What three Higgs doublets can do for you

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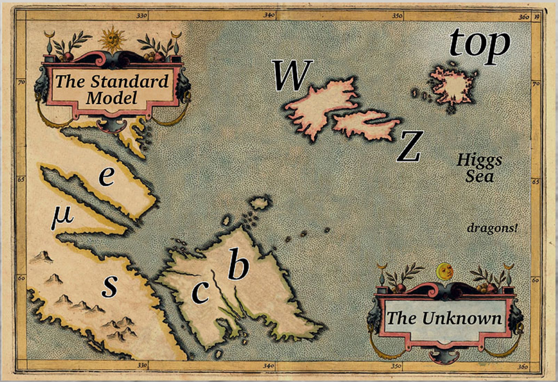
New Physics



Standard Model

Non-minimal Higgs sectors

Non-minimal Higgs sectors: a conservative approach to New Physics.

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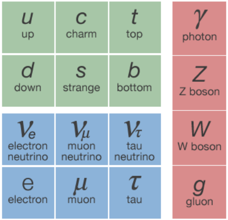

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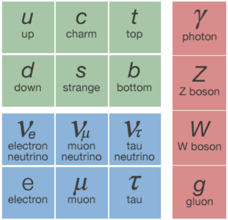
Multi-Higgs models



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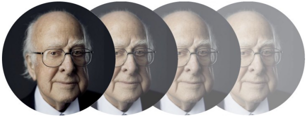


Several Higgs generations

Higgses can come in generations → N-Higgs-doublet models (NHDMs).

- T.D. Lee, 1973: 2HDM as a new source of CP-violation (CPV);
- Weinberg, 1976: 3HDM with natural flavour conservation and CPV;
- Intense activity in 70–80's: trying to reconstruct hierarchical quark and lepton masses and mixing patterns from symmetries and their breaking;
- 1990–2000's: MSSM requires two Higgs doublets;
- Around 2000's: cosmological consequences of extra scalar fields: scalar dark matter candidates protected by residual symmetries and strong first-order phase transitions → baryogenesis.
- In total, $\mathcal{O}(10^4)$ papers over 40 years.

3HDM vs 2HDM

- more scalars: $3 \times 4 - 3 = 9$ real d.o.f.; 5 neutral H_i + 2 charged $H_i^{\pm}$;
- more options for model-building (scalar and fermion) $\Rightarrow$ richer pheno;
  - many symmetry options [classic papers];
  - automatic scalar alignment from large symmetry groups;
  - new options for CP violation [Branco, Gerard, Grimus, 1984];
  - exotic CP symmetry of order 4 [Ivanov, Silva, 2015];
  - combining features of 2HDM: NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009].
- astroparticle consequnces:
  - more options for scalar dark matter and dark sectors, e.g. two inert doublets [Cordero et al, 2017];
  - new options for baryon asymmetry [Davoudiasl, Lewis, Sullivan, 2019];
  - many minima $\to$ multi-step phase transitions $\to$ GW signals.

3HDM in 2021

After decades of the 2HDM efforts, the community turns to 3HDMs:

- 2106.11977 (Boto, Romao, Silva): phenomenology of Type-Z 3HDM with $\mathbb{Z}_3$ symmetry
- 2106.06425 (Das et al): BGL-like 3HDM with naturally suppressed FCNC
- 2106.03159 (Darvishi, Masouminia, Pilaftsis): maximally symmetric 3HDMs
- 2104.13075 (Hundi, Sethi): neutrino masses from A_4 3HDM + 6 triplets
- 2104.11440 (Ivanov, Obodenko): further constraints of CP4 3HDM
- 2104.11428 (Buskin, Ivanov): stability of A_4 3HDM
- 2104.08146 (Charkaborti et al): $\mathbb{Z}_3$ 3HDM ith light charged scalars
- 2104.07047 (Cárcamo Hernández, Kovalenko, Maniatis, Schmidt): fermion masses and $g-2$ anomalies in 3HDMs
- 2103.16635 (Jurčiukonis, Lavoura): $Z\bar{b}b$ vertex in 2HDM and 3HDM
- 2103.12089 (Davoudiasl, Lewis, Sullivan): Higgs Troika model for 3HDM baryogenesis
- 2102.02800 (Gómez-Bock, Mondragón, Pérez-Martínez): S_3 3HDM

Symmetries in the 3HDM

NHDM in a nutshell

N Higgs doublets ϕ_a, $a = 1, \dots, N$, with equal quantum numbers.

- The general NHDM potential

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)\,,$$

  with $N^2(N^2+3)/2$ free parameters (14 for the 2HDM, 54 for the 3HDM).

- Quark Yukawa sector

$$-\mathcal{L}_Y = \bar{Q}_{Li}\Gamma_{ij}^{(a)}\phi_a d_{Rj} + \bar{Q}_{Li}\Delta_{ij}^{(a)}\tilde{\phi}_a u_{Rj} + h.c.$$

  Substituting vevs $\langle \phi_a^0 \rangle = v_a/\sqrt{2}$, we get

$$M_d = \frac{1}{\sqrt{2}}\sum_a \Gamma^{(a)} v_a\,, \quad M_u = \frac{1}{\sqrt{2}}\sum_a \Delta^{(a)} v_a^*\,,$$

  and eventually to m_q, V_{CKM}, and FCNCs.

Symmetries in NHDM

The NHDM potential

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

can be invariant under global symmetries:

- family symmetries: $\phi_a \to U_{ab}\phi_b$, with $U \in U(N)$,
- generalized CP symmetries (GCPs): $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, with $X \in U(N)$.

Can be extended to the Yukawa sector (many irrep options).
Symmetry group G is (partially) broken by vevs $\to$ characteristic phenomenology (scalars, DM candidates, fermion masses, mixing, sources of CPV, etc).

Symmetries in 3HDM: flavour physics connection

- Impose group G $\to$ reduce free parameters in scalar and Yukawa sectors.
- Pick up a G-breaking minimum $\to$ compute m_q and V_{CKM}.
- Possible relations among m_q and V_{CKM}?

Some history:

- Late 1970's: permutations S_3, S_4 [Pakvasa, Sugawara, 1978, 1979, + Yamanaka, 1982]; rephasing + permutations $\Delta(54)$ [Segre, Weldon, Weyers, 1979].
- Typical predictions: $m_t \sim 20$–40 GeV, no simple solution found.
- Trial and error; no systematic study existed of G + irreps + vev options.

Which symmetry groups G are possible within 3HDM?

The lessons of 2HDM

- $\mathbb{Z}_2$ symmetry: invariance under $\phi_1 \to \phi_1$, $\phi_2 \to -\phi_2$. If $\langle \phi_2^0 \rangle = 0$, we get a DM candidate $\to$ inert doublet model [Deshpande, Ma, 1978].
- Softly broken $\mathbb{Z}_2$ (Type I, Type II, etc): an evergreen bSM playground.
- Explicit CP conservation: $\phi_i(\vec{r}, t) \to \phi_i^*(-\vec{r}, t)$ forces all coefs to be real. Vevs can have a relative phase $\to$ spontaneous CP-violation [TDLee, 1973].
- Other options for the 2HDM scalar sector [Ivanov, 2005]:

$$\mathbb{Z}_2, \quad \mathbb{Z}_2 \times \mathbb{Z}_2 \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad SU(2)\,.$$

- Scalar + Yukawa: extra opportunities $\mathbb{Z}_3$ and S_3 [Kajiyama, Okada, Yagyu, 2013; Johansen, Sher 2015; Cogollo, Silva, 2015, etc].

Symmetries in 3HDM

Full classification of symmetries in the 3HDM scalar sector:

- abelian groups: [Ferreira, Silva, 1012.2874; Ivanov, Keus, Vdovin, 1112.1660]

$$\mathbb{Z}_2, \quad \mathbb{Z}_3, \quad \mathbb{Z}_4, \quad \mathbb{Z}_2 \times \mathbb{Z}_2, \quad U(1), \quad U(1) \times \mathbb{Z}_2, \quad U(1) \times U(1)\,.$$

- discrete non-abelian groups: [Ivanov, Vdovin, 1210.6553]:

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

- The classification is exhaustive: imposing any other discrete group in the 3HDM scalar sector will produce an accidental continuous symmetry.
- symmetry breaking patterns $G \to G_v$: [Ivanov, Nishi, 1410.6139]
- interplay between G and CP [many classical works].
- accidental symmetries of the potential beyond $SU(3)$: [Darvishi, Pilaftsis, 1912.00887].

CP4 3HDM

Freedom of defining CP

In QFT, CP is not uniquely defined *a priori*.

- phase factors $\phi(\vec{r},t) \xrightarrow{CP} e^{i\alpha}\phi^*(-\vec{r},t)$ [Feinberg, Weinberg, 1959],
- with N scalar fields ϕ_i, the general CP transformation is

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X \in U(N)\,.$$

  If $\mathcal{L}$ is invariant under such J with whatever fancy X, it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

- NB: The "standard" convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent!

Freedom of defining CP

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^*\,, \quad X \in U(N)\,,$$

Applying J twice leads to family transformation $J^2 = XX^*$ which may be non-trivial. It may happen than only $J^k = \mathbb{I}$ ($k =$ power of 2).

CP-symmetry does not have to be of order 2

The usual CP = CP2, the first non-trivial is CP4, then CP8, CP16, etc.

Models with higher-order GCP were known in 2HDM [Ferreira, Haber, Maniatis, Nachtmann, Silva, 2011] but they always included the usual CP.

CP4 3HDM

CP4 3HDM [Ivanov, Silva, 2015], which is physically distinct from the usual *CP* [Haber, Ogreid, Osland, Rebelo, 2018].

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)\,,$$

with all parameters real, and

$$V_1 = \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

with real λ_6 and complex $\lambda_{8,9}$. It is invariant under CP4 $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad \mathrm{CP4}^2 = \mathrm{diag}(1,\,-1,\,-1)\,, \quad \mathrm{CP4}^4 = \mathbb{I}\,.$$

Phenomenology of CP4 3HDM

If vevs conserve CP4, scalar DM candidates stabilized by CP4, with peculiar properties [Koepke, 2018; Ivanov, Laletin, 2018].

flavored CP4 3HDM:

- CP4 can be extended to the Yukawa sector, four realizations found [Aranda, Ivanov, Jimenez, 2017; Ferreira et al, 2017];
- CP4 must be spontaneously broken → peculiar patterns in the flavor sector;
- Parameter space scan of [Ferreira et al, 2017] identified many points compatible with theory constraints, EWPT, fermion masses and mixing, several meson oscillation parameters.
- However, the scan of [Ferreira et al, 2017] produced many points with $H_i^\pm$ lighter than top, leading to

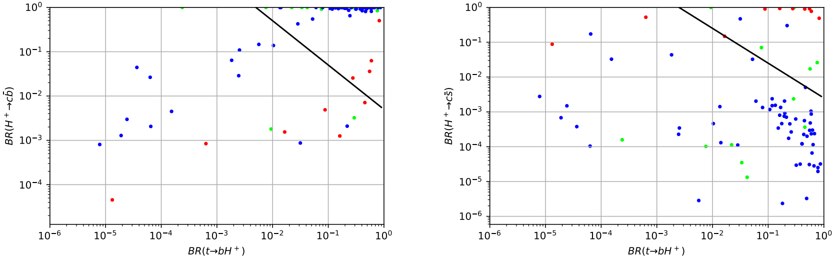
$$t \to H^+ d_i\,, \quad H^+ \to \bar{d}_i u_j\,,$$

with a variety of $H^+ d_i u_j$ coupling patterns.

Phenomenology of CP4 3HDM

In [Ivanov, Obodenko, 2021] we took all these points and checked for

- the total $\Gamma_t = 1.42^{+0.19}_{-0.15}$ GeV [PDG];
- $Br(t \to H^+ b) \times Br(H^+ \to c\bar{b}) < 0.5\%$ based on [CMS, 2018];
- $Br(t \to H^+ b) \times Br(H^+ \to c\bar{s}) < 0.25\%$ based on [CMS, 2020].



Almost all points were excluded. Exotic cases survived: $H^+ \to u\bar{b}$ as the dominant decay mode or $t \to H^+ s$ as the main production mode.

Single assumption → numerous consequences → requires further study.

3HDMs with approximate symmetries

The symmetry dilemma of the 3HDM

- The original idea from 1970's: pick up a large G, extend it to the fermion sector, observe $G \to G_v$ at the minimum $\to$ derive masses/mixing/CPV.
- Many combinations of G + irreps + vevs were tested $\to$ severe problems in the quark sector; A_4, S_4 illustrations in [Gonzales Felipe et al, 2013].
- The fundamental obstacle [Leurer, Nir, Seiberg, 1993]:

  If the (active) Higgs sector is equipped with G, vevs must break G completely in order to produce physical m_q's and CKM.
- For large G, this is algebraically impossible [Gonzales Felipe et al, 2014]
- For small G, too many free parameters $\to$ poor predictive power.

3HDMs with approximate symmetries seem to be perfecty viable candidates.

Need to learn how to work efficiently in the entire parameter space.

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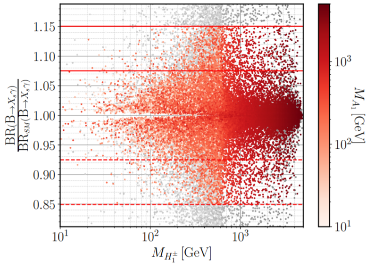
Navigating 3HDM parameter space

How would you get collider/astroparticle predictions in the general 3HDM?

- If all parameters are given, predictions can be computed and tested with publicly available tools (e.g. SARAH/SPheno, HiggsBounds/HiggsSignals, MadGraph ...)
- In specific models with several free parameters, just do numerical scan and explore predictions with scatter plots (example from [Das et al, 2106.06425])



But what to do with 54 free parameters (general 3HDM scalar sector)?
Two values for each parameter lead to $2^{54} \sim 10^{16}$ different models.

No chance to cover the full 3HDM parameter space with a random scan!

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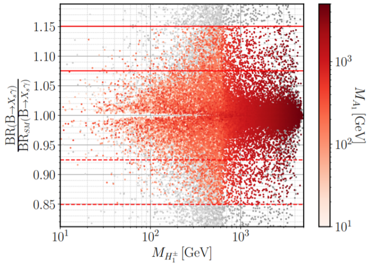

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Navigating 3HDM parameter space

If one wants to go beyond isolated examples, one must learn how to navigate in the entire parameter space. This task preceeds collider predictions!

A possible strategy

- Within the multi-dimensional parameter space, identify reference manifolds: models with clear pheno consequences (e.g. 3HDM with symmetries).
- In the vicinity of each reference manifold, identify which directions affect which observables, how deviations depend on the distance, and how correlated they are.

The goal:

- qualitative and quantitative understanding where to look for benchmark models with desired phenomenological features.

An illustration: 3HDM with softly broken $\Sigma(36)$

$\Sigma(36)$ is the largest discrete group for the 3HDM scalar sector:

$$V = -m^2(\phi_a^\dagger \phi_a) + V_4\,,$$

where V_4 contains just λ_1, λ_2, λ_3 [Ivanov, Vdovin, 2013]. The model leads to

- rigid vev alignments: $(1,0,0)$, $(1,1,1)$, etc.;
- pairwise mass degenerate Higgses with the extra relation $m_H^2 = 3m_h^2$;
- exact scalar alignment;
- no CP violation in the scalar sector;
- scalar DM candidates.

An illustration: 3HDM with softly broken $\Sigma(36)$

Soft breaking terms $m^2_{ab}(\phi^\dagger_a \phi_b)$ can violate all these features.

But are these 9 free parameters m^2_{ab} on equal footing?

5 free parameters preserve the vev alignment:

- can be found explicitly for any choice of the minimum;
- induce mass splitting of degenerate Higgses;
- control the choice of local vs. global minimum;
- lead to loop-induced decays of previously stable scalars;
- do not spoil the scalar alignment.

The remaining 4 free parameters shift the vev alignment:

- break the exact scalar alignment;
- can be parametrized via $H_i WW$ couplings of 4 extra neutral Higgses H_i.

We know how to build a 3HDM with softly broken $\Sigma(36)$ with a desired phenomenological pattern [Ivanov, Levy, Varzielas], work in progress.

An illustration: 3HDM with softly broken $\Sigma(36)$

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Basis-invariant methods

Extra challenge: large freedom of basis changes: $\phi_a \mapsto U_{ab}\phi_b$, $U \in U(N)$.

Physics does not change upon basis changes!

Two models can look different but lead to the same physics $\to$ challenge!

A symmetry can be evident in one basis and hidden in another $\to$ challenge!

Basis-independent methods

Detecting structural properties of the NHDMs scalar sector in any basis.

General recipe [Botella, Silva, 1995]:

- write down all couplings as tensors under basis changes,
- take products, contract all indices $\to$ basis invariants J_k,
- find algebraically independent J_k and link them to observables.

Explicit CP conservation in 2HDM

Checking explicit CP-conservation [Davidson, Haber, 2005; Gunion, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005]:

- $V = Y_{ab}(\phi_a^\dagger \phi_b) + Z_{ab,cd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$.
- There exists of a basis with all coefs real $\to$ symmetry $\phi_a \mapsto \phi_a^*$.
- Basis-independent criterion: check the following four invariants

$$\mathrm{Im}(Z^{(1)}_{ac} Z^{(1)}_{eb} Z_{be,cd} Y_{da}) = 0\,, \qquad \mathrm{Im}(Y_{ab} Y_{cd} Z_{ba,df} Z^{(1)}_{fc}) = 0\,,$$
$$\mathrm{Im}(Z_{ab,cd} Z^{(1)}_{bf} Z^{(1)}_{dh} Z_{fa,jk} Z_{kj,mn} Z_{nm,hc}) = 0\,,$$
$$\mathrm{Im}(Z_{ac,bd} Z_{ce,dg} Z_{eh,fq} Y_{ga} Y_{hb} Y_{qf}) = 0\,, \quad \text{where} \quad Z^{(1)}_{ac} \equiv Z_{ab,bc}\,.$$

A whole machinery of basis-invariant approach to the 2HDM pheno based on such invariants (+ new ones with v_a): [Haber + collaborators, 2005, 2006, 2011, 2020].

Explicit *CP* conservation via basis invariants

Drawbacks:

- non-intuitive, relied heavily on computer algebra (see however [Trautner, 1812.02614]);
- exceedingly complicated beyond 2HDM; conditions for *CP* symmetry in 3HDM via basis invariants still not found [Varzielas et al, 1603.06942].
- not all information can be easily retrieved! *CP*-odd basis invariants cannot distinguish the usual *CP* from CP4 (order-4 *CP* symmetry).

Beyond basis invariants

one can recover basis-invariant statements from basis-covariant objects.

- 2HDM: geometric constructions in the bilinear space → answers to all symmetry related questions [Nachtmann et al, 2004–2007; Ivanov, 2006–2007; Nishi, 2006–2008].

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Navigating the entire 3HDM parameter space

In [Ivanov, Varzielas, 1903.11110], the idea was applied to all symmetries of the 3HDM scalar sector.

> Presence of any symmetry in the 3HDM scalar sector can now be algorithmically detected in any basis.

A secure way to navigate in the full parameter space of the 3HDM scalar sector.

Next steps:

- Implement the algorithms in a working computer code;
- Define and quantify distance of any 3HDM to the nearest symmetric manifold.

Conclusions

- In terms of phenomenological signals, 3HDMs can offer much more than 2HDMs. But they were mostly explored for isolated 3HDM examples.
- Navigating the huge parameter space of the general 3HDM is challenging and requires methods beyond straightforward numerical scans.
- We now have
  - the complete classification of 3HDM symmetries and their breaking;
  - basis-invariant methods to detect symmetric situations.

  They open up a way to systematic exploration of phenomenologically distinct situations of the general 3HDM up to collider and astroparticle predictions.

This will be my project at SYSU, Zhuhai, in the next few years.

We will be glad to collaborate with anyone interested.

Extra slides

Classifying discrete symmetry groups in the 3HDM

"Abelian LEGO" strategy for discrete symmetries:

- Derive all allowed abelian Higgs-family groups A via Smith normal form technique [Ivanov, Keus, Vdovin, 1112.1660]:

$$\mathbb{Z}_2\,, \quad \mathbb{Z}_3\,, \quad \mathbb{Z}_4\,, \quad \mathbb{Z}_2\times\mathbb{Z}_2\,, \quad \mathbb{Z}_3\times\mathbb{Z}_3\,.$$

- Non-abelian group G can contain only these abelian subgroups $\to$ $|G| = 2^p 3^q$ $\to$ using Burnside's theorem and applying some finite group theory, we obtain the key result:

$$G = A \rtimes K\,, \quad K \subseteq Aut(A)\,.$$

- Checking all A's, we get all possible G's [Ivanov, Vdovin, 1210.6553]:

$$S_3, \quad D_4, \quad A_4, \quad S_4, \quad \Delta(54), \quad \Sigma(36)\,.$$

Symmetry breaking in 3HDM

Strongest and weakest breaking of discrete symmetries in 3HDM and spontaneous CPV [Ivanov, Nishi, 1410.6139].

group	$\|G\|$	$\|G_v\|_{min}$	$\|G_v\|_{max}$	sCPV possible?
abelian	2, 3, 4, 8	1	$\|G\|$	yes
$\mathbb{Z}_3 \rtimes CP$	6	1	6	yes
S_3	6	1	6	—
$\mathbb{Z}_4 \rtimes CP$	8	2	8	no
$S_3 \times CP$	12	2	12	yes
$D_4 \times CP$	16	2	16	no
$A_4 \rtimes CP$	24	4	8	no
$S_4 \times CP$	48	6	16	no
$\Delta(54)$	18	6	6	—
$\Delta(54) \rtimes CP$	36	6	12	yes
$\Sigma(36) \times CP$	72	12	12	no