Thermal Relic of Self-Interacting Dark Matter in Decoupled Sector

Bin Zhu

Department of Physics, Chung-Ang University University, 2103.06050 with Murat Abdughani

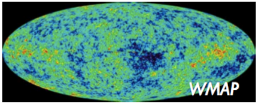
May 17, 2021

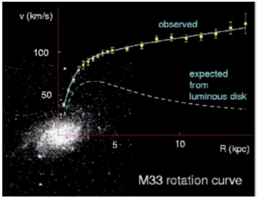
Outline

Why Light Mediator?

Cold Collisionless Dark Matter Paradigm

- 27% of the Universe is dark matter (DM)
- All evidence for DM comes through its gravitational effect in the Universe
- Large scale structure of the Universe well described by cold, collisionless DM – i.e. typical WIMP-Neutralino



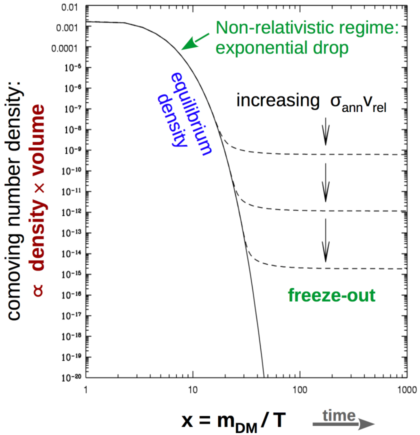


Relic density of DM with contact interactions

- Early unvierse: Hot theraml bath of elementary particles: $n_\chi = n_\chi(T)$ kept in chemical equilibrium via annihilation, $\chi + \bar{\chi} \to f + \bar{f}$
- As universe expands and cools, annihilations become inefficient. Exponential decrease of $n_\chi(T) \to$ freeze-out yields relic density

$$\Omega_\chi = 0.26 \times \left(\frac{3 \times 10^{-26} cm^3/s}{\langle \sigma v \rangle} \right)$$

velocity independent cross section



Small Scale Anomalies: Self-interacting DM

failure of CCDM-only simulations

- Self-interacting DM can solve all problems
  - Core vs cusp: energy transfer from outer to inner halo
  - Diversity: different rotation curve
  - Too big to fail: rotation curves modified by self-interactions
- The required scattering cross section is huge

$$\frac{\sigma}{m_\chi} \sim 1\text{cm}^2/g$$

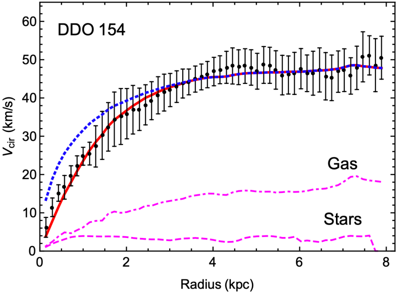
Short-range DM Tension-1

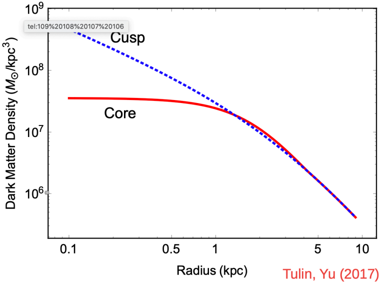
Self-interacting DM usually suggests a ultra-light force mediator to boost dark matter self-scattering → Long-range interaction

Supplementary Materials of Small Scale Anomalies

Core-Cusp Problem

This is the seemingly mass deficit observed in objects such as dwarf galaxies when compared to the predictions of collisionless dark matter. Moore (1994)



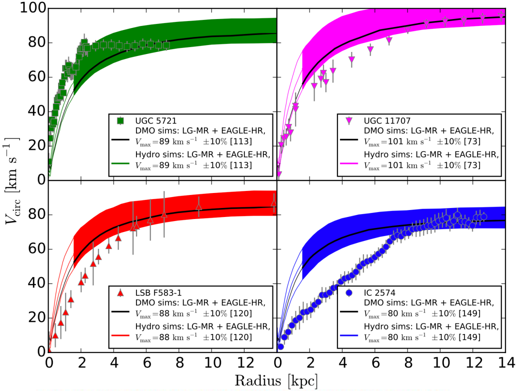


Tulin, Yu (2017)

Supplementary Materials of Small Scale Anomalies

Diversity Problem

Cosmological structure formation is predicted to be a self-similar process with a remarkably little scatter in density profiles for halos of a given mass. However, disk galaxies with the same maximal circular velocity exhibit a much larger scatter in their interiors and inferred core densities vary by a factor of order ten.



The unexpected diversity of dwarf galaxy rotation curves 2015

Approaches to Small Scale Anomalies

Recent ideas for models of SIDM

Forget debate on astrophysical solutions

Baryons, Supernova feedback; Tidal force;

Logic of particle solutions

Postulate dark matter interactions that become relevant at small scales, without modifying the physics at large scales.

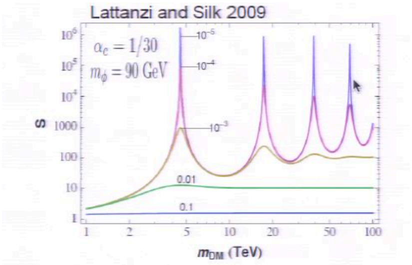
Similar processes found in nature and implication on model building

- Scattering of nucleons $\rightarrow$ pions act as light mediators
  our concern in this work
- n-p Scattering $\rightarrow$ finite size effect
  Chu, Cely, Murayama (2019)
- Scattering of alpha particles $\rightarrow HeHe \rightarrow Be \rightarrow HeHe \rightarrow$ Resonant scattering
  Chu, Cely, Murayama (2018), McGehee, Murayama, Tsai (2019)
- Fusion process $\rightarrow$ Constant cross-section
  McDermott (2017)

Cosmic Ray Anomalies: Sommerfeld Effect

failure of DM with velocity independent cross section

- WIMP such as neutralino annihilation can explain cosmic ray anomalies but large cross section 100 times over thermal relic value.
- A new force mediator acting between WIMPs enhances the cross section via sommerfeld enhancement

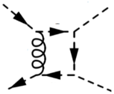


Short-range DM Tension-2

Sommerfeld enhancement requires light mediator $\rightarrow$ Long-range interaction

Effects Impacting the Relic Abundance

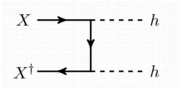
Higher order corrections



$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{NLO}} v_{\text{rel}}$

can lead to corrections of around 20% to the DM abundance

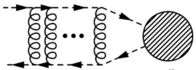
Born level annihilation



$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{tree}} v_{\text{rel}}$

usual DM codes include *only* born level calculation

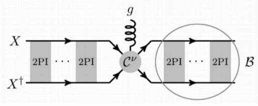
Sommerfeld enhancement



$$\left(\frac{\alpha}{v_{\text{rel}}}\right)^n \sim 1$$

$\sigma_{\text{eff}} v_{\text{rel}} = \sigma^{\text{tree}} v_{\text{rel}} \times S_0$

Bound state formation



$\langle \sigma_{\text{eff}} v_{\text{rel}} \rangle = \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle + \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle_{\text{eff}}$

bound state formation and subsequent decay open up a new effective DM annihilation channel

Julia.Harz

What is Missing?

Direct Detection

Theorem

Over the past two decades, direct detection experiments have performed better than we had any right to expect, and yet no WIMPs have appeared

Proof.

1. Co-annihilations between the dark matter and another state
2. Annihilations to W, Z and/or Higgs bosons; scattering with nuclei only through highly suppressed loop diagrams
3. Interactions which suppress elastic scattering with nuclei by powers of velocity or momentum
4. Dark matter is lighter than a few GeV
5. Dark Matter is in decoupled hidden sector

□

Hidden Sector Freeze-out

Hidden Section: Existence of Mediator

Motivation

Why do we need new mediator besides SM?

- Self-interacting dark matter
- Light dark matter (sub-GeV)
  Null results of WIMP direct detection $\rightarrow$ There is huge room for light dark matter detection $\rightarrow$ Can we go lower in DM mass? $\rightarrow$ Correct relic density requires light mediator
- Muon g-2 (If HVP in lattice is wrong)

$$\Delta a_\mu = a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = (2.51 \pm 0.59) \times 10^{-9}$$

- Proton radius puzzle (If true)
  Measuring R_p using electrons: 0.88 fm, using muons: 0.84 fm

Classification of Mediators

EFT: coupling hidden sector to SM via portals according to spin

- Higgs-singlet scalar interactions (Higgs portal)

$$H^+ H \left(\lambda S^2 + AS\right)$$

- Kinetic mixing with additional $U(1)$ group (Dark photon portal)

$$\epsilon B^{\mu\nu} F'_{\mu\nu}$$

- Neutrino Yukawa coupling (Neutrino portal)

$$y_\nu LHN$$

- Axion coupling to current (Axion Portal)

$$C_f / f_a \partial_\mu a J^\mu$$

Small portal coupling leads to effectively hidden sector

Here we focus on scalar portal as a decoupled hidden sector

Equilibrium Condition

Definition of hidden sector

What is equilibrium?

$df_i/dt = 0, \text{for any } f_i$ but universe is expanding $H \neq 0$

We have equilibrium if $\Gamma > H(t)$ in terms of grand canonical ensemble

$$f_i = \frac{1}{\exp(E_i - \mu_i)/T_i \pm 1}$$

- T_i, mean kinetic energy of species
- μ_i, characterizes particle number $n_i - n_{\bar{i}} \sim \mu_i$

As a result, we have two definitions of thermal equilibrium:

- Inelastic, usually annihilation leads to chemical equilibrium. n_f is in equilibrium
- Elastic, usually scattering leads to kinetic equilibrium. p_f similar

Freeze-out considers kinetic but not chemical equilibrium

$$f_i = \beta f_i^{\text{eq}}$$

Two approaches of Kinetic Decoupling

- Early kinetic and chemical equilibrium decoupling $\rightarrow$ Our concern in this paper $\rightarrow$ Two sectors!

$$\Gamma_\phi < H(m_\phi)$$

  χ and ϕ remains in kinetic and chemical equilibrium with each other, but not with SM.

- Late kinetic equilibrium decoupling $\rightarrow$ Future study

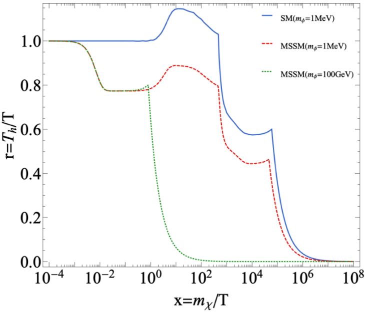
$$|\mathcal{M}|^2_{\chi\phi\leftrightarrow\chi\phi} = \frac{8g_\chi^4\beta(\omega,t)}{\left(2m_\chi\omega + m_\phi^2\right)^2\left(2m_\chi\omega - m_\phi^2 + t\right)^2}$$

Standard Freeze-out falis in both scenarios

Need to conisder one level up for n_χ in the Boltzmann hierarchy i.e. dark sector temperature $T_\chi = T_h$

Thermodynamics of a Hidden Sector

- As the universe expands, the temperatures of the two sectors remain independent of each other, and evolve according to entropy conservation
- The details of this evolution depend on the masses of the particles involved, and on whether or not chemical equilibrium is maintained
- When mediators ultimately decay, they reheat the universe, diluting the abundances of any relics



Evolution of Degrees of Freedom

Determine ratio between hidden and visible sector

- Temperature of hidden and visible sector

$$r = T_h/T, \quad r_{\text{inf}} = T^h_{\text{inf}}/T_{\text{inf}}$$

- Entropy conserves separately:

$$s = \left(2\pi^2/45\right) g_{*_s}(T)T^3$$
$$s_h = \left(2\pi^2/45\right) g^h_{*_s}\left(T_h\right) T_h^3$$
$$d\left(sa^3\right)/dt = 0$$
$$r = \left(\frac{g_{*_s}(T)}{g_{*_s,\text{inf}}}\right)^{1/3} \left(\frac{g^h_* s{,}\text{inf}}{g^h_{*_s}(T_h)}\right)^{1/3} r_{\text{inf}}$$

- If hidden sector consists of a Majorana dark matter and a complex scalar mediator

$$g_{*s,\text{ inf}} = 3.75$$

SM: $g_{*s,inf} \simeq 106.75$

MSSM: $g_{*s,\text{ inf}} \simeq 228.75$

- Evolution is determined by

$$g_{*\epsilon} = \frac{15g}{4\pi^4}\int_z^\infty \frac{u^2\sqrt{u^2-z^2}}{e^u \pm 1} du,$$
$$g_{*p} = \frac{15g}{4\pi^4}\int_z^\infty \frac{\left(u^2-z^2\right)^{3/2}}{e^u \pm 1} du,$$
$$g_{*s} = \frac{3g_{*\epsilon}+g_{*p}}{4}, z = m/T$$

Freeze-out in Hidden Sector

Assumption on hidden sector interaction strength

Start with a Majorana fermion χ and a complex scalar ϕ in hidden sector. Assume χ and ϕ have a strong interaction so that both of them lie in chemical and kinetic equilibrium. While ϕ has tiny portal coupling to SM so that hidden sector is realized.

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle_{\chi\chi\to\phi\phi} \left[n_\chi^2 - n_\phi^2 \left(\frac{n_\chi^{\rm eq}}{n_\phi^{\rm eq}} \right)^2 \right]$$

$$\dot{n}_\phi + 3Hn_\phi = \langle \sigma v \rangle_{\chi\chi\to\phi\phi} \left[n_\chi^2 - n_\phi^2 \left(\frac{n_\chi^{\rm eq}}{n_\phi^{\rm eq}} \right)^2 \right] - \Gamma_\phi \left[n_\phi - n_\phi^{\rm eq} \right]$$

- $m_\chi \approx m_\phi$ and portal coupling is small $\to$ Co-decaying DM 1607.03110
- Portal coupling is large $\to$ Secluded DM 0711.4866
- $m_\chi \geq m_\phi$ and portal coupling is large $\to$ Forbidden DM 1505.07107
- $m_\chi \gg m_\phi$ and portal coupling is small $\to$ Our concern 2103.06050

Modified Boltzmann Equation

Additional Assumption: ϕ freeze-out relativistically without Cannibal process $\phi\phi\phi \to \phi\phi$ since $m_\chi \gg m_\phi$

Thermodynamics Effect

What will change when we consider hidden sector thermodynamics seriously?

Almost massless ϕ lies in thermal equilibrium so Boltzmann equation reduces to 1

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle_{\chi\chi \to \phi\phi} \left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2\right)$$

- One is that the thermal average of cross section $\langle \sigma v \rangle$ must be performed in T_h rather than T
- Thermal equilibrium distribution is evaluated at hidden sector temperature

$$n_{\chi,\mathrm{eq}} = g_\chi \frac{\left(m_\chi^2 T_h\right)}{2\pi^2} K_2 \left[\frac{m_\chi}{T_h}\right] = g_\chi \frac{\left(m_\chi^2 rT\right)}{2\pi^2} K_2 \left[\frac{m_\chi}{rT}\right]$$

- Hubble expansion includes the contribution from hidden sector

$$H^2 = \frac{8\pi}{3m_{\mathrm{pl}}^2}(\rho + \rho_h) = \frac{8\pi}{3m_{\mathrm{pl}}^2}\frac{\pi^2}{30}\left(g_{*\epsilon}(T)T^4 + g_{*\epsilon}^h(T_h)T_h^4\right) \equiv \frac{4\pi^3}{45}\frac{m_\chi^4}{m_{\mathrm{pl}}^2}\frac{g_*^{\mathrm{eff}}(T)}{x^4}$$

Byproduct of Hidden Sector

Realization of SIDM

- In semi-classical regime $\eta = k/m_\phi > 1$, semi-analytical formula is derived in 2011.04679

$$\sigma_V^{\text{Semi-Classical}} = \frac{\pi}{m_\phi^2} \times \begin{cases} 4\beta^2 \zeta_n(\eta, 2\beta) & \beta \leq 0.1 \\ 4\beta^2 \zeta_n(\eta, 2\beta) e^{0.67(\beta - 0.1)} & 0.1 < \beta \leq 0.5 \\ 2.5 \log(\beta + 1.05) & 0.5 < \beta < 25 \\ \frac{1}{2}\left(1 + \log\beta - \frac{1}{2\log\beta}\right)^2 & \beta \geq 25 \end{cases}$$

where

$$\zeta_n(\eta, \beta) = \frac{\max(n, \beta\eta)^2 - n^2}{2\eta^2\beta^2} + \eta\left(\frac{\max(n, \beta\eta)}{\eta}\right)$$

$$\eta(x) = x^2 \left[-K_1(x)^2 + K_0(x) K_2(x)\right]$$

- In quantum regime $\eta < 0.4$, Hulthen potential approximation is

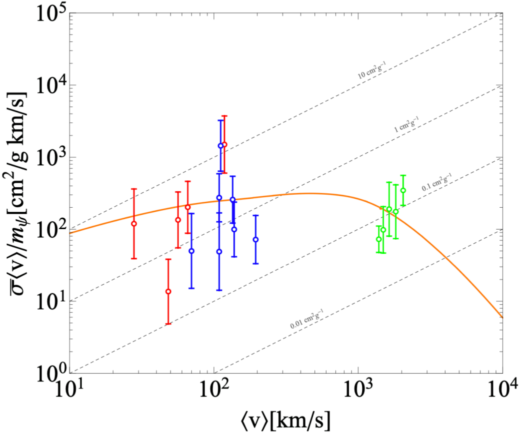
$$\sigma_{\rm V}^{\text{Hulthen}} = \frac{4\pi}{\eta^2} \frac{\sin^2\delta}{\pi}$$

- Overlap regime $0.4 < \eta < 1$

$$\sigma_V^{\text{Interpolation}} = (1-\eta)/0.6\sigma_V^{\text{Hulthen}} + (\eta - 0.4)/0.6\sigma_V^{\text{Semi-Classical}}$$

Observable: Self-interacting dark matter

It works easily



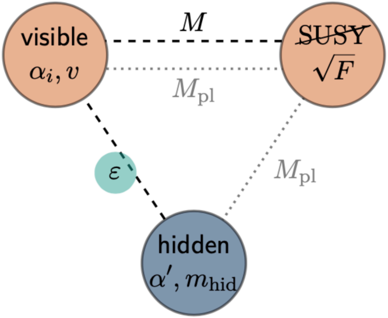
Benchmark point: $m_\chi = 200\text{GeV}, m_\phi = 5\text{MeV}, \alpha = 0.3$

Benchmark Model: General NMSSM and BSF

Why General NMSSM?

Naturally Higgs Portal DM without Fine-tuning Problem

- $\lambda H_u H_d$ can be ignored for direct detection
- Singlino + Singlet composes an effectively hidden sector
- Mass hierarchy between singlino and singlet comes from supergravity effect



NMSSM Interactions

Superpotential

Superpotential determines the supersymmetric interaction between matter superfields

$$
\begin{aligned}
W =& Y_u^{ij} \hat{U}_i \hat{Q}_j \hat{H}_u - Y_d^{ij} \hat{D}_i \hat{Q}_j \hat{H}_d - Y_e^{ij} \hat{E}_i \hat{L}_j \hat{H}_d + \mu \hat{H}_u \hat{H}_d \\
& \frac{1}{2} \mu_s \hat{S}^2 + \frac{1}{3} \kappa \hat{S}^3 + \epsilon \hat{S} \hat{H}_u \hat{H}_d
\end{aligned}
$$

Soft Terms

Soft terms provide non-supersymmetric interaction between singlets, which play a crucial role in BSF

$$
\mathcal{L}_{NMSSM} = \mathcal{L}_{MSSM} + \frac{1}{3} A_\kappa S^3 + \cdots
$$

Long-range interaction in NMSSM

Mediator Classification: CP-even and CP-odd singlet

$S = \phi + i\sigma$

Why only consider CP-even mediator ϕ long-range force carrier, since CP-odd mediator interaction is spin dependent.

Resulting Potential

$V = -\frac{\alpha e^{-m_\phi r}}{r}$ and $\alpha = \frac{\kappa^2}{4\pi}$

Real Issue? 2003.00021

CP-odd might lead to interesting singular potential which will be studied in future

Cross-Section

Tree-Level

When m_χ is much heavier than m_ϕ and m_σ, the resulting tree-level cross section is easy to identify

$$\sigma^{\rm ann}(\chi\chi \to \phi\phi)v_{\rm rel} \simeq \frac{17}{256\pi}\frac{\kappa^4}{m_\chi^2}\left(1 - \frac{22}{51}\frac{A_\kappa}{\kappa m_\chi} + \frac{1}{17}\frac{A_\kappa^2}{\kappa^2 m_\chi^2}\right)v_{\rm rel}^2$$
$$\sigma^{\rm ann}(\chi\chi \to aa)v_{\rm rel} \simeq \frac{9}{256\pi}\frac{\kappa^4}{m_\chi^2}\left(1 - \frac{14}{27}\frac{A_\kappa}{\kappa m_\chi} + \frac{1}{9}\frac{A_\kappa^2}{\kappa^2 m_\chi^2}\right)v_{\rm rel}^2$$
$$\sigma^{\rm ann}(\chi\chi \to \phi a)v_{\rm rel} \simeq \frac{9}{64\pi}\frac{\kappa^4}{m_\chi^2}\left(1 + \frac{2}{3}\frac{A_\kappa}{\kappa m_\chi} + \frac{1}{9}\frac{A_\kappa^2}{\kappa^2 m_\chi^2}\right)$$

S-wave and P-wave

Usually s-wave is constrained by CMB, but ours is hidden sector without too much energy injection

Cross-Section

Sommerfeld Enhancement

Resummation in ladder approximation

$$\left[-\frac{\nabla^2}{2\mu} + V_{\text{scatt}}(\mathbf{r})\right]\phi_{\mathbf{k}}(\mathbf{r}) = \mathcal{E}_{\mathbf{k}}\phi_{\mathbf{k}}(\mathbf{r})$$

At the limit of Hulthen potential, the sommerfeld factor can be derived analytically

$$S(v) = \frac{2\pi\alpha}{v}\frac{\sinh(2\pi\hat{p}/\delta)}{\cosh(2\pi\hat{p}/\delta) - \cos\left(2\pi\sqrt{\delta - \hat{p}^2}/\delta\right)}$$

with

$$\delta = \frac{m_*}{2\alpha\mu}, \quad \hat{p} = \frac{p}{2\alpha\mu}, \quad m_* = \left(\pi^2/6\right)m_\phi$$

Factorization not always hold $\sigma = S(v)\sigma_{\text{Tree}}$ for partial wave unitarity 1603.01383

Need to consider matching condition for regularization factor

$$\sigma v \simeq \frac{\sigma v_0 S(v)}{\left|1+\left(\eta\sqrt{\frac{\mu^2\sigma_{sc,0}}{4\pi}-\left(\frac{\mu^2\sigma v_0}{4\pi}\right)^2}-i\frac{\mu^2\sigma v_0}{4\pi}\right)(T(v)+iS(v))v\right|^2}$$

Something Forgotten?

Radiative Bound State Formation in Long Range Potential: $\chi\chi \to \phi + \mathcal{B}(\chi\chi)$

$$\left[-\frac{\nabla^2}{2\mu} + V_{\text{bound}}\,(\mathbf{r})\right]\psi_{n\ell m}(\mathbf{r}) = \mathcal{E}_{n\ell}\psi_{n\ell m}(\mathbf{r})$$

Is there additional dark matter candidate from bound state?

Wait! There is also dissociation (reverse process)! So shouldn't the bound-states get immediately destroyed, back to the scattering states?

- Ionization $\mathcal{B}(\chi\chi) + \phi \to \chi + \chi$
- Decay $\mathcal{B} \to \phi\phi, \phi\sigma$

Modified Boltzmann Equation Again

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \left[n_\chi^2 - \left(n_\chi^{\text{eq}} \right)^2 \right] - \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle n_\chi^2 + \Gamma_{\text{ion}} n_B$$
$$\frac{dn_B}{dt} + 3Hn_B = \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle n_\chi^2 - \Gamma_{\text{ion}} n_B - \Gamma_{\text{dec}} \left(n_B - n_B^{\text{eq}} \right)$$

Alternatively, it is possible to compute it to a very good approximation by solving a single Boltzmann equation, with an effective annihilation cross-section

$$\langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle_{\text{eff}} = \langle \sigma_{\text{BSF}} v_{\text{rel}} \rangle \times \left(\frac{\Gamma_{\text{dec}}}{\Gamma_{\text{dec}} + \Gamma_{\text{ion}}} \right)$$

Reduces the system to one degree of freedom again.

$$\dot{n}_\chi + 3Hn_\chi = -\langle \sigma v \rangle_{\text{eff}} \left(n_\chi^2 - n_{\chi,\text{eq}}^2 \right)$$

Including Bound State Formation

The effective bound state formation cross-section is related with ionisation and decay process

$$\langle \sigma_{\rm BSF} v_{\rm rel} \rangle_{\rm eff} = \langle \sigma_{\rm BSF} v_{\rm rel} \rangle \times \left(\frac{\Gamma_{\rm dec}}{\Gamma_{\rm dec} + \Gamma_{\rm ion}} \right)$$

- BSF

$$\sigma_{\rm BSF} v_{\rm rel} = 2 \frac{\alpha^2}{\mu^2} \left(\frac{2A_\kappa}{\mu \alpha^2} \right)^2 \sqrt{1 - \left(\frac{2m_\phi}{\left(\alpha^2 + v_{\rm rel}{}^2\right)} \right)^2} S_0^{\rm BSF}(\zeta, \xi) \left(\frac{\zeta^2}{1+\zeta^2} \right)^2 \exp(-4\zeta \,\mathrm{arccot}(\zeta))$$

It is slightly different from vector mediator. Scalar only has self-scattering A_κ contribution with $\zeta = \alpha / v_{\rm rel}$ and $\xi = \mu\alpha / 0.84 m_\phi$

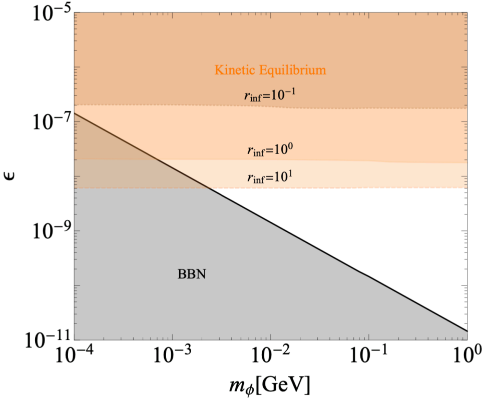
- Decay

$$\Gamma_{\rm dec} \simeq \left| \psi_{100}^{\chi\chi}(0) \right|^2 (\sigma_{\rm ann} v_{\rm rel})^{\rm tree}$$

- Ionization

$$\Gamma_{\rm ion} \simeq \langle \sigma_{\rm BSF} v_{\rm rel} \rangle \left(\frac{m_X T}{4\pi} \right)^{3/2} e^{-|E_{\rm B}|/T}$$

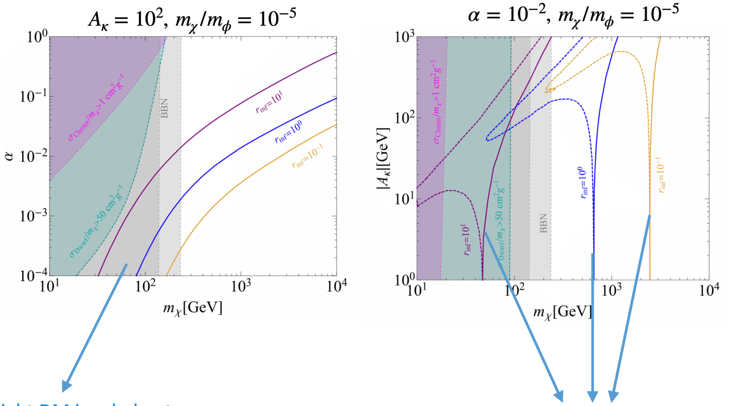
Constraints for the Mediator



Two Important Constraints

BBN and Kinetic equilibirum decoupling

Results



Light DM is ruled out.

Initial conditions affect the result.

Future Direction

Is higher partial wave important?

Discussion

- What we have done:
  Long-range effect in Hidden Sector
- What we have not done:
  - P-wave contribution
  - Late Kinetic Decoupling
  - Number violation process for mediator
  - Singular potential effect in long range potential