1. 谐振子:

a) 理论: 生成函数 $Z=\int[Dx(t)]\,e^{-S[x(t)]/\hbar}$ ← $\langle x_b t_b | x_a t_a\rangle$

算符矩阵元: $\langle O\rangle = \frac{1}{Z}\int[Dx(t)]\,O(x)\,e^{-\frac{S[x]}{\hbar}}$

$\langle x^2\rangle = \frac{1}{Z}\int[Dx(t)]\,x^2\,e^{-\frac{S[x]}{\hbar}}$

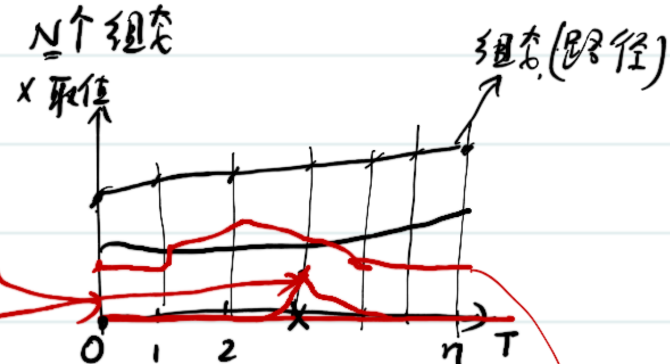
$dx(t=0)\quad dx(t=a)\quad dx(t=2a)\quad \cdots\, d(x=iva)$

方案

b) 计算: configuration: $P(x)=\frac{1}{Z}e^{-\frac{S[x]}{\hbar}}$

N个点 $\langle O\rangle=\frac{1}{N}\sum_{i=1}^{N}O(x^{(i)})$

a) 生成组态: $P(x)=\frac{1}{Z}e^{-\frac{S(x)}{\hbar}}$ N个组态



Metropolis Algorithm:

1. $x(t=0,1,2,\cdots n)=0$
2. $x'(t=i)=x(t=i)+\delta x * r$

   $x(t=3)$
3. $S[x]\to S[x']$

   如果 $S[x']<S[x]$ ✓ ok

   如果 $S[x']>S[x]$, $e^{-(S[x']-S[x])}>0.5$ ✓ ok (0.5: 随机)

   否则 $S[x]$.
4. repeat 2, 3 ⇒ (0 n) 第2条
5. repeat 2. 3. 4 N次

b) 计算 $\langle O\rangle$, $\langle x^2\rangle$ $\langle x(t=3a)\,x(t=0a)\rangle$

1+1 QED gauge field theory

$E^i \cdot B^i \to \underline{\underline{A_\mu}}$

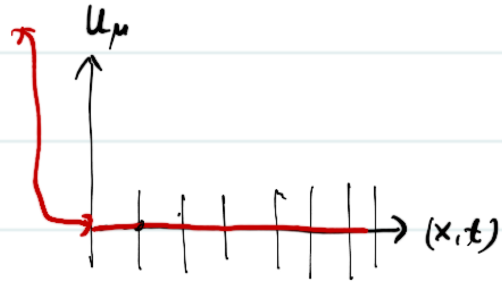
gauge field configuration:

谐振子 x	规范场 $A_\mu \to U_\mu$
t	(x,t)
$\underline{\underline{x}}$	$A_\mu(x,t)$
$S(x)$	$S(A_\mu)$

Step 1: $\underline{U_\mu(x,t)} = \underline{1+0i}$ $(\mu = 0, 1)$

Step 2: $U'_\mu(x,t) = X\, U_\mu(x,t)$

随机选一个(x,t) $X = e^{i\phi}$ $\underline{\phi \in [-\pi, \pi]}$



Step 3: $S[u'] < S[u]$ ✓OK

$S[u'] > S[u]$. $e^{-(S[u'] - S[u])}$ >0.5 ✓OK

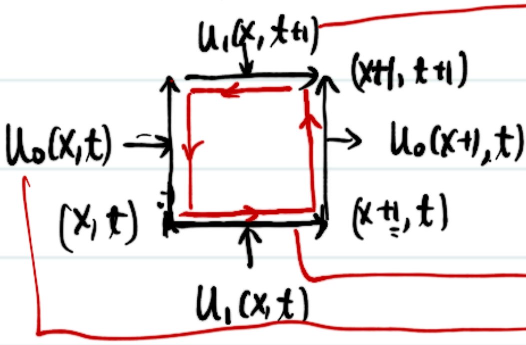
<0.5 $S[U_\mu]$ ✓

Step 4: repeat 2、3 第2条路径:⇒组态

step 5: repeat 2、3、4 $N = 100$ 个组态

作用量:

$\underline{U_\mu(x,t)} \approx \underline{A_\mu(x,t)\, a}$



$\underline{p(x,t)} = U_1(x,t)\, U_0(x+1,t)\, [U_1(x,t+1)]^\dagger \times [U_0(x,t)]^\dagger$

$$S = -\frac{1}{\beta^2}\sum_{(x,t)} P(x,t) \quad \checkmark$$

$a=0.1 \quad n_t = 100, \quad n_x = 50, \quad \beta = 1$

$$\langle O \rangle \Rightarrow \langle P(x,t) \rangle \qquad\qquad \langle x^2(t) \rangle = \sum_{i=1}^{N} \frac{x^2(t=ia)}{N}$$

$$U_0(x+1,t) \to U_0((x+1)\%n_x,\ t)$$

① 生成组态: ✓ ✓

② [illegible] (x,t) ✓ ✓

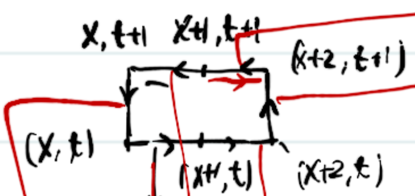
③

$\otimes$ t_+

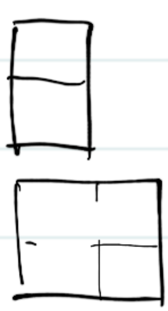
t

$$e^{-tV(r)}$$

$$\langle W_{1\times2}(x,t) \rangle = \frac{1}{N_x \times N_t}\sum_{x,t} W_{1\times2}(x,t)$$



$$W_{1\times2}(x,t) = U_1(x,t)\, U_1(x+1,t)\, U_0(x+2,t) \times [U_1(x+1,t+1)]^\dagger \times [U_1(x,t+1)]^\dagger \times [U_0(x,t)]^\dagger$$



$\underline{x=1}$.  $\Rightarrow e^{-t\,V(x=1a)} \Rightarrow V(x=1a)$

$\underline{x=2}$.  $\Rightarrow \quad \Rightarrow e^{-t\,V(x=2a)} \Rightarrow V(x=2a)$

$\vdots$

$V(x= \quad)$?

$$\boxed{S = -\frac{1}{\beta^2} \sum_{x,t} \mathrm{Re}[p(x,t)]}$$

生成组态:

Step 1. $U_\mu(x,t) = 1+0i$ default

step 2. $U'_\mu(x,t) = X U_\mu$

$X = e^{i\phi}$ $\phi \in [-\pi,\pi]*0.7$

step 3: $S[u'] < S[u]$ 选择 u'

$S[u'] > S[u]$: $e^{-(S[u']-S[u])}$ >0.5 选择 u'

<0.5 选择 u

Step 4 重复 2.3 直到(x,t). 第2个组态

Step 5 重复 2.3.4 N次 N个组态

作用量 $S = -\frac{1}{\beta^2}\sum_{x,t} P(x,t)$

$P(x,t) = U_1(x,t)\, U_0(x+1,t)\, [U_1(x,t+1)]^\dagger [U_0(x,t)]^\dagger$

作用量：1. $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ 是光子/电磁场拉氏量.

2. $S = \int \mathcal{L} d^4x \rightarrow -\frac{1}{\beta^2} \sum_{x,t} p(x,t)$.

1. $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ $\quad \mu, \nu = 0, 1, 2, 3$

$A^\mu = (\phi, \vec{A}^i)$

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad a_\mu = g_{\mu\nu} a^\mu$

运动方程：Maxwell 方程组.

最小作用量： $S = \int d^4x \, \mathcal{L}$

$\delta S = 0$

Euler-Lagrange $\rightarrow$ $\frac{\partial \mathcal{L}}{\partial A_\nu} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} = 0 \leftarrow \frac{\partial \mathcal{L}}{\partial x} - \partial_t \frac{\partial \mathcal{L}}{\partial(\partial_t x)} = 0$

$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$

$\frac{\partial \mathcal{L}}{\partial A_\beta} = 0$

$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\beta)} = -\frac{1}{4} \frac{\partial(F_{\mu\nu} F^{\mu\nu})}{\partial(\partial_\alpha A_\beta)} = -F^{\alpha\beta}$

$\frac{\partial F_{\mu\nu}}{\partial(\partial_\alpha A_\beta)} = \frac{\partial(\partial_\mu A_\nu - \partial_\nu A_\mu)}{\partial(\partial_\alpha A_\beta)} = g^\alpha_\mu g^\beta_\nu - g^\alpha_\nu g^\beta_\mu$

$\boxed{\partial_\alpha F^{\alpha\beta} = 0} \rightarrow$ 2个 Maxwell 方程

$E^i = -F^{0i} = -(\partial^0 A^i - \partial^i A^0) \qquad \rightarrow \partial^i A^0 = \partial^i \phi$

$\vec{B} = \nabla \times \vec{A} \rightarrow B^z = \partial^x A^y - \partial^y A^x . \quad \rightarrow B^k = \frac{1}{2} \varepsilon^{ijk} F^{ik}$

$\rightarrow F^{ij} = -\varepsilon^{ijk} B^k$

① $\partial_\alpha F^{\alpha\beta} = 0$ 取 $\beta = 0$ $\quad \partial_\alpha \underline{F^{\alpha 0}} = 0 \rightarrow \underline{\partial_i E^i = 0} \rightarrow \nabla\cdot\vec{E} = 0$ 。

② $\beta \neq 0 \; \beta = i$, $\partial_0 F^{0i} + \partial_j F^{ji} = 0$

$$\partial^0 F^{0i} - \partial^j F^{ji} = 0$$

$$\rightarrow -\partial^0 E^i + \partial^j \varepsilon^{ijk} B^k = 0 \longrightarrow \partial^0 E^i + \varepsilon^{ijk}\partial^j B^k = 0$$

$$\nabla\times\vec{B} = -\frac{\partial\vec{E}}{\partial t}$$

$$\varepsilon^{\mu\nu\alpha\beta}\partial_\nu F_{\alpha\beta} = 0 \qquad \vec{B} = \nabla\times\vec{A} \rightarrow \nabla\cdot\vec{B} = 0$$

2. $S = \int d^4x \, \mathcal{L} \longrightarrow S = -\frac{1}{\beta^2}\sum_{x,t} \mathrm{Re}[P(x,t)]$

A_μ

$$\underline{U_\mu(x,t\rightarrow x+1,t) = \exp\left[i\int A \, dx_\mu\right]} = \underline{\exp[ia A_\mu(x,t)]} \quad a^2$$

$A_\mu(x+\frac{a}{2},t)\cdot a$

$\boxed{e^\alpha e^\beta = e^{\alpha+\beta}}$

$x+1$

$$P(x,t) = U_1(x,t)\, U_0(x+1,t)\, [U_1(x,t+1)]^\dagger [U_0(x,t)]^\dagger$$

$$= \exp[ia A_1(x,t) + ia A_0(x+1,t) - ia A_1(x,t+1) - ia A_0(x,t)].$$

$$= \exp(ia A_1(x,t) + ia A_0(x,t) + ia\cdot a\cdot \partial_1 A_0(x,t) - ia A_1(x,t) - ia a \partial_0 A_1(x,t)$$

$$- ia A_0(x,t)]$$

$a \quad a^2$

$$= \exp[ia^2(\partial_1 A_0 - \partial_0 A_1)] \qquad \boxed{F_{10} = \partial_1 A_0 - \partial_0 A_1}$$

$$= \exp[ia^2 F_{\mu\nu}]$$

$$= 1 + ia^2 F_{\mu\nu} + (ia^2 F_{\mu\nu})^2 + \cdots$$

$$\mathrm{Re}[P(x,t)] = 1 - a^4 (F_{\mu\nu} F_{\mu\nu})$$

$$\underline{S = \int \mathcal{L}\, d^4x}$$

无任何影响

$$S_L = \underline{-\frac{1}{\beta^2}\sum_{x,t}\mathrm{Re}[P(x,t)]} = -\frac{1}{\beta^2}\sum_{x,t}\left[1 - a^4(F_{\mu\nu}F^{\mu\nu})\right]$$

$$= \frac{a^4}{\beta^2} \sum_{x,t} F_{\mu\nu} F_{\mu\nu}$$

β

e^{iS}

e^{-S}

$$S = \int \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) d^4x = \sum \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) a^4$$

$$\boxed{U = P\exp\left[ig \int ds\, n \cdot A\right]}$$

$n = n_\mu$ U_μ A_μ