift-invariant interaction

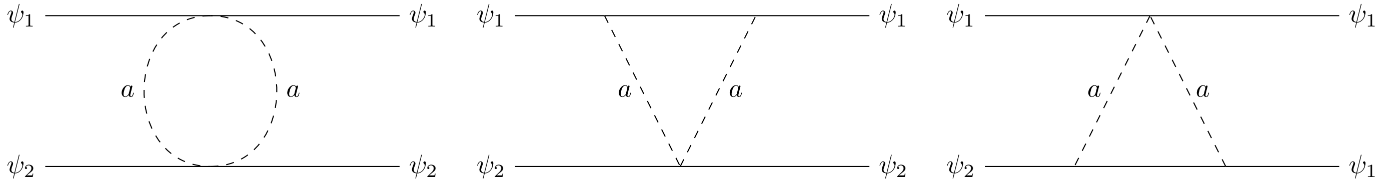
$$\mathcal{L} \supset \frac{\partial_\mu a}{f_a} \bar{\psi}\gamma^\mu\gamma^5\psi$$



$$V_{2a}(r) \sim 1/(f_a^4 r^5)$$

Grifols & Tortosa, PLB (1994)

Order $1/r^3$ is cancelled due to shift symmetry



- Include shift-symmetry-breaking interaction

$$\mathcal{L} \supset \frac{\partial_\mu a}{f_a} \bar{\psi}\gamma^\mu\gamma^5\psi + \frac{\mu}{f_a^2} a^2 \bar{\psi}\psi$$

$$\text{for } r \ll m_a^{-1}: \quad V_{2a}(r) \sim \mu^2 / \left(f_a^4 r^3\right)$$

$$\text{for } r \gg m_a^{-1}: \quad V_{2a}(r) \sim e^{-2m_a r}$$

Shift symmetry breaking enhances the two-axion force:

$$\boldsymbol{1/r^5 \to 1/r^3}$$

Ferrer & Nowakowski, PRD (1999)

Bauer & Rostagni, PRL (2024)

Axion force at one-loop level

Is it possible to get a $1/r$ axion force?

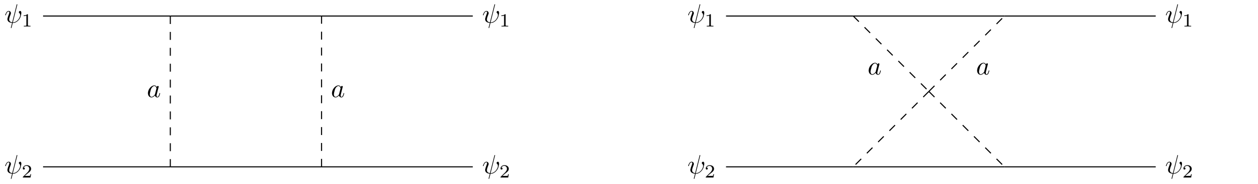
Yes, adding the axion background!

- Shift-invariant interaction

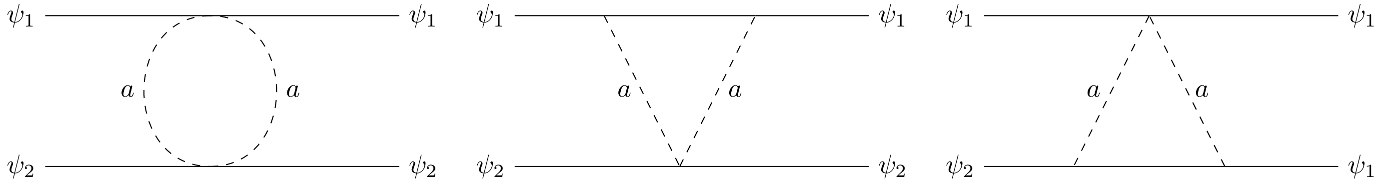
$$\mathcal{L} \supset \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi$$

$$V_{2a}(r) \sim 1/(f_a^4 r^5)$$

Grifols & Tortosa, PLB (1994)



Order $1/r^3$ is cancelled due to shift symmetry



- Include shift-symmetry-breaking interaction

$$\mathcal{L} \supset \frac{\partial_\mu a}{f_a} \bar{\psi} \gamma^\mu \gamma^5 \psi + \frac{\mu}{f_a^2} a^2 \bar{\psi} \psi$$

for $r \ll m_a^{-1}$: $\quad V_{2a}(r) \sim \mu^2 / \left(f_a^4 r^3\right)$

for $r \gg m_a^{-1}$: $\quad V_{2a}(r) \sim e^{-2 m_a r}$

Shift symmetry breaking enhances the two-axion force:

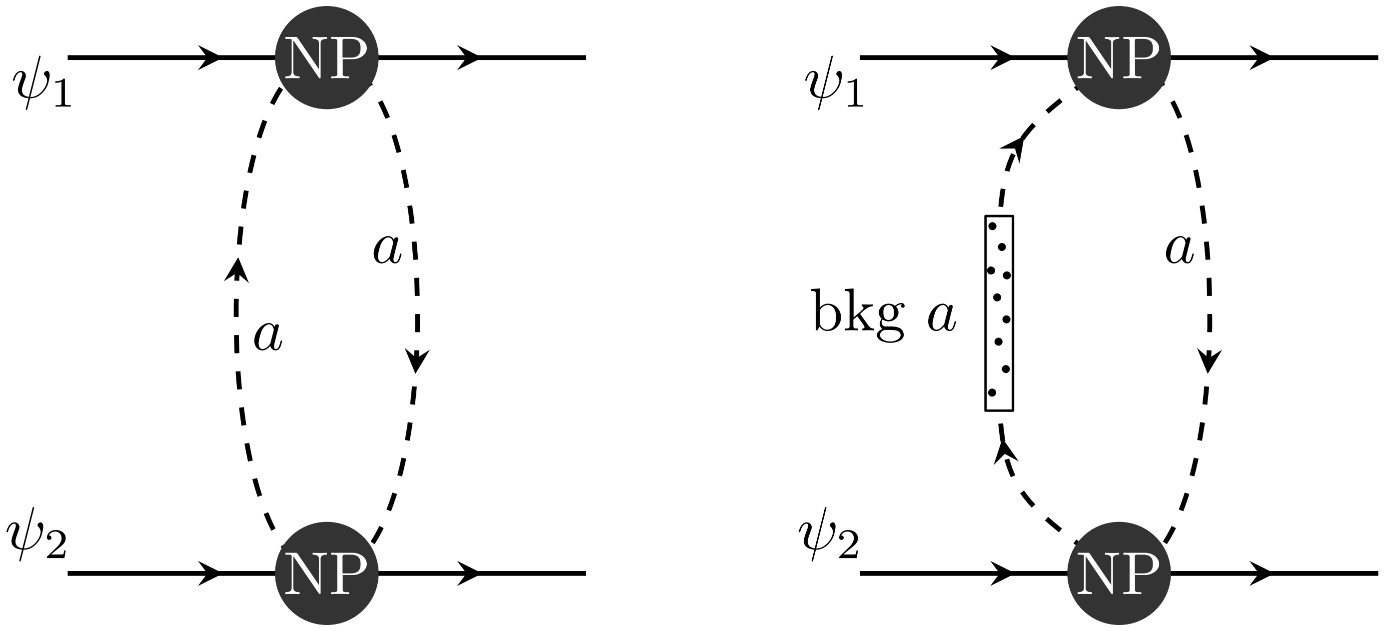
$$\mathbf{1/r^5 \to 1/r^3}$$

Ferrer & Nowakowski, PRD (1999)

Bauer & Rostagni, PRL (2024)

Coherent enhancement from axion background

in vacuum — in axion background



Grossman, **BY** & Zhou, [2504.00104]

$$\frac{i}{k^2 - m_a^2} \quad \rightarrow \quad \frac{i}{k^2 - m_a^2} + \underline{2\pi\delta\left(k^2 - m_a^2\right) f(\mathbf{k})}$$

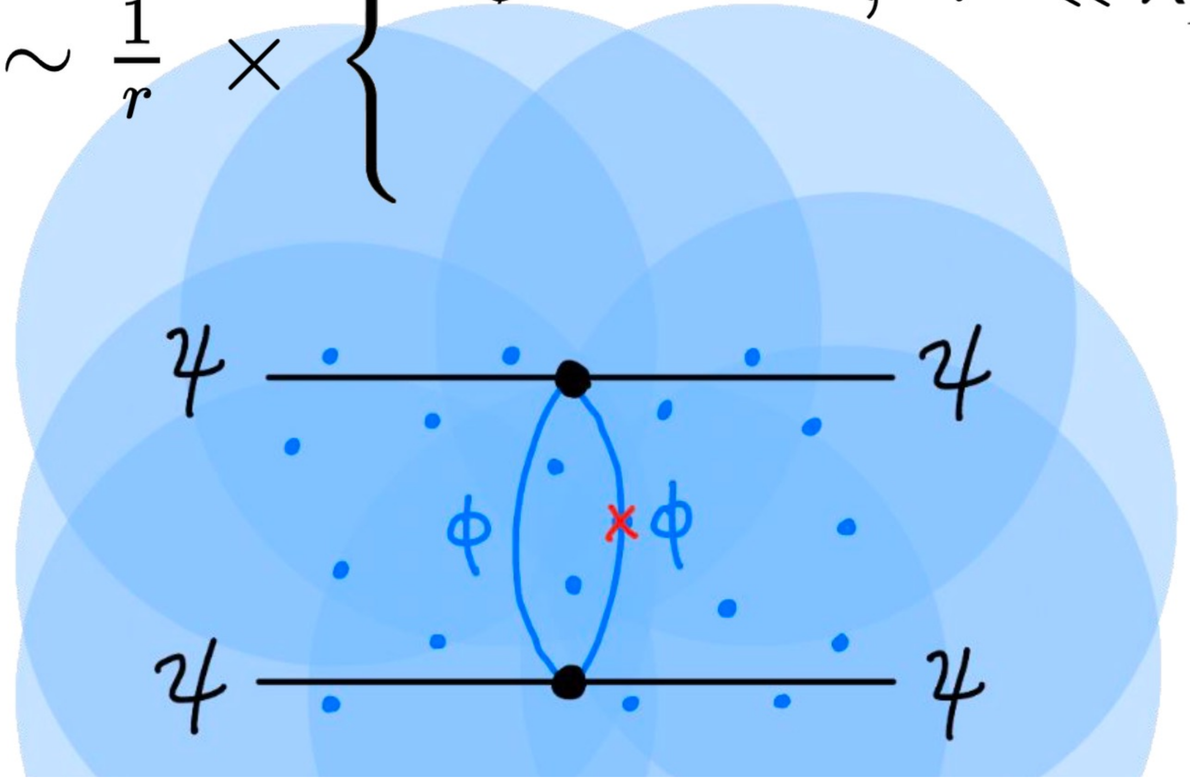
extra term ∝ number density

Background-induced force

Generic form:

$$V_{\rm bkg}(r) \sim \frac{1}{r} \times \begin{cases} n_\phi & , \quad r \ll \lambda_{\rm DB} \end{cases}$$

"coherent limit"

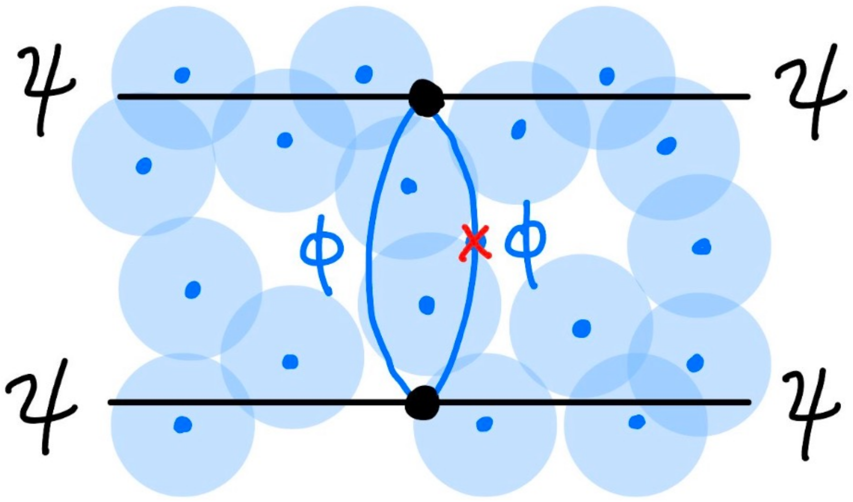


Background-induced force

Generic form:

$$V_{\text{bkg}}(r) \sim \frac{1}{r} \times \begin{cases} n_\phi & , \quad r \ll \lambda_{\text{DB}} \\ n_\phi \left(\frac{\lambda_{\text{DB}}}{r}\right)^n & , \quad r \gg \lambda_{\text{DB}} \end{cases}$$

“decoherence effect”



Background-induced force

Generic form:

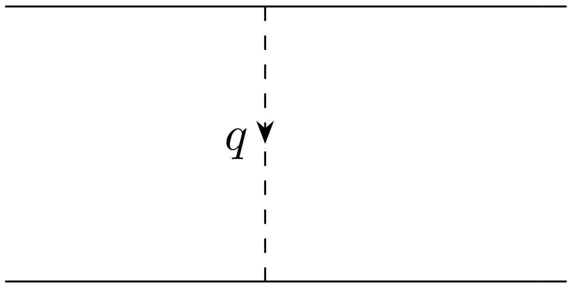
$$V_{\text{bkg}}(r) \sim \frac{1}{r} \times \begin{cases} n_\phi & , \quad r \ll \lambda_{\text{DB}} \\ n_\phi \left(\dfrac{\lambda_{\text{DB}}}{r}\right)^n & , \quad r \gg \lambda_{\text{DB}} \end{cases}$$

"decoherence effect"

power index n depends on specific distribution

No Yukawa suppression at large r !!

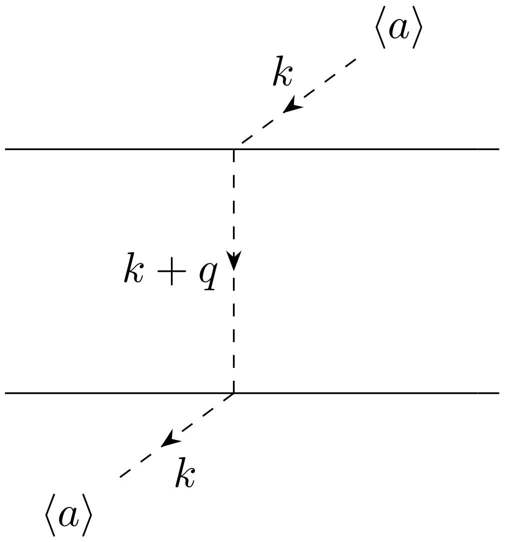
Why NO Yukawa suppression for $V_{\mathbf{bkg}}$?



$$\text{vacuum}: \frac{1}{q^2 - m^2} = \frac{-1}{\mathbf{q}^2 + m^2} \xRightarrow{\text{Fourier transform}} \frac{e^{-mr}}{r}$$

propagator in vacuum is off-shell

exponential suppression corresponds to "off-shellness"

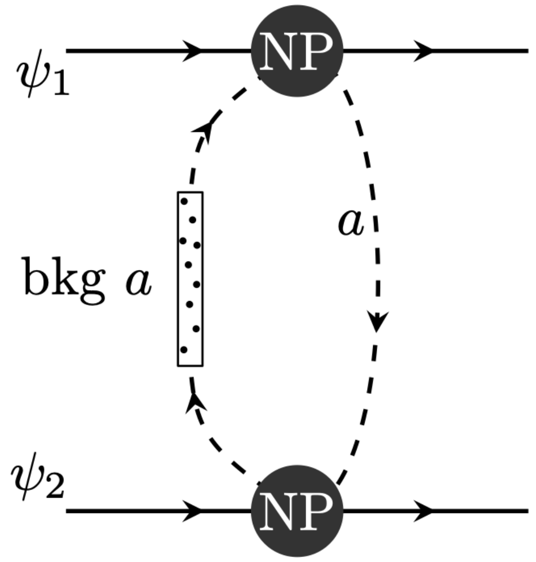


$$\text{background}: \frac{1}{(k+q)^2 - m^2} = \frac{-1}{\mathbf{q}^2 + 2\mathbf{k}\cdot\mathbf{q}} \xRightarrow{\text{Fourier transform}} \frac{e^{i|\mathbf{k}|r}}{r}$$

propagator in background is "nearly on-shell"

no exponential suppression

Background-induced axion force



$$\mathcal{L} \supset \frac{\partial_\mu a}{f_a}\bar{\psi}\gamma^\mu\gamma^5\psi + \frac{\mu}{f_a^2}a^2\bar{\psi}\psi$$

$$V_{\text{bkg}}^{\text{shift}}(\mathbf{r};\mu) = -\frac{\mu^2}{4\pi r f_a^4}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f(\mathbf{k})}{2E_{\mathbf{k}}}\underline{\left[\cos\left(|\mathbf{k}|\, r - \mathbf{k}\cdot\mathbf{r}\right) + \cos\left(|\mathbf{k}|\, r + \mathbf{k}\cdot\mathbf{r}\right)\right]}$$

decoherence factor

- Coherent limit $r \ll 1/k$, cosine terms → 1, we get a $1/r$ potential

$$V_{\text{bkg}}(r) = -\frac{\mu^2}{4\pi f_a^4} \times \frac{\tilde{n}_a}{m_a} \times \frac{1}{r}$$

$$\tilde{n}_a \equiv \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\frac{f(\mathbf{k})}{E_{\mathbf{k}}/m_a}$$

effective number density

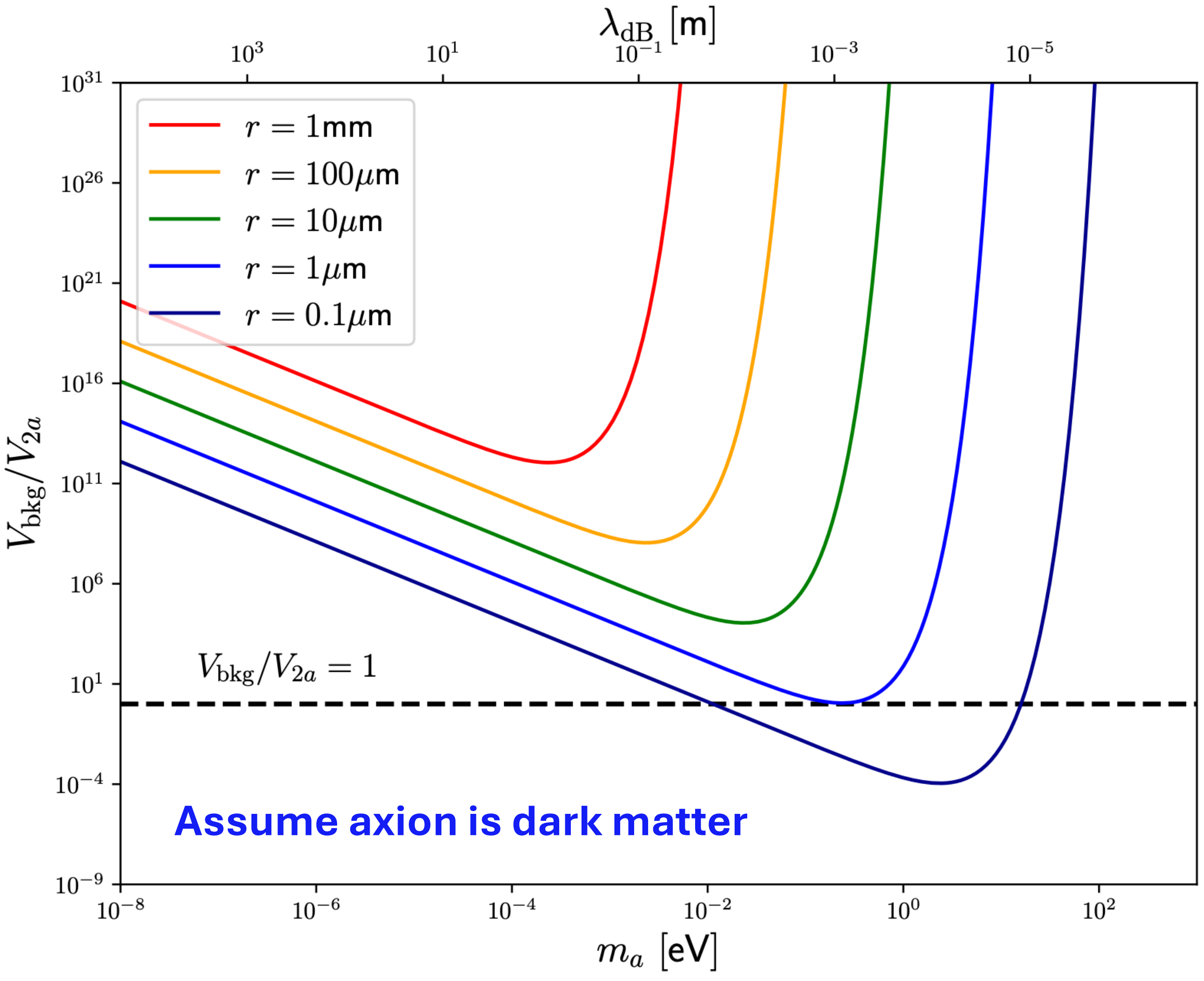
- Decoherence region $r \gg 1/k \sim \lambda_{\text{dB}}$, oscillation kills the leading $1/r$ term

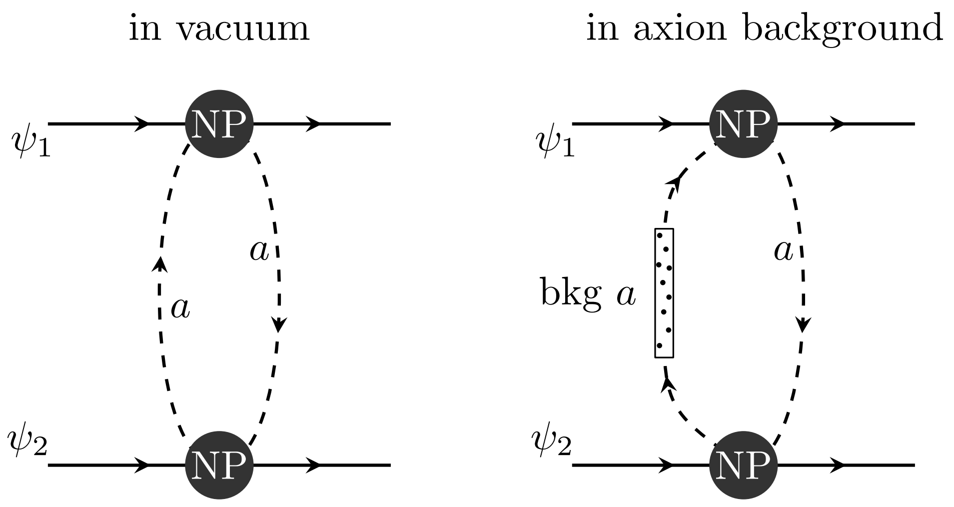
$$V_{\text{bkg}}(r) = -\frac{\mu^2}{4\pi f_a^4} \times \frac{\tilde{n}_a}{m_a} \times \frac{1}{r} \times \left(\frac{\lambda_{\text{dB}}}{r}\right)^n$$

index n depends on specific axion background

($n = 2$ for axion dark matter in galaxy)

Vacuum vs. background axion force





$$V_{2a}(r) = -\frac{\mu^2}{32\pi^3 f_a^4}\frac{m_a}{r^2}K_1(2m_a r)$$

$$V_{\mathrm{bkg}}(r) = -\frac{\mu^2}{4\pi f_a^4} \times \frac{\rho_a}{m_a^2} \times \frac{1}{r}$$

Probe axion force via fifth-force experiments

$$V(r) = V_{\rm grav}(r) + V_{2a}(r) + V_{\rm bkg}(r) \equiv V_{\rm grav}(r) + \delta V(r)$$

Experiments	$\delta V / V_{\rm grav}$	$\langle r \rangle$	Refs
MICROSCOPE	3.8×10^{-15}	~ 7100km	[64]
Washington2007	1.8×10^{-13}	~ 6400km	[65]
Washington1999	3.0×10^{-9}	~ 0.3m	[66]
Irvine1985	7×10^{-5}	$2-5$cm	[67]
HUST2012	10^{-3}	~ 2mm	[68]
Eöt-Wash2006	2×10^{-3}	~ 0.5mm	[69]
HUST2020	3×10^{-3}	~ 0.3mm	[70]
Washington2020	~ 1	52μm	[71]
Future levitated optomechanics	$\sim 10^4$	1μm	[72]
IUPUI2014	2.7×10^7	560nm	[73]

Assume axion is dark matter, and satisfies boosted Maxwell-Boltzmann distribution in the local galaxy

$$f_{\rm BMB}(\mathbf{k}) = n_a\,(2\pi)^{3/2}\,\kappa_0^{-3}\,e^{-|\mathbf{k}-m_a\mathbf{v}_a|^2/(2\kappa_0^2)}$$

momentum dispersion: $\kappa_0 \sim m_a v_a$ with $v_a \sim 10^{-3}$

coherent length: $\lambda_{\rm dB} \sim 1/\kappa_0 \sim 10^3/m_a$

The coherent length can reach macroscopic scale

$\Rightarrow$ a real $1/r$ potential !

Probe axion force via fifth-force experiments

$$V(r) = V_{\rm grav}(r) + V_{2a}(r) + V_{\rm bkg}(r) \equiv V_{\rm grav}(r) + \delta V(r)$$

Grossman, **BY** & Zhou, [2504.00104]

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MICROSCOPE	3.8×10^{-15}	~ 7100km	[64]
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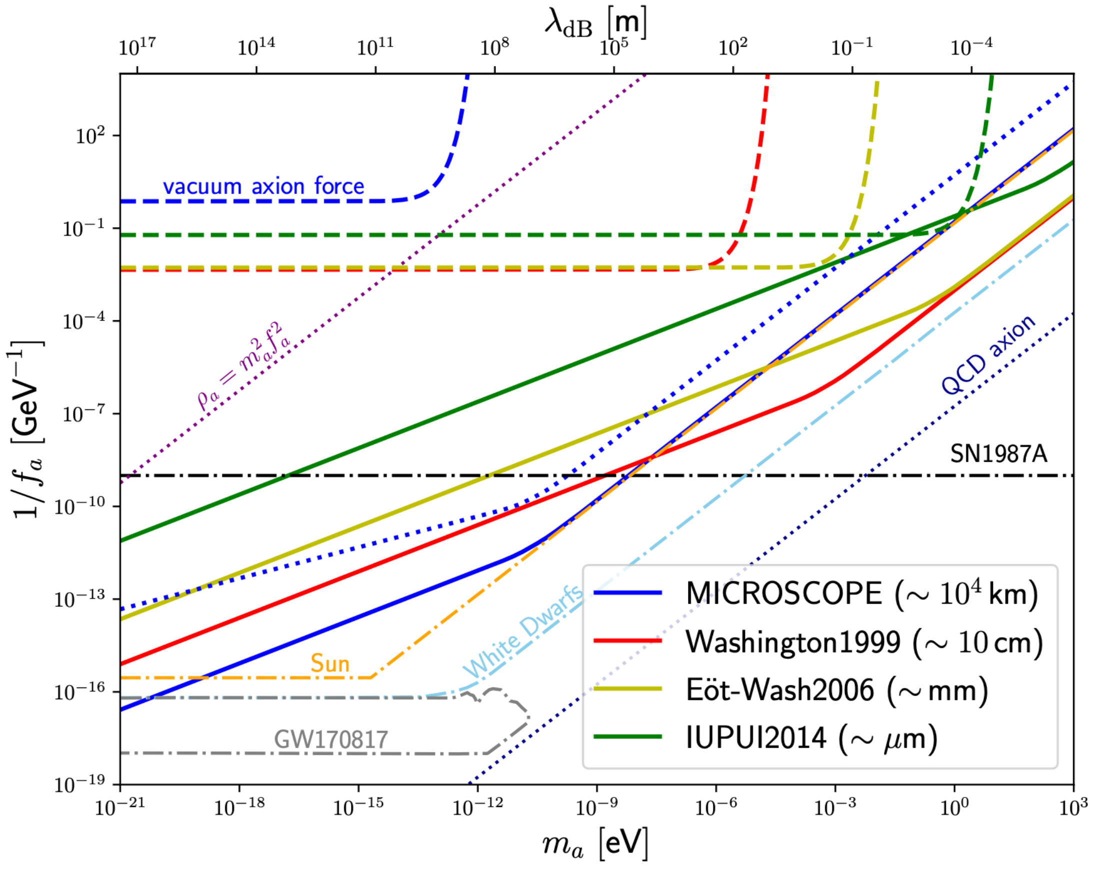


Axion force – quick summary

- Axion-mediated quantum force is enhanced in a cold axion background

$$1/r^3 \quad \to \quad 1/r \times n_a/m_a$$

- This enhancement is very significant thanks to the **large occupation number** and **macroscopic coherent length** of background axions

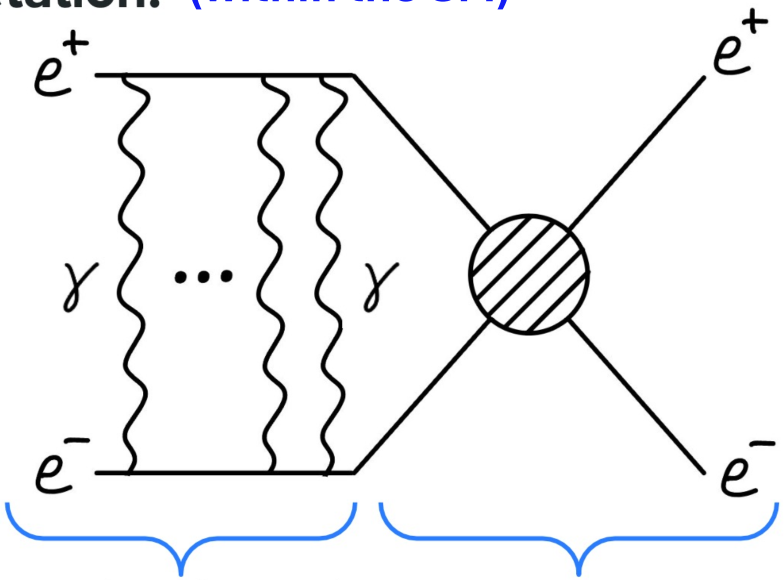
- This effect can be probed by fifth-force search experiments

2 Sommerfeld enhancement from quantum forces for dark matter

Based on Ferrante, Perelstein & **BY**, [2507.12522]

Sommerfeld enhancement (SE)

Diagrammatic Interpretation: (within the SM)

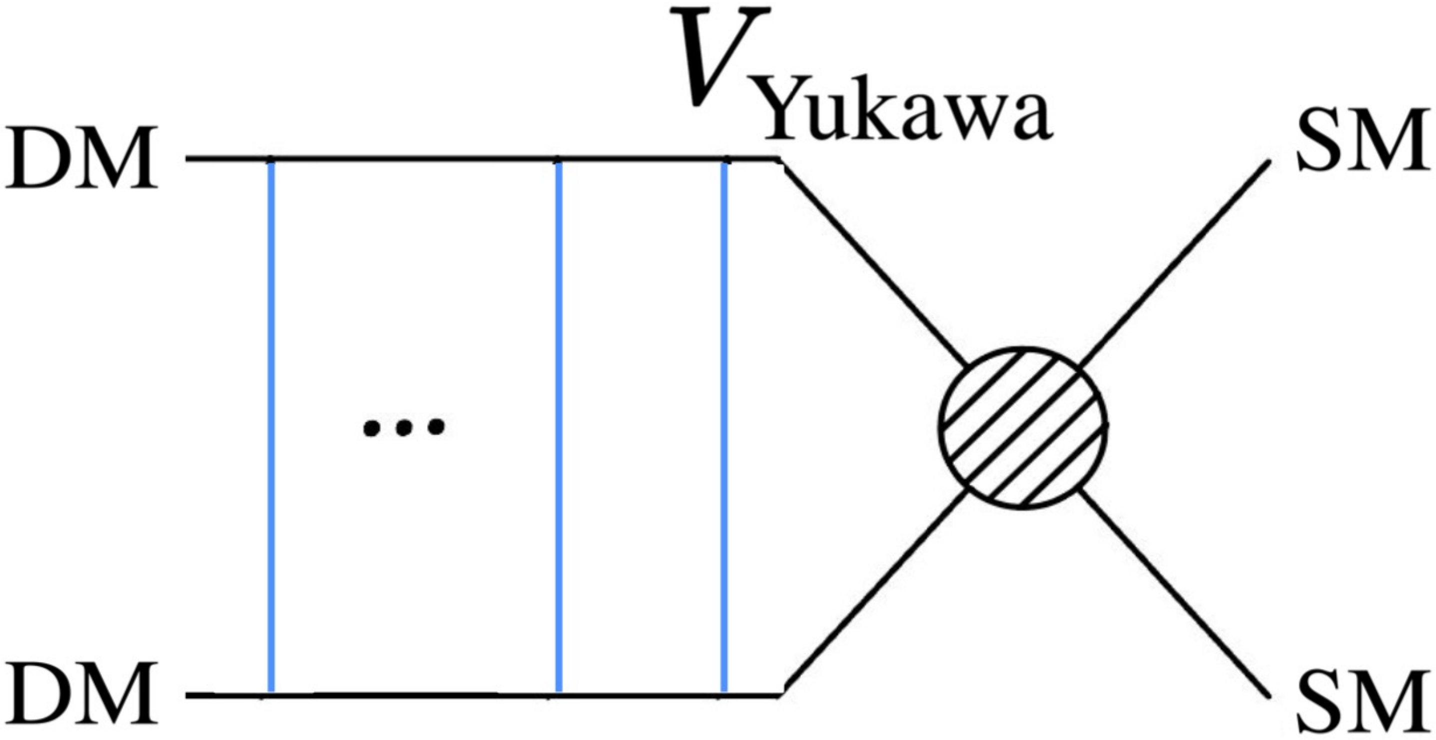


Long-range, Nonperturbative (QM/resum)

Short-range, Perturbative (QFT)

Lepage et al (1994), 9407339

SE from classical forces

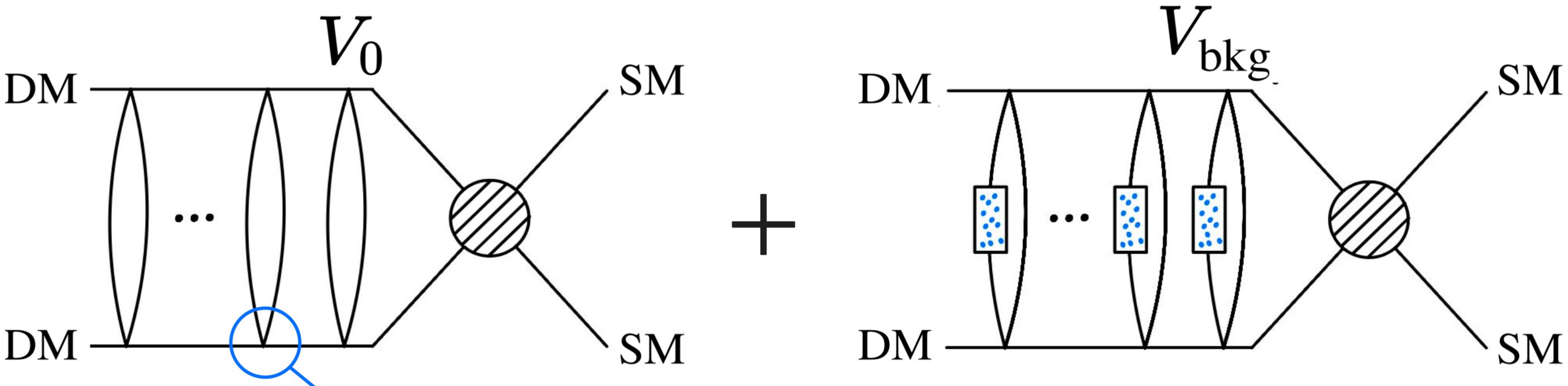


Hisano et al (2003-2007)
0212022, 0412403,
0511118, 0610249

and many other works…

This effect is generic for non-relativistic dark matter in the presence of long-range forces

SE from quantum forces (& background effects)



Start by choosing a generic set of quadratic operators

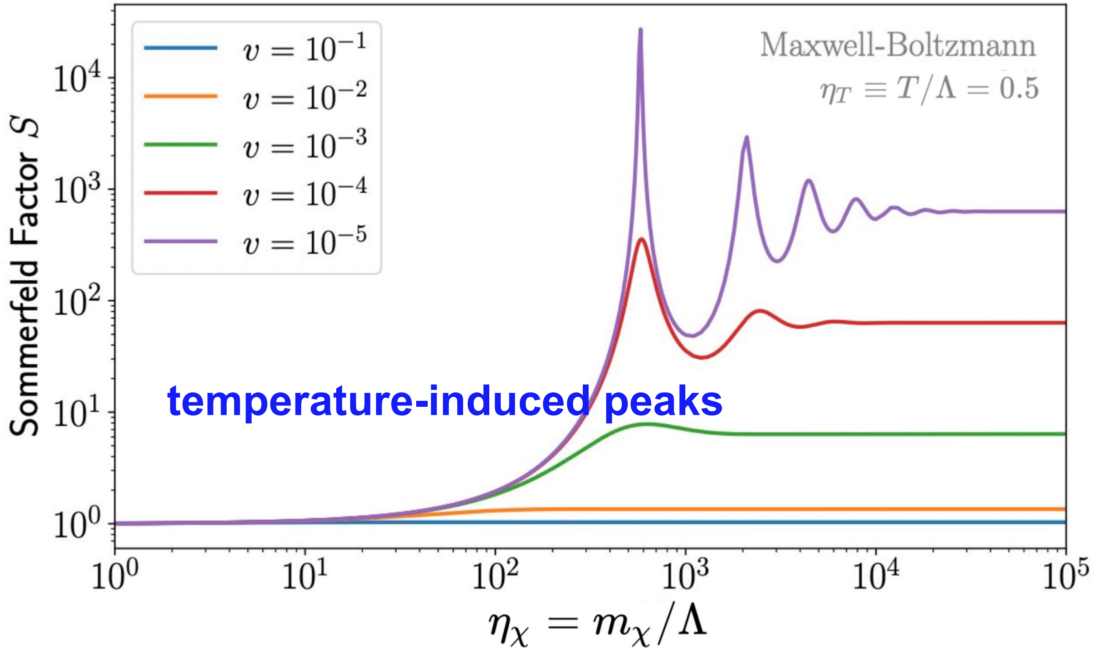
$$\mathcal{O}_S = \frac{1}{\Lambda}\overline{\chi}\chi\phi^2$$

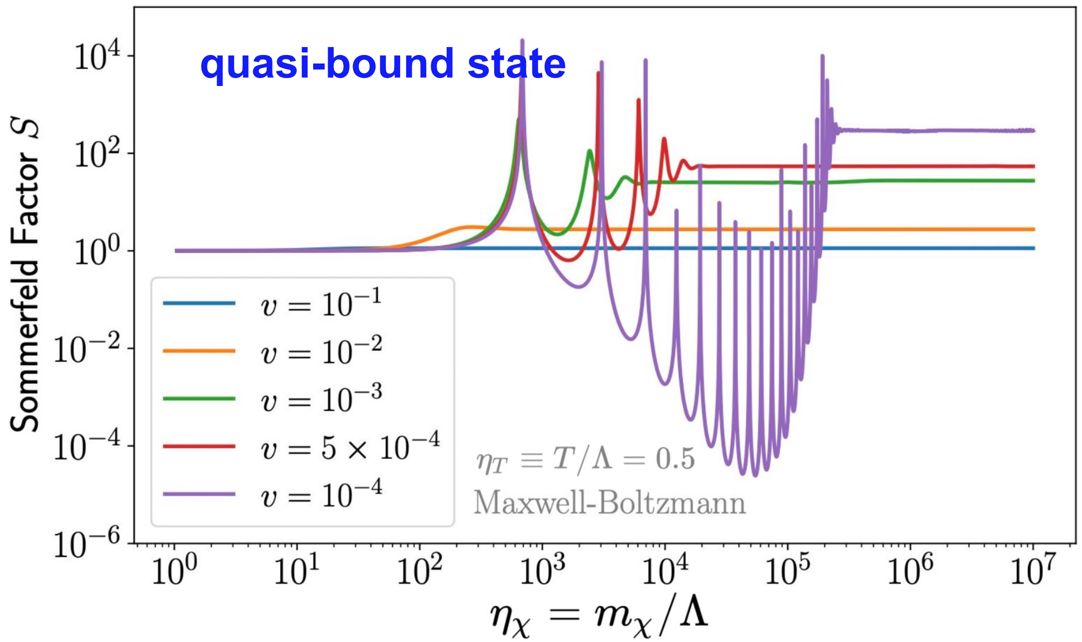
$$\mathcal{O}_F = \frac{1}{\Lambda^2}\overline{\chi}\chi\overline{\psi}\psi$$

$$\widetilde{\mathcal{O}}_F = \frac{1}{\Lambda^2}\overline{\chi}\gamma^\mu\chi\overline{\psi}\gamma_\mu\psi$$

Ferrante, Perelstein & **BY**, [2507.12522]

Novel structures for SE from quantum forces

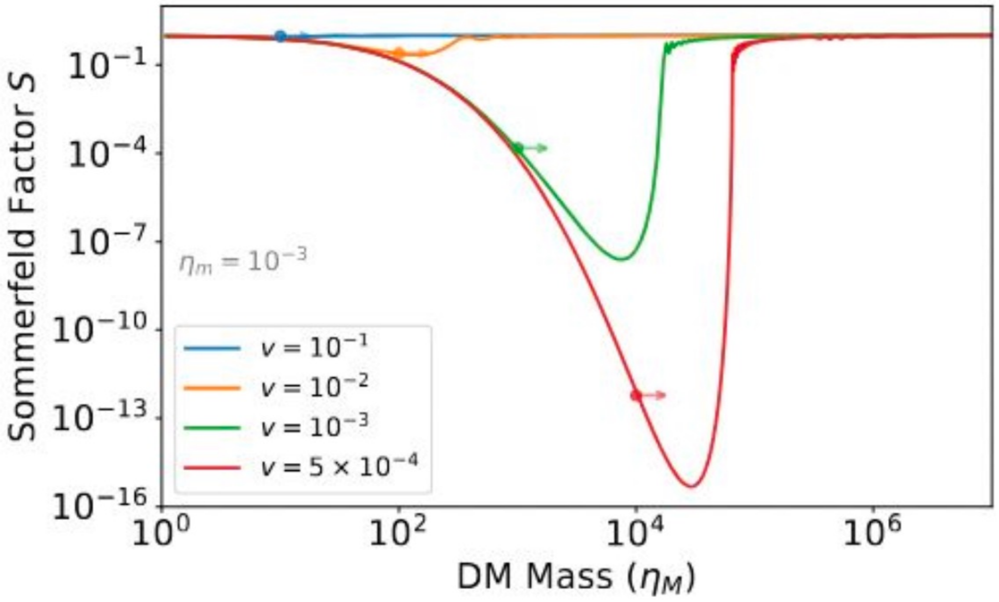




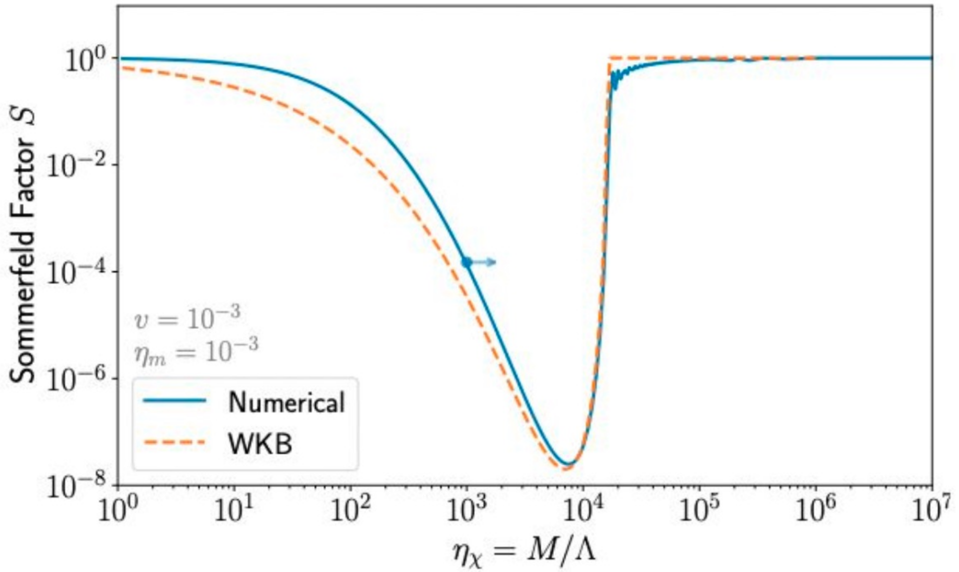
$$V^S_{\rm bkg} \sim \frac{-1}{\Lambda^2 r}\frac{T^2}{4r^2T^2+1}$$

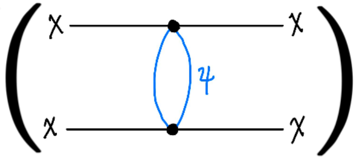
$$V^F_{\rm bkg} \sim \frac{-T^4}{\Lambda^4 r}\frac{1-12r^2T^2}{(1+4r^2T^2)^3}$$

Sommerfeld suppression from repulsive quantum force



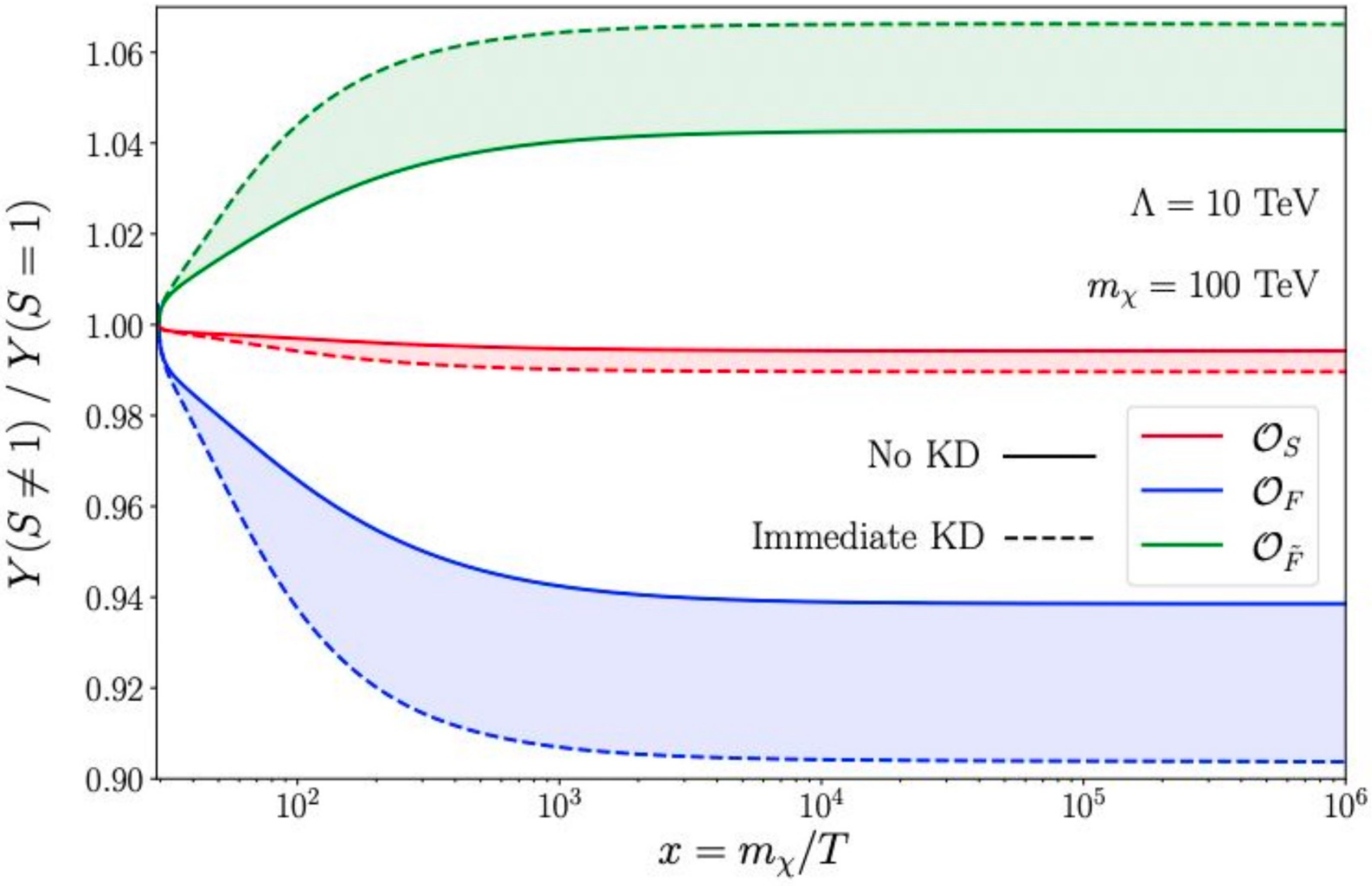
WKB



$$V_0^{\widetilde{F}}\Big($$


$$\Big) \sim \frac{+m_\psi^2}{\Lambda^4}\,\frac{K_2 + m_\psi r K_1}{r^3} \overset{r \ll 1/\Lambda}{\sim} \frac{+1}{r^5}$$

Effects on thermal freeze-out

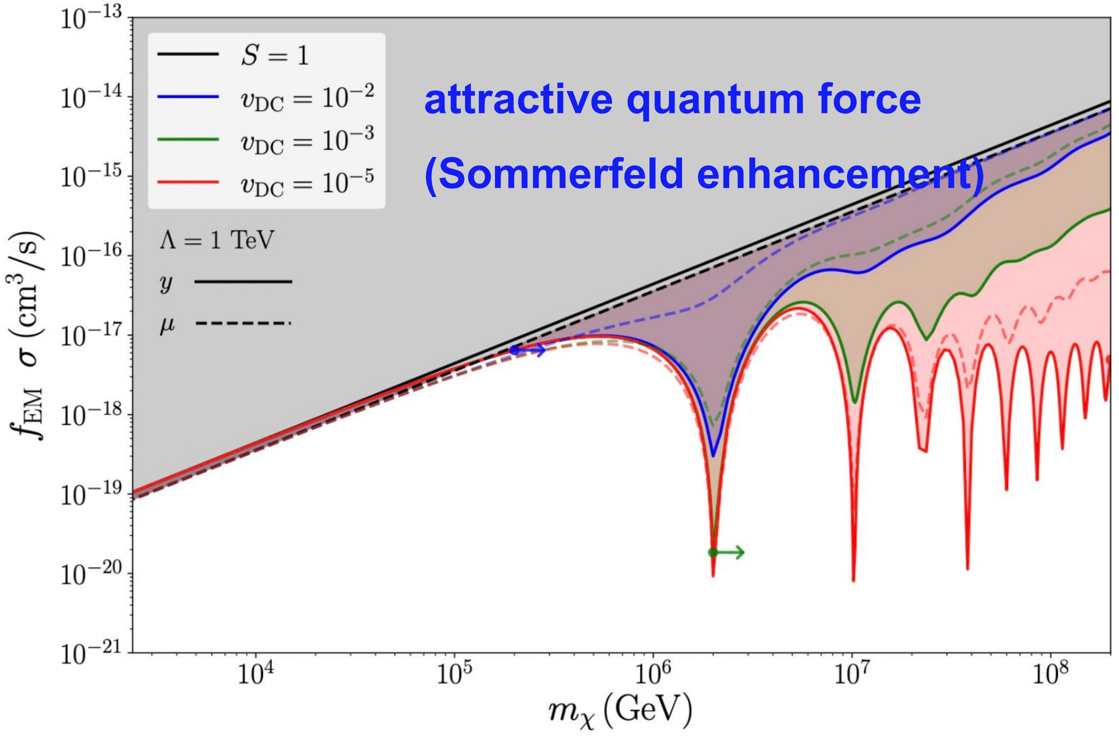


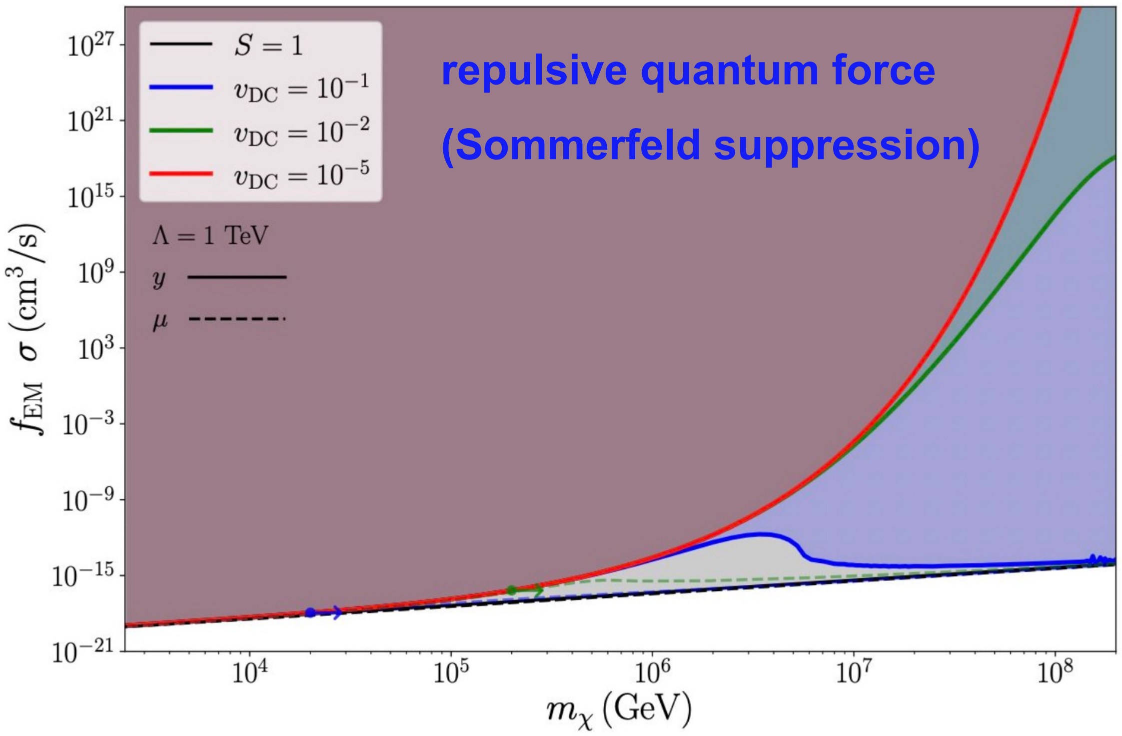
typical effects of order 1% to 10%

limited by not so small velocity of dark matter at freeze-out

Effects on CMB spectral distortion

redshift $z \sim 10^3 - 10^6$

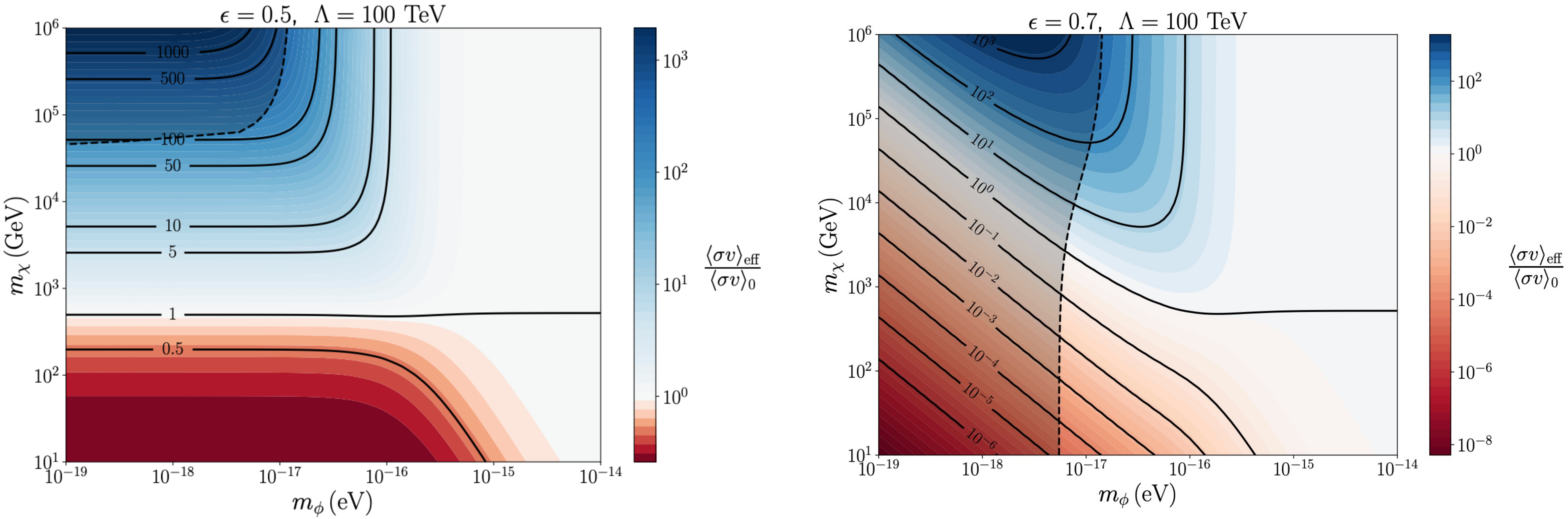




change several orders of magnitude in both directions

significant due to small velocity of dark matter

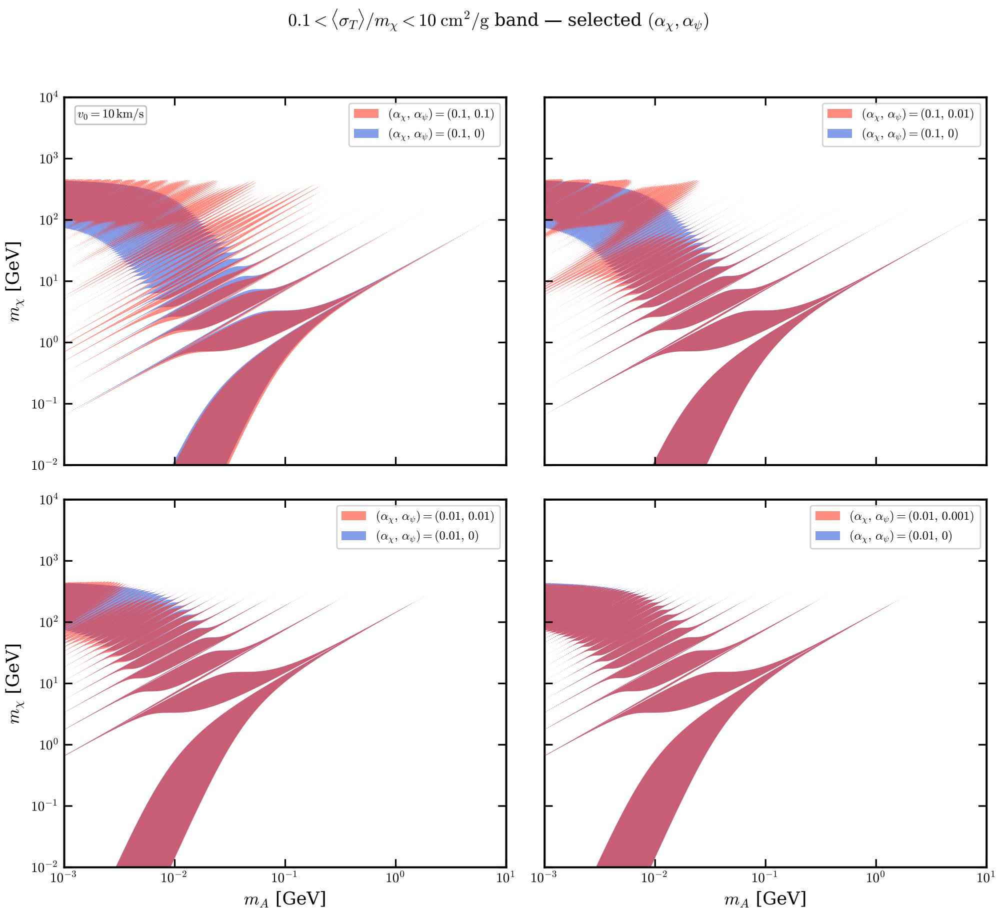
Effects on dark matter indirect detection



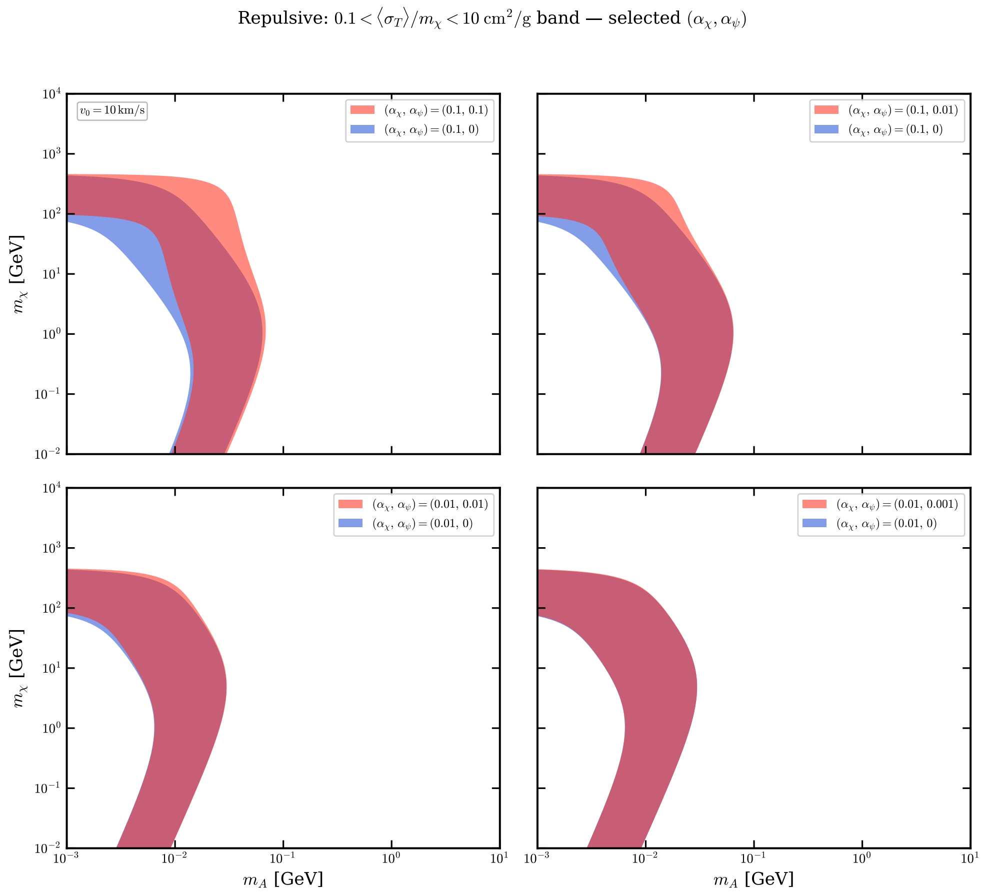
either enhance or suppress DM annihilation cross section by
2-3 orders of magnitude (non-thermal background effect)

Dark matter self-interaction

Ferrante, Perelstein, Sieng & **BY**, to appear soon



attractive quantum forces



repulsive quantum forces

A sketch of today's talk

- Introduction of quantum forces
- Neutrino-mediated quantum force
- Quantum forces in the dark sector
- **Summary**

Summary

- Quantum forces provide a unified framework to describe/probe feebly coupled light degrees of freedom in **precision frontier** and in **cosmology**

- In the SM, the neutrino-mediated quantum force has a sizable effect in **atomic systems**, relevant for precision test of the SM at low energies

- Quantum forces in the dark sector are relevant for **axion searches** and the **cosmic evolution of dark matter**

Thank you for your attention!

Outlook

- More precise calculation of the neutrino force in **heavy atoms** and **laboratory neutrino beams**

  -> experimental verification of the neutrino force

- Quantum forces in the **non-perturbative** regime (e.g., in dense astrophysical environments)

  -> identify new scaling behavior beyond Born, relevant for neutrinos in astro environments

- Neutrino - DM interactions beyond the Standard Model

  -> neutrino force may affect DM **self-interaction** and DM **(in)direct detection**

Backup slides

Dispersion technique for quantum forces

- Amplitude from QFT ⇒ extract the imaginary part ⇒ effective potential

$$V_{\text{eff}}(r) = \frac{1}{8\pi^2 r} \int_0^\infty \mathrm{d}t \ \mathrm{Disc}\left[i\mathcal{A}_{\text{NR}}(t)\right] e^{-\sqrt{t}r}$$

Feinberg, Sucher & Au, 1989

$$\mathrm{Disc}\left[i\mathcal{A}_{\text{NR}}(t)\right] \equiv i\mathcal{A}_{\text{NR}}(t+i\epsilon) - i\mathcal{A}_{\text{NR}}(t-i\epsilon) = -2\,\mathrm{Im}\left[\mathcal{A}_{\text{NR}}(t)\right] \quad (\text{with } \epsilon \to 0^+)$$

The discontinuity part is always finite according to the optical theorem

Electric-dipole forbidden transition

$|A\rangle$ and $|B\rangle$: eigenstates with same parity

$|\Delta\rangle$: eigenstate with opposite parity

electric-dipole transition is forbidden by selection rule: $\langle A|d_e|B\rangle \equiv 0$

Parity-violating potential perturbs the eigenstates:

$$|A\rangle \to |A'\rangle = |A\rangle + C_{A\Delta}|\Delta\rangle, \qquad C_{A\Delta} = \tfrac{\langle\Delta|V_{\mathrm{PV}}|A\rangle}{E_A - E_\Delta}$$

$$|B\rangle \to |B'\rangle = |B\rangle + C_{B\Delta}|\Delta\rangle, \qquad C_{B\Delta} = \tfrac{\langle\Delta|V_{\mathrm{PV}}|B\rangle}{E_B - E_\Delta}$$

$$\langle A'|d_e|B'\rangle = \sum_{\Delta} \frac{\langle A|d_e|\Delta\rangle\langle\Delta|V_{\mathrm{PV}}|B\rangle}{E_B - E_\Delta} + (A \leftrightarrow B)$$

Radiative corrections to APV

- The 1-loop radiative correction up to $O(\alpha G_F)$ was computed long ago

Radiative corrections to atomic parity violation

W. J. Marciano
Department of Physics, Brookhaven National Laboratory, Upton, Long Island, New York 11973

A. Sirlin
Department of Physics, New York University, New York, New York 10003
(Received 16 July 1982)

Parity violation in atoms induced by radiative corrections

W. J. Marciano and A. I. Sanda
Department of Physics, The Rockefeller University, New York, New York 10021
(Received 26 January 1978)

- The $O(G_F^2)$ neutrino contribution was neglected previously

$$G_F^2 \sim G_F q^2/m_Z^2 \to 0$$

The amplitude of neutrino loop vanishes when $q^2 \to 0$

But this is only correct for δ-function approximation ($m_Z^2 \to \infty$)

Therefore, terms of order $G_F\alpha(m^2/M_W{}^2)$ are regarded as being of order $G_F{}^2$ and are neglected in our analysis.[11]]

The neutrino force has a range much larger than *Z* force, so the δ-function approximation is no longer applicable!

How does long-range interaction contribute?

When there exists **long-range** PV interaction, the correct way is:

1) Perform Fourier transform of the amplitude to get the effective potential (instead of taking $q^2 \to 0$ at the amplitude level)

2) Convolve the parity-violating potential with the atomic wave functions

Lesson: The zero-momentum transfer limit ($q^2 \to 0$) cannot capture the long-range contribution to the APV matrix elements

Neutrino effects on extracting $\sin^2\theta_W$

$$\frac{-g^2}{64\pi\cos^2\theta'_W}Q'_W\frac{e^{-m_Z r}}{r} \equiv \frac{-g^2}{64\pi\cos^2\theta_W}Q_W\frac{e^{-m_Z r}}{r} + \text{loop contributions}$$

$$Q_W^{(\prime)} = \mathcal{Z}\left(1-4\sin^2\theta_W^{(\prime)}\right) - \mathcal{N}$$

$\mathcal{Z}:$ number of protons

$\mathcal{N}:$ number of neutrons

$\theta_W:$ true value

$\theta'_W:$ extracted value assuming only Z force

- θ_W and θ'_W are connected by:

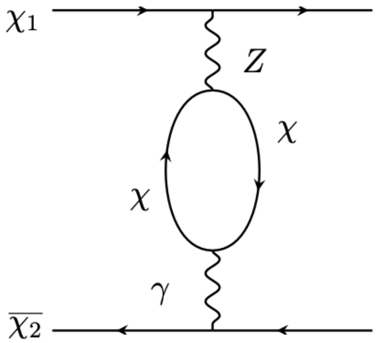
$$\frac{\cos^2\theta'_W}{Q'_W}C_{AB} = \frac{\cos^2\theta_W}{Q_W}C^Z_{AB} \propto \frac{g^2}{64\pi}\langle B|\frac{e^{-m_Z r}}{r}|A\rangle$$

$$C^j_{AB} = \frac{\langle B|V^j_{\mathrm{PV}}|A\rangle}{E_A - E_B}$$

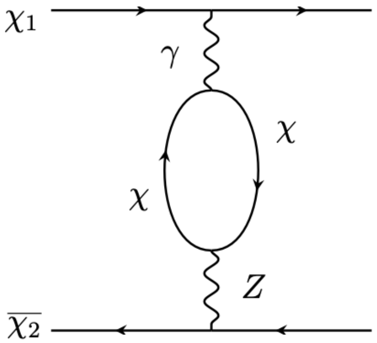
$$C_{AB} = C^Z_{AB}(1+\eta)$$

$$\boxed{\frac{\sin^2\theta'_W - \sin^2\theta_W}{\sin^2\theta_W} \approx \frac{\eta}{\mathcal{N}+3\mathcal{Z}}\left[3\mathcal{N} + (4\mathcal{N}-3\mathcal{Z})\left(1-4\sin^2\theta_W\right)\right]}$$

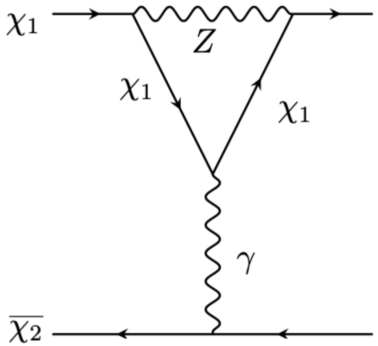
Photon penguin diagrams



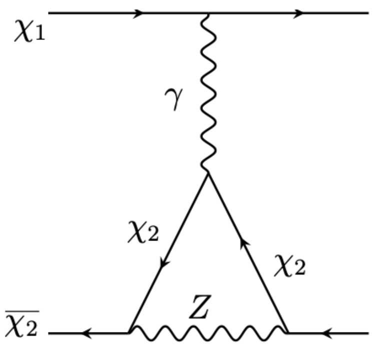
(a) Self-energy (γSE$_1$)



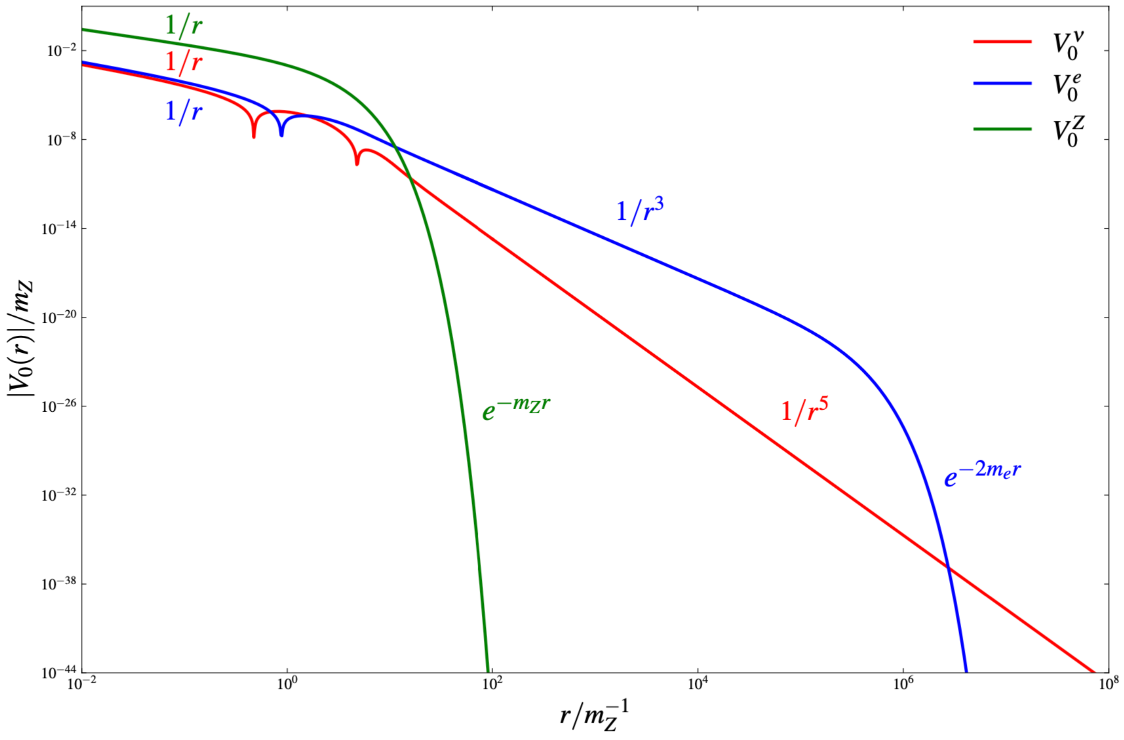
(b) Self-energy (γSE$_2$)



(c) Penguin (γPG$_1$)



(d) Penguin (γPG$_2$)



$$S_F(x-y) \equiv \langle 0 | T\psi(x)\overline{\psi(y)} | 0 \rangle \xrightarrow{\text{finite-T}} S_F(x-y) \equiv \langle \text{bkg} | T\psi(x)\overline{\psi(y)} | \text{bkg} \rangle$$

$$\langle ①, ②, ③ \cdots | \int_{\mathbf{p}} \int_{\mathbf{k}} \cdots \left[a_{\mathbf{p}} \cdots + b^\dagger_{\mathbf{p}} \cdots \right] \left[a^\dagger_{\mathbf{k}} \cdots + b_{\mathbf{k}} \cdots \right] | ①, ②, ③ \cdots \rangle$$

contraction 1

contraction 2

coherent scattering

with ① or ② or ③ ...?
indistinguishable = coherency

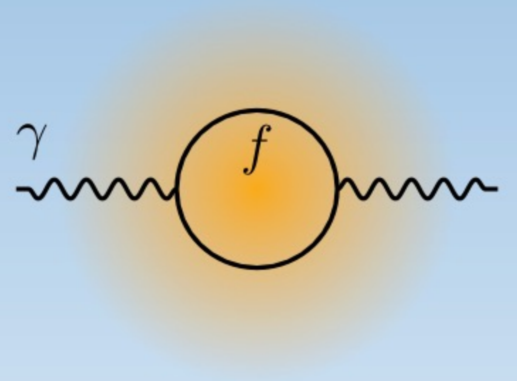
$$\text{propagator} = \boxed{\frac{i}{p^2 - m^2}} - \boxed{(2\pi)\delta\left(p^2 - m^2\right)\left[\Theta\left(p^0\right) n_+(\mathbf{p}) + \Theta\left(-p^0\right) n_-(\mathbf{p})\right]}$$

extra term $\propto n_\pm$ (number density)

Interesting example: the finite-T photon mass

$$m_\gamma^2 = 4\pi\alpha \frac{Q_f^2 n_f}{m_f}$$

finite-T QFT



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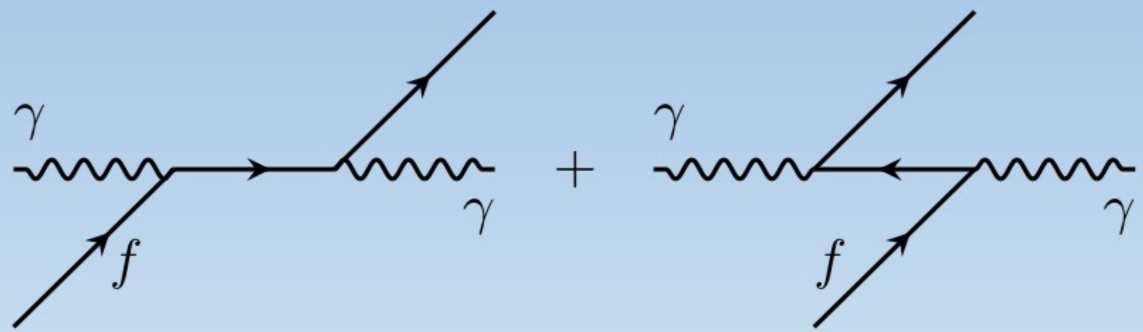
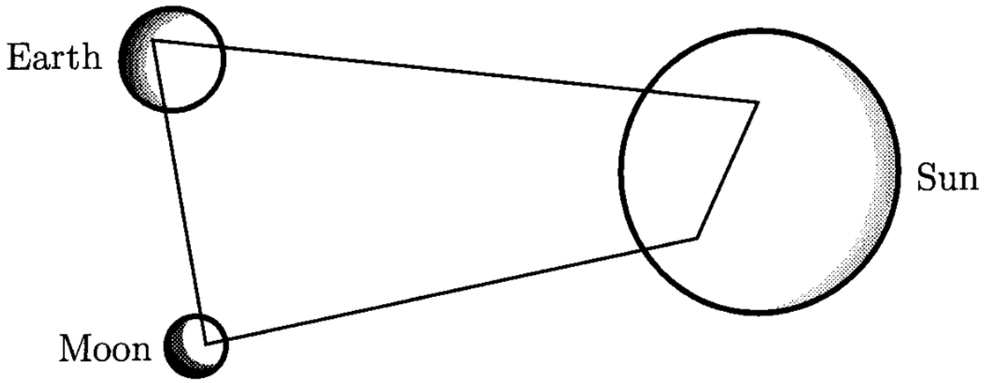
coherent scattering



Loop-level result in finite-T QFT = Tree-level result in zero-T QFT

Recall Feynman's trick to get 1/r potential



$$E = -G'^3\, m_1 m_2 m_3\, \pi^2 \frac{1}{(r_{12} + r_{23} + r_{13}) r_{12} r_{23} r_{13}}. \qquad (2.4.4)$$

If one of the masses, say mass 3, is far away so that r_{13} is much larger than r_{12}, we do get that the interaction between masses 1 and 2 is inversely proportional to r_{12}.

What is this mass m_3? It evidently will be some effective average over all other masses in the universe. The effect of faraway masses spherically distributed about masses 1 and 2 would appear as an integral over an average density; we would have

background contribution

$$E = -\frac{G'^3\, m_1 m_2\, \pi^2}{r_{12}} \boxed{\int \frac{4\pi\rho(R) R^2 dR}{2R^3}}, \qquad (2.4.5)$$

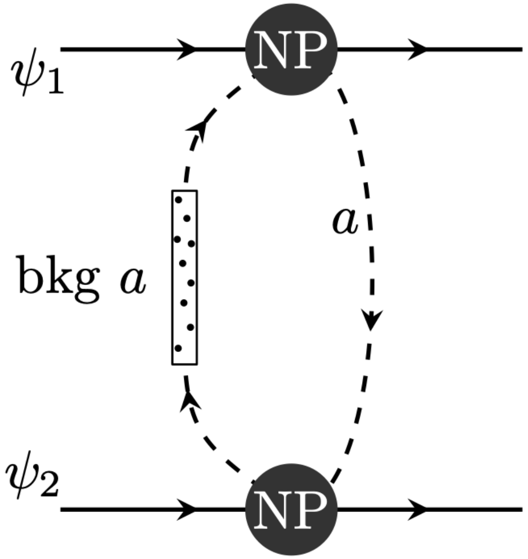
- To mimic the $1/r$ gravity from neutrino force,
  Feynman: $1/r^4 \to 1/r$
  Our background force: $1/r^5 \to 1/r$

- Why $1/r$: Background puts one of the neutrino propagators on-shell, so this force is essentially a **classical** effect

Anisotropic backgrounds could significantly change the scaling behavior of long-range forces

This fact is crucial for the DM search

Background axion force: generic form



$$\boxed{V_{\rm bkg}(r) \sim \frac{1}{r} \times \frac{n_a}{4\pi} \times \frac{m_1 m_2}{f_a^4} \times \frac{\text{order parameter}}{\text{background scale}} \times \text{decoherence factor}}$$

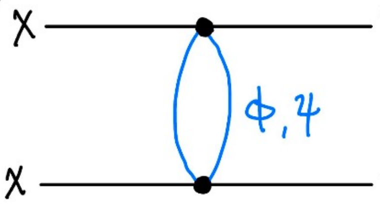
- Why $1/r$: one propagator is on-shell (classical effect)
- Why ∝ number density: coherent effect
- Why need shift symmetry breaking: nonzero classical field value
- Decoherence region: more suppressed than $1/r$ at $r > 1/k \sim \lambda_{\rm dB}$

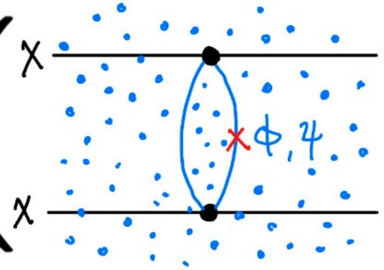
How to break shift symmetry?

- $m_a \neq 0$
- **Explicit breaking** $a^2\bar{\psi}\psi$
- **Background effect** $k \gg m_a$

Strategy for calculating SE from quantum forces

- Calculate Potential:

$V = V_0\Big($ 

$\Big) + V_{\mathrm{bkg}}\Big($ 

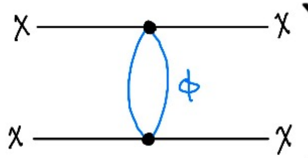
$\Big)$

- Substitute into Schrödinger equation and calculate Sommerfeld factor:

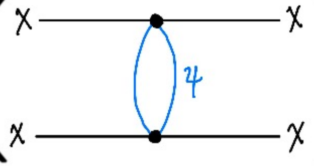
$$\frac{-1}{2m_\chi}\psi'' + V\psi = \frac{m_\chi v^2}{2}\psi$$

Sommerfeld Enhancement of Vacuum Forces

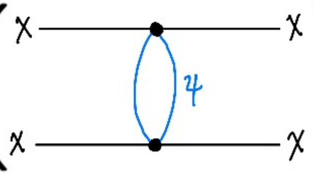
- V0 for scalar, fermion, vector-coupled fermion

$V_0^S\Big($ 

$\Big) \sim \dfrac{-m_\phi}{\Lambda^2}\dfrac{K_1(2m_\phi r)}{r^2} \overset{r \ll 1/\Lambda}{\sim} \dfrac{-1}{r^3}$

$V_0^F\Big($ 

$\Big) \sim \dfrac{-m_\psi^2}{\Lambda^4}\dfrac{K_2(2m_\phi r)}{r^3} \overset{r \ll 1/\Lambda}{\sim} \dfrac{-1}{r^5}$

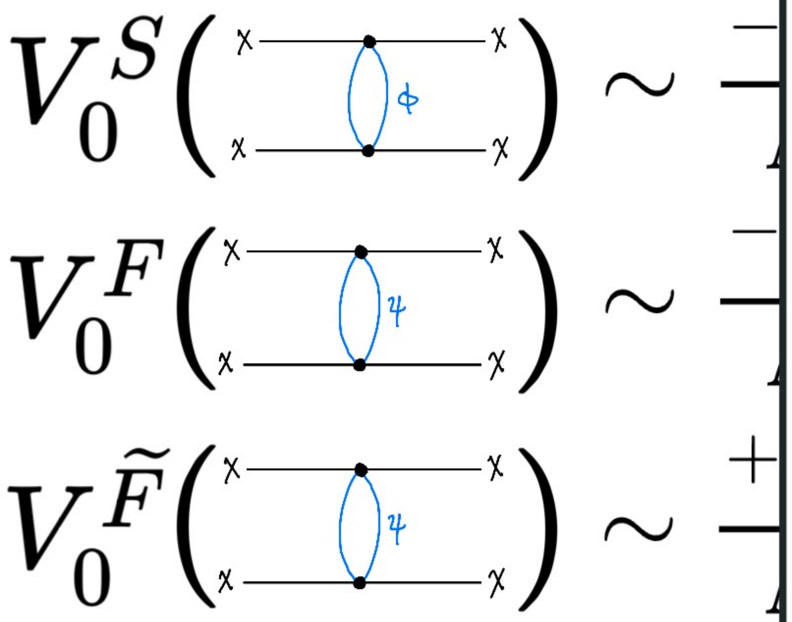
$V_0^{\widetilde{F}}\Big($ 

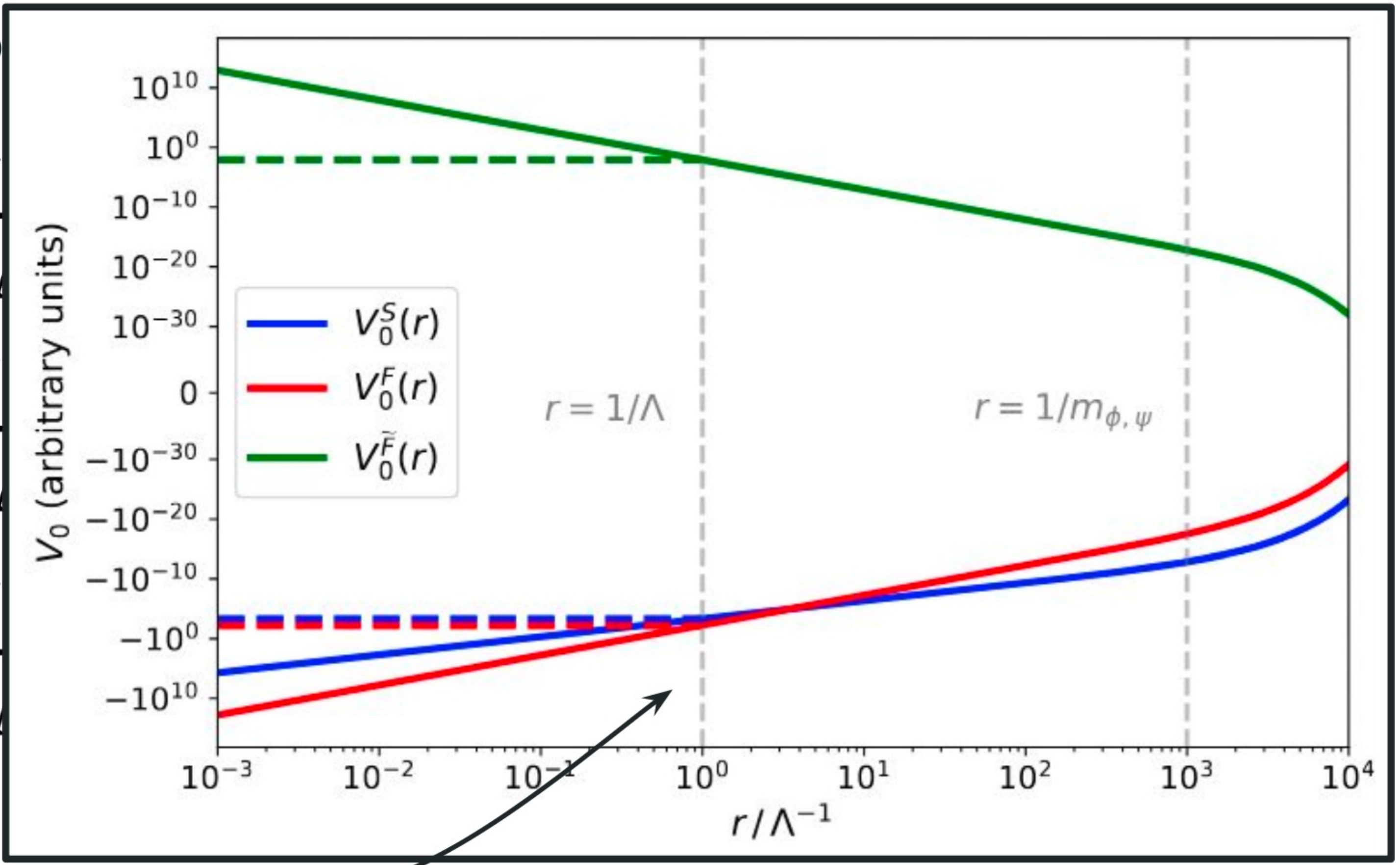
$\Big) \sim \dfrac{+m_\psi^2}{\Lambda^4}\dfrac{K_2 + m_\psi r K_1}{r^3} \overset{r \ll 1/\Lambda}{\sim} \dfrac{+1}{r^5}$

- Singular, need to regulate!
- Impose a cutoff at $r \sim 1/\Lambda$

Sommerfeld Enhancement of Vacuum Forces

- V0 for scalar, fermion, vecto





- Singular, need to regulate!
- Impose a cutoff at $r \sim 1/\Lambda$

Sommerfeld Enhancement from V_{bkg} (setting $m_\phi = 0$)

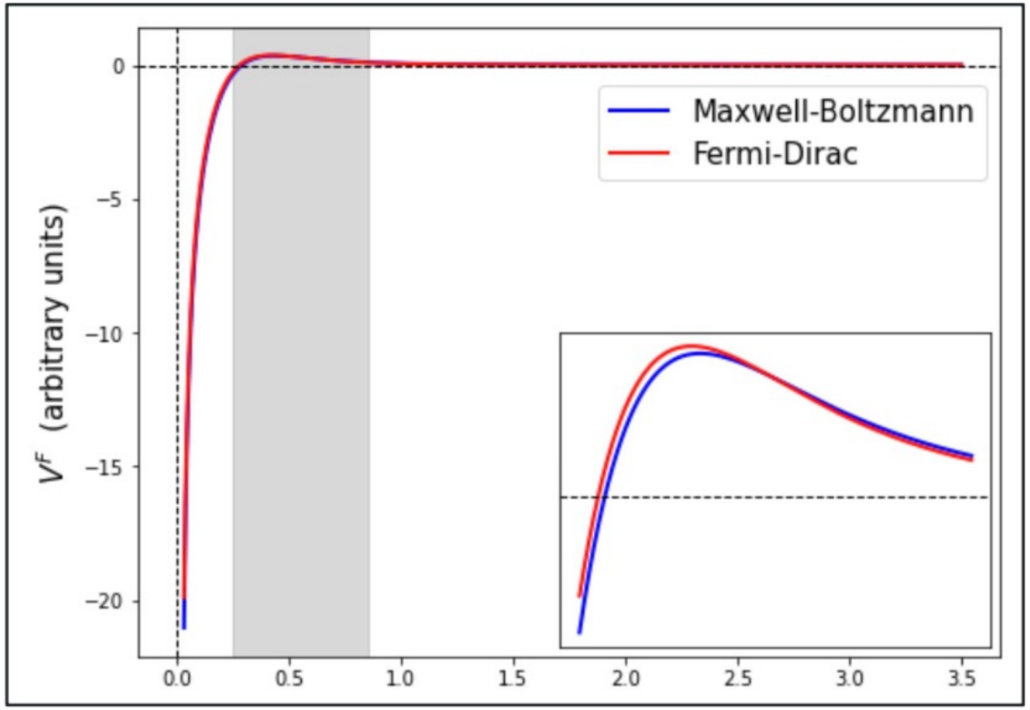
- V_{bkg} for a Maxwell-Boltzmann background:

$$V^S_{\text{bkg}} \sim \frac{-1}{\Lambda^2 r}\frac{T^2}{4r^2T^2+1} \sim \begin{cases} -1/r \,, & r \ll T^{-1} \\ -1/r^3 \,, & r \gg T^{-1} \end{cases}$$

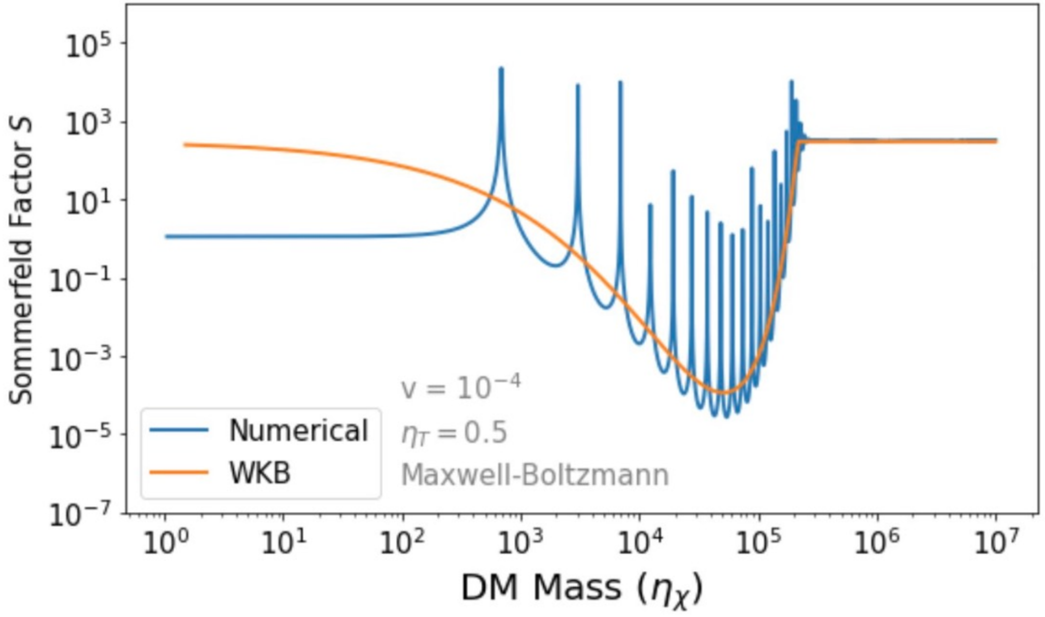
$$V^F_{\text{bkg}} \sim \frac{-T^4}{\Lambda^4 r}\frac{1-12r^2T^2}{(1+4r^2T^2)^3} \sim \begin{cases} -1/r \,, & r \ll T^{-1} \\ +1/r^5 \,, & r \gg T^{-1} \end{cases}$$

- No need to regulate!
- Temperature acts as regulator

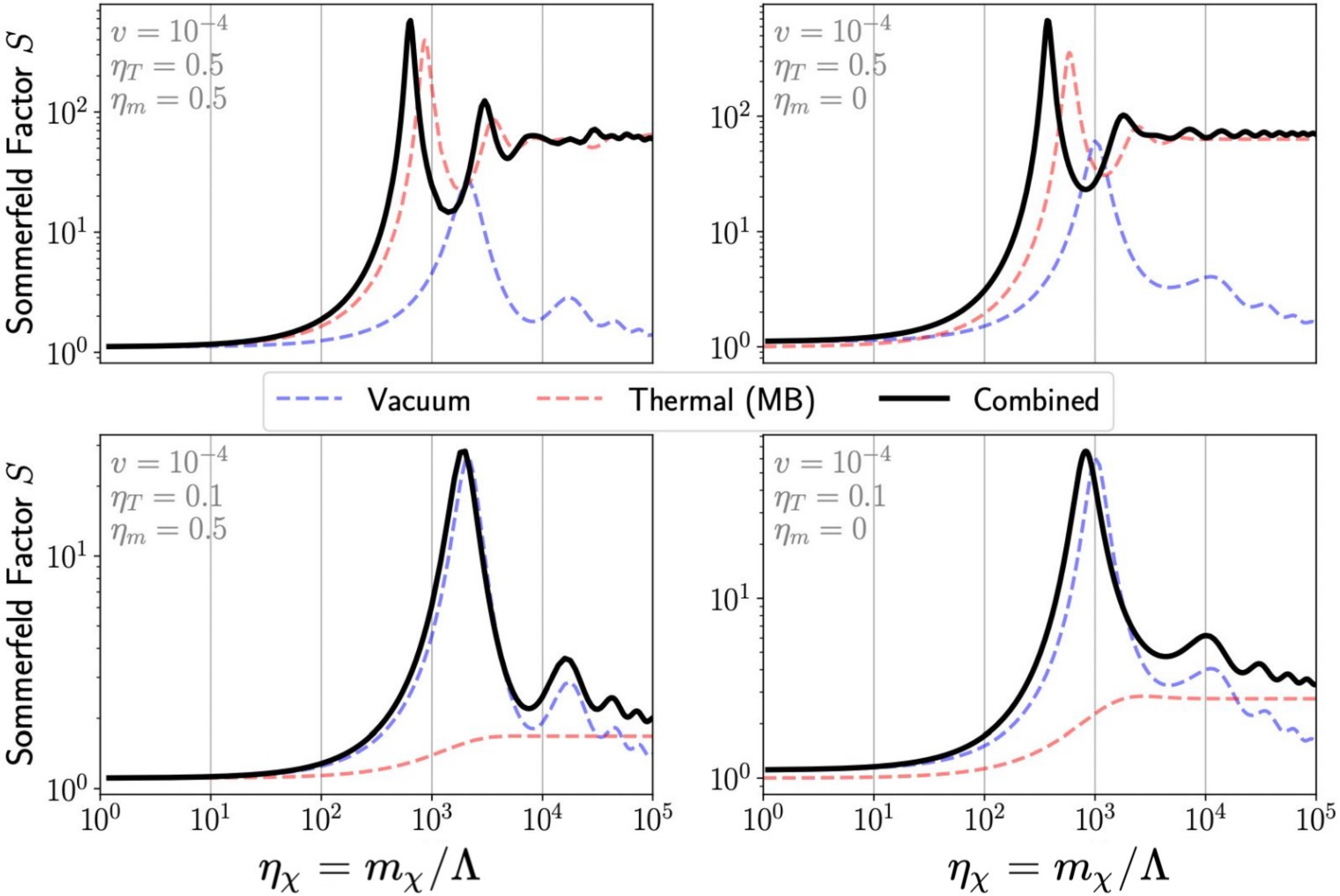
SE from fermionic background force



$$V^F_{\text{bkg}} \sim \frac{-T^4}{\Lambda^4 r} \frac{1-12r^2T^2}{(1+4r^2T^2)^3}$$



Comparison between V_0 and V_{bkg} for SE (scalar mediator)

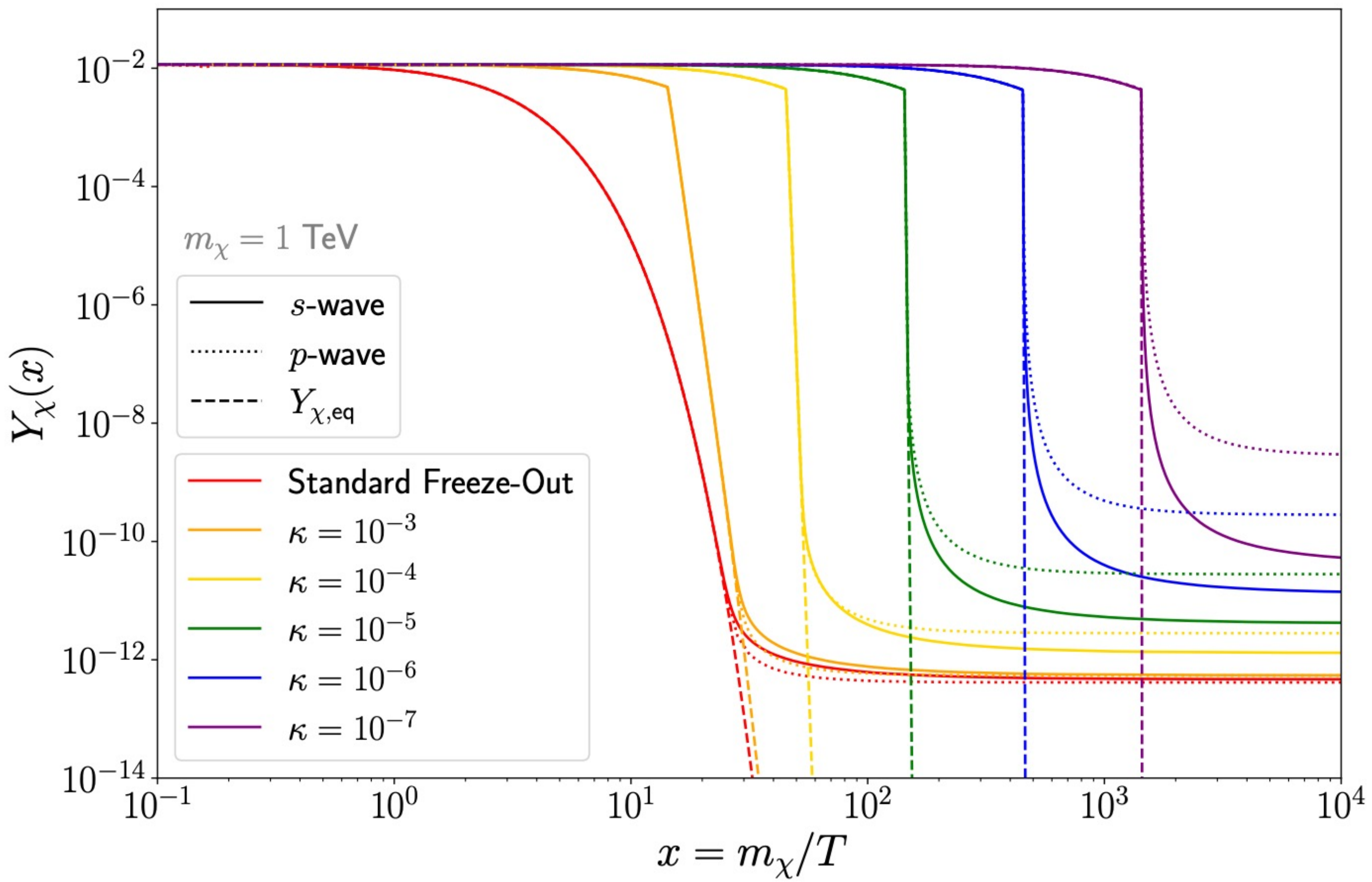


$$\eta_T = \frac{T}{\Lambda} = 0.5$$

$$\eta_T = \frac{T}{\Lambda} = 0.1$$

Coherent freeze-out

“WIMP Meets ALP”



$$\mathcal{L}_{\text{eff}} \supset \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

Ferrante, Perelstein & **BY**, [2511.16731]