# Split Invariant Curves In Rotating Bar Potentials


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## Abstract


Invariant curves are generally closed curves in the Poincaré surface of section. Here we study an interesting dynamical phenomenon, first discovered by Binney (1985) in a rotating Kepler potential, where an invariant curve of the surface of section can split into two disconnected line segments under certain conditions, which is distinctively different from the islands of resonant orbits. We first demonstrate the existence of split invariant curves in the Freeman bar model, where all orbits can be described analytically. We find that the split phenomenon occurs when orbits are nearly tangent to the minor/major axis of the bar potential. Moreover, the split phenomenon seems "necessary" to avoid invariant curves intersecting with each other. Such a phenomenon appears only in rotating potentials, and we demonstrate its universal existence in other general rotating bar potentials. It also implies that actions are no longer proportional to the area bounded by an invariant curve if the split occurs, but they can still be computed by other means.


## Introduction

The surface of section (SoS) was first introduced by Poincaré (1892), which is the cross section of the 4D phase space cut at $x = 0$ with $\dot{x} > 0$. The morphology of the surface of section is affected by integrals of motion, which are any function of the phase space coordinates and constant along an orbit. Regular orbits of the same energy form invariant curves in the surface of section because of the second/third integral of motion in addition to the energy and the angular momentum. Irregular (chaotic) orbits, which have no additional integrals of motion, distribute as discrete points within the region bounded by the zero velocity curve (Binney & Tremaine 2008).

Invariant curves are generally closed curves surrounding a periodic parent orbi. However, there exist some invariant curves which split into two disconnected line segments under certain conditions. Binney (1985) first noticed this phenomenon for a rotating Kepler potential when they demonstrated that the phase-space volume is not necessarily proportional to the area within an invariant curve unless the recurrence time is fixed. While the action cannot be calculated simply by the area of an invariant curve when the split occurs, one can still turn to line integrals to complete the calculations.

We study the properties of split invariant curves of the planar Freeman bar potential. The Freeman bar model was proposed by Freeman (1966). and all the orbits in this model have analytical expressions. It is simple yet illustrative, thus it is often used to draw general conclusions of rotating bar potential. The model differs from other more realistic bar models mainly on two aspects: Firstly, it does not have a co-rotation radius; Secondly, it contains only regular orbits, and has no chaotic orbits.

## Methodology

Each orbit in a Freeman bar model is the superposition of two elliptic motions, which is named $\alpha$-motion and $\beta$-motion respectively. $X_\alpha$ and $X_\beta$ in the following plot characterize their sizes.

The SoS of the Freeman bar potential is shown in Figure 1. The cyan curve is the split invariant curve. We also plot its corresponding orbit.

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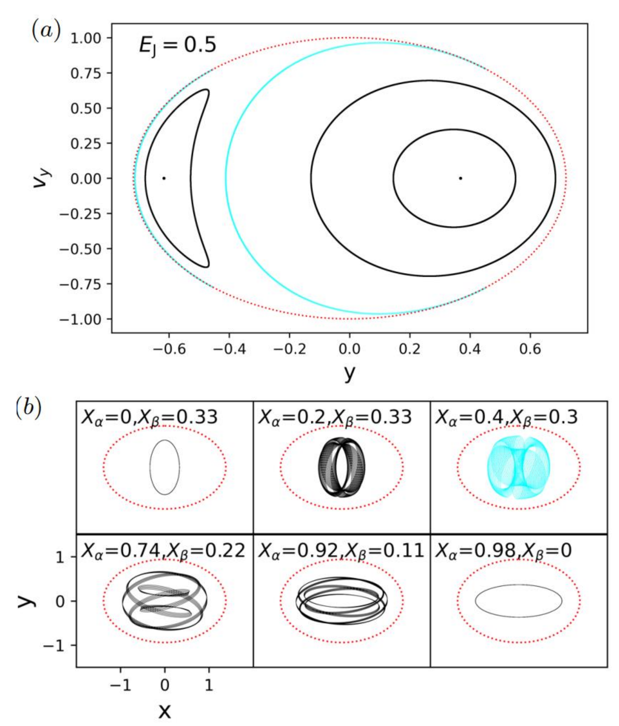


Figure 1

Note that a SoS records the value of $(y, \dot{y})$ when an orbit crosses the bar's minor($y$) axis when $\dot{x} < 0$. The split only occurs when the orbit is nearly tangent to the $y$-axis, which is shown in Figure 2. While Figure 2a, b. d show properties of unsplit invariant curves, Figure 2c shows how a split invariant curve occurs under certain conditions.

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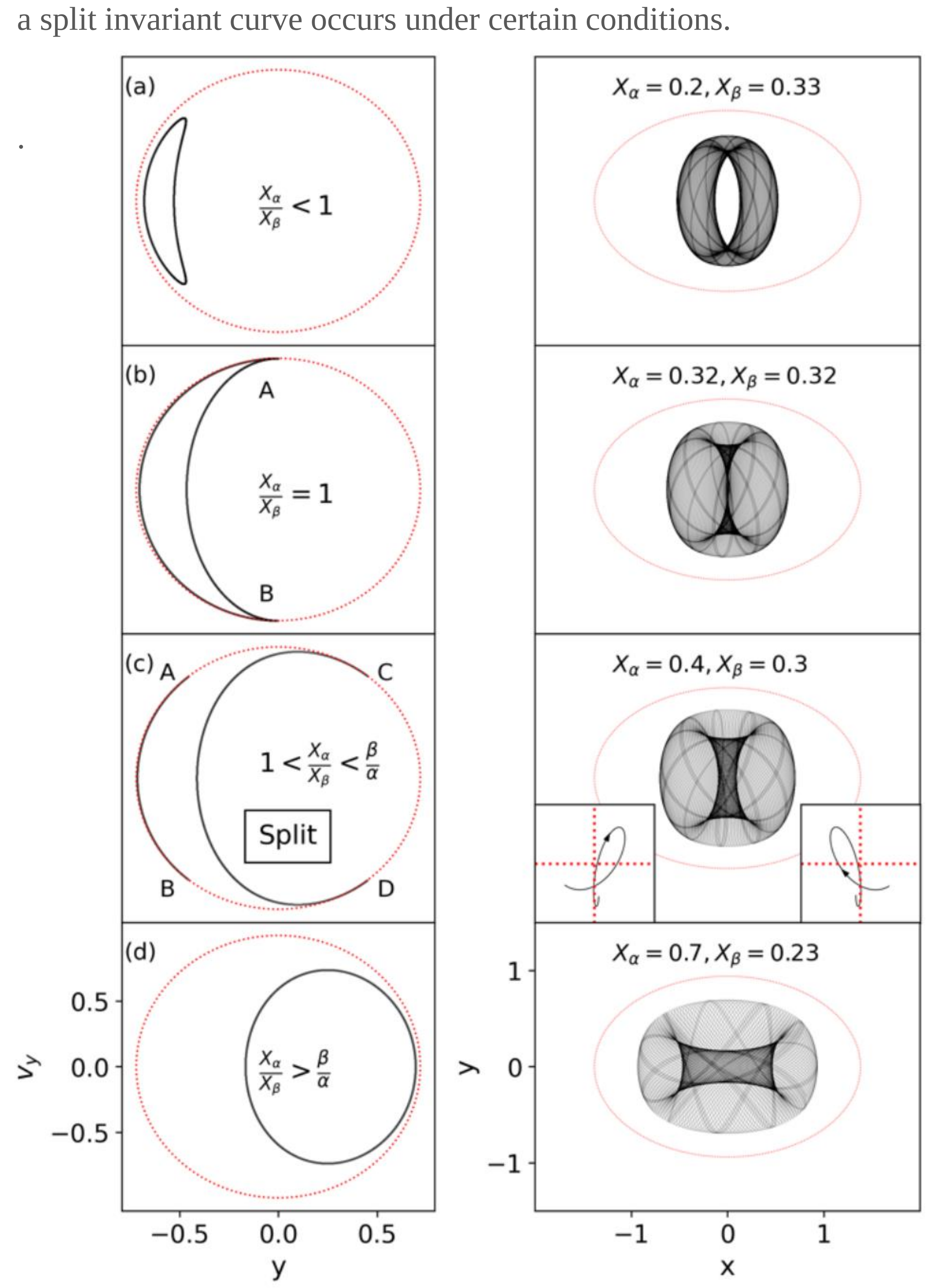


Figure 2

## Results

We notice that the condition of split invariant curves is related to the pattern speed of the potential. The split phenomenon is more pronounced in more rapidly rotating systems . We also find that no split invariant curve exists in a non-rotating potential, but there are always split invariant curves in rotating ones.

If the left part of the split invariant curve is mirror-symmetrized to the right side with respect to the $y=0$ line, the mirror-symmetrized curve forms a closed curve with the right part of the invariant curve (the cyan curve in Figure 3). If the invariant curve were not split, the closed curve would have intersected with another invariant curve (the red curve in Figure 3), which ought not to happen. In other words, the split phenomenon is "necessary" to avoid invariant curves intersecting with each another.

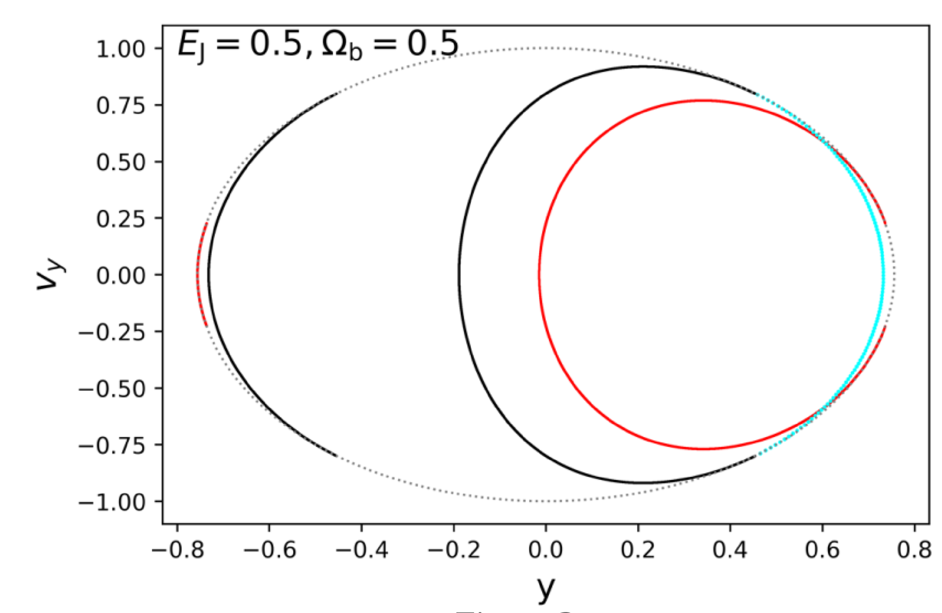


Figure 3

The Freeman bar potential is quadratic, thus one may wonder if split invariant curves occur only in such an idealized potential. We have tested a more general logarithmic potential. As shown in Figure 4, similar split invariant curves also exist in such a more general potential.

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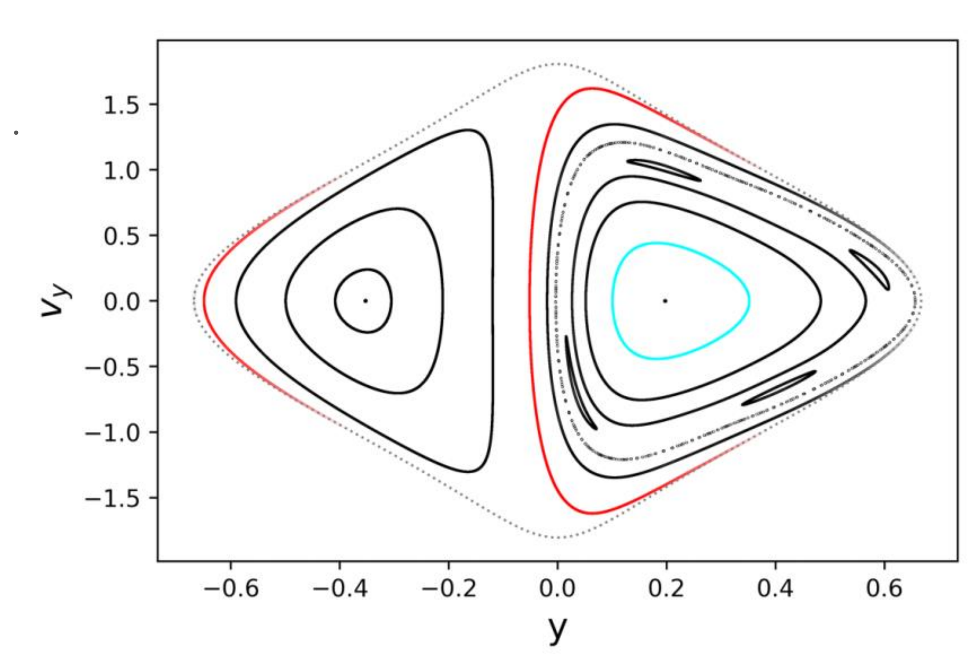


Figure 3

we believe that split invariant curves differ distinctively from those disconnected islands. Firstly, the disconnected islands are always closed "curves", not disconnected line segments. Secondly, resonance usually marks the separation between chaotic and regular orbits, while the cyan split invariant curve in Figure 3 occurs even between the regular orbits belonging to the same orbital family, along with the clearly disconnected islands formed by resonant orbits. Moreover, The Freeman bar is an analytical model without chaos or resonant orbits, thus split invariant curves may not be related to resonant orbits.

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## Conclusion

While invariant curves are generally closed curves in the surface of section, Binney (1985) first noticed the existence of split invariant curves in a rotating Kepler potential. In this work, we study the properties of such split invariant curves in a Freeman bar, where all the orbits can be described analytically. We confirm that some invariant curves in a rotating potential split into two disconnected parts under certain conditions, which differs from the disconnected islands formed by resonant orbits. We show that the orbits are nearly tangent to y-axis (or x-axis), or $L_z = 0$ with $y \neq 0$ (or $x \neq 0$), when the phenomenon occurs.

We find that the split invariant curves appear in the both x-v_x and y-v_y sections of SoS, but occur only in rotating potentials. We also confirm the existence of the split phenomenon in more generic potentials, such as a logarithmic potential. The split phenomenon shows that actions are no longer proportional to the area bounded by an invariant curve if the split occurs, but they can still be computed by other means (Binney 1985).

Although we have worked out how split invariant curves emerge in the Freeman bar, some questions still remain unsolved, such as the additional implications of split invariant curves, and the properties of the split phenomenon in the presence of addition orbital families other than $x_1$ and $x_4$. Thus, more future studies are needed to better understand this phenomenon and its implications.

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