Production of Doubly Heavy Hadrons

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Outline

1. Background（Bc, doubly heavy baryons）

2. The production at e^+e^- colliders

3. Fragmentation functions

4. Conclusions

1. Background for Bc ($c\bar{b}$)

➢ Only meson state with two different heavy flavors

- Only weak decay is possible => weak interaction

➢ Its production can be described by NRQCD factorization

- A lot of the dynamics can be calculated perturbatively
- The production mechanism of Bc is simpler than that of heavy quarkonium

➢ It was first observed by CDF collaboration in 1998

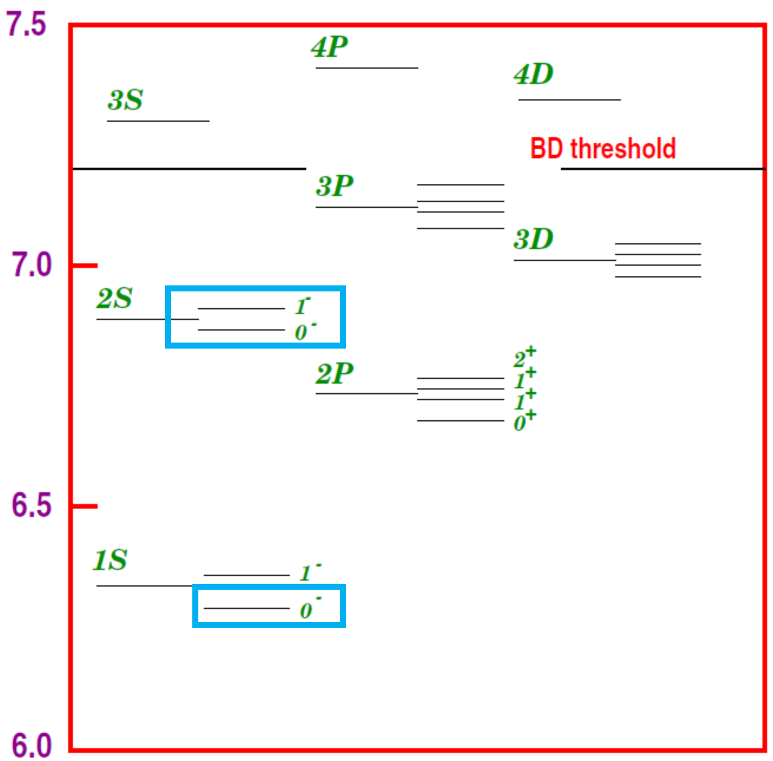
(u,d,s-1963; c-1974; b-1977; t-1995)

1. Background for Bc

➢ **Many Bc excited states have not been observed experimentally**

Bc(2S) and Bc*(2S) were observed in 2019
PRL122,132001(2019,CMS); PRL122,232001(2019,LHCb)

Excited Bc states



The mass spectrum of $c\bar{b}$ states

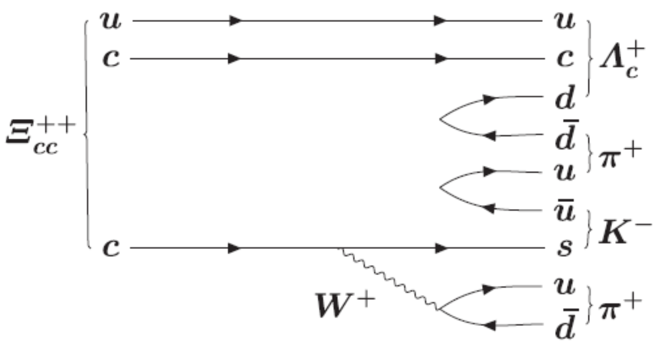
1. Background for doubly heavy baryons

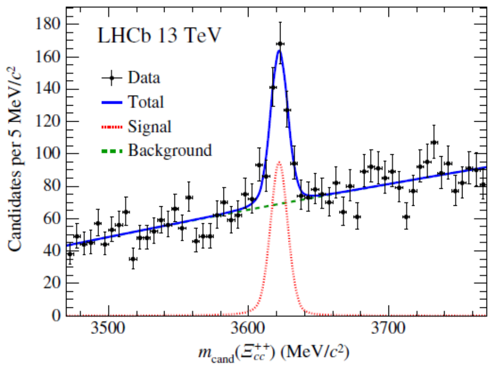
- **They provide a good platform for studying strong and weak interactions**

  Decay => weak interaction

  Production=> strong interaction=>pQCD, NRQCD

- **Ξcc was first observed by LHCb collaboration in 2017**





LHCb Collabration,PRL119, 112001 (2017)

2. The production at e^+e^- colliders

- **Advantages of the production at e^+e^- colliders**
  - **The center-of-mass system of the process is known**

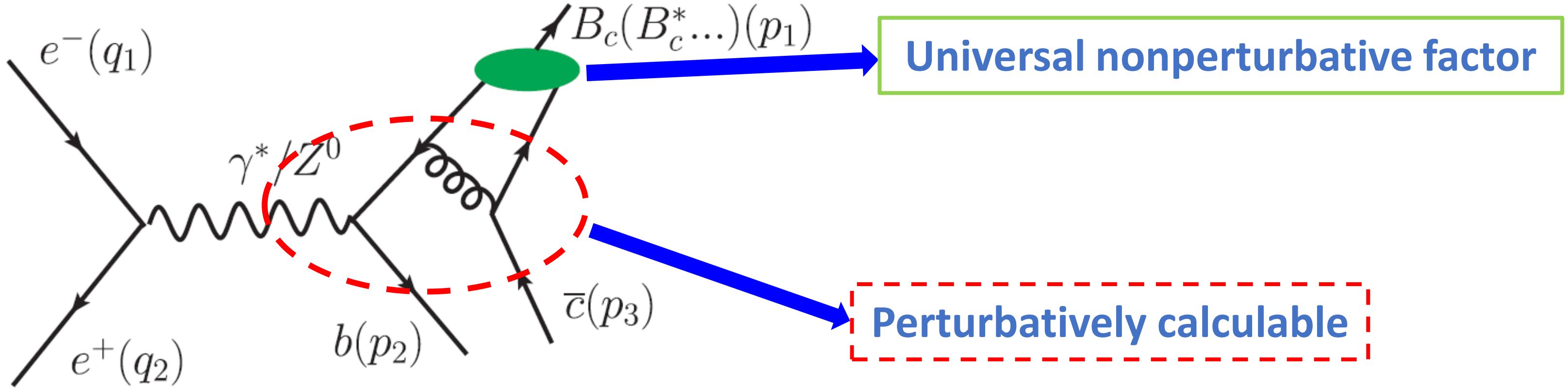
    Angle distributions and forward-backward asymmetry of doubly heavy hadrons have proper meaning in understanding the production.

  - **There are less backgrounds at an e^+e^- collider**

    A good platform for precision measurements.

- **Running at the Z pole**
  - **Z-resonance effect**
  - **CEPC**

LO calculation for Bc



$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\hat{\sigma}(e^+ + e^- \to c\bar{b}[n] + b + \bar{c})\langle O^{Bc}(n)\rangle$$

NRQCD factorization

Short-distance coefficients

Long-distance matrix elements

B-factories cannot produce the Bc meson because the beam energy is not enough for the Bc production.

Numerical results

Phys. Rev. D 93, 034019, (2016), X.-C. Zheng, C.-H. Chang et al.

States	σ(**pb**)	Events/year
$B_c\ ({}^1S_0)$	2.73	2.7×10^6
$B_c^*({}^3S_1)$	3.82	3.8×10^6
$B_c^{**}({}^1P_1)$	0.27	2.7×10^5
$B_c^{**}({}^3P_1)$	0.16	1.6×10^5
$B_c^{**}({}^3P_2)$	0.34	3.4×10^5
$B_c^{**}({}^3P_2)$	0.37	3.7×10^5

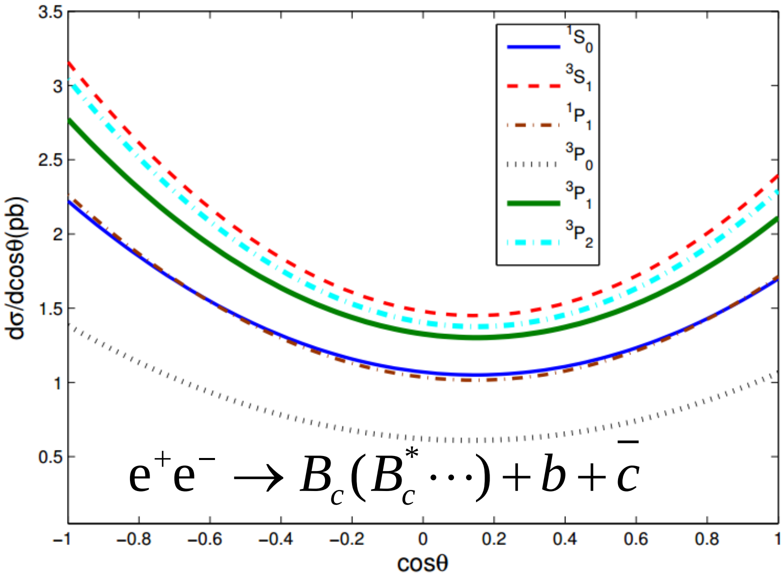
Cross sections at **the Z pole** with $\mathrm{L}=10^{35}cm^{-2}s^{-1}$

States	σ(**fb**)	Events/year
$B_c\ ({}^1S_0)$	0.47	4.7×10^2
$B_c^*({}^3S_1)$	0.72	7.2×10^2
$B_c^{**}({}^1P_1)$	0.05	50
$B_c^{**}({}^3P_1)$	0.03	30
$B_c^{**}({}^3P_2)$	0.07	70
$B_c^{**}({}^3P_2)$	0.07	70

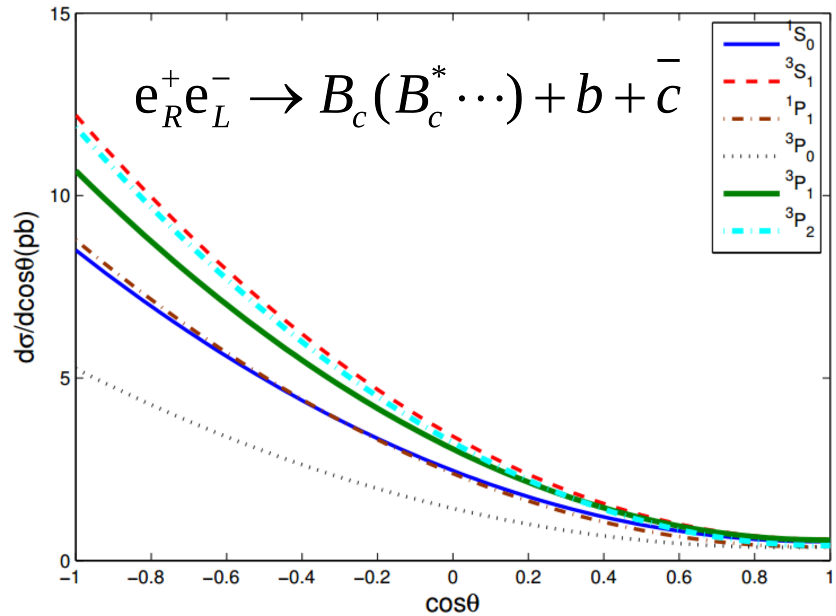
Cross sections at $\sqrt{s}=250GeV$ with $\mathrm{L}=10^{35}cm^{-2}s^{-1}$

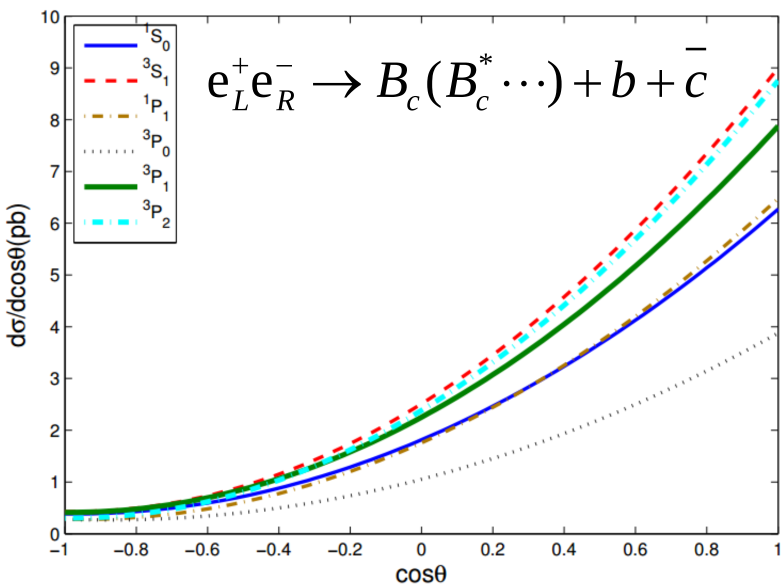
- **The Z-resonance effect is important for studying Bc and its excited states**
- **The luminosity of the e^+e^- collider should be $10^{35-36}\mathrm{cm}^{-2}\mathrm{s}^{-1}$**

The production at e+e- colliders



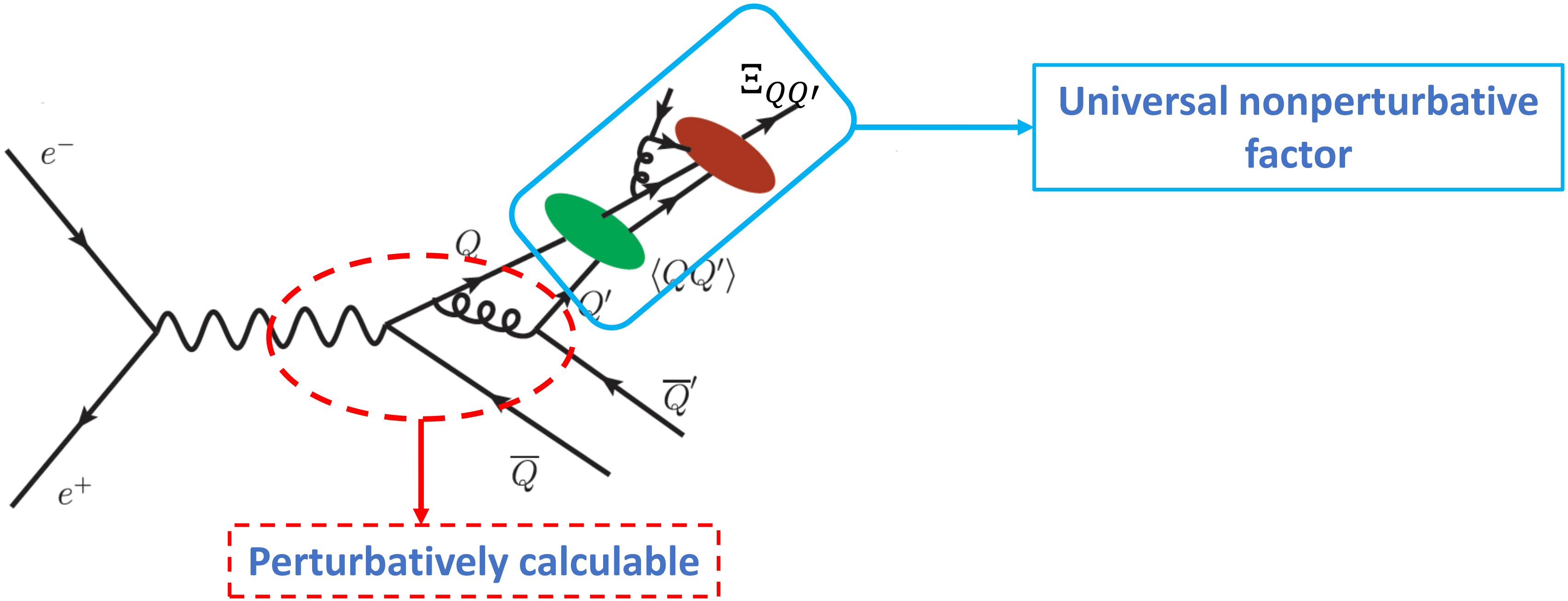
Phys. Rev. D 93, 034019, (2016), X.-C. Zheng, C.-H. Chang et al.





The angle distributions are forward-backward asymmetric.

➢ Production of doubly heavy baryons



1) Production of diquark (in color $\overline{3}$ state)

The calculation is similar to the Bc case

2) The diquark fragments into the doubly heavy baryon

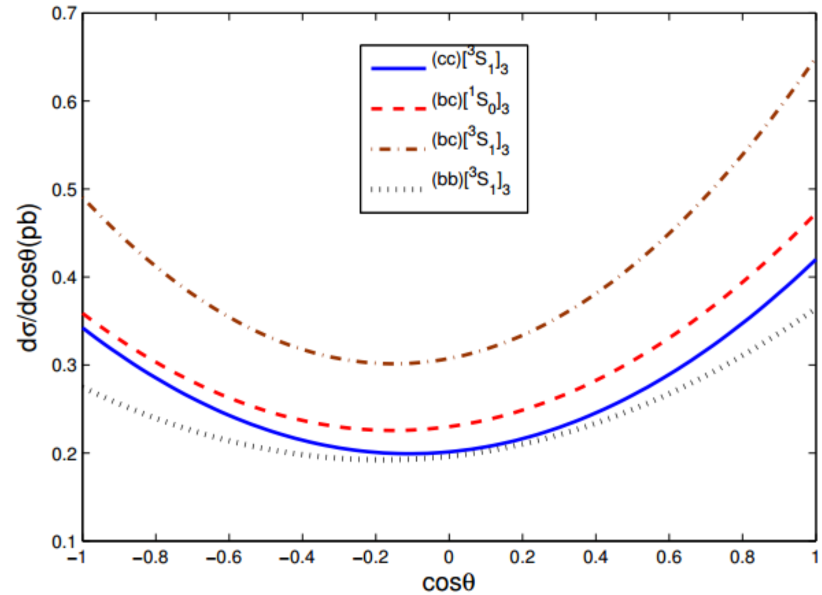
$\delta(1-z)$, Peterson model (a diquark to a doubly heavy baryon **is similar to a heavy quark to a heavy meson**)

➢ Production of doubly heavy baryons

Phys. Rev. D 93, 034019, (2016), X.-C. Zheng, C.-H. Chang et al.

States	σ(**pb**)	Events/year
Ξ_{cc}	0.52	5.2×10^5
Ξ_{bc}	1.37	1.4×10^6
Ξ_{bb}	0.05	5.0×10^4

Cross sections at **the Z pole** with $L=10^{35}cm^{-2}s^{-1}$



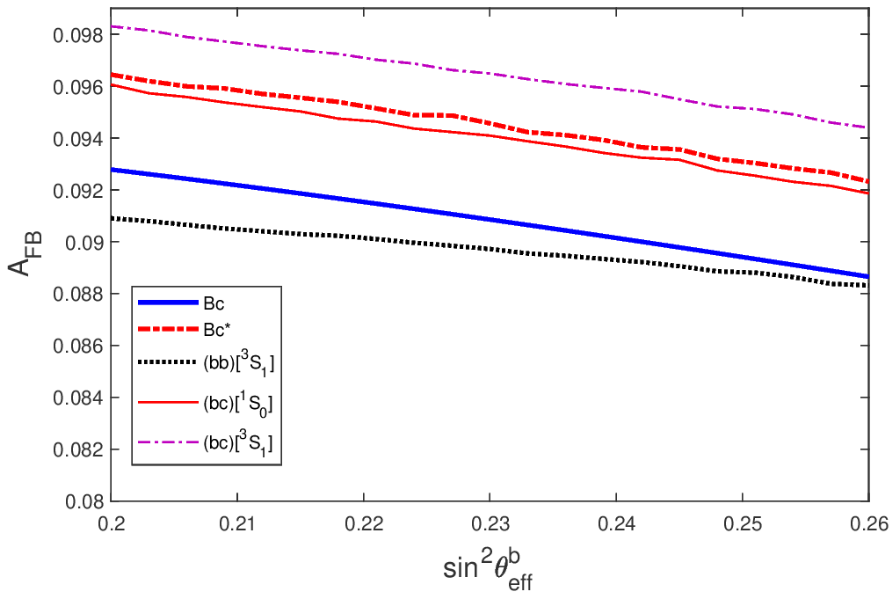
Differential angle distribution

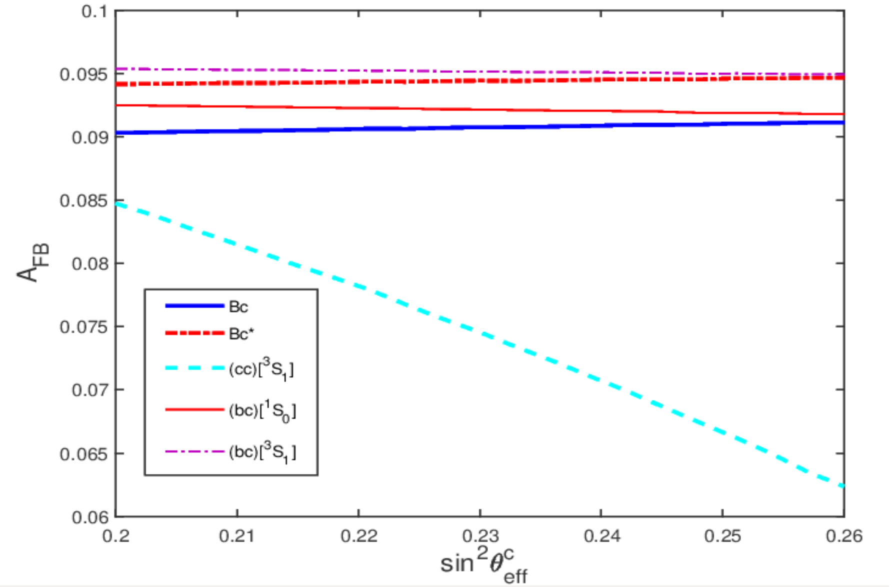
The angle distributions are also forward-backward asymmetric.

Forward-backward asymmetry:

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}.$$

Sci. China-Phys.Mech. Astron. 63, 281011,(2020), X.-C. Zheng, C.-H. Chang et al.



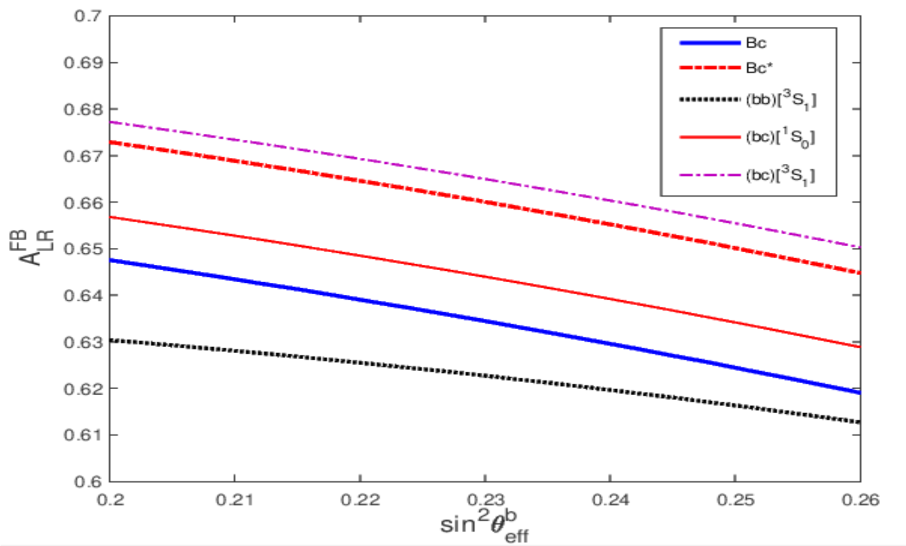


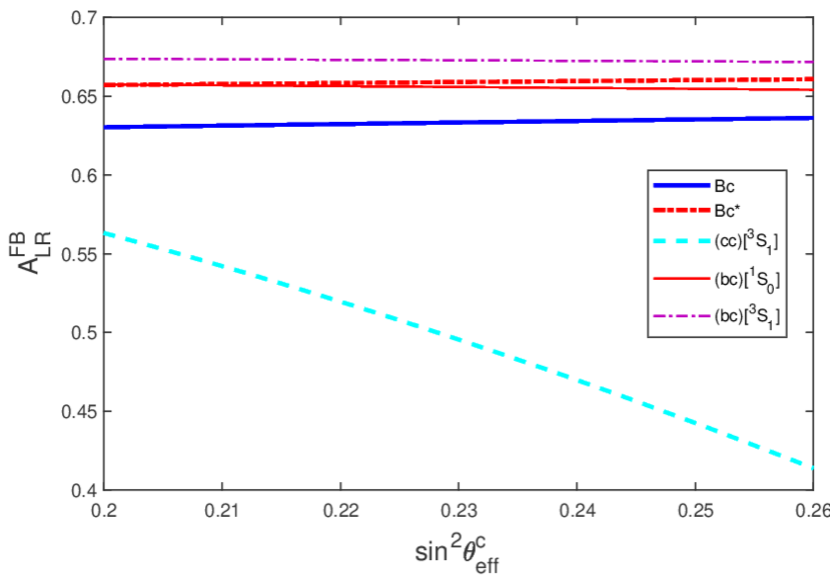
$\sin^2\theta^f_{eff}$ can be determined through measuring the forward-backward asymmetry of the doubly heavy-flavored hadrons.

Left-right forward-backward asymmetry:

$$A_{LR}^{FB} = \frac{\sigma_{LF} - \sigma_{LB} - \sigma_{RF} + \sigma_{RB}}{\sigma_{LF} + \sigma_{LB} + \sigma_{RF} + \sigma_{RB}}.$$

Sci. China-Phys.Mech. Astron. 63, 281011,(2020), X.-C. Zheng, C.-H. Chang et al.

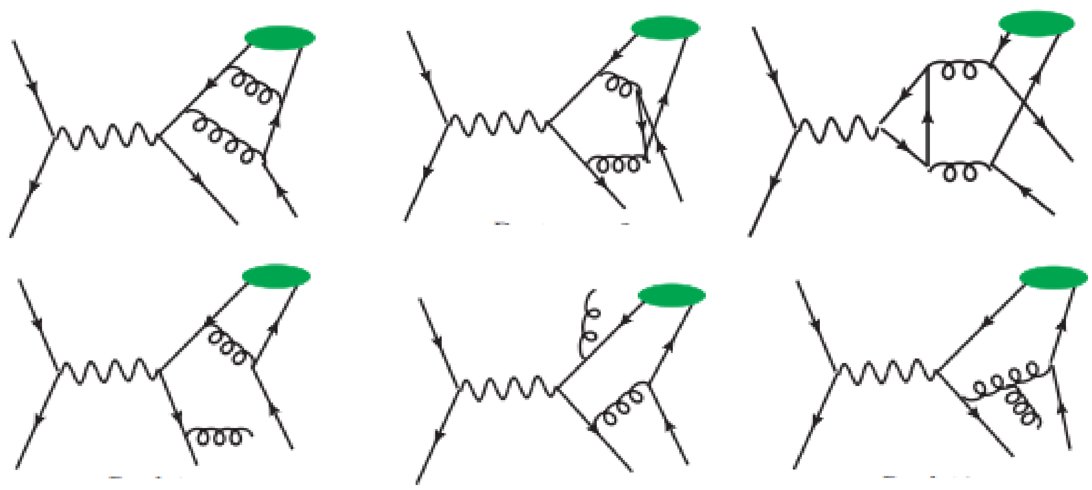




$\sin^2\theta^f_{eff}$ can be determined through measuring the left-right-forward-backward asymmetry of the doubly heavy-flavored hadrons.

➢ NLO calculations for Bc and Bc*

- To see the changes of the physical observables from the LO calculations to the NLO calculations.
- To see how the dependence on the renormalization scale changes after including the NLO QCD corrections.



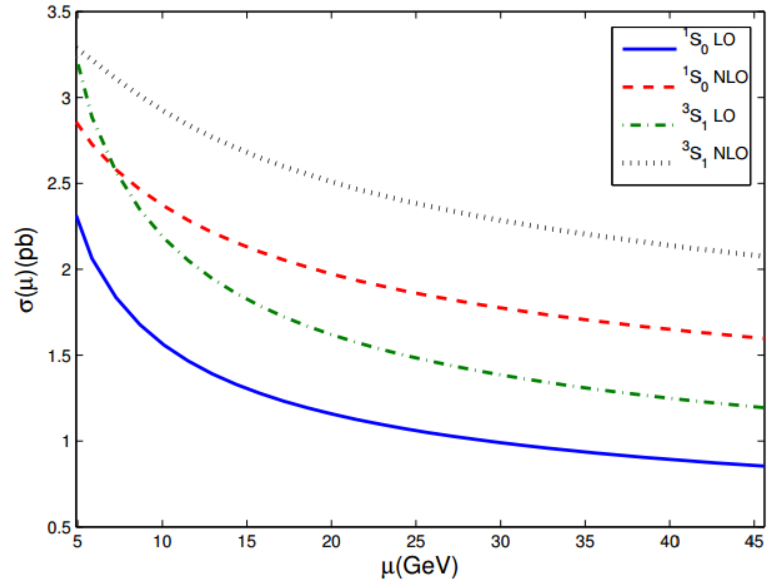
84 Feynman diagrams for the virtual correction, 24 Feynman diagrams for the real correction.

Numerical results

Sci. China-Phys.Mech. Astron. 61, 031012,(2018), X.-C. Zheng, C.-H. Chang et al.

μ	$\alpha_s(\mu)$	$\sigma_{\rm LO}$(pb)	$\sigma_{\rm NLO}$(pb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
$2m_b$	0.180	1.58	2.38	1.51
$m_z/2$	0.132	0.85	1.58	1.86

Cross section of Bc



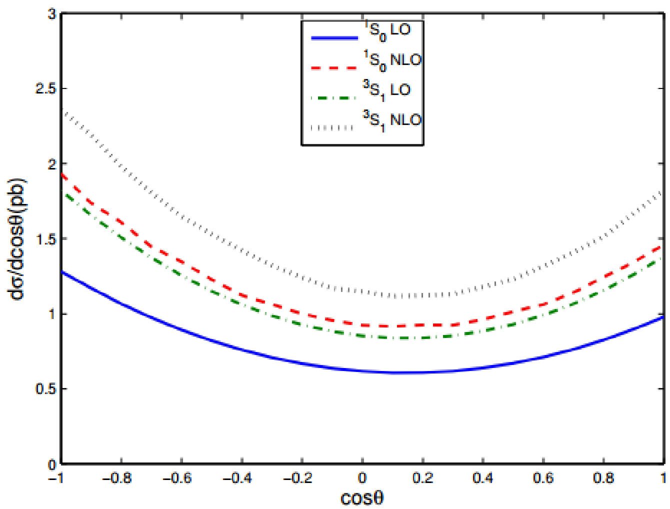
μ	$\sigma_{\rm LO}$(pb)	$\sigma_{\rm NLO}$(pb)	$\sigma_{\rm NLO}/\sigma_{\rm LO}$
$2m_b$	2.20	2.93	1.33
$m_z/2$	1.18	2.06	1.74

Cross section of Bc*

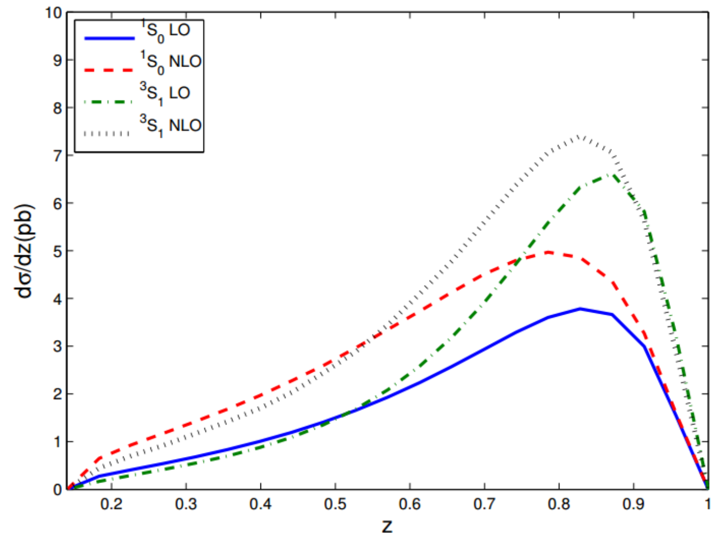
The dependence on μ is weaken significantly due to NLO corrections.

The NLO corrections are significant!

Sci. China-Phys.Mech. Astron. 61, 031012,(2018), X.-C. Zheng, C.-H. Chang et al.



Differential angle distribution



Differential energy distribution

- The K-factor changes very little with different θ;
- The NLO corrections change the energy distribution significantly.

3. Fragmentation functions

- NRQCD factorization

$$d\sigma(e^+ + e^- \to Bc + b + \bar{c}) = \sum_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c}) \langle O^{\mathrm{Bc}}(n) \rangle$$

- Fragmentation mechanism

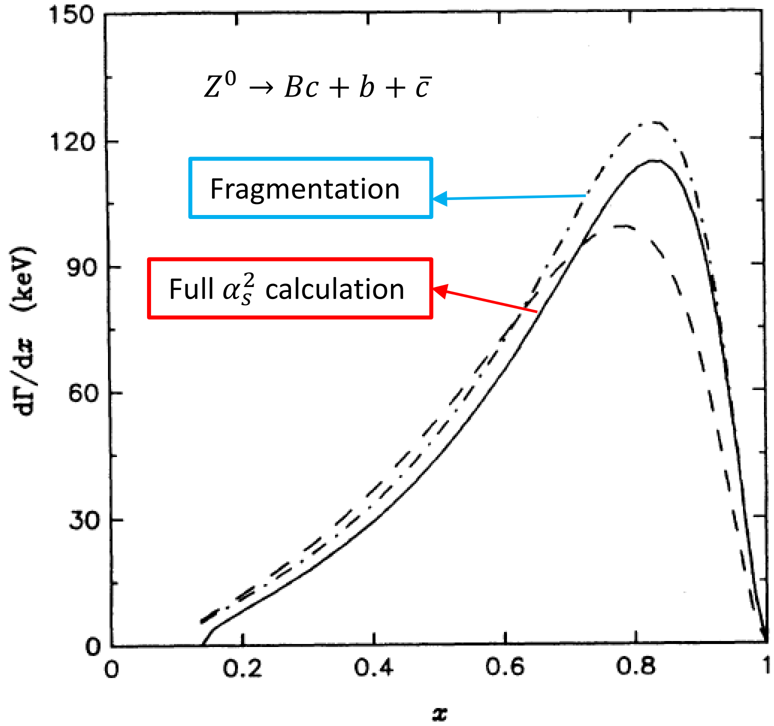
$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c}) = \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + \mathrm{O}(m_Q^2/s)$$

Fragmentation function

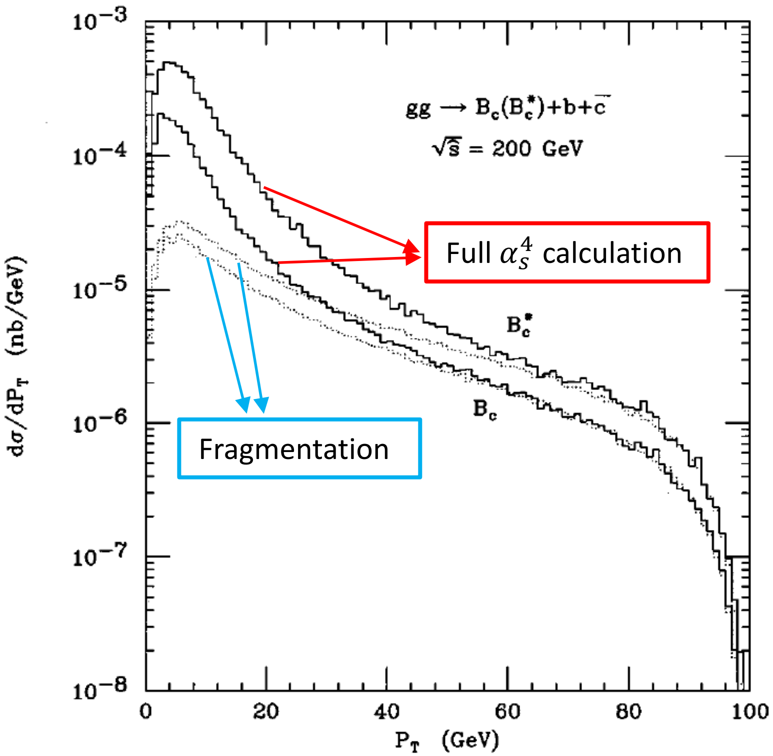
Partonic production cross section

- The production of the Bc meson is dominated by the fragmentation mechanism when $s \gg m_Q^2$

Comparison of the fragmentation approach and the full QCD calculation



C.-H. Chang and Y.-Q. Chen, PRD46,3845(1992)



C.-H. Chang, Y.-Q. Chen and R. J. Oakes, PRD54,4344(1996)

- NRQCD factorization

$$d\sigma(e^+ + e^- \to Bc + b + \bar{c})$$
$$= \sum_n d\sigma(e^+ + e^- \to (c\bar{b})[n] + b + \bar{c})\langle O^{Bc}(n)\rangle$$

Energy scales:
$\sqrt{s}, m_Q$

Log-terms appear in short-distance coefficients:

$$\alpha_s^m \sum_{n=0}^{\infty} \alpha_s^n \ln^n(s/m_Q^2)$$

Collinear gluon emission

Spoil or weak the convergence of the series

$\ln(p_t^2/m_Q^2)$ appearing in the production at a hadron collider

- Fragmentation approach

$$d\sigma(e^+ + e^- \to Bc(p) + b + \bar{c})$$
$$= \sum_i d\hat{\sigma}(e^+ + e^- \to i + X)(p/z, \mu_F) \otimes D_{i\to Bc}(z, \mu_F) + \mathrm{O}(m_Q^2/s)$$

Involving $\ln(s/\mu_F^2)$

$\mu_F = O(\sqrt{s})$

NRQCD factorization:

$$D_{i\to Bc}(z, \mu_{F0}) = \sum_n d_{i\to c\bar{b}[n]}(z, \mu_{F0}) \langle O^{Bc}(n) \rangle$$

$\mu_{F0} = O(m_Q)$

Involving $\ln(\mu_{F0}^2/m_Q^2)$

Evolution of fragmentation functions

$$\frac{d}{d\ln\mu_F^2} D_{i\to Bc}(z, \mu_F) = \sum_j P_{ij}(z/y, \ \alpha_s(\mu_F)) \otimes D_{j\to Bc}(y, \mu_F)$$

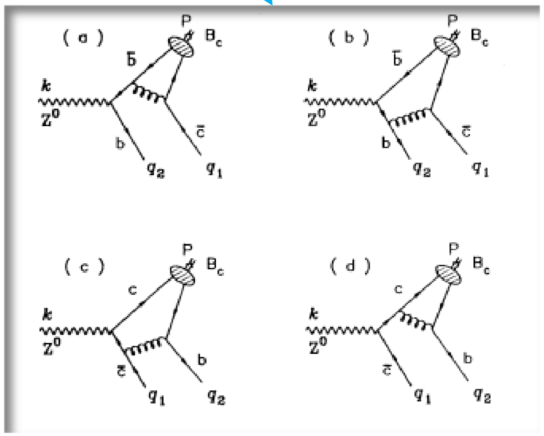
$$P_{ij}(z, \ \alpha_s(\mu_F)) = P_{ij}^{(0)}(z)\frac{\alpha_s(\mu_F)}{2\pi} + P_{ij}^{(1)}(z)\left(\frac{\alpha_s(\mu_F)}{2\pi}\right)^2 + \mathrm{O}(\alpha_s^3)$$

Collinear log-terms have been resumed through the DGLAP evolution.

➢ **LO fragmentation functions** for the Bc production

- Extracting from the LO calculation of process $Z^0 \rightarrow Bc + b + \overline{c}$

C.-H. Chang, Y.-Q. Chen, Phys. Rev. D 46, 3845, (1992);



- Calculating from the definition:

J.-P. Ma, Phys. Lett. B 332, 398, (1994);

◆ There were no NLO results for $D_{i\rightarrow Bc}(z, \mu_F)$ before our calculation.

◆ In order to obtain the theoretical predictions under the fragmentation approach up to NLO QCD accuracy, the NLO results for $D_{i\rightarrow Bc}(z, \mu_F)$ are needed.

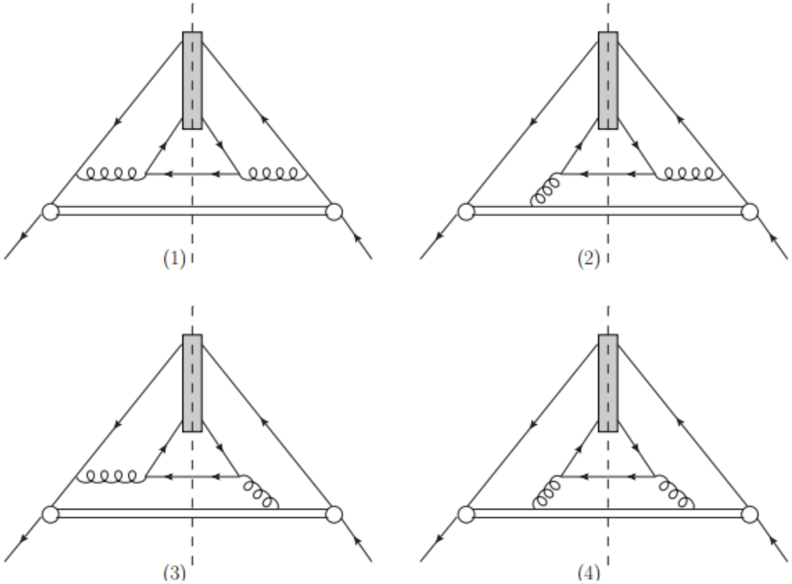
Fragmentation function calculation

Based on the definition of FFs by **Collins and Soper.**

Nucl. Phys. B 194, 445, (1982).

Process independent approach

LO cut diagrams:



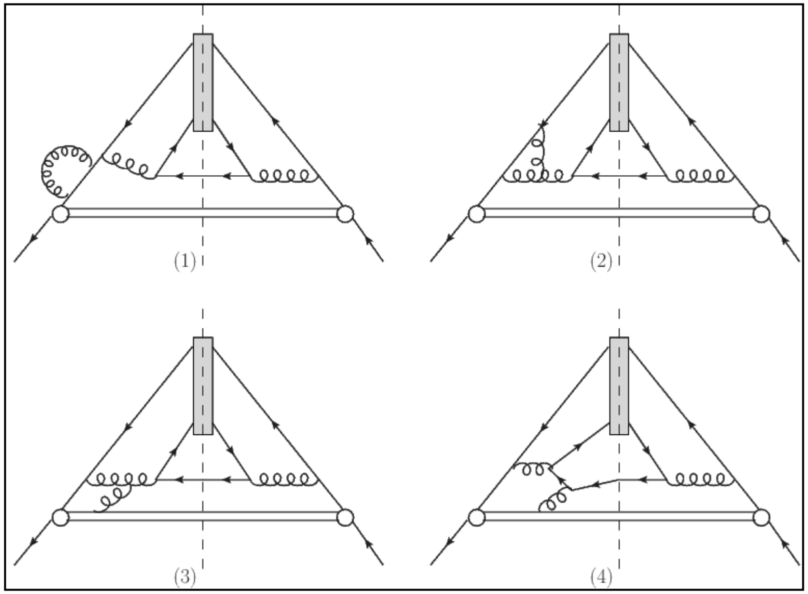
LO fragmentation functions:

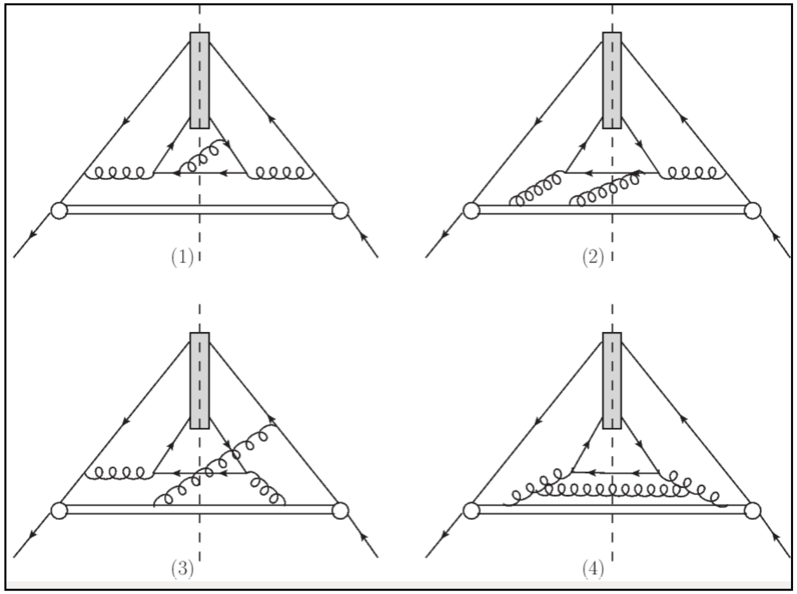
$$D^{\rm LO}_{\bar{b}\to B_c}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{81\pi r_c^2(1-r_b z)^6 M^3}[6-18(1-2r_c)z+(21-74r_c+68r_c^2)z^2 -2r_b(6-19r_c+18r_c^2)z^3+3r_b^2(1-2r_c+2r_c^2)z^4],$$

$$D^{\rm LO}_{\bar{b}\to B_c^*}(z) = \frac{2\alpha_s^2 z(1-z)^2|R_S(0)|^2}{27\pi r_c^2(1-r_b z)^6 M^3}[2-2(3-2r_c)z+3(3-2r_c+4r_c^2)z^2 -2r_b(4-r_c+2r_c^2)z^3+r_b^2(3-2r_c+2r_c^2)z^4].$$

NLO corrections

Sample NLO cut diagrams





54 virtual cut diagrams， 72 real cut diagrams.

➢ Virtual corrections

Tensor reduction, IBP reduction

Many integrals containing an eikonal line, e.g,

$$\int \frac{d^D l}{[(l-p_1)^2-m_1^2+i\varepsilon][(l-p_2)^2-m_2^2+i\varepsilon][(l-p_3)^2-m_3^2+i\varepsilon](l\cdot n+i\varepsilon)}$$

➢ Real corrections

UV and IR divergences!

$$\mathrm{D}^{real}_{\bar{b}\to Bc}(z)=\int N_{CS}\,d\phi_{real}(A_{real}-A_S)+\int N_{CS}\,d\phi_{real}A_S$$

Calculated in 4 dimensions

Calculated in d dimensions

Various types of subtraction terms need to be integrated!

➢ Renormalization

Renormalization of QCD:

$$\delta Z_2^{OS} = -C_F \frac{\alpha_s(\mu_R)}{4\pi}\left[\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\ \gamma_E + 3\ \ln\frac{4\pi\mu_R^2}{m^2} + 4\right],$$

$$\delta Z_m^{OS} = -3\ C_F \frac{\alpha_s(\mu_R)}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu_R^2}{m^2} + \frac{4}{3}\right],$$

$$\delta Z_3^{OS} = \frac{\alpha_s(\mu_R)}{4\pi}\left[(\beta_0' - 2C_A)\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right) - \frac{4}{3}T_F\left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu_R^2}{m_c^2}\right) - \frac{4}{3}T_F\left(\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\frac{4\pi\mu_R^2}{m_b^2}\right)\right],$$

$$\delta Z_g^{\overline{MS}} = -\frac{\beta_0}{2}\frac{\alpha_s(\mu_R)}{4\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\ (4\pi)\right], \qquad (71)$$

Renormalization of the operator:

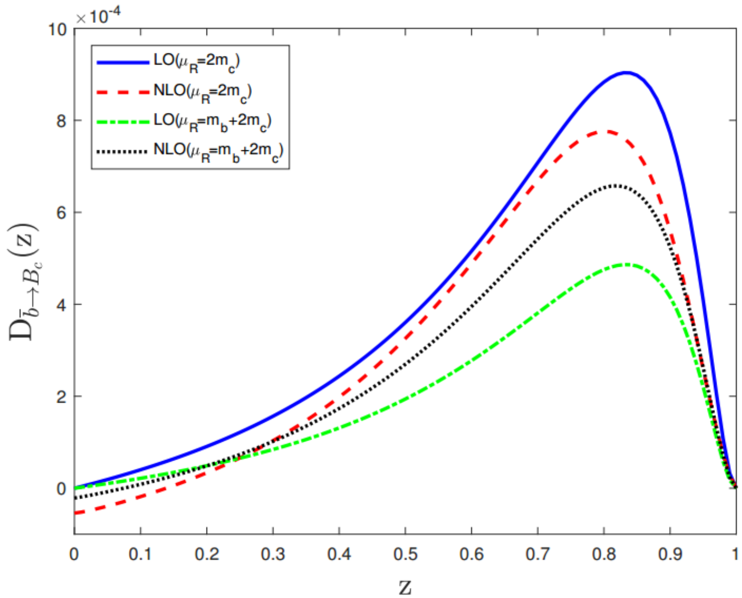
$$D^{\rm operator}_{\bar{b}\to c\bar{b}[n]}(z) = -\frac{\alpha_s(\mu_R)}{2\pi}\left[\frac{1}{\epsilon_{UV}} - \gamma_E + \ln\ (4\pi) + \ln\frac{\mu_R^2}{\mu_F^2}\right] \times \int_z^1 \frac{dy}{y} P_{\bar{b}\bar{b}}(y) D^{\rm LO}_{\bar{b}\to c\bar{b}[n]}(z/y),$$

Note: Fragmentation functions are factorization scheme dependent, the most common used factorization scheme is the $\overline{MS}$ scheme.

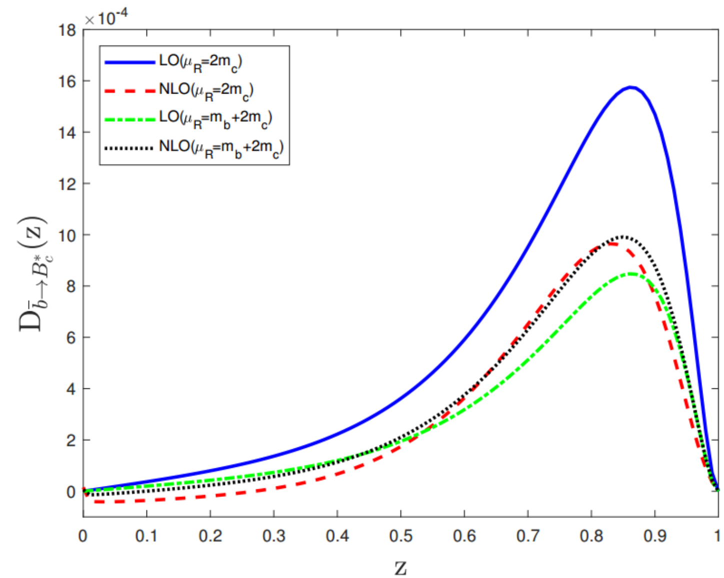
NLO results

Phys. Rev. D 100, 034004, (2019), X.-C. Zheng, C.-H. Chang, X.-G. Wu.

NLO fragmentation functions for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$



$$D_{\bar{b}\to Bc}(z, \mu_{F0} = m_b + 2m_c)$$

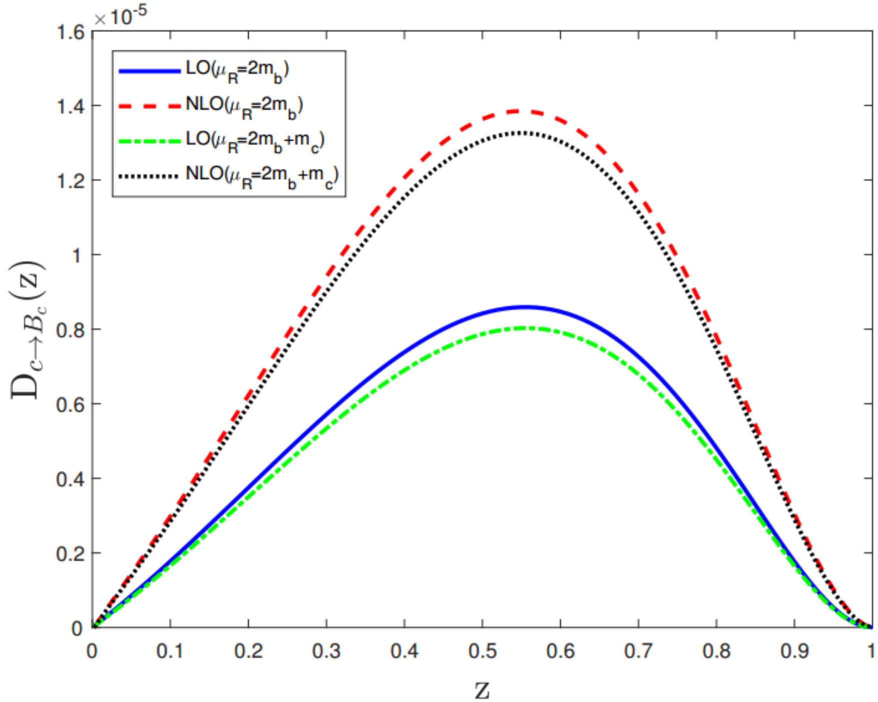


$$D_{\bar{b}\to B_c^*}(z, \mu_{F0} = m_b + 2m_c)$$

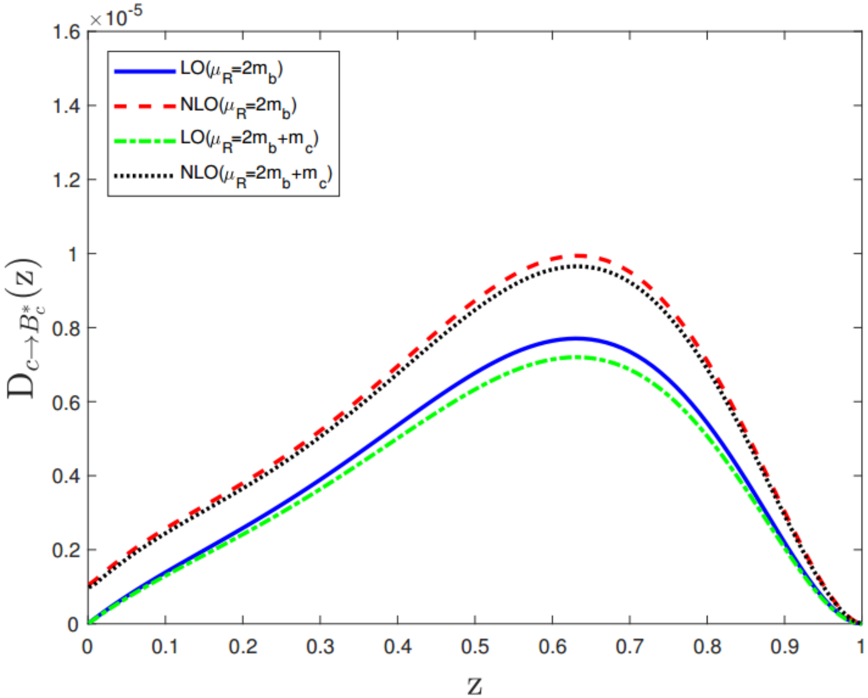
These fragmentation functions can be studied at high-energy colliders.

NLO results

NLO fragmentation functions for $c \to B_c$ and $c \to B_c^*$



$D_{c\to Bc}(z, \mu_{F0} = m_b + 2m_c)$



$D_{c\to B_c^*}(z, \mu_{F0} = m_b + 2m_c)$

NLO results

Fragmentation probability and average value of z

$$P = \int_0^1 dz\, D(z), \qquad \langle z \rangle = \frac{\int_0^1 dz\, z\, D(z)}{\int_0^1 dz\, D(z)},$$

$\bar{b} \to Bc$

μ_R	$P \times 10^4$(LO)	$P \times 10^4$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_c$	3.82	3.14	0.68	0.70
$m_b + 2m_c$	2.05	2.73	0.68	0.69

$\bar{b} \to B_c^*$

μ_R	$P \times 10^4$(LO)	$P \times 10^4$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_c$	5.36	2.91	0.73	0.77
$m_b + 2m_c$	2.89	3.25	0.73	0.74

$c \to Bc$

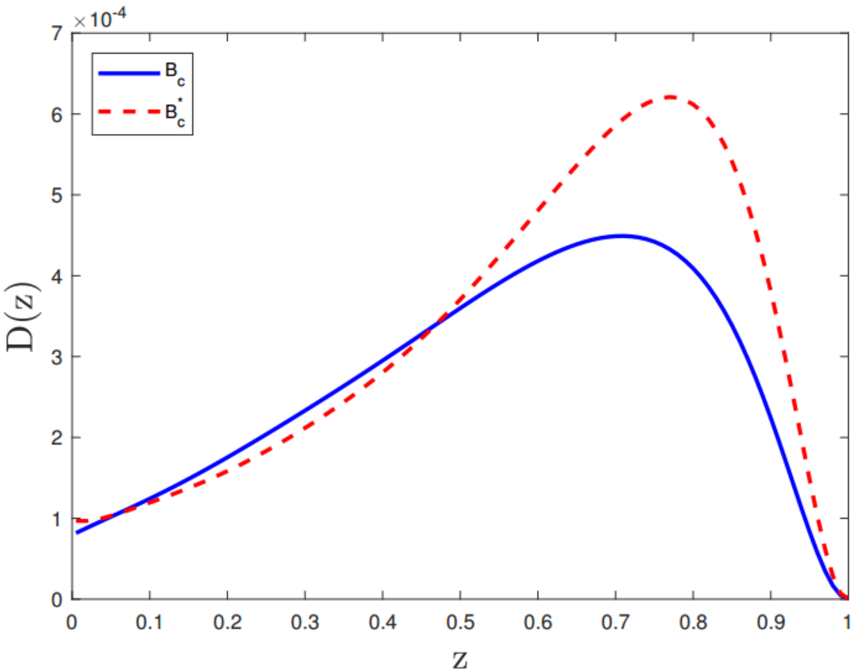
μ_R	$P \times 10^6$(LO)	$P \times 10^6$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_b$	4.95	8.07	0.51	0.51
$2m_b + m_c$	4.63	7.72	0.51	0.51

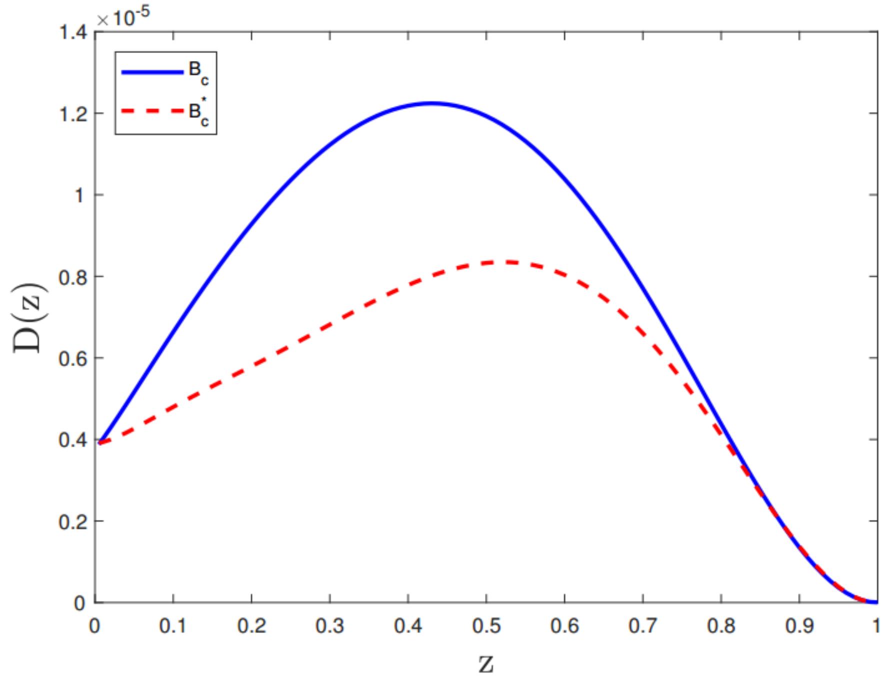
$c \to B_c^*$

μ_R	$P \times 10^6$(LO)	$P \times 10^6$(NLO)	$\langle z \rangle$(LO)	$\langle z \rangle$(NLO)
$2m_b$	4.28	5.75	0.55	0.54
$2m_b + m_c$	4.00	5.57	0.55	0.54

NLO results

The fragmentation functions at the scale of m_Z

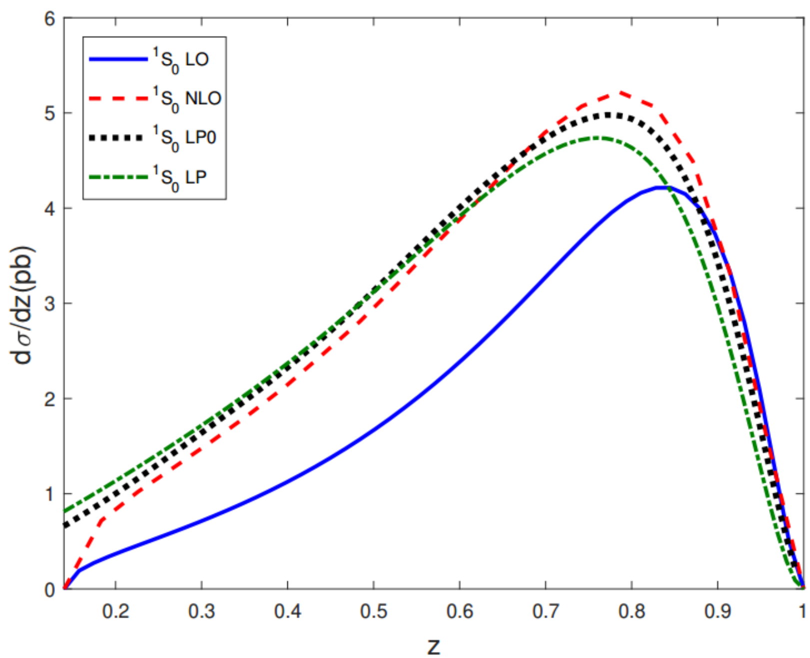




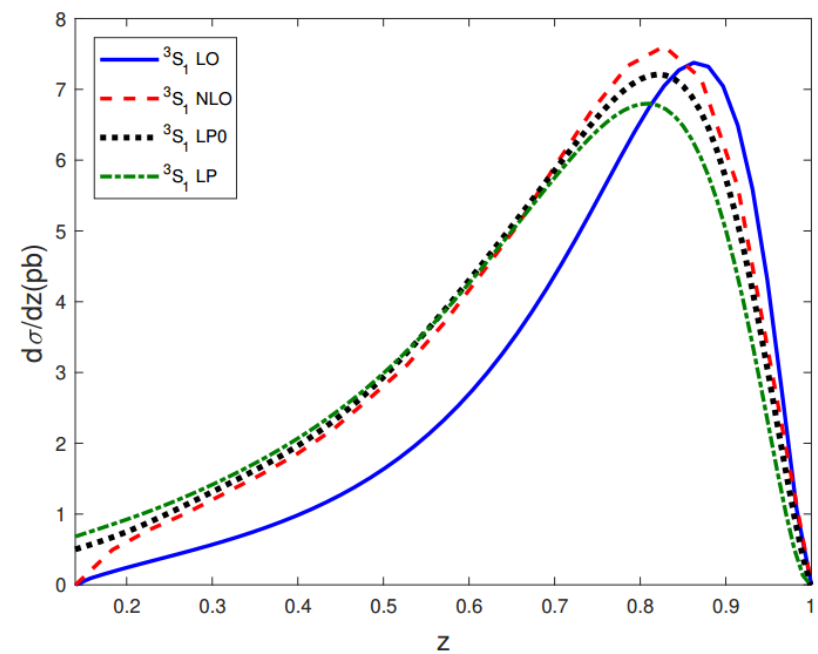
$D_{\bar{b}\to Bc(Bc^*)}(z,\mu_F=m_Z)$ $D_{c\to(Bc)B_c^*}(z,\mu_F=m_Z)$

- Obtained through the DGLAP evolution from the fragmentation functions at the initial factorization scale.

Bc and Bc* production at a Z-factory



$d\sigma / dz(Bc)$



$d\sigma / dz(Bc^*)$

LO,NLO: fixed-order approach.

LP0： fragmentation approach, no DGLAP evolution.

LP： fragmentation approach, evolved with DGLAP equation.

➢ Quarkonium FFs at NLO

E. Braaten et al, NPB 586, 427, (2000) , $g \to Q\bar{Q}({}^3S_1^{[8]})$

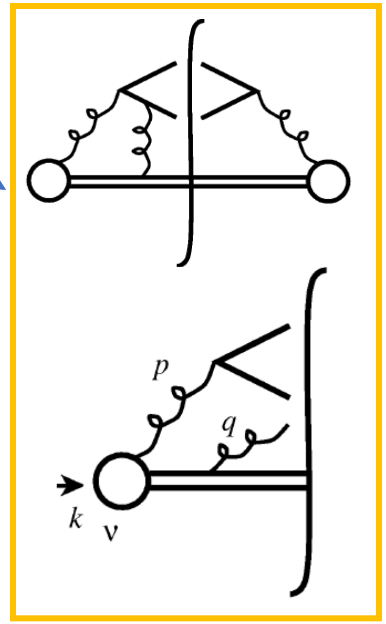
E. Braaten et al, JHEP 04, 121, (2015) , $g \to Q\bar{Q}({}^1S_0^{[1]})$

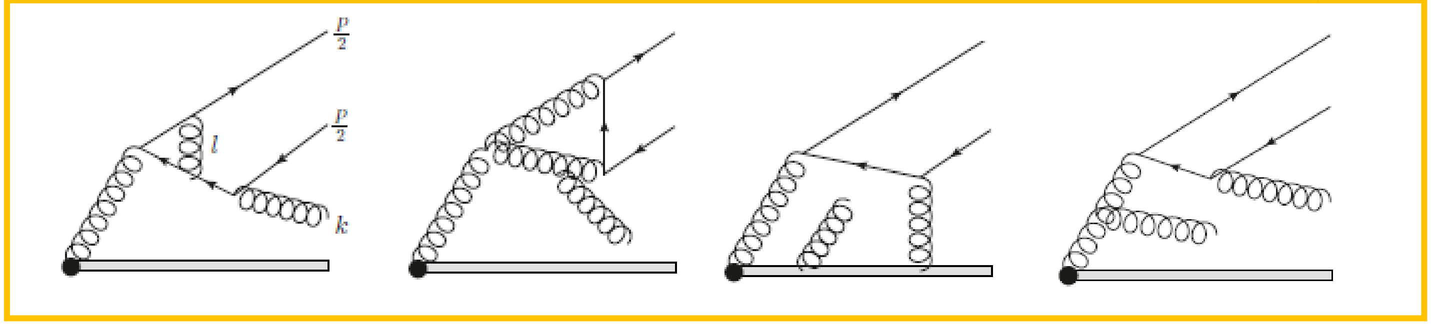
E. Braaten et al, JHEP 01, 227, (2019) , $g \to Q\bar{Q}({}^1S_0^{[8]})$

Y. Jia et al, arXiv:1810.04138(2018) , $g \to Q\bar{Q}({}^1S_0^{[1,8]})$

Y. –Q. Ma et al, JHEP 04, 116, (2019) , $g \to Q\bar{Q}({}^1S_0^{[1,8]})$

Y. –Q. Ma et al, JHEP 08, 111, (2021) , $g \to Q\bar{Q}({}^3P_J^{[1,8]})$



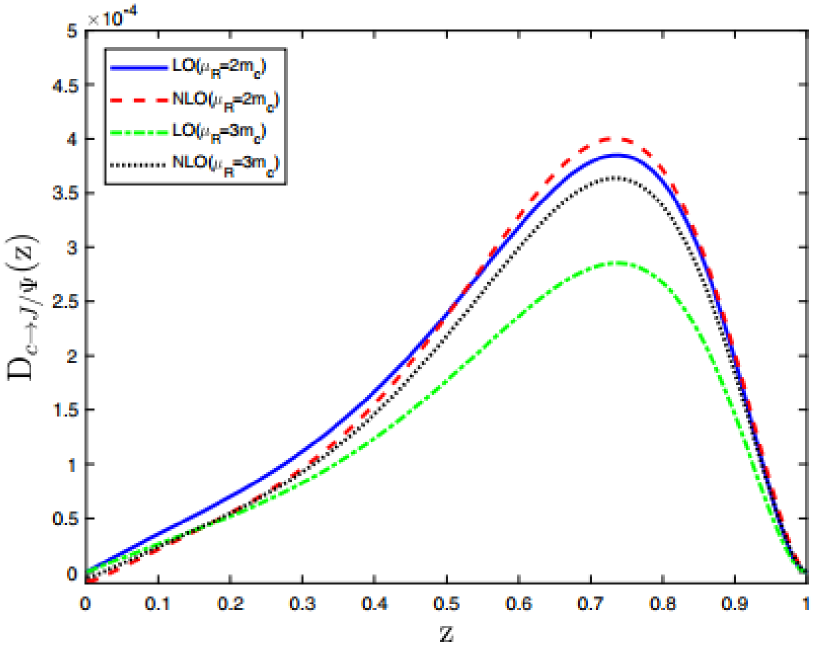


Several typical diagrams

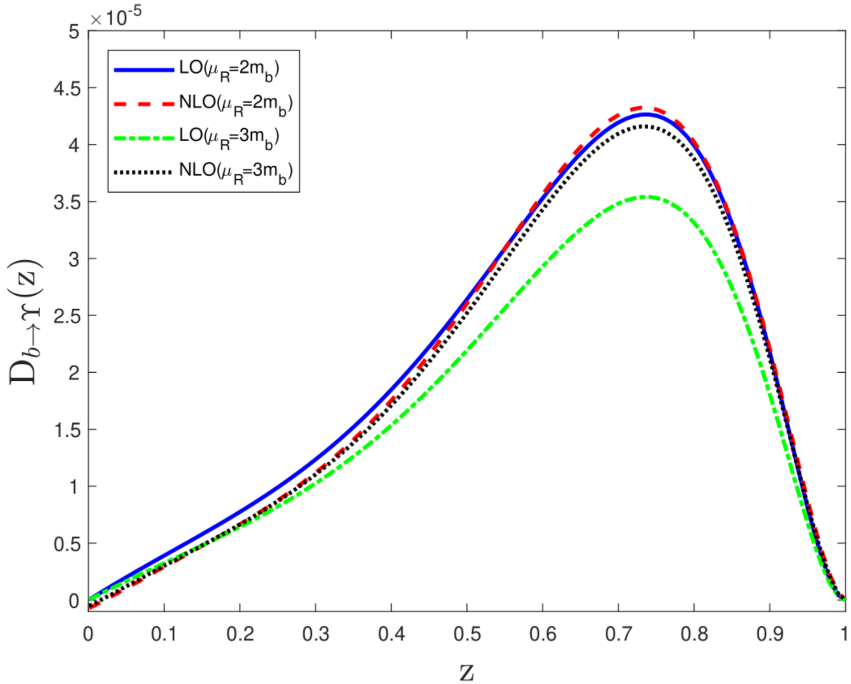
Quarkonium FFs

Phys. Rev. D 100, 014005, (2019),
X.-C. Zheng, C.-H. Chang, X.-G. Wu.

NLO fragmentation functions for $c \to J/\psi$ and $b \to \Upsilon$



$$D_{c\to J/\psi}(z, \mu_{F0} = 3m_c)$$

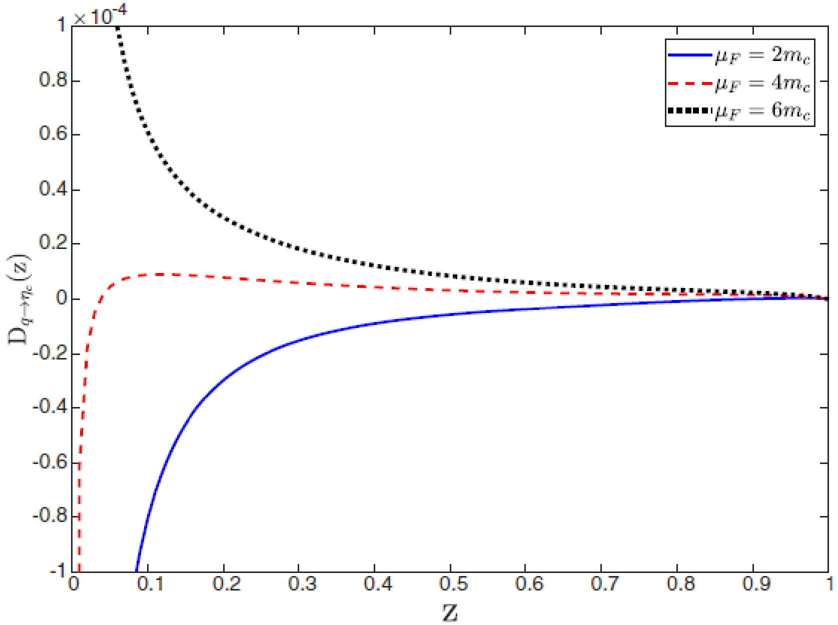


$$D_{b\to\Upsilon}(z, \mu_F = 3m_b)$$

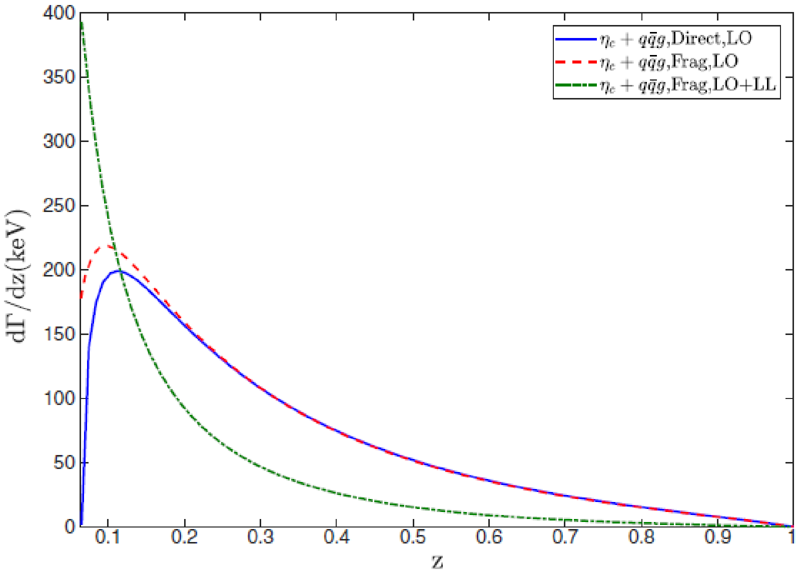
Quarkonium FFs

Phys. Rev. D 103, 074004, (2021),
X.-C. Zheng, Z.-Y. Zhang, X.-G. Wu.

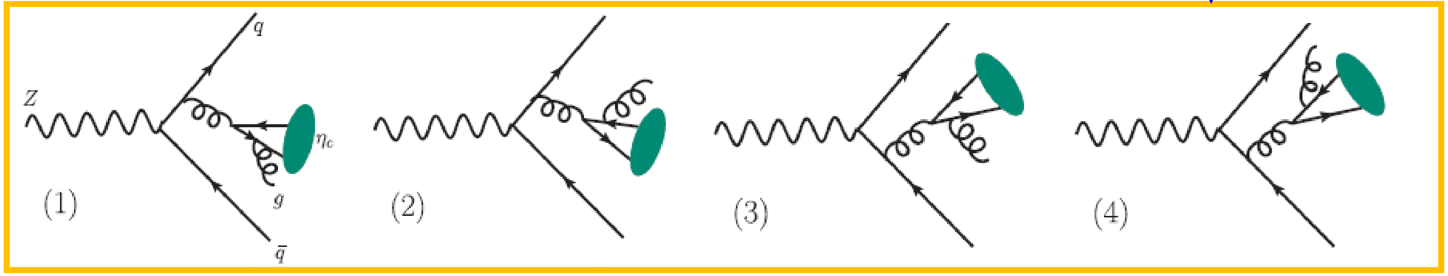
Fragmentation functions for $q \to \eta_Q (q \neq Q)$



$D_{q\to\eta_c}(z, \mu_F)$



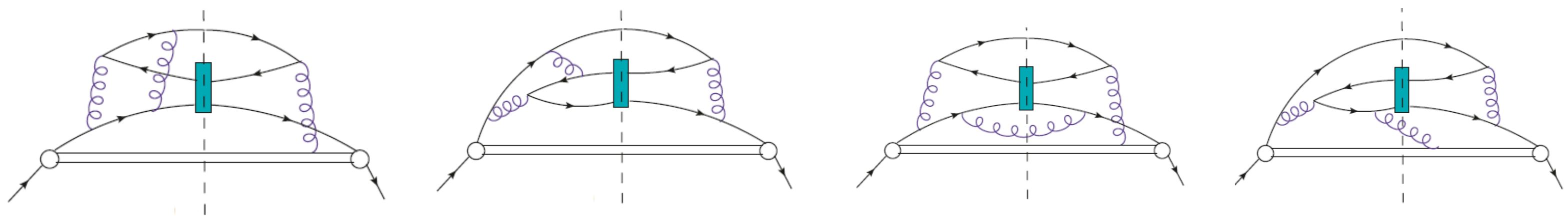
$d\Gamma/dz$ for $Z \to \eta_c + q\bar{q}g$

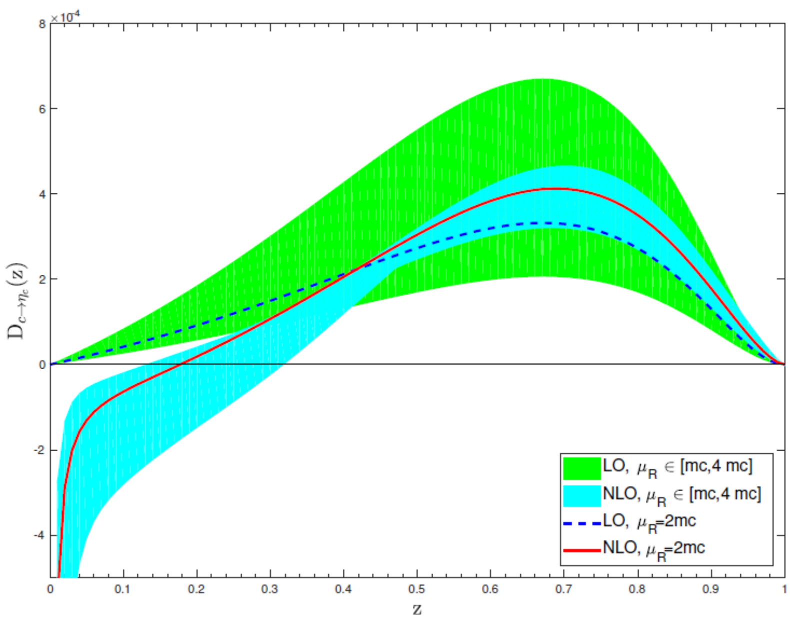


Quarkonium FFs

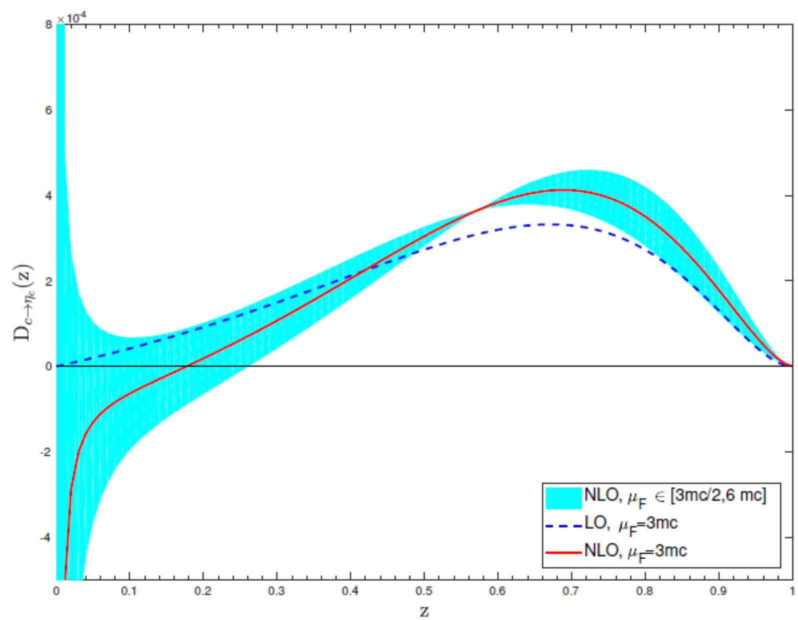
JHEP 07, 014, (2021),
X.-C. Zheng, X.-G. Wu, X.-D. Huang.

NLO fragmentation functions for $Q \to \eta_Q$





$D_{c\to\eta_c}(z,\mu_F)$



$D_{c\to\eta_c}(z,\mu_F)$

Quarkonium FFs

Eur. Phys. J. C (2021) 81:597
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Regular Article - Theoretical Physics

Next-to-leading-order QCD corrections to heavy quark fragmentation into ${}^1S_0^{(1,8)}$ quarkonia

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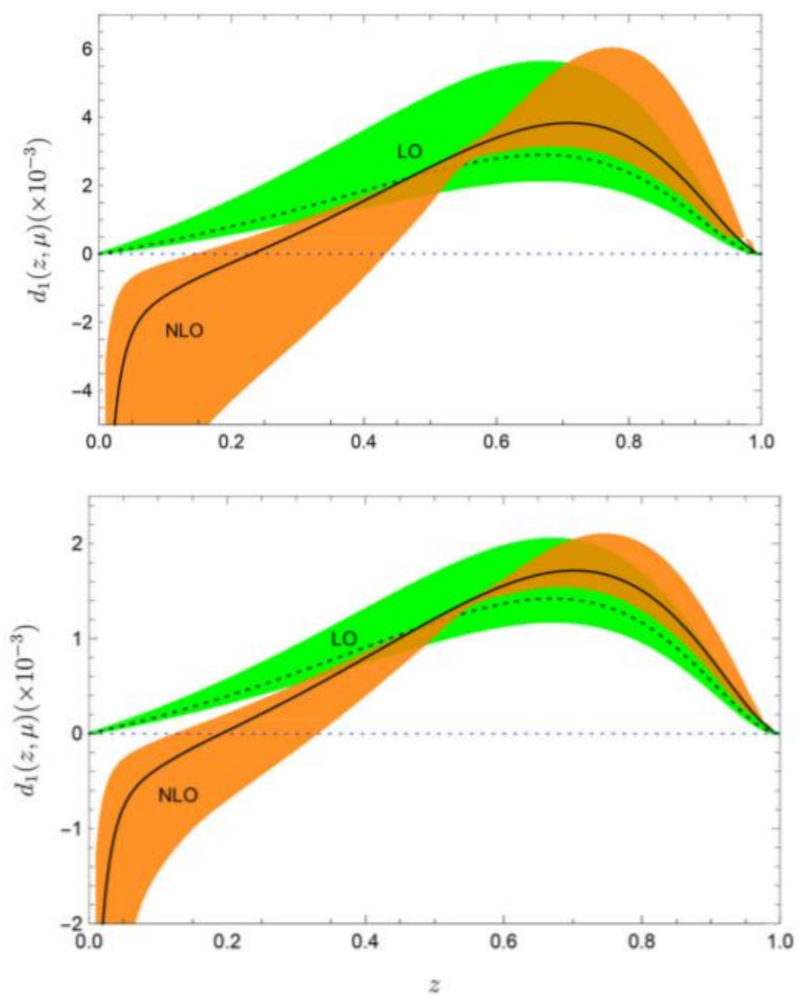
Abstract Within NRQCD factorization framework, in this work we compute, at the lowest order in velocity expansion, the next-to-leading-order (NLO) perturbative corrections to the short-distance coefficients associated with heavy quark fragmentation into the ${}^1S_0^{(1,8)}$ components of a heavy quarkonium. Starting from the Collins and Soper's operator definition of the quark fragmentation function, we apply the sector decomposition method to facilitate the numerical manipulation. It is found that the NLO QCD corrections have a significant impact.

hadronization mechanism. Similar to parton distribution functions, the scale dependence of FFs is governed by the celebrated Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equation:

$$\frac{d}{d\ln\mu^2} D_{i\to H}(z,\mu) = \sum_i \int_z^1 \frac{dy}{y} P_{ji}(y,\alpha_s(\mu)) D_{j\to H}\left(\frac{z}{y},\mu\right). \quad (2)$$

Note added After this work was completed and while we were preparing the manuscript, very recently a preprint [52] has appeared, which also computes the NLO perturbative corrections to the heavy quark fragmentation into a ${}^1S_0^{(1)}$ quarkonium. Their numerical results appear to be compatible with ours in this color-singlet channel.



Conclusions

- A **super Z factory** can provide a good platform for studying the properties of doubly heavy hadrons;
- The **NLO fragmentation functions** for a quark into a doubly heavy meson (Bc, J/ψ, Υ, η_c, η_b) have been obtained.
- The experimental studies on these **fragmentation functions** are expected.

Thank you！