Quantum corrections, neutrino masses and new oscillation phenomena

Pedro Machado

December, 2021

Based on Babu, Brdar, de Gouvêa, M 2108.11961

Outline

What are quantum corrections?

Where do neutrino masses come from?

How can quantum corrections and the mechanism of neutrino masses leave an imprint on oscillation phenomenology?

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Question 1:

What are quantum corrections?

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What are quantum corrections?

How do we predict phenomena?

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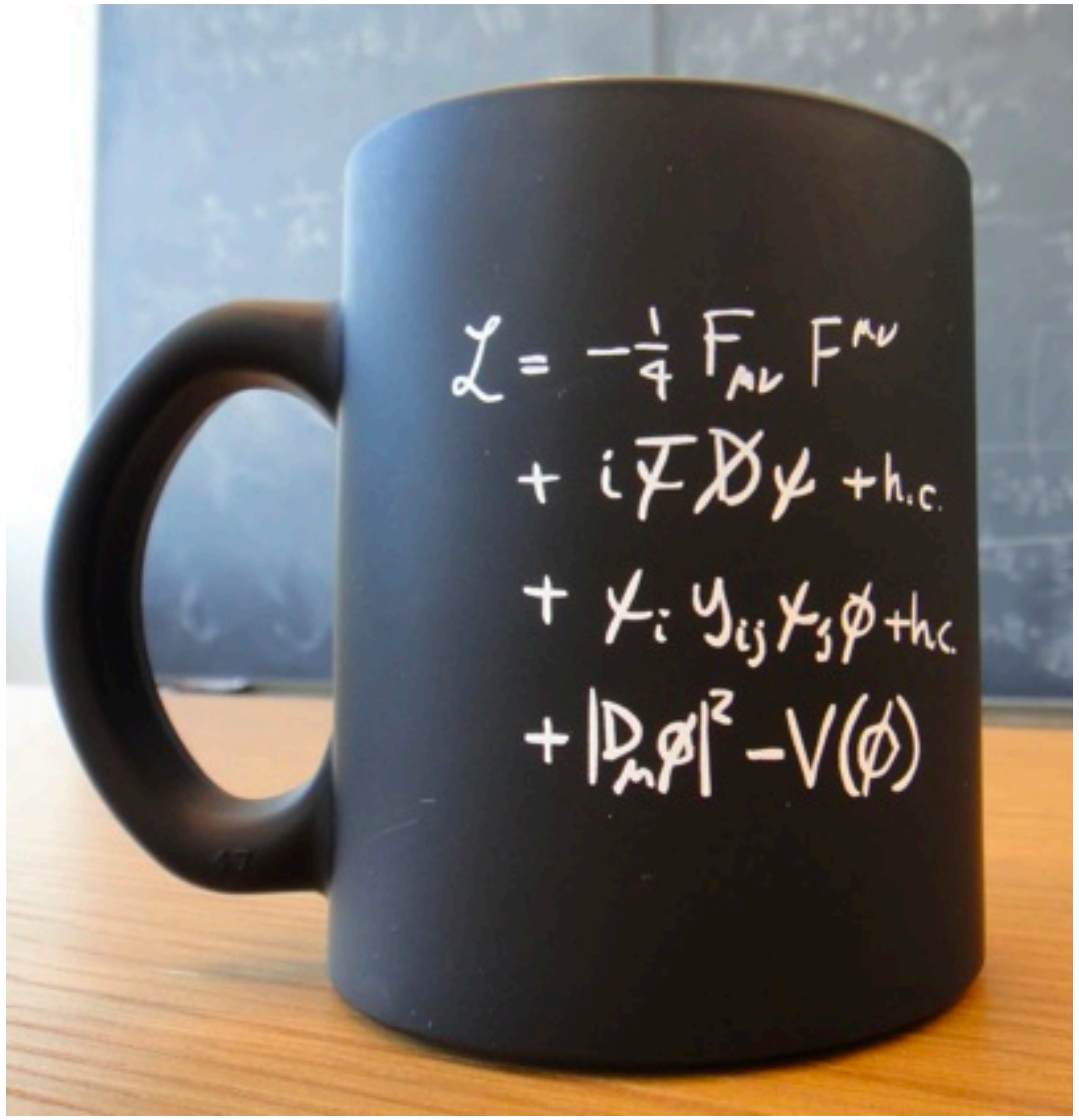
Start with a “master equation”



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What are quantum corrections?

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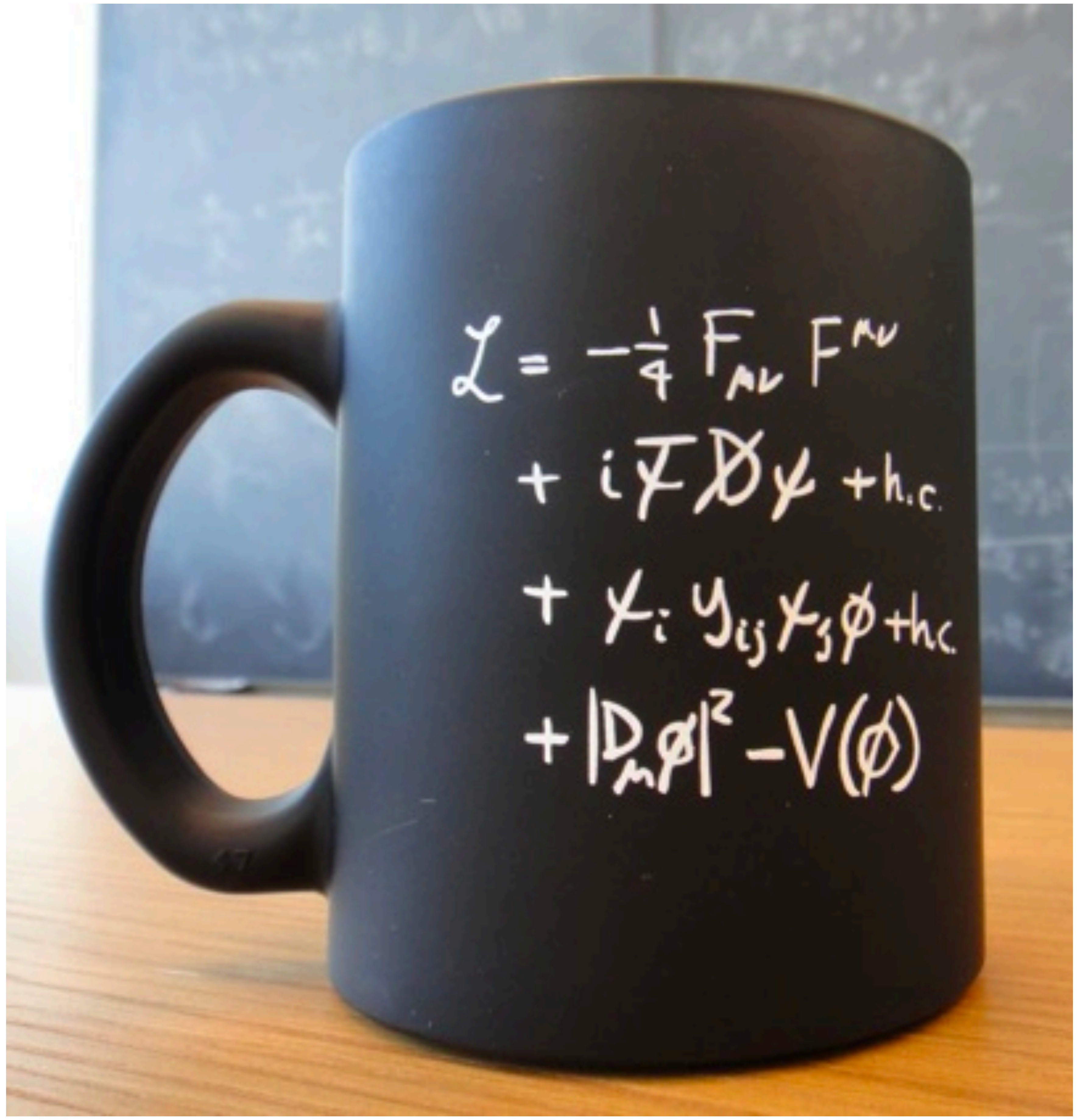

Now, solve it

Evolve $|a\rangle$ to $|b\rangle$:

$$\langle b | \exp(-iHt) | a \rangle = U(a, b; t)$$

$$U(a,b;t) = \int \mathcal{D}x(t) \exp\left(i \int dt' \mathcal{L}\right)$$

All possible paths

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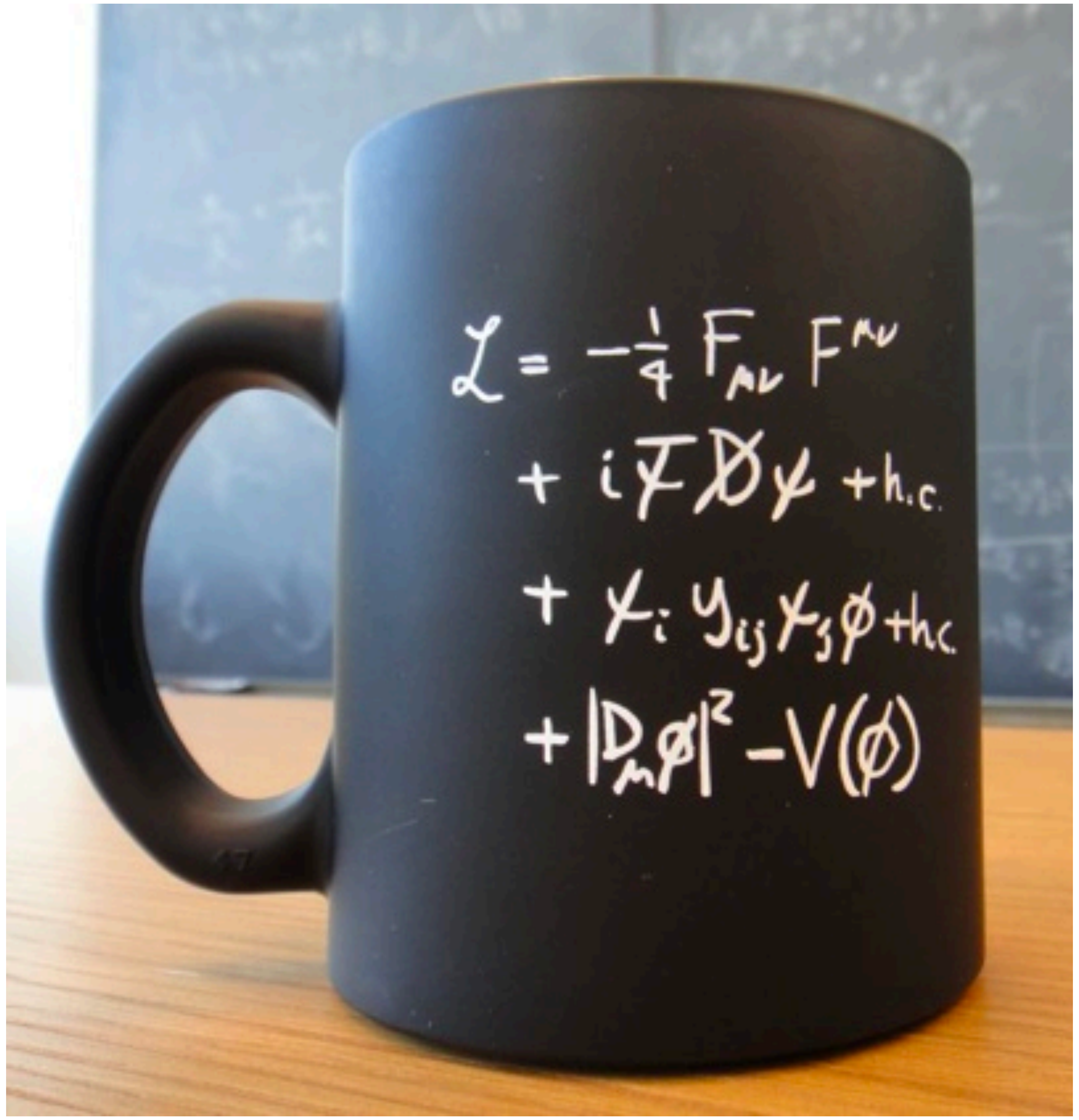

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All possible paths

We can’t do it exactly, we’ve got to rely on **perturbation theory**

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What are quantum corrections?

The perturbation theory we use is similar to a Taylor expansion
From the Lagrangian to the “Taylor expansion” terms is not obvious.

Take electron-electron scattering

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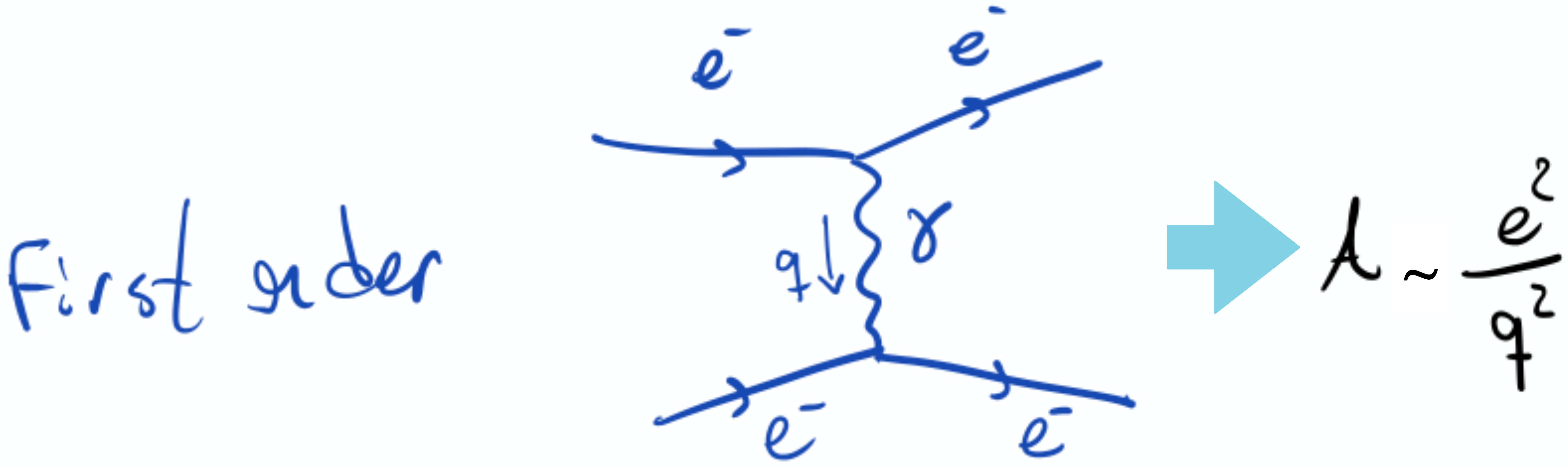

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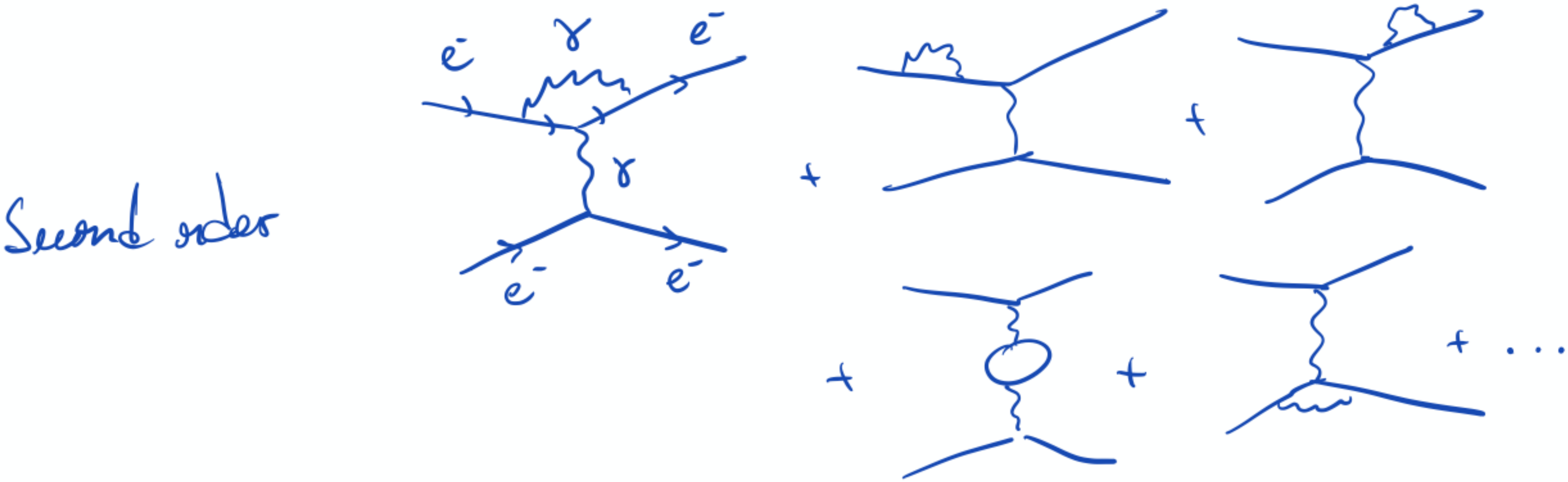

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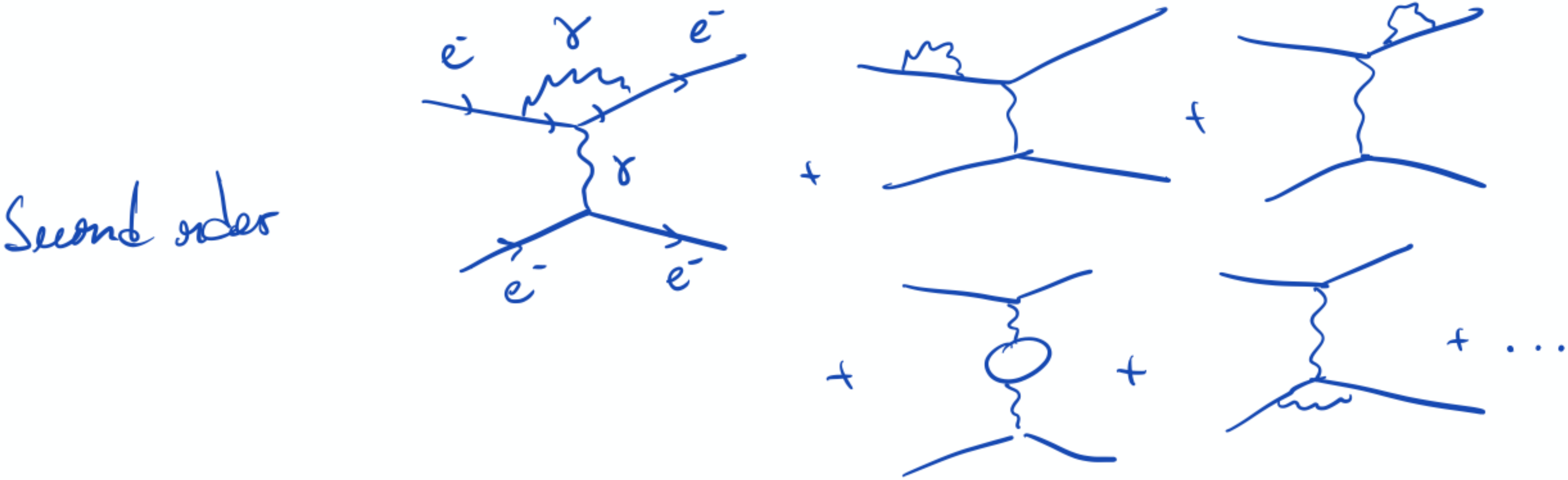
What are quantum corrections?

These higher order corrections on the "Taylor expansion" are the **quantum corrections**



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What are quantum corrections?

Now, it would be great to redefine our Lagrangian in a way that can easily account for quantum corrections

$$A(1st + 2nd + \dots) \sim \frac{e^2}{q^2}$$

We can redefine constants to absorb the higher order effects

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an organization principle to deal with quantum corrections

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an organization principle to deal with quantum corrections

The crux: quantum corrections depend on the scale of the process

What are quantum corrections?

By measuring observables at several different scales,
we have confirmed the *running* of constants

Here are three examples

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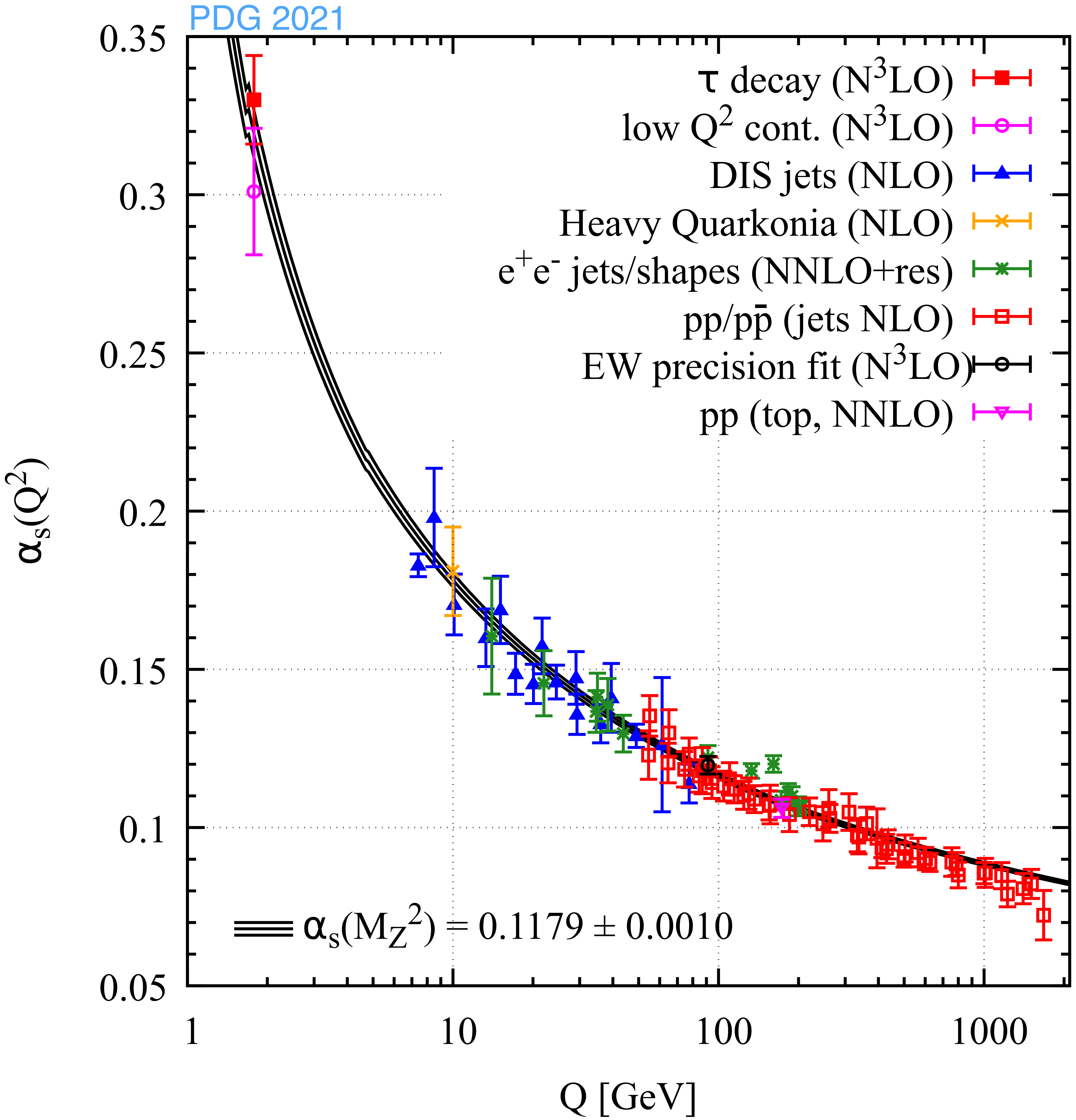
Strong interaction coupling

How do we see it?

- QCD production at e+e− colliders
- Deep inelastic scattering observables
- QCD jet production at hadron colliders
- …



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What are quantum corrections?

Weak mixing angle

How do we see it?
- Weak interaction cross sections
- Ratio between Z and W boson masses
- Parity violation observables
- …

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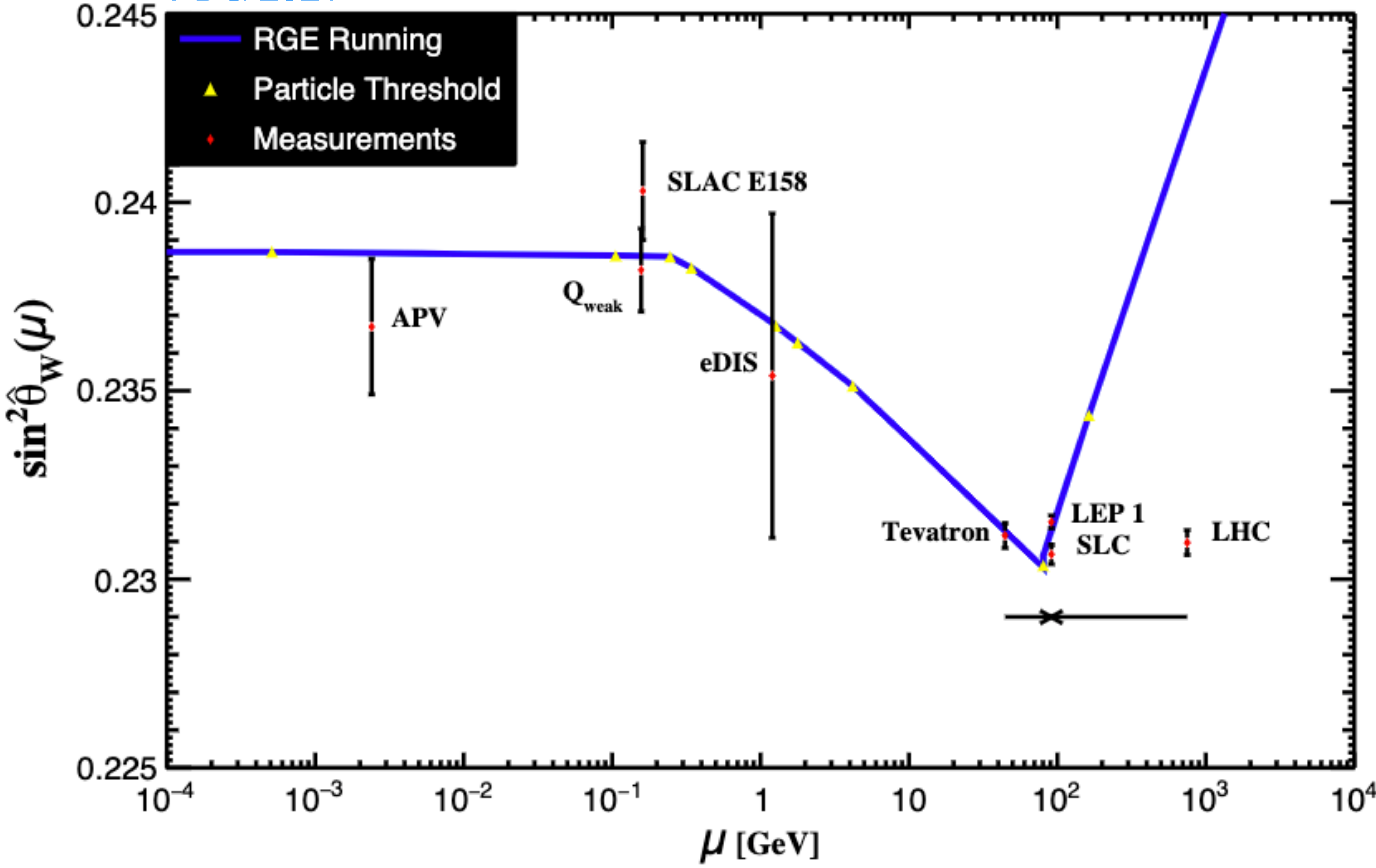

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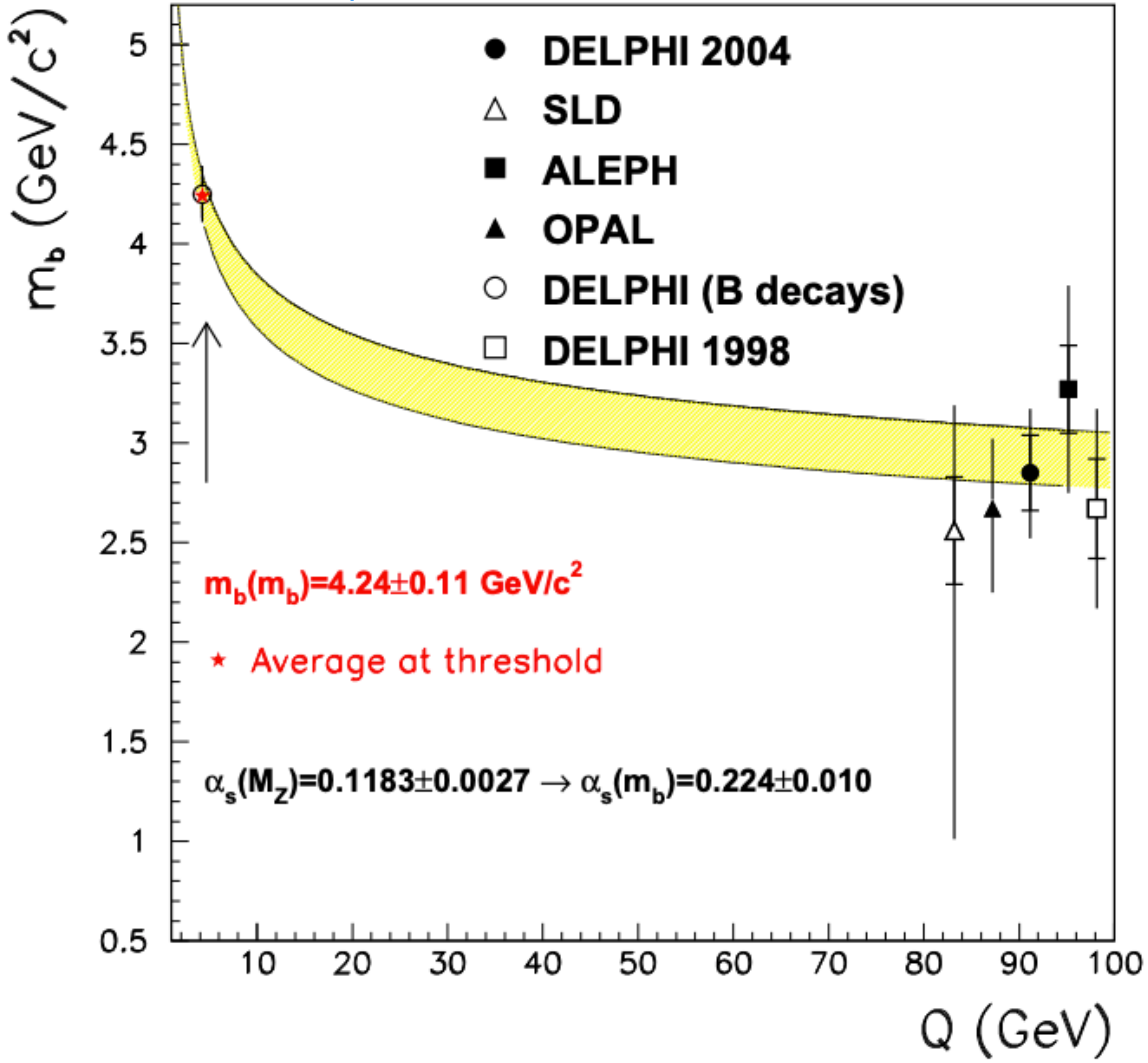
What are quantum corrections?

Mass of the b quark

How do we see it?

- b-jet observables near the Z mass scale

DELPHI hep-ex/0603046



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What are quantum corrections?

"Constants" actually run

Their values depend on the scale at which we are measuring some physical process

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Question 2:

Where do neutrino masses come from?

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Where do neutrino masses come from?

Simply put: we don’t know.

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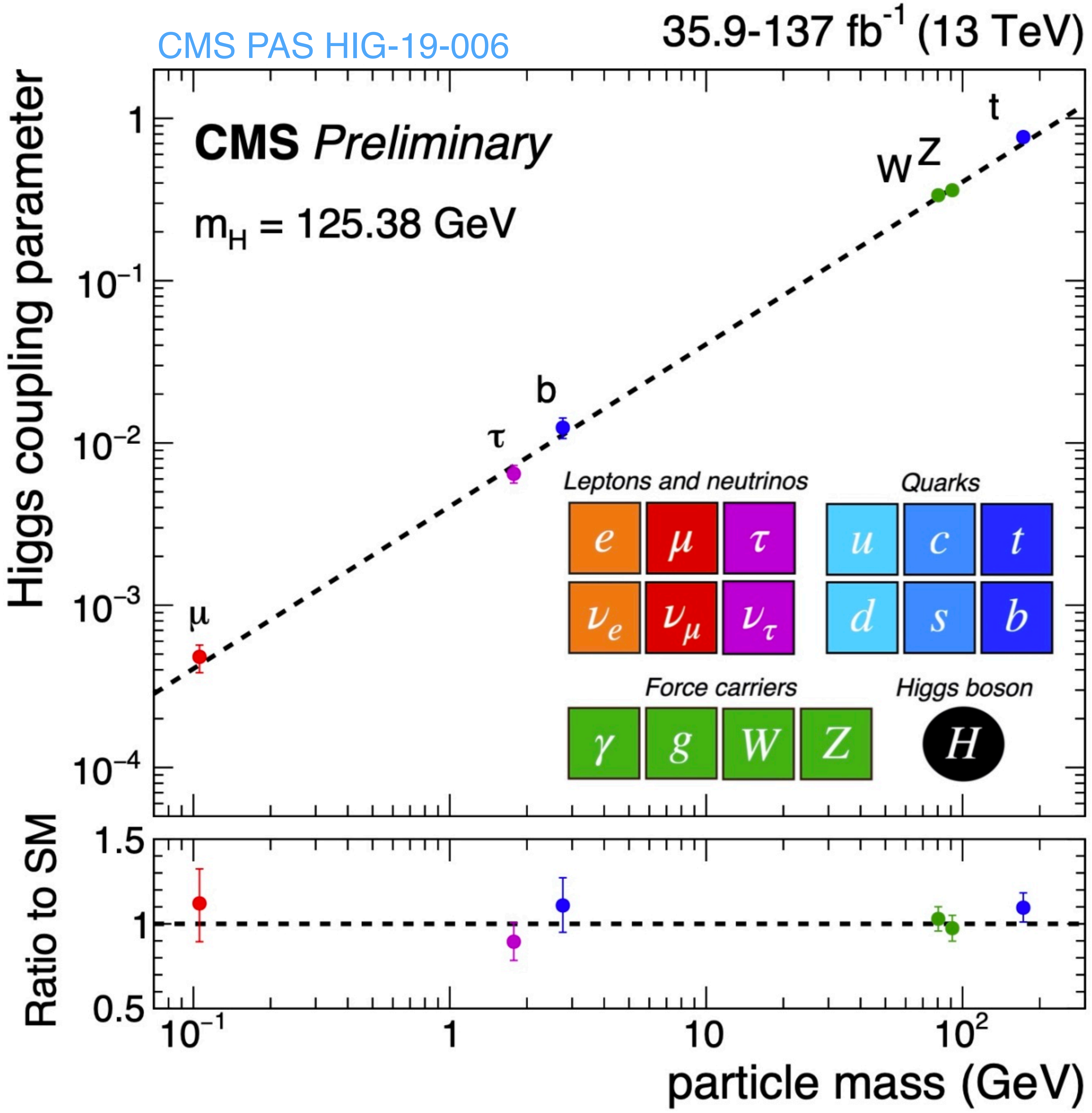
Where do neutrino masses come from?

It seems that the Higgs mechanism gives mass to the gauge bosons and third family charged fermions

Same mechanism can be present for all other charged fermions

What about neutrinos?



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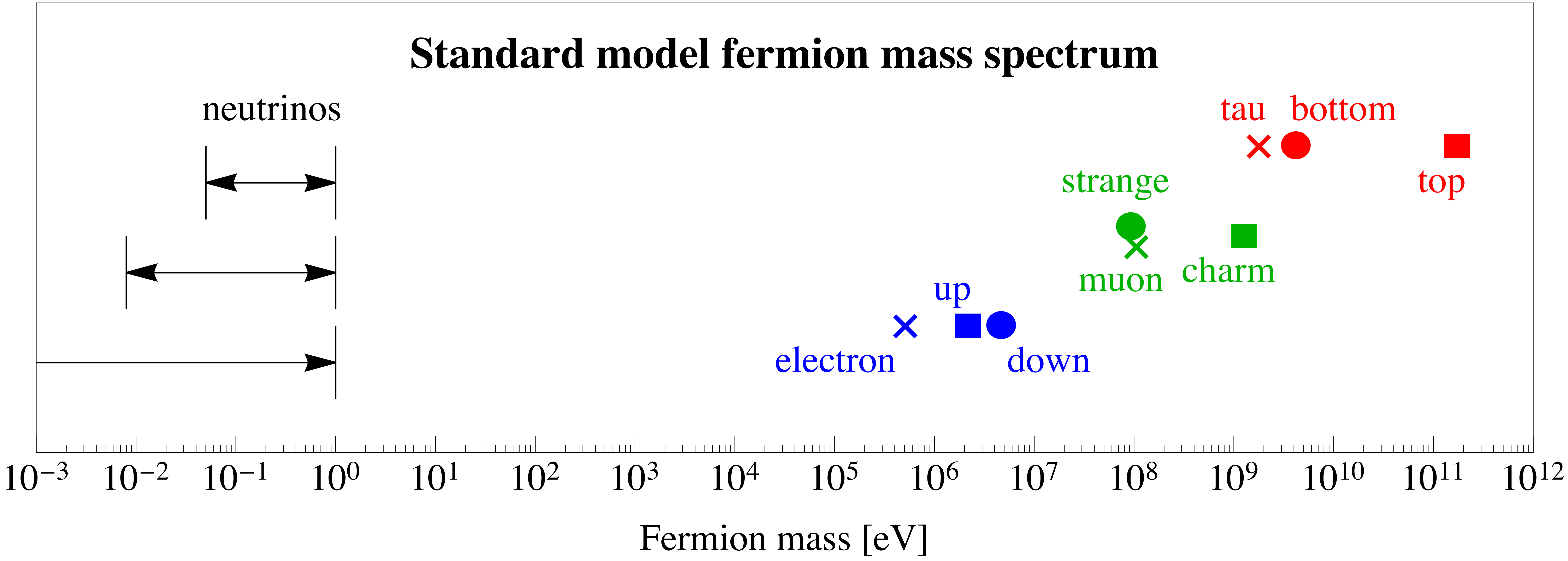
Where do neutrino masses come from?



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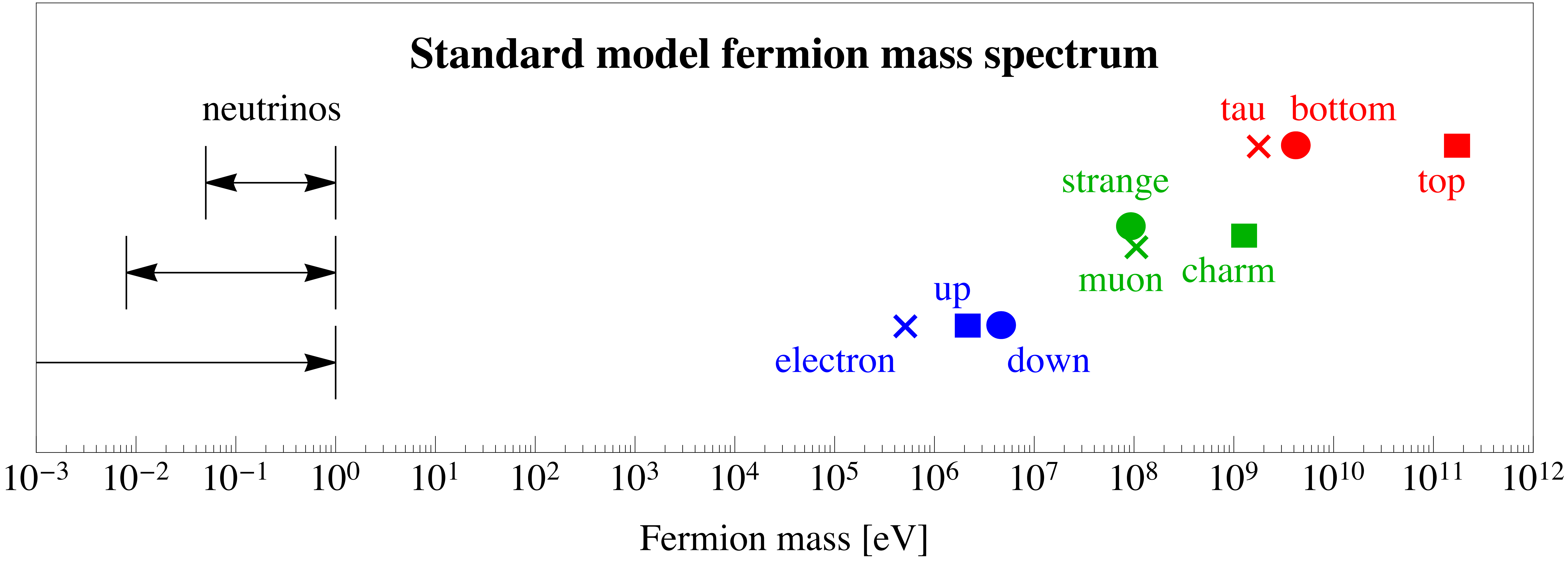

Maybe neutrino masses are small because they are suppressed by a large scale

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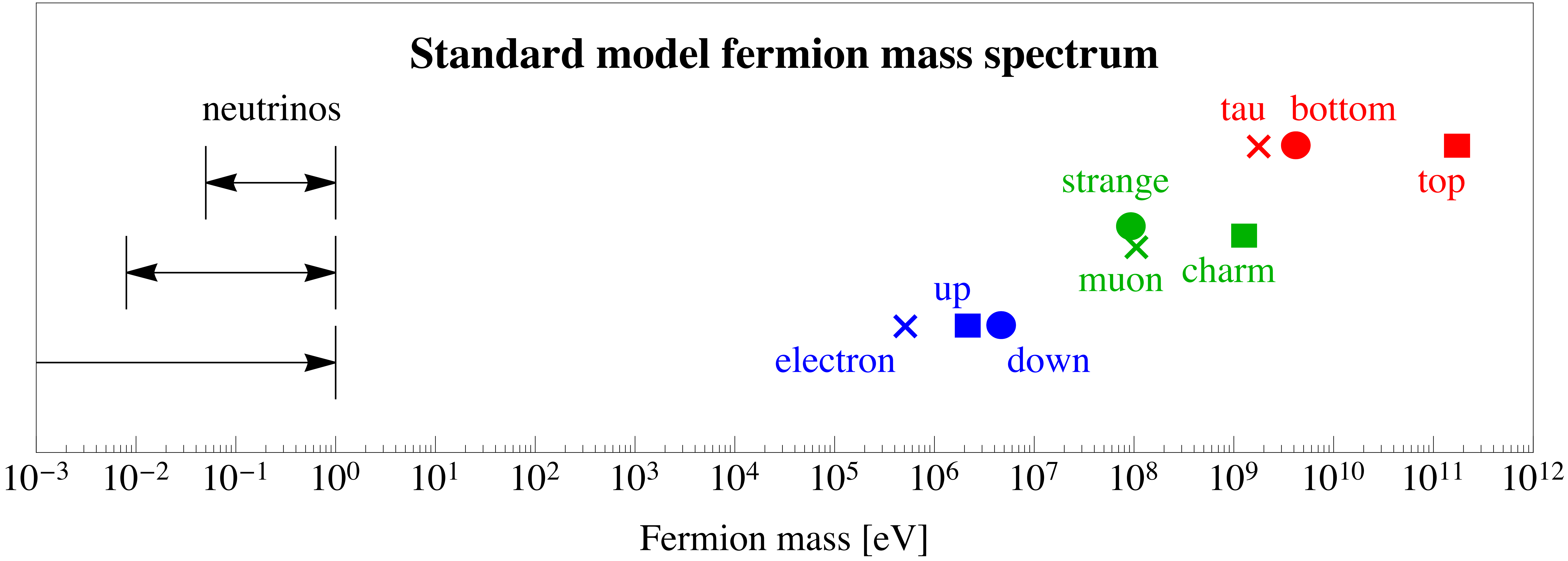
Where do neutrino masses come from?



Maybe neutrino masses are small because they are suppressed by a large scale

or maybe neutrino masses are small because the scale of the mass mechanism is low

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Where do neutrino masses come from?

Regardless, neutrino masses are nonzero

since neutrinos mix and oscillate

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Where do neutrino masses come from?

Mixing phenomenon arrives from a mismatch
between the weak interaction and the mass mechanism

$$\mathcal{L}_{weak} \sim \frac{g}{\sqrt{2}} \bar{\nu}_{\alpha L} \slashed{W}^{+} \ell_{\alpha} + h.c.$$

$$\mathcal{L}_{mass} \sim \bar{\nu}_{\alpha} M^{\nu}_{\alpha\beta} \nu_{\beta} + \bar{\ell}_{\alpha} M^{\ell}_{\alpha\beta} \ell_{\beta} + h.c.$$

Not diagonal

$$\Rightarrow \nu_{\alpha} = \sum_i U_{\alpha i} \nu_i$$

PMNS matrix

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Where do neutrino masses come from?

Mixing phenomenon arrives from a mismatch
between the weak interaction and the mass mechanism

$$U = R_{23} V_{13} R_{12}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{bmatrix}$$

$$R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

$$V_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}$$

$$R_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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Question 3:

How can quantum corrections
and the mechanism of neutrino masses
leave an imprint on oscillation phenomenology?

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How can quantum corrections and the mechanism of neutrino masses leave an imprint on oscillation phenomenology?

If the neutrino mass mechanism takes place at low scales,
there could be significant running of the PMNS matrix

Precision neutrino oscillation physics can be a portal to one of the outstanding questions of the standard model: the mechanism of neutrino masses

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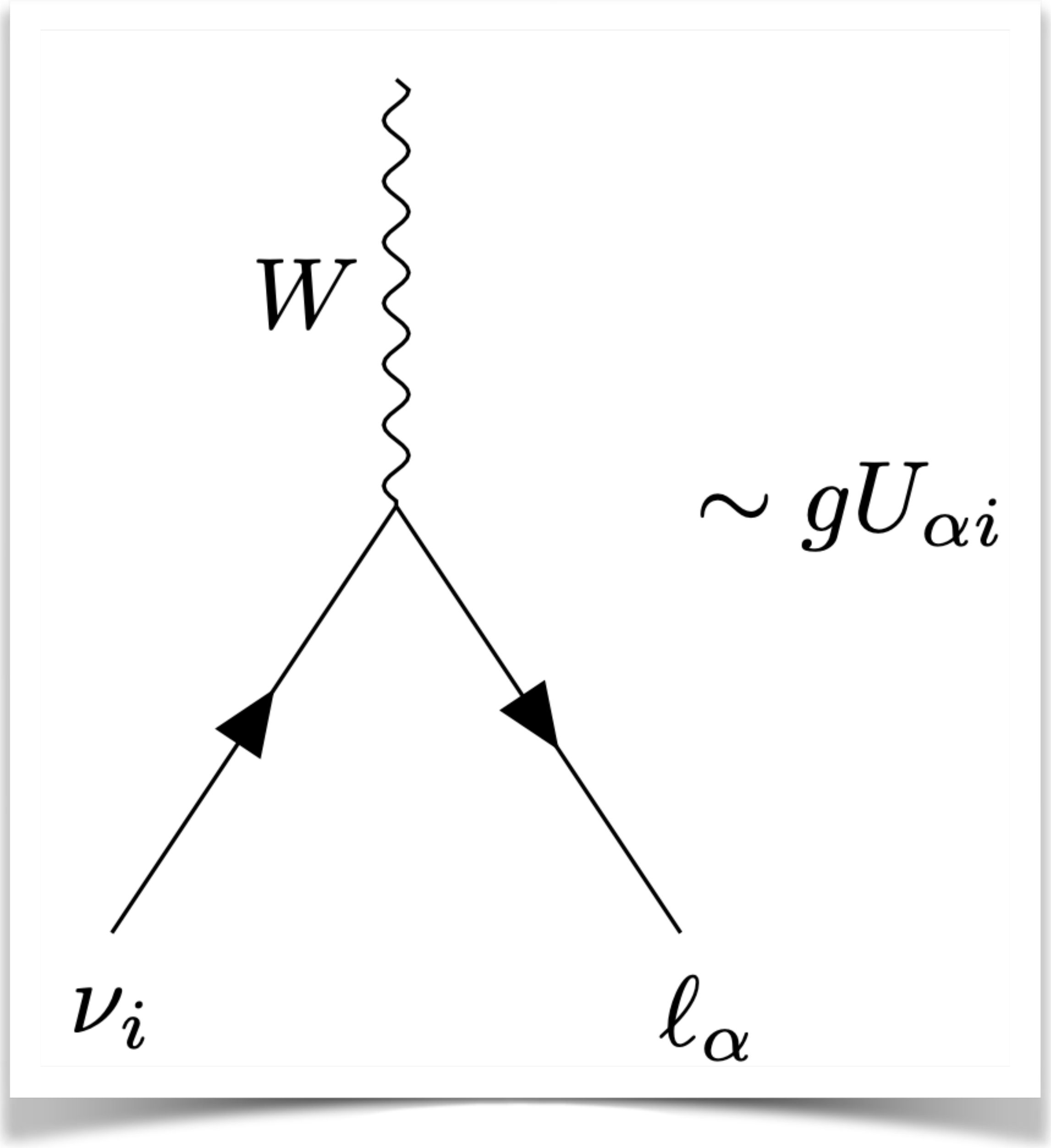
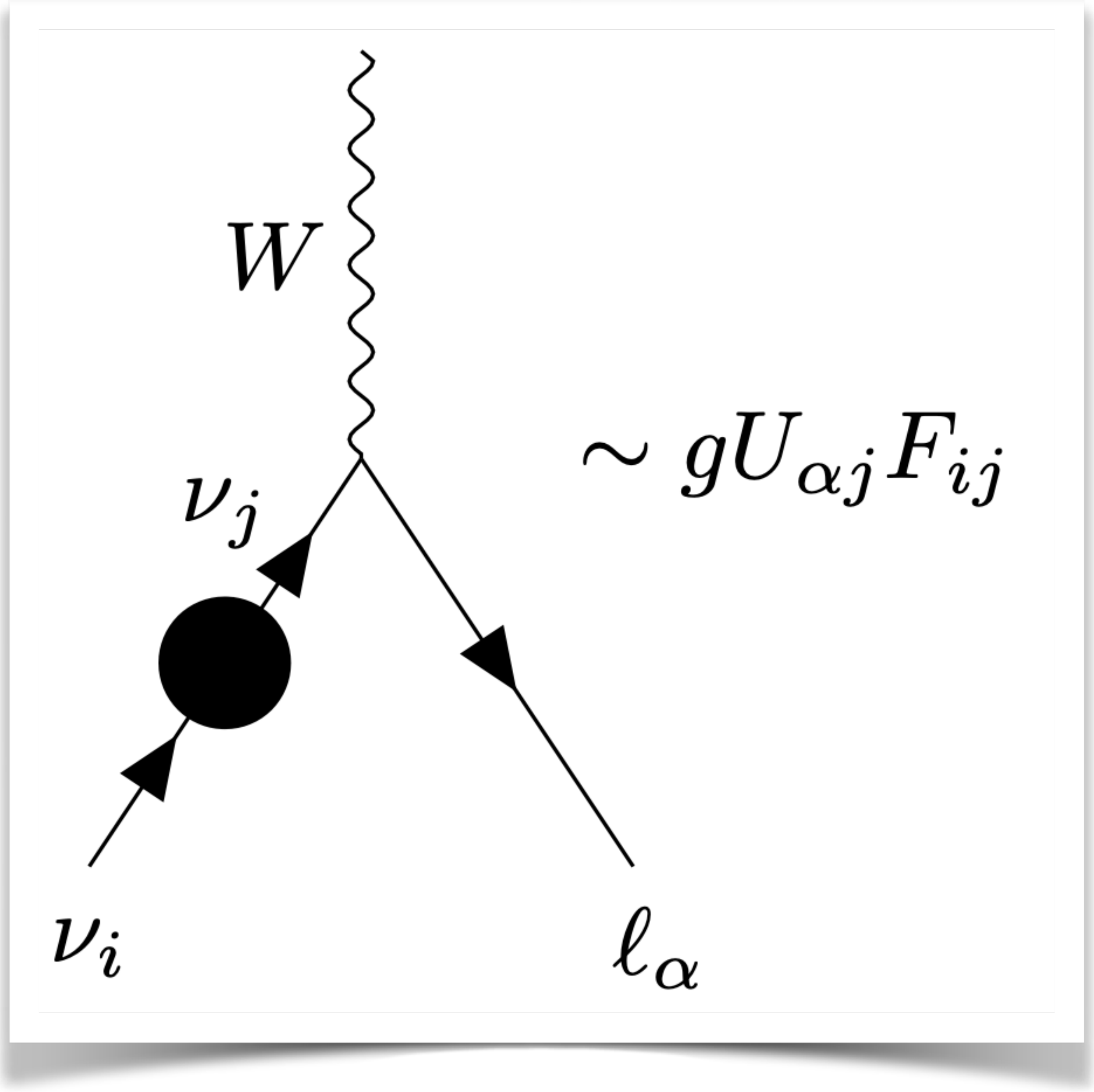
Energy dependent neutrino mixing





If there are significant quantum corrections to the neutrino mass matrix at low scales, the PMNS matrix becomes scale dependent.

This means that production and detection of neutrinos may not go via the same PMNS matrices.

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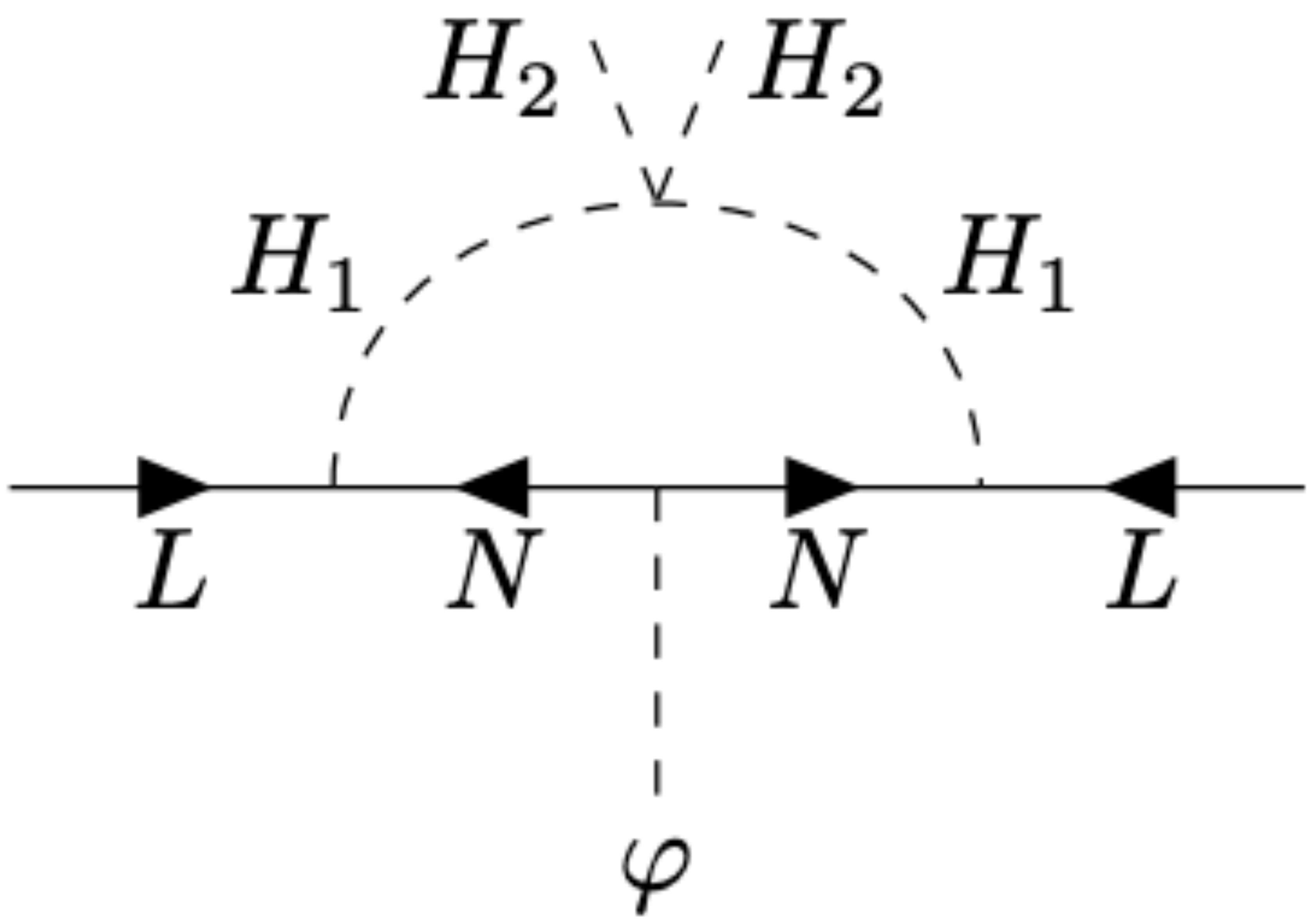
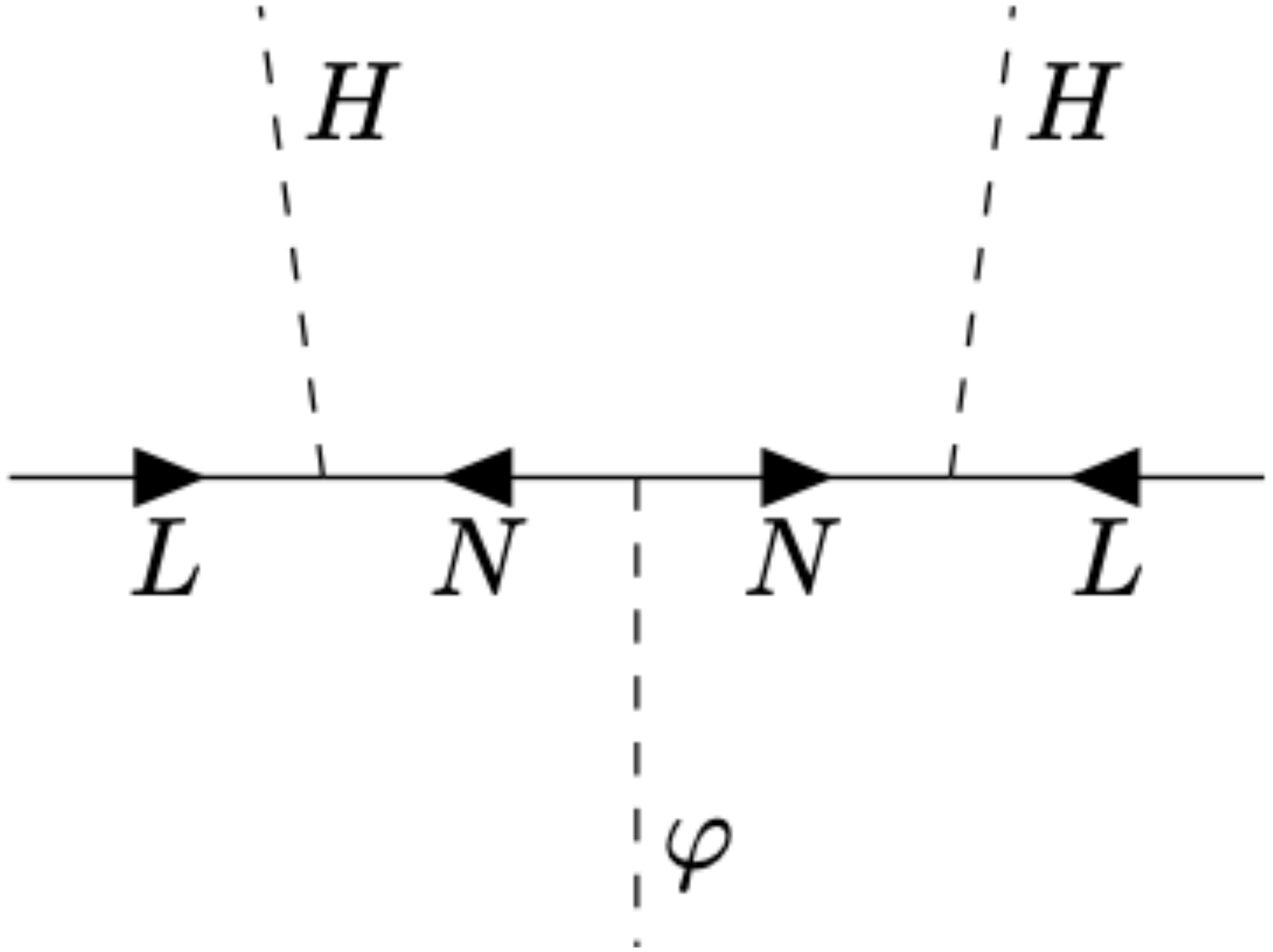
Energy dependent neutrino mixing

Possible mass mechanisms

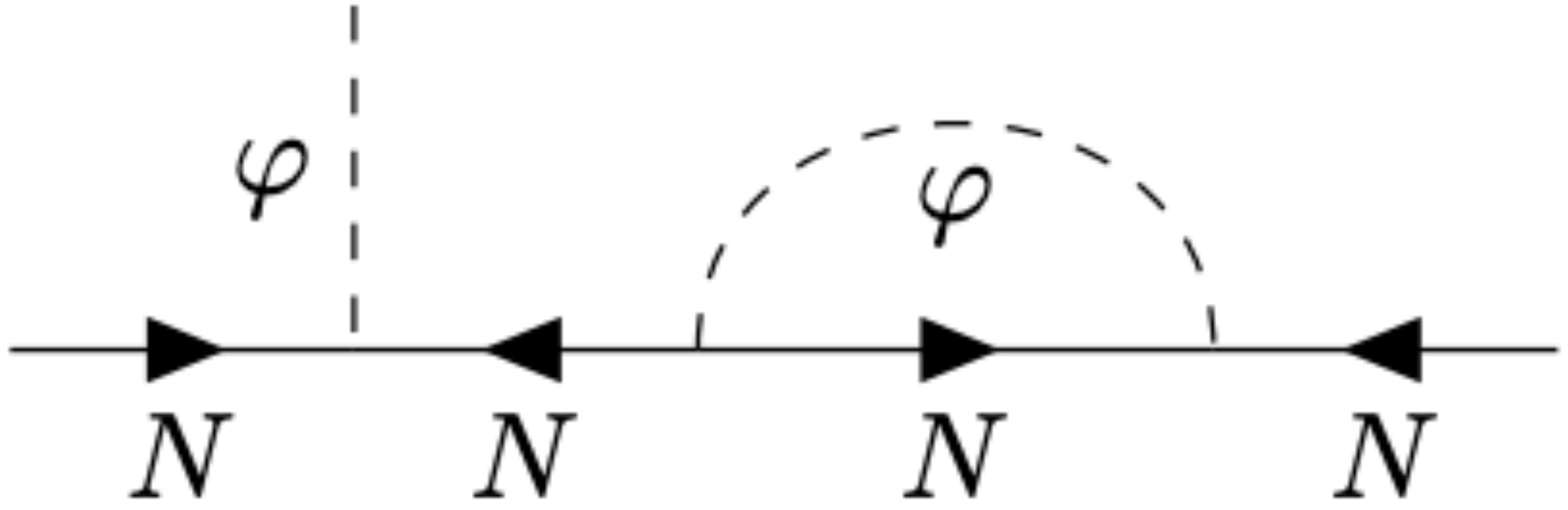




$$M_\nu \simeq \frac{v_\varphi}{16\sqrt{2}\pi^2} Y_\nu Y_N Y_\nu^T \ln \frac{M_H^2}{M_A^2}$$

$$16\pi^2 \frac{dY_N}{d\ln|Q|} = 4Y_N \left[Y_N^2 + \frac{1}{2}\mathrm{Tr}(Y_N^2) \right]$$

Diagram contributing to the running



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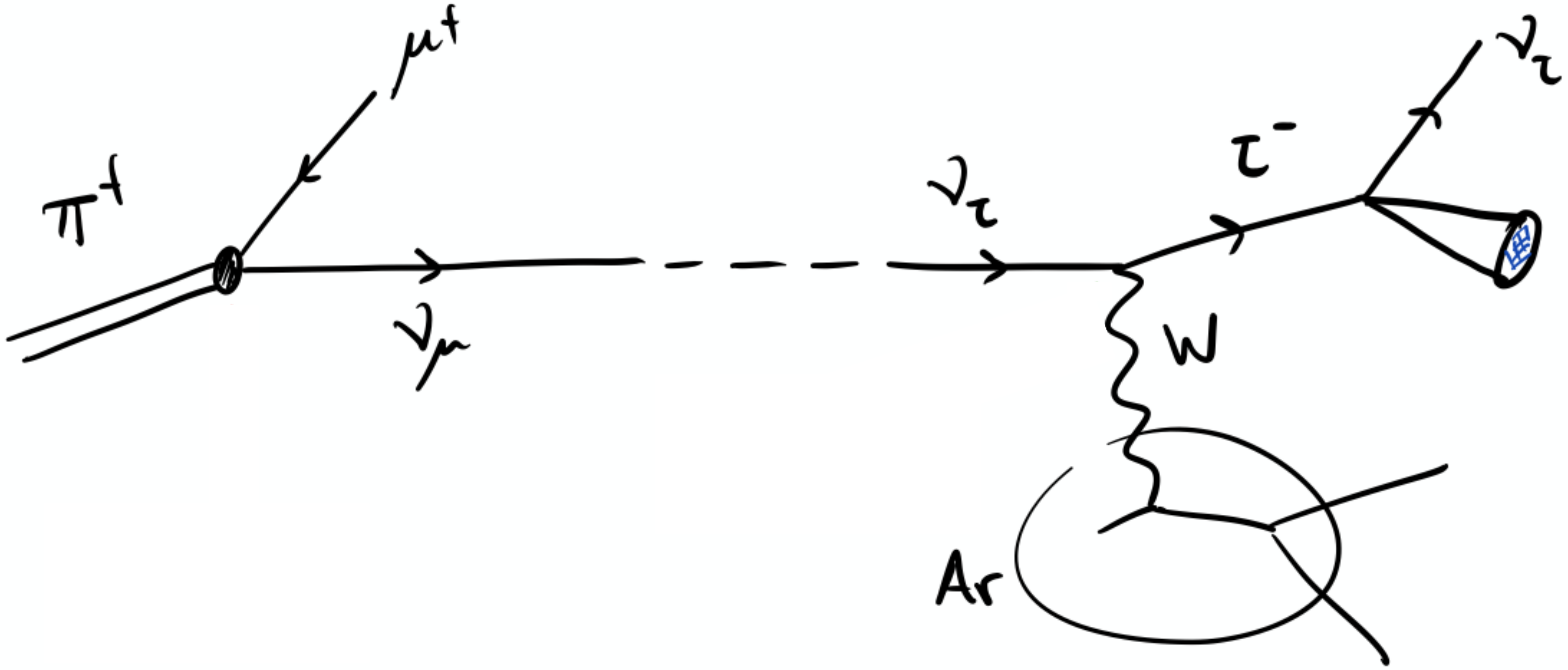

Standard case

$$A(\nu_\mu \to \nu_\tau) = \langle \nu_\tau | \exp(-iHL) | \nu_\mu \rangle$$
$$= \sum_i U_{\tau i} U^*_{\mu i} \exp\left(-\frac{i m_i^2 L}{2E}\right)$$

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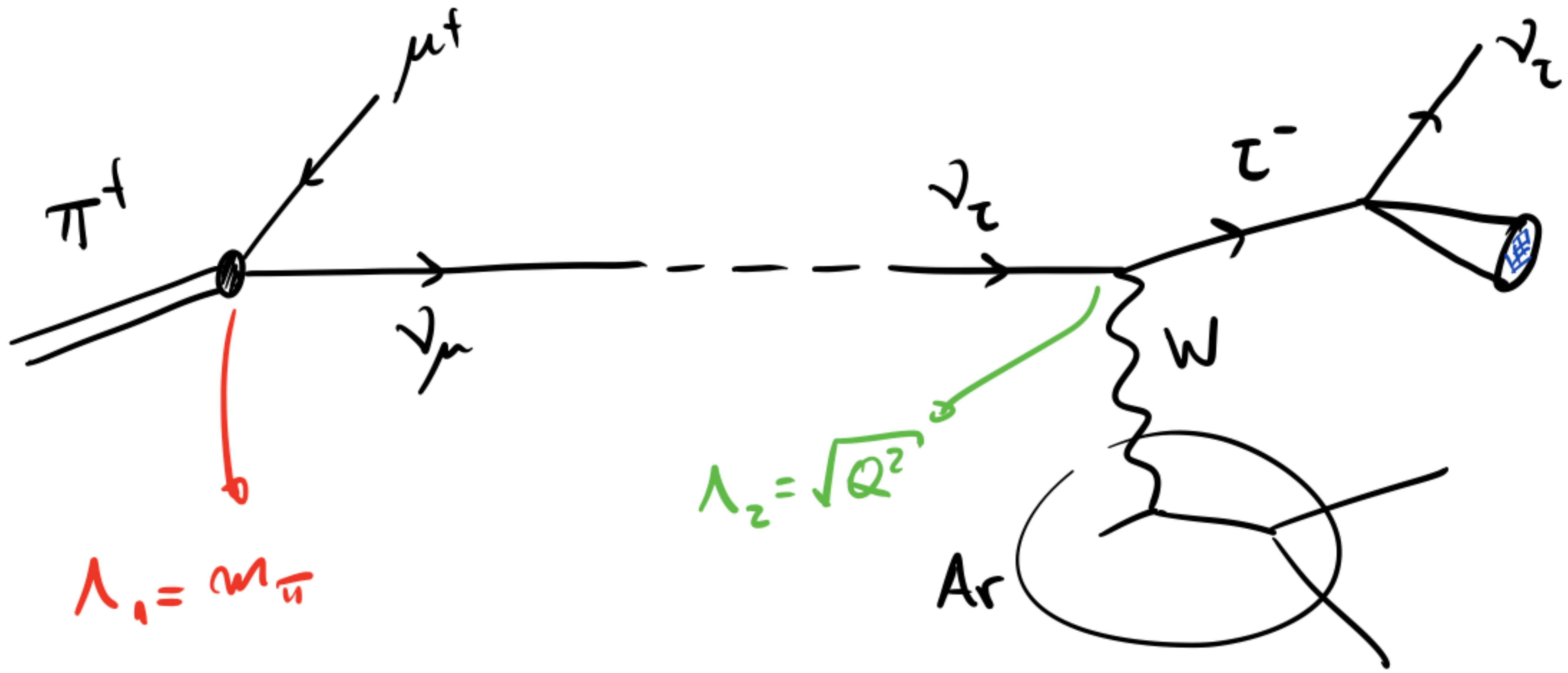
Energy dependent neutrino mixing



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With E dependent effects:

$$\mathcal{A}(\nu_\mu \to \nu_\tau) = \langle \nu_\tau, Q_2^2 | \exp(-iHL) | \nu_\mu, Q_1^2 \rangle = \sum_i U_{\tau i}(Q_2^2)\, U^*_{\mu i}(Q_1^2) \exp\left(-\frac{i m_i^2 L}{2E}\right)$$

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Energy dependent neutrino mixing

What are the effects we would be looking for?

I will use two flavor oscillations to show simplified formulae

$$P_{e\mu} = P_{\mu e} = \sin^2(\theta_p - \theta_d) + \sin 2\theta_p \sin 2\theta_d \sin^2\left(\frac{\Delta m^2 L}{4E} + \frac{\beta}{2}\right)$$

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production (θ_p) detection (θ_d) novel phase (β)

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production detection novel phase

1) Zero distance transitions: β is induced due to PMNS mismatch (though it is 2nd order)

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production (θ_p) detection (θ_d) novel phase ($\beta/2$)

1) Zero distance transitions: β is induced due to PMNS mismatch (though it is 2nd order)
2) Finite L: $P_{\alpha\beta} \neq P_{\bar{\alpha}\bar{\beta}}$, as the phase flips sign (*apparent* CPT violation)

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3) Distortions of oscillation probability, since $\theta_{p,d}$ depend on energy

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Energy dependent neutrino mixing

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production (θ_p) detection (θ_d) novel phase (β)

1) Zero distance transitions: β is induced due to PMNS mismatch (though it is 2nd order)
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3) Distortions of oscillation probability, since $\theta_{p,d}$ depend on energy
4) New sources of CP violation

$$P_{\mu e} - P_{\bar{\mu}\bar{e}} \simeq -8J\Delta_{21}\sin^2\left(\frac{\Delta_{31}}{2}\right)\left[1 + \left(2\frac{\epsilon_{12}}{\sin 2\theta_{12}} + \epsilon_\alpha \frac{c_\delta}{s_\delta}\right)\frac{\cot(\Delta_{31}/2)}{\Delta_{21}}\right]$$

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Energy dependent neutrino mixing

What are the effects we would be looking for?

Short baseline constraints:

Experiment	E (GeV)	$\sqrt{Q_d^2}$ (GeV)	channel	constraint
ICARUS [64]	17	3.94	$\nu_\mu \to \nu_e$	3.4×10^{-3}
CHARM-II [65]	24	4.70	$\nu_\mu \to \nu_e$	2.8×10^{-3}
NOMAD [61–63]	47.5	6.64	$\nu_\mu \to \nu_e$	7.4×10^{-3}
			$\nu_\mu \to \nu_\tau$	1.63×10^{-4}
NuTeV [66, 67]	250	15.30	$\nu_\mu \to \nu_e$	5.5×10^{-4}
			$\nu_e \to \nu_\tau$	0.1
			$\nu_\mu \to \nu_\tau$	9×10^{-3}

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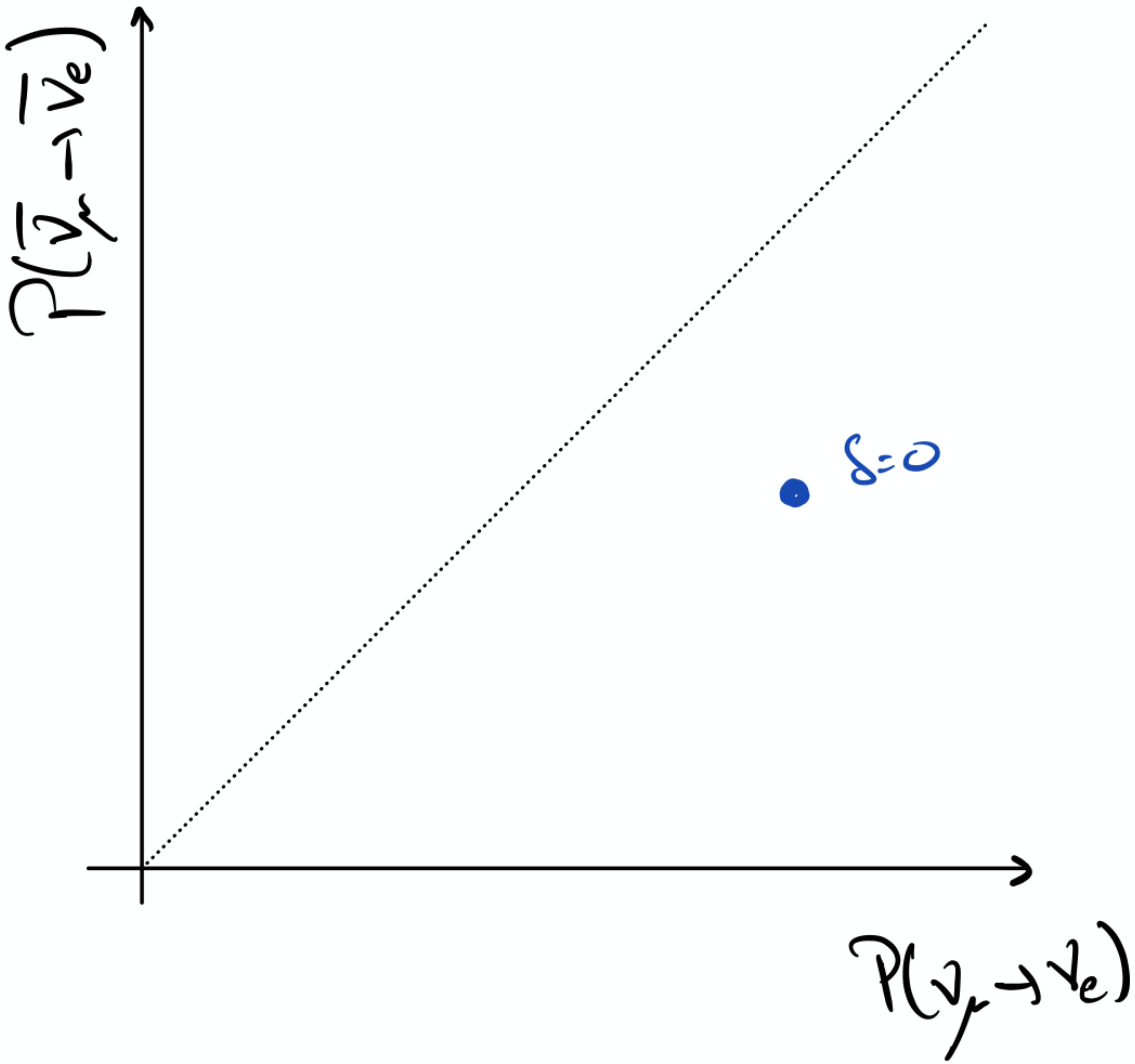
Bi-probability plots



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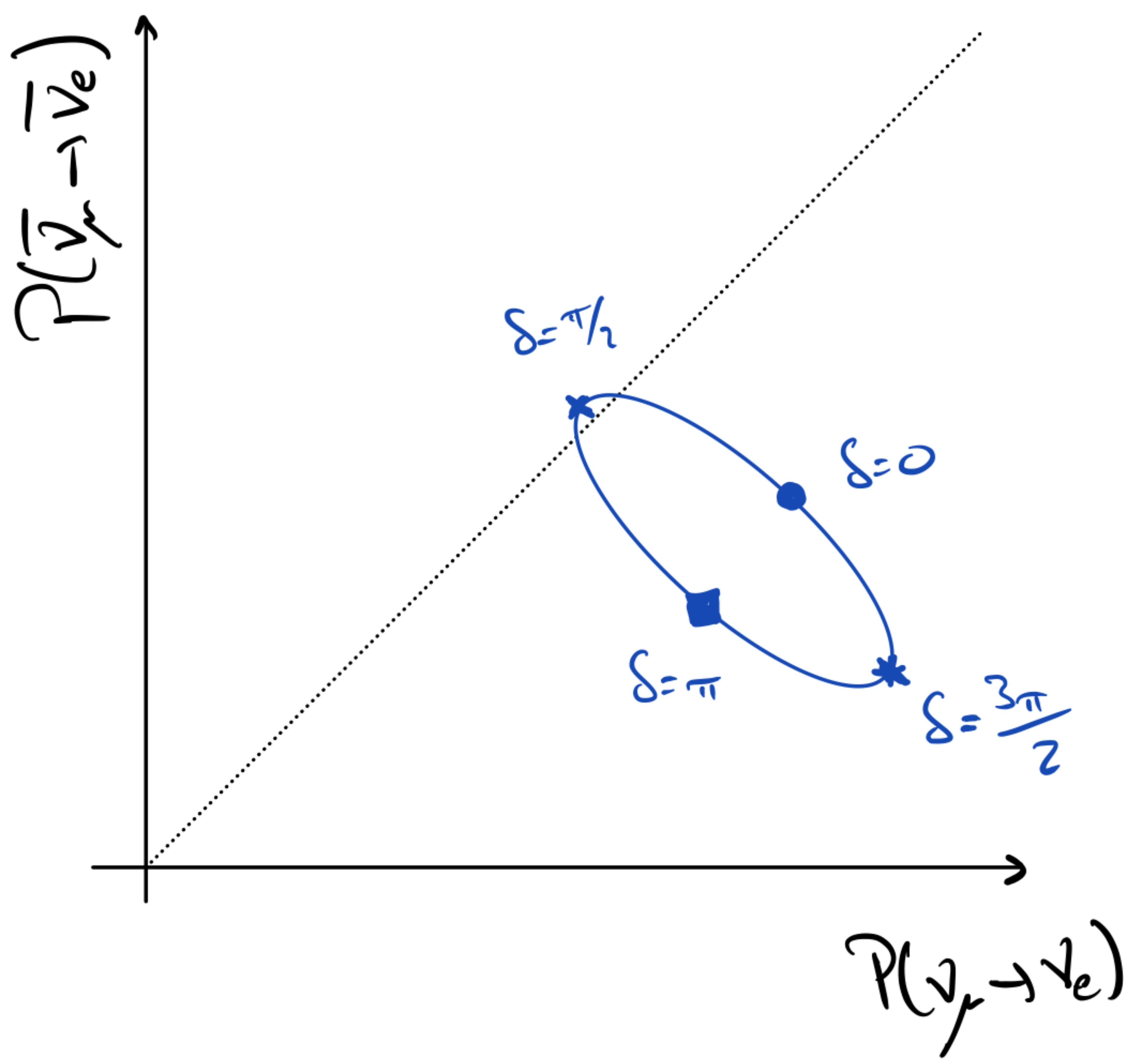

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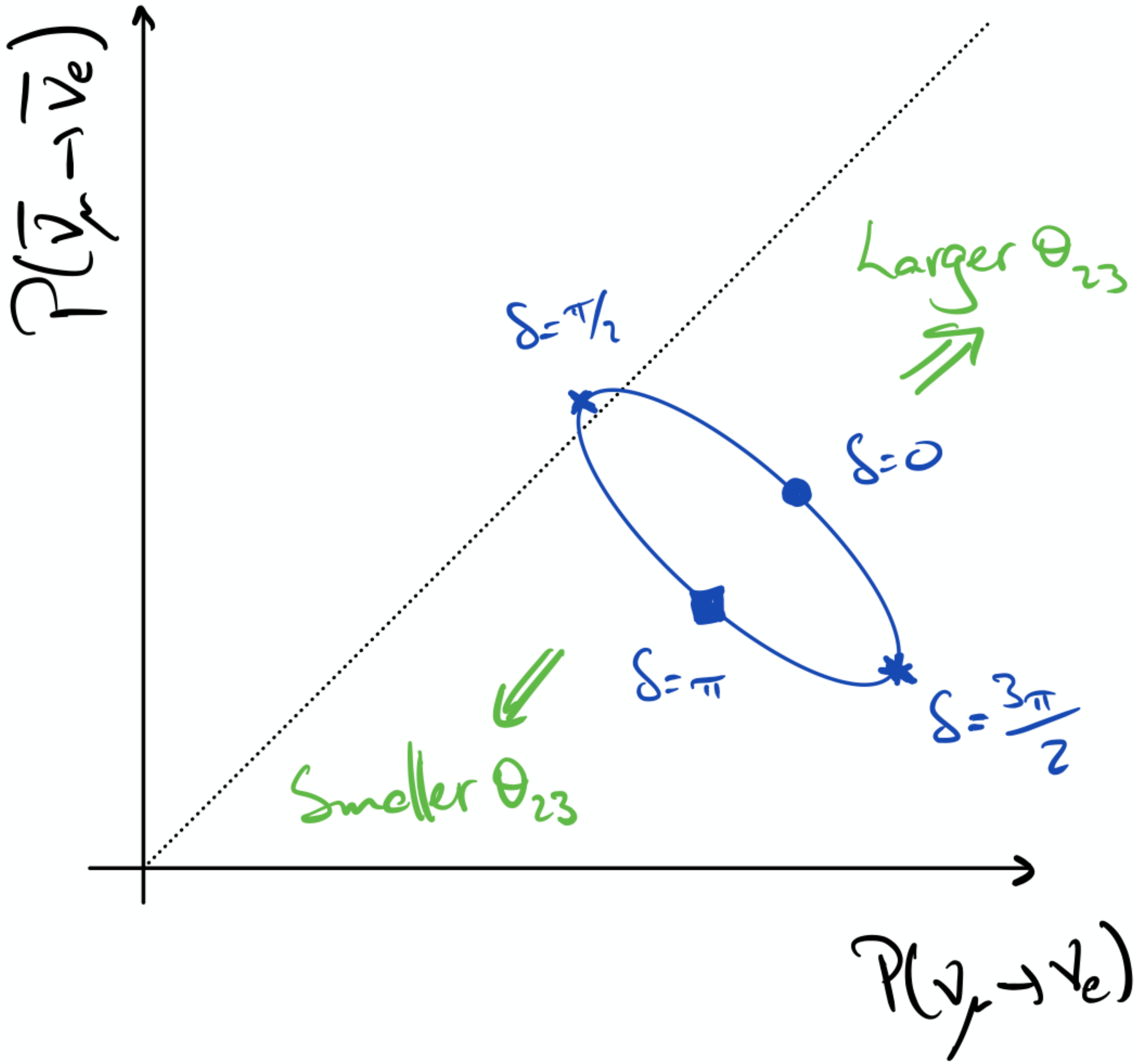

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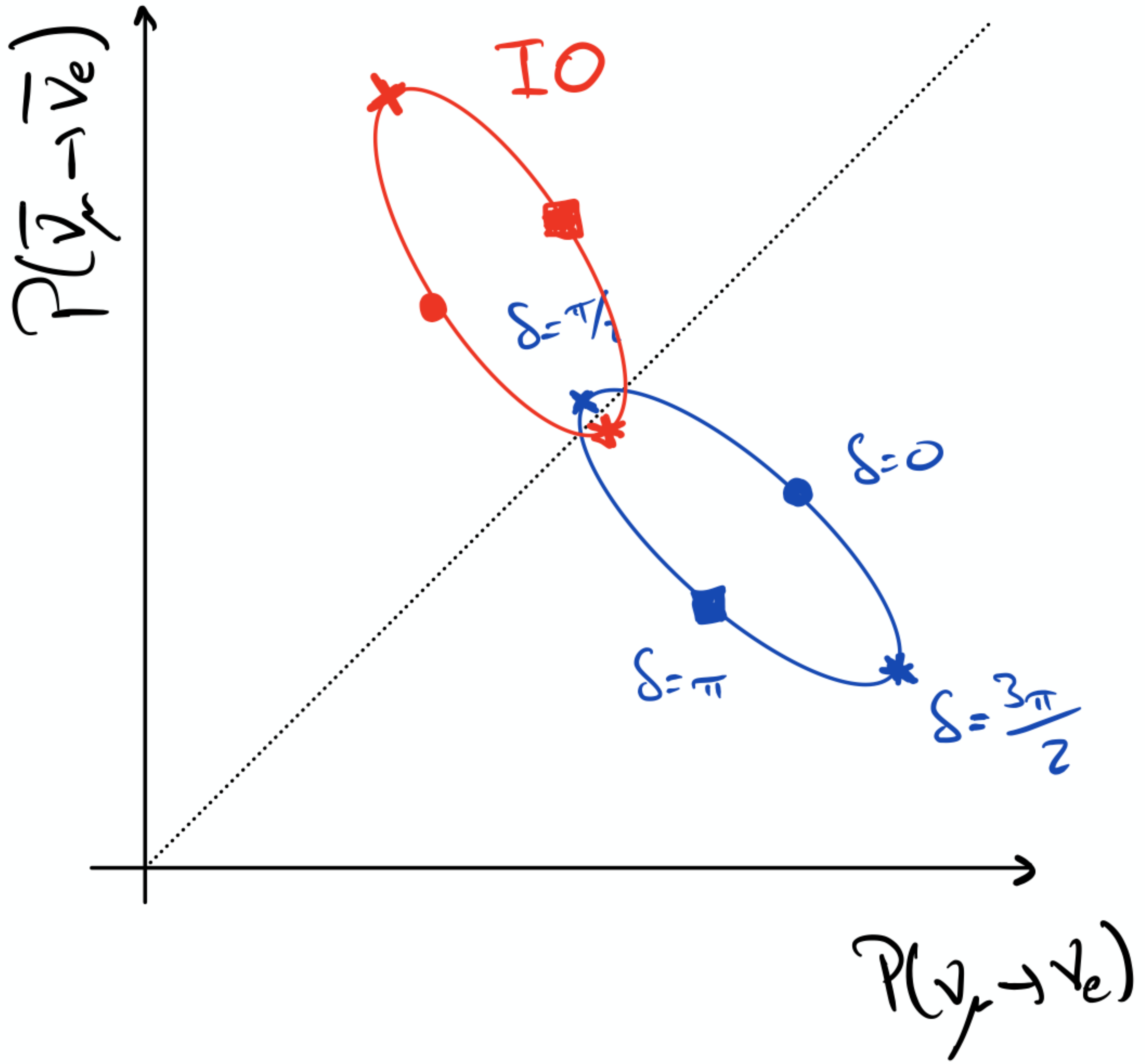

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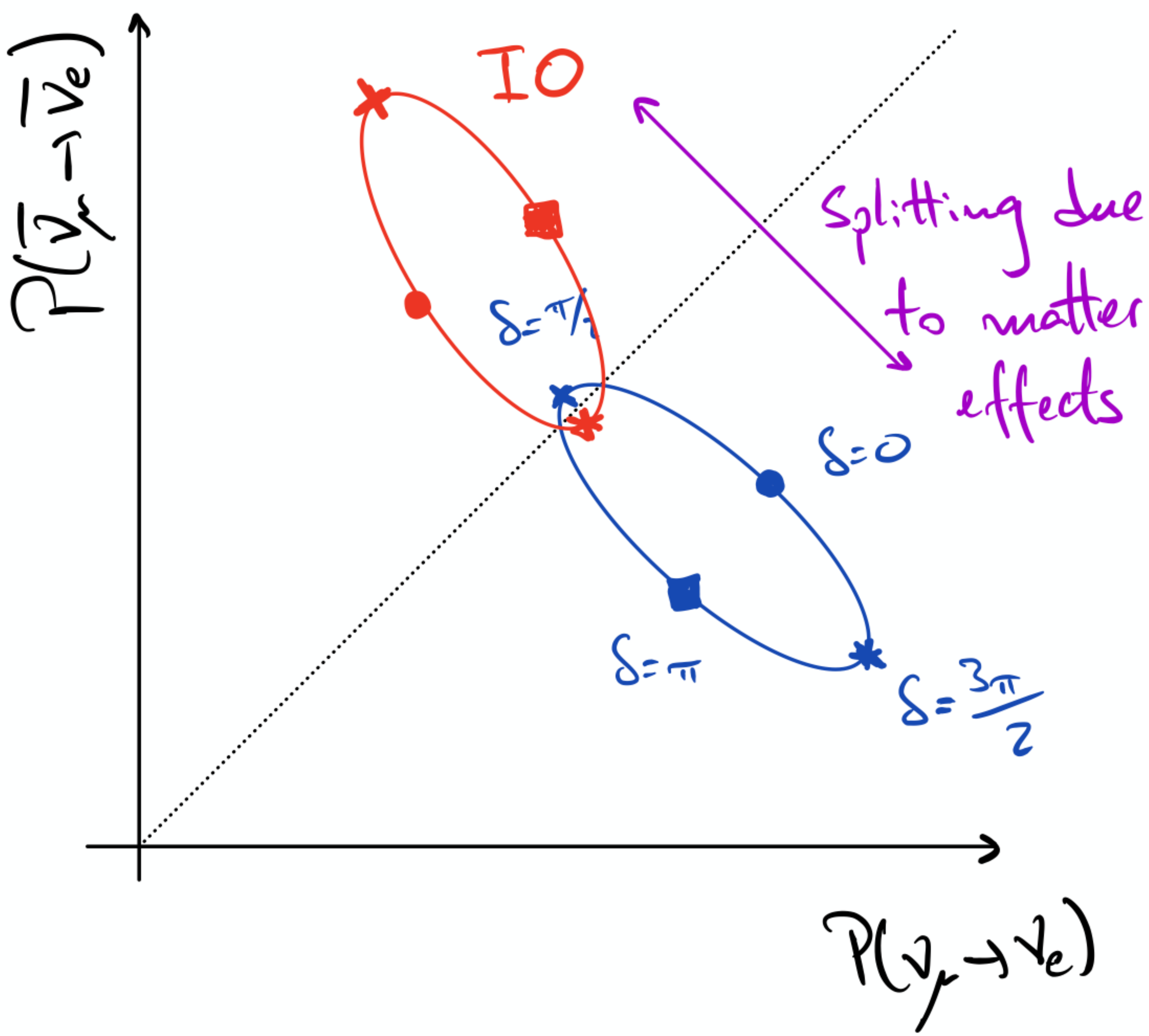

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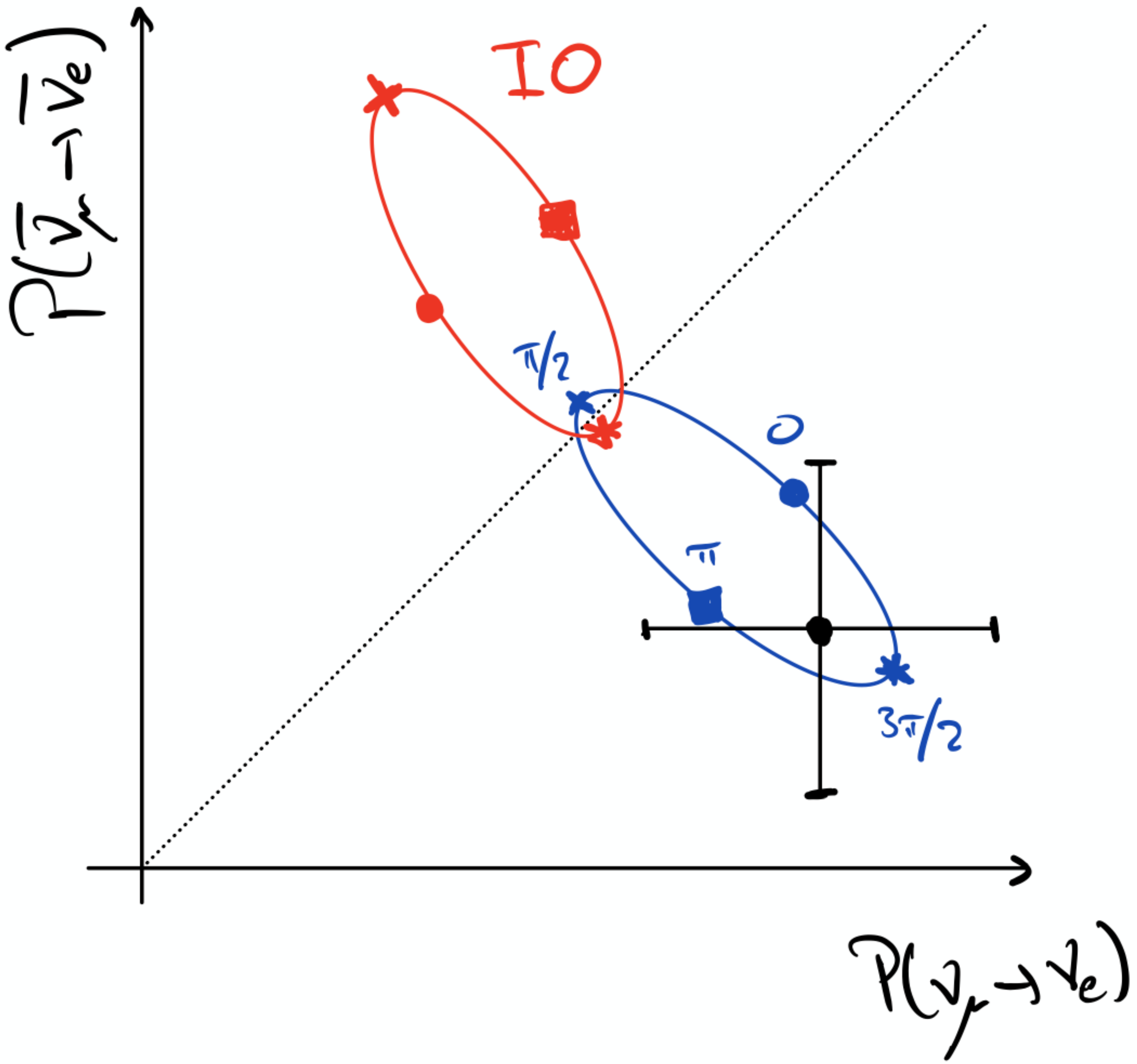

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Energy dependent neutrino mixing

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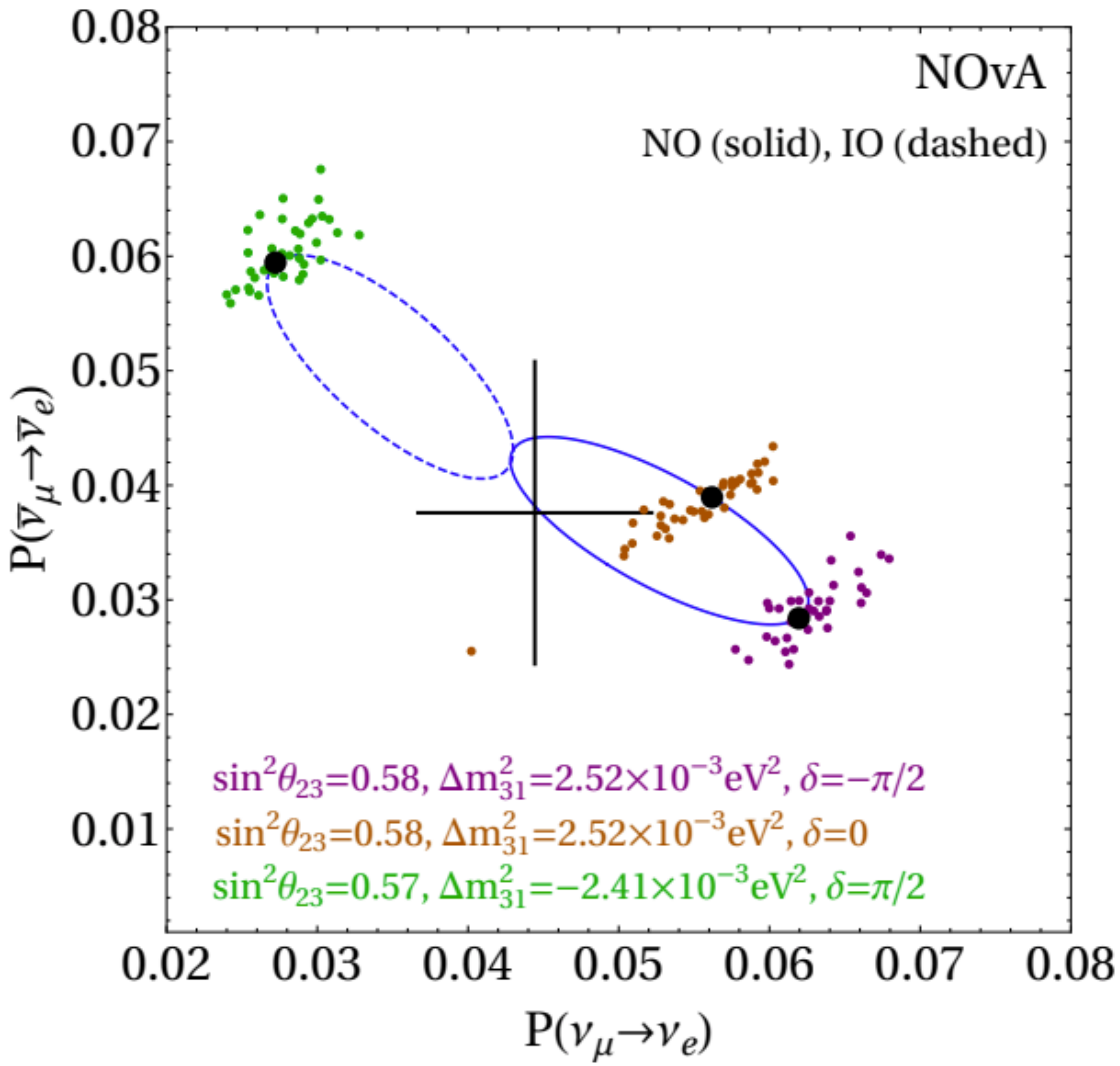

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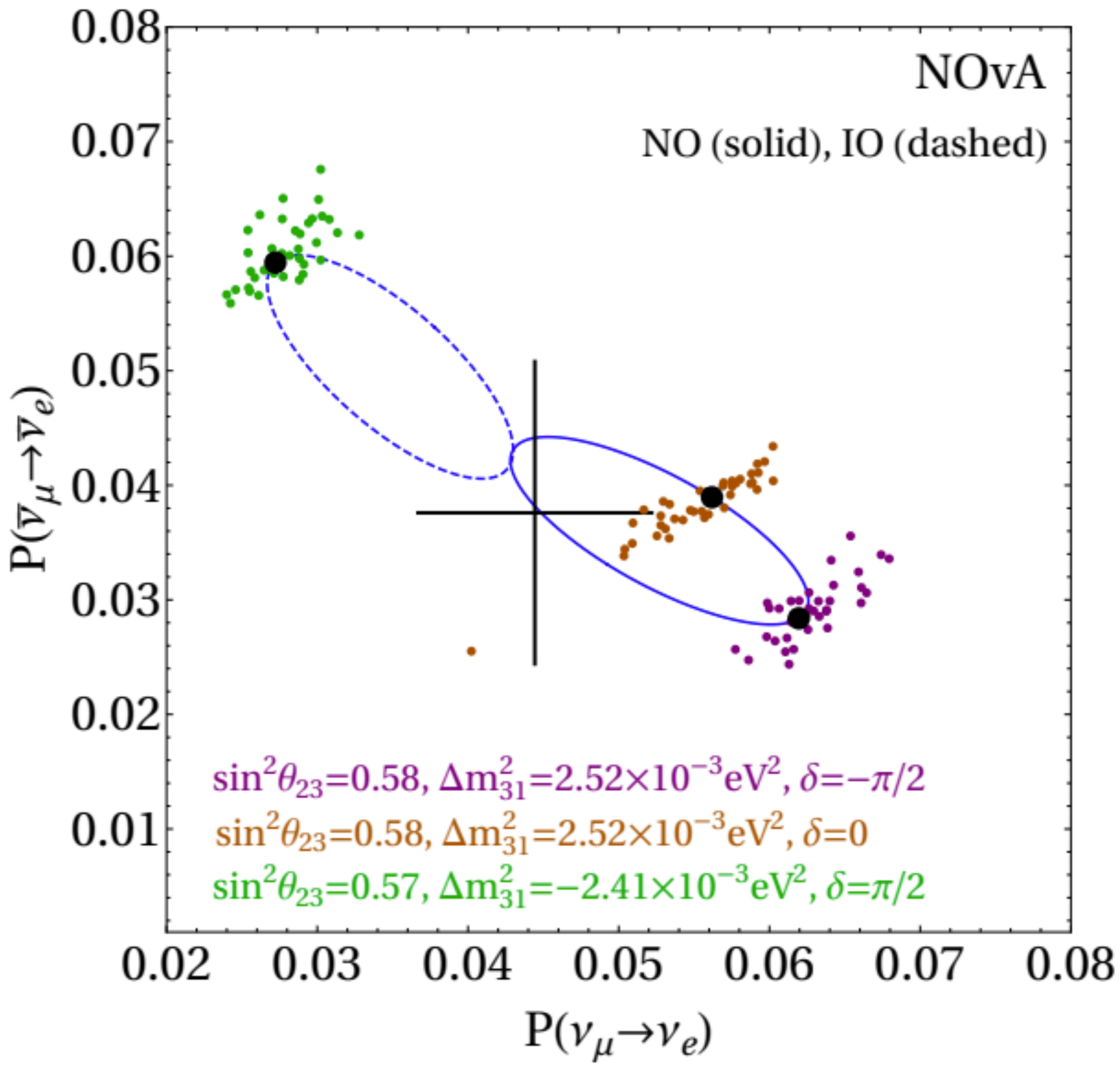

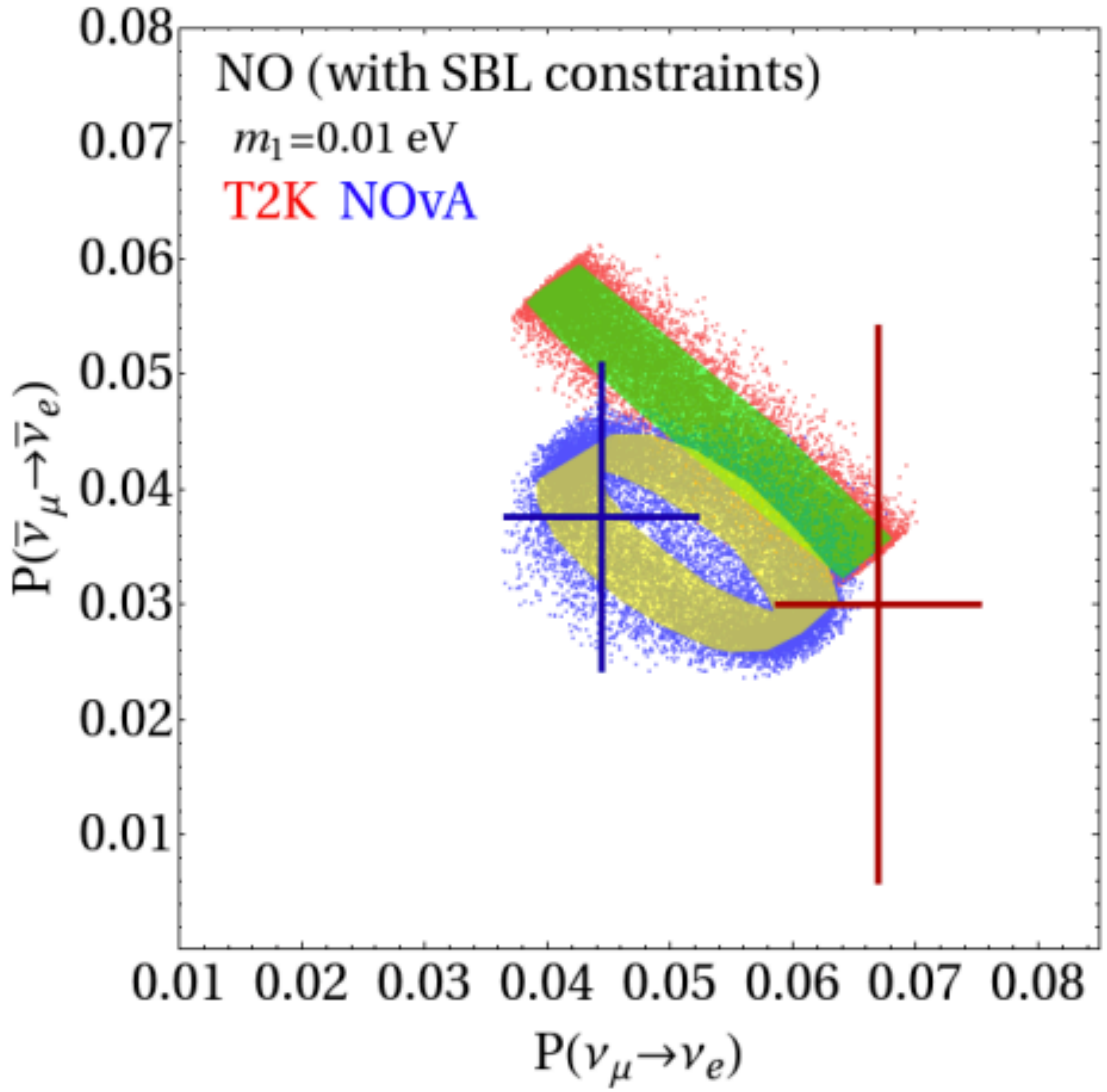

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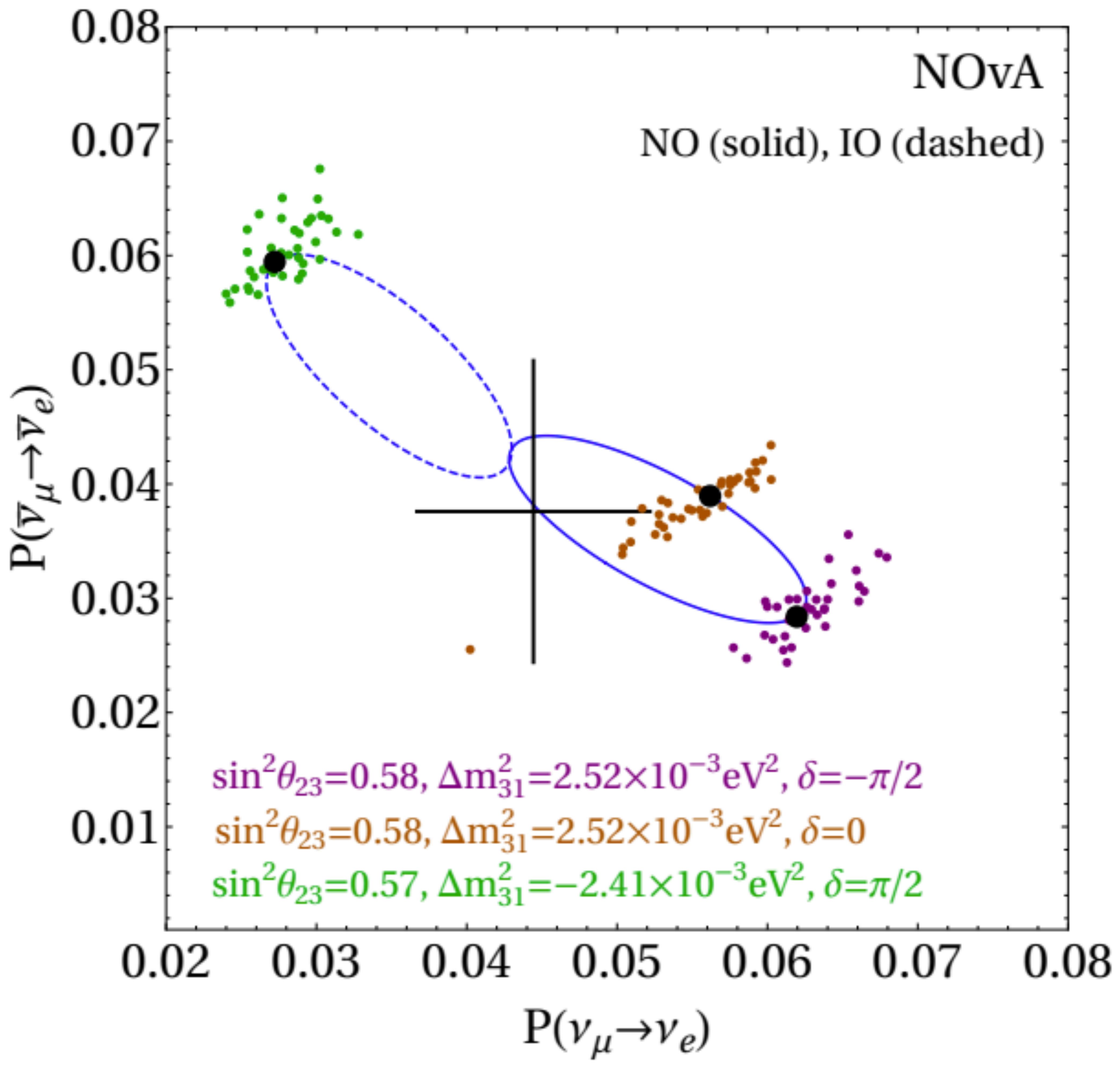

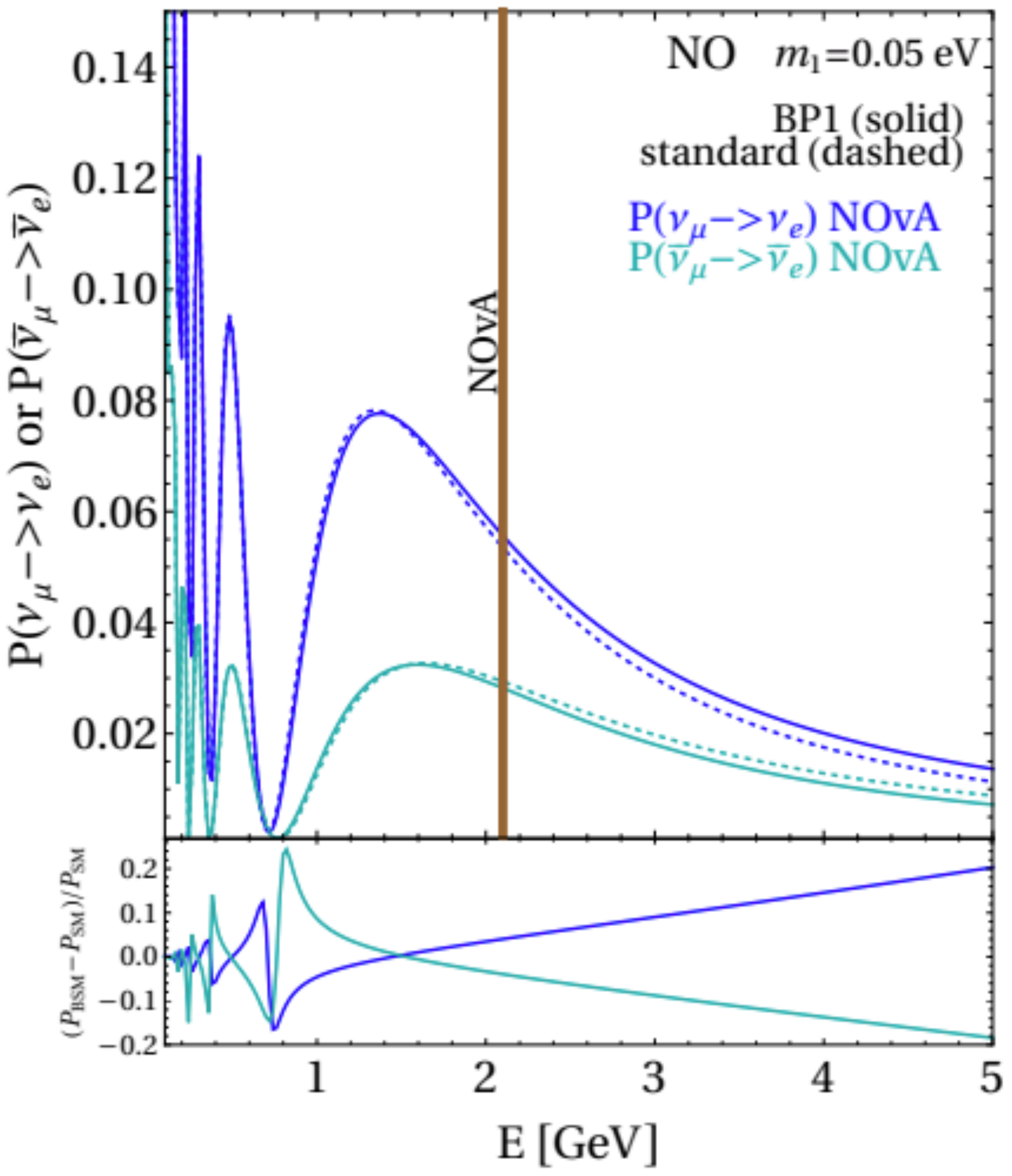

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Energy dependent neutrino mixing

Cosmogenic neutrinos: flavor composition

$$P_{\nu_\alpha \to \nu_\beta} = P_{\nu_\beta \to \nu_\alpha} = \delta_{\alpha\beta} - 2\sum_{k>j} \mathrm{Re}\left[U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\right] = \sum_{j=1}^{n} \left|U_{\alpha j}\right|^2 \left|U_{\beta j}\right|^2$$

These neutrinos come from so far that they decohere

Even if we do not know the flavor composition at the source, the possible flavor composition at detection is constrained and is related to the mixing matrix

$$P_{\nu_\alpha \to \nu_\beta} = \sum_{j=1}^{3} \left|U_{\alpha j}(Q_p^2)\right|^2 \left|U_{\beta j}(Q_d^2)\right|^2$$

$$X_\beta = \sum_\alpha P_{\nu_\alpha \to \nu_\beta} X_\alpha^{\mathrm{prod}}$$

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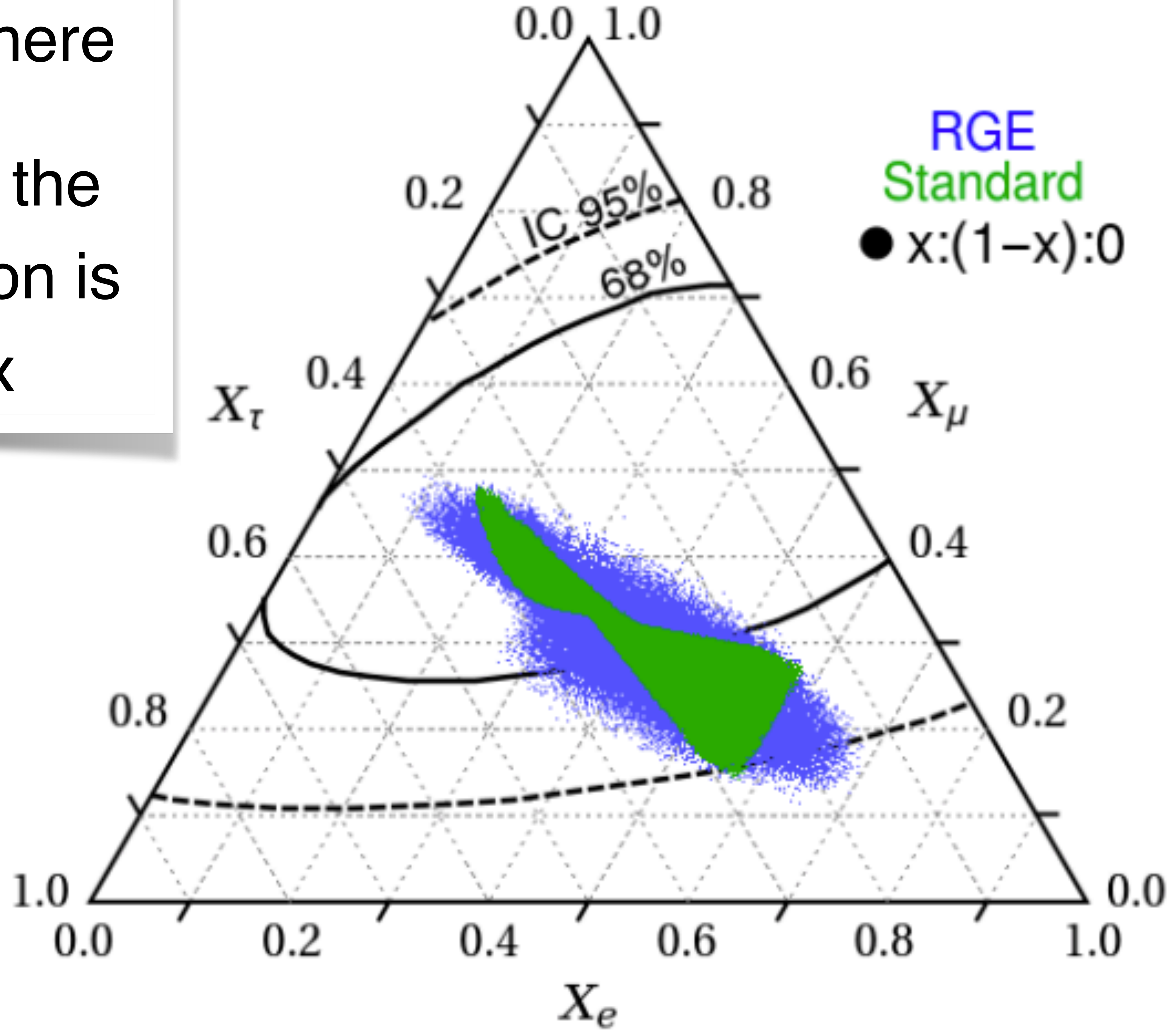
$$P_{\nu_\alpha \to \nu_\beta} = \sum_{j=1}^{3} \left|U_{\alpha j}(Q_p^2)\right|^2 \left|U_{\beta j}(Q_d^2)\right|^2$$

$$X_\beta = \sum_\alpha P_{\nu_\alpha \to \nu_\beta} X_\alpha^{\mathrm{prod}}$$



Conclusions

**If the neutrino mass model takes place at low scales,
it can induce quantum corrections that affect neutrino oscillations**

This boils down to producing and detecting neutrinos via different mixing matrices

Several effects are present: zero baseline transitions, apparent CPT violation, enhanced CP violation, overall distortions on the oscillation probabilities, changes on flavor composition of cosmogenic neutrinos

DUNE and IceCube-gen2 are in a very special position to probe this framework

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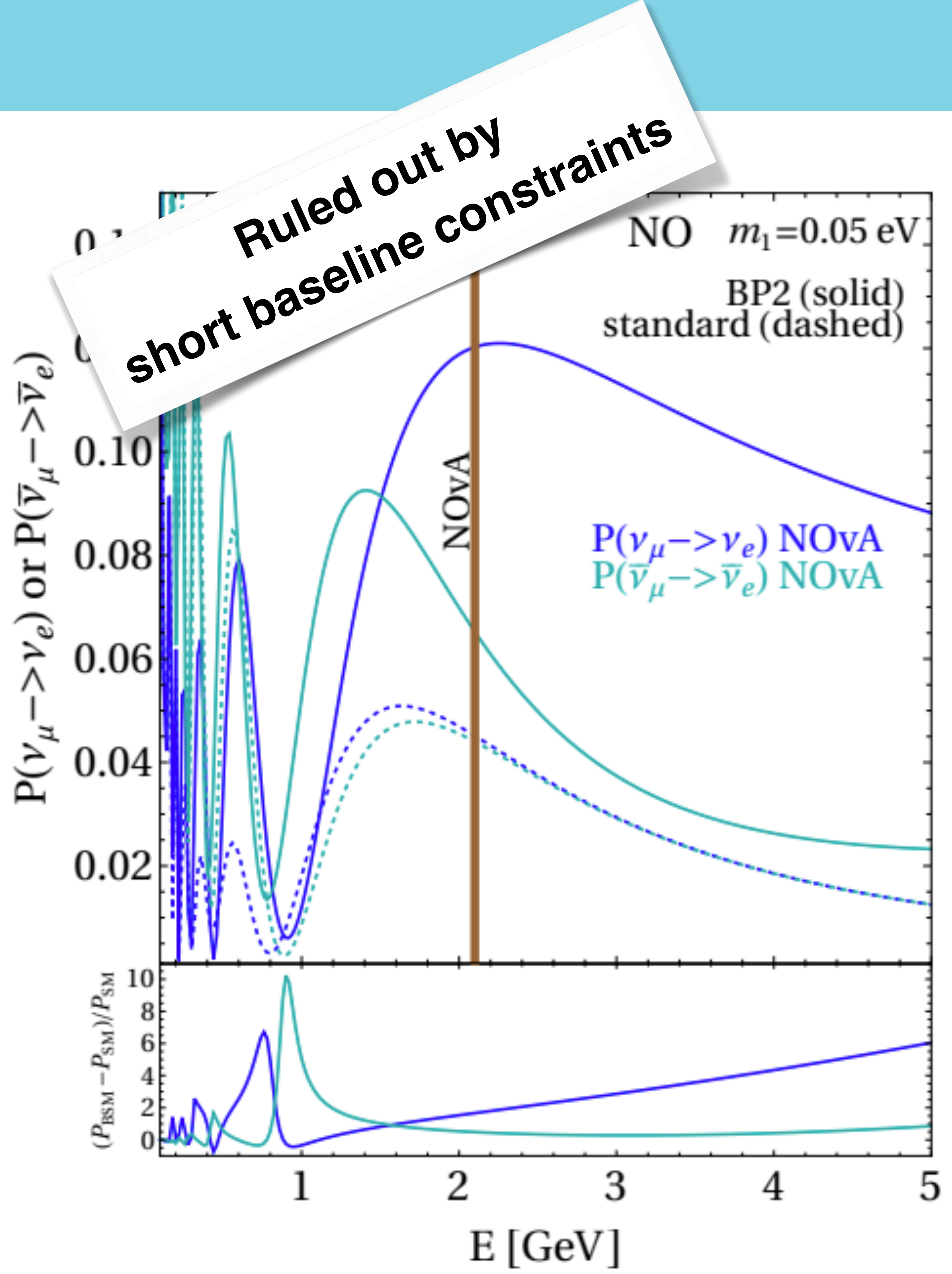

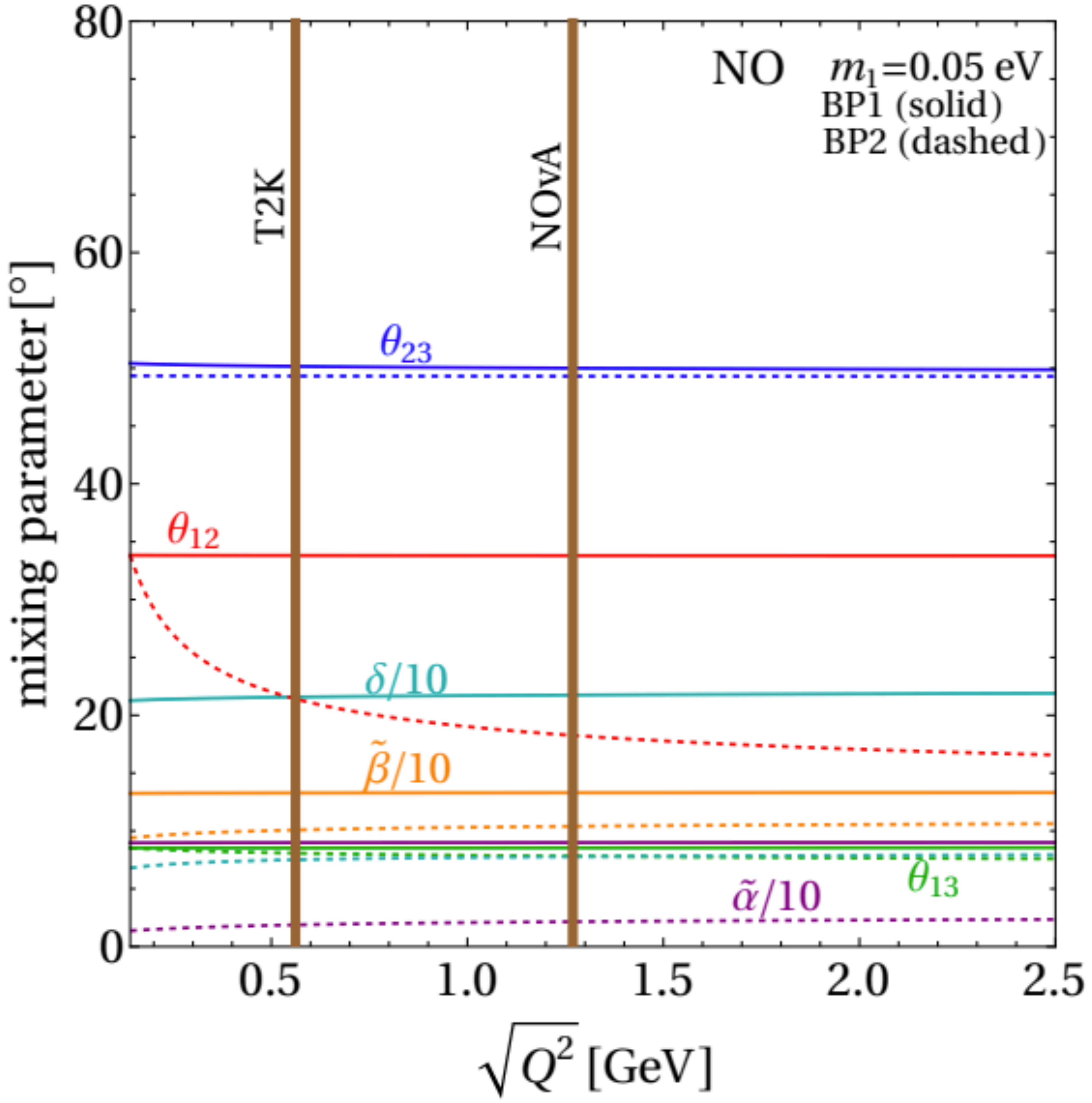

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$$P_{\mu\tau} \simeq \left|\epsilon_{23} + \frac{i}{2}\left(\epsilon_\alpha c_{12}^2 - \epsilon_\beta\right)\right|^2 = \epsilon_{23}^2 + \frac{1}{4}(\epsilon_\alpha c_{12}^2 - \epsilon_\beta)^2$$

$$P_{\mu e} \simeq \frac{1}{2}\left|\epsilon_{13} + e^{i\delta}\left(\epsilon_{12} + i\,\epsilon_\alpha c_{12} s_{12}\right)\right|^2 \simeq \frac{1}{8}\epsilon_\alpha^2 \sin^2 2\theta_{12} - \frac{\epsilon_\alpha \epsilon_{13}}{2}\sin 2\theta_{12} s_\delta + \frac{\epsilon_{12}^2 + \epsilon_{13}^2}{2} + \epsilon_{12}\epsilon_{13} c_\delta$$

$$P_{\mu\mu} - P_{\bar\mu\bar\mu} \simeq \left\{-(\epsilon_\alpha c_{12}^2 - \epsilon_\beta)\sin^2 2\theta_{23} + 8\epsilon_{12} c_{13}^2 s_{13} c_{23} s_{23}^3 s_\delta - \epsilon_\delta s_{23}^4 \sin^2 2\theta_{13}\right\} \sin\Delta_{31} - \\ - \epsilon_{13}\sin 2\theta_{12}\sin 2\theta_{23} s_\delta \left(1 + s_{23}^2 \cos\Delta_{31}\right)\sin\Delta_{21}\,.$$

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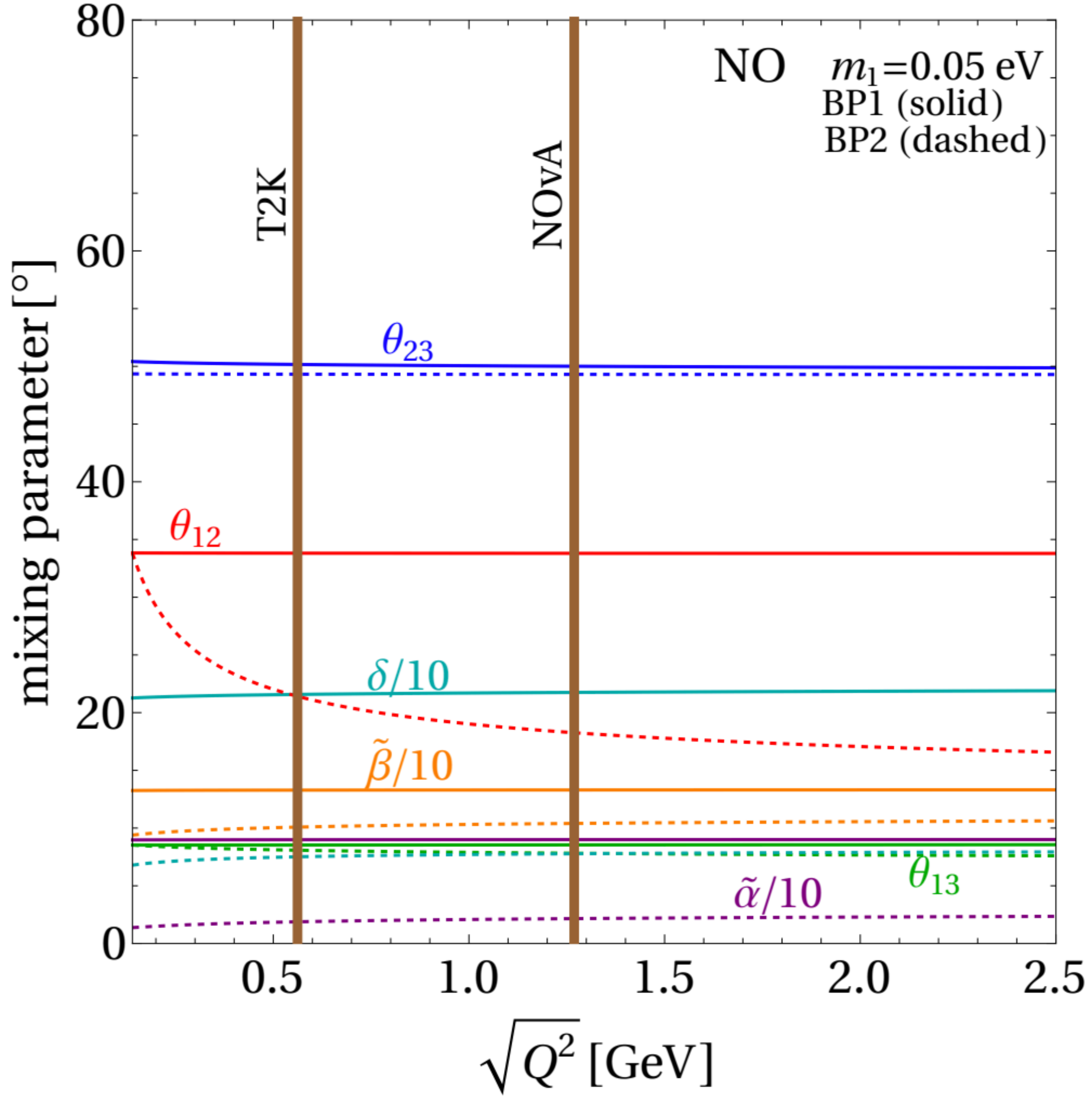

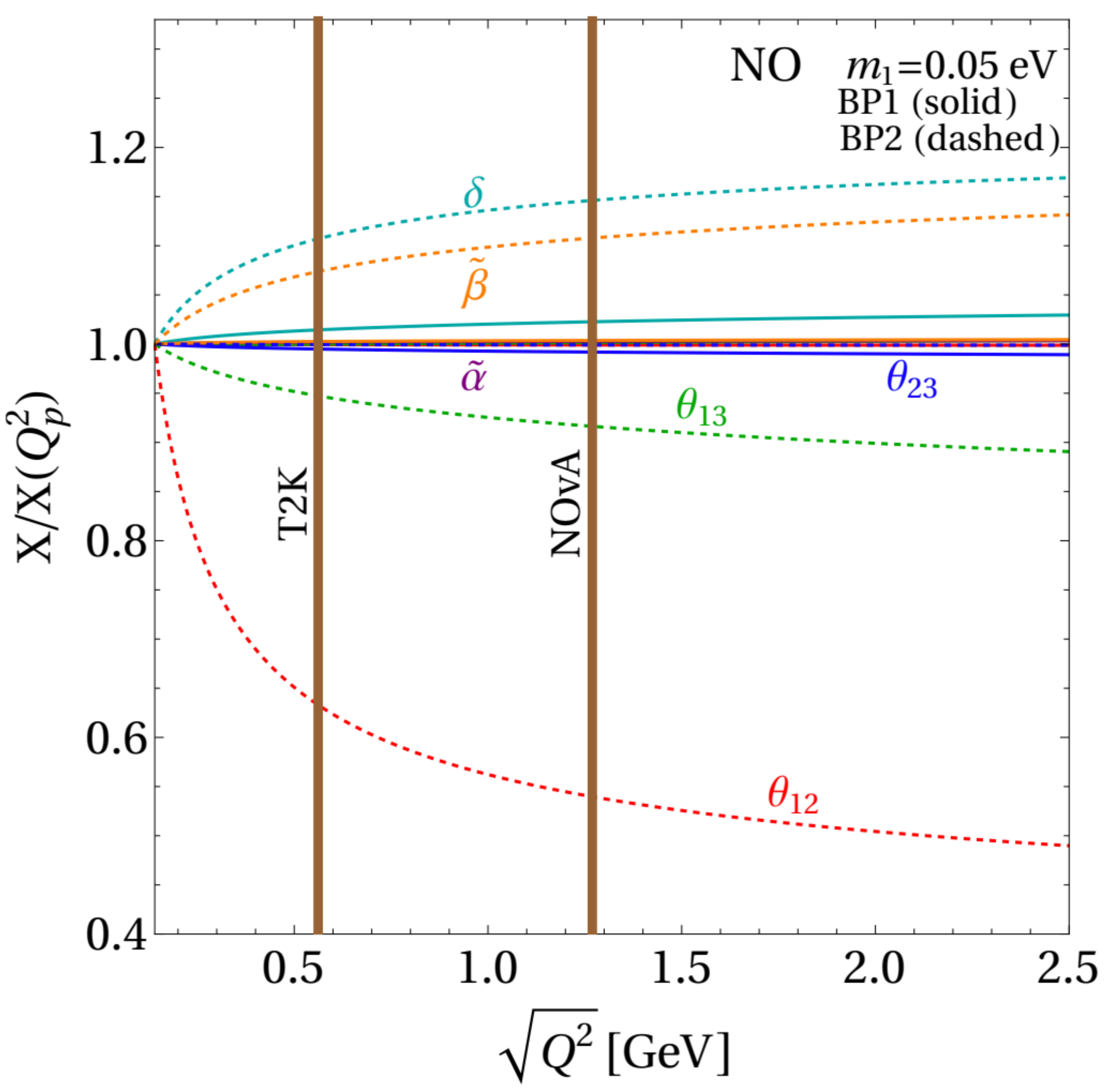

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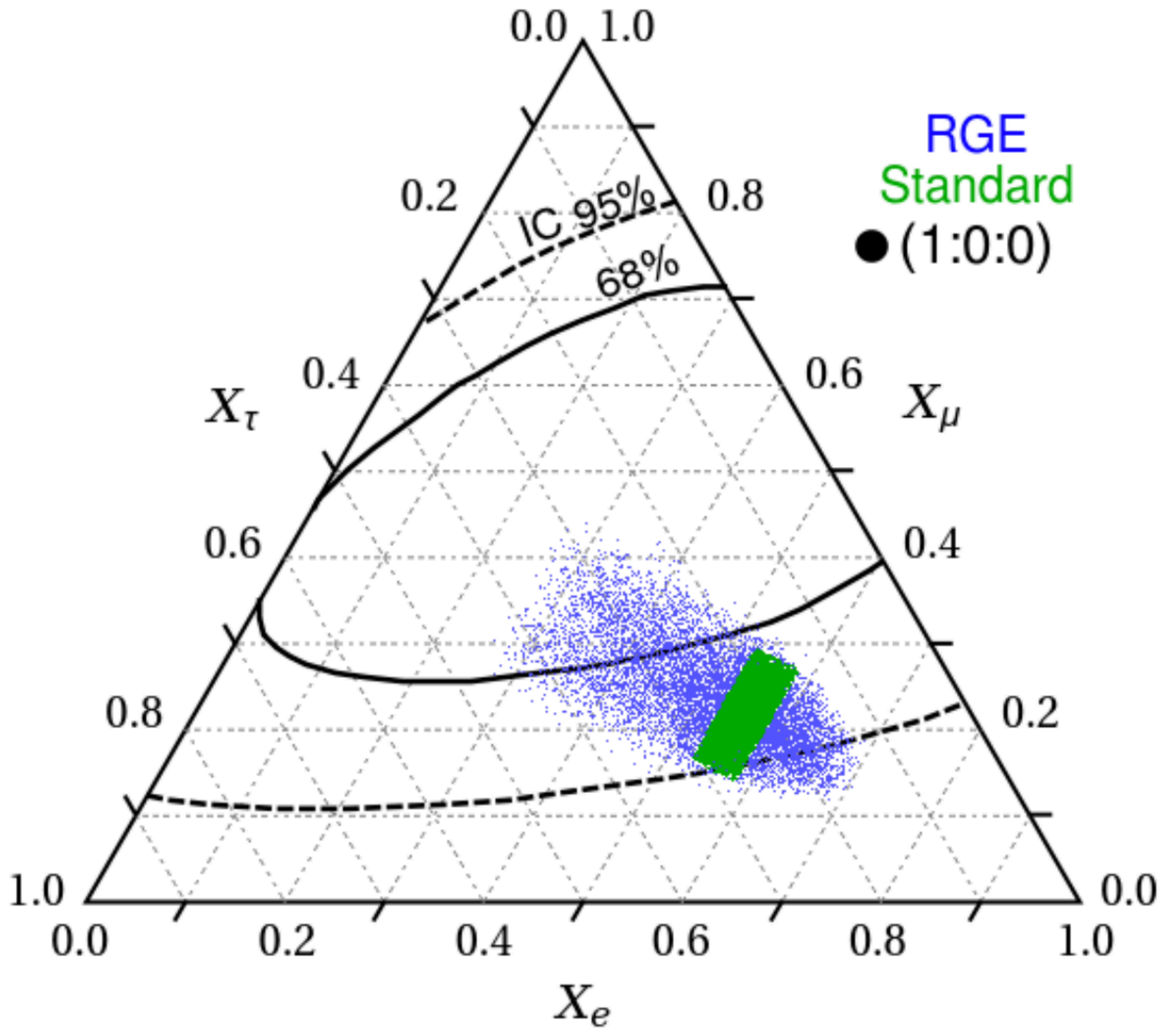

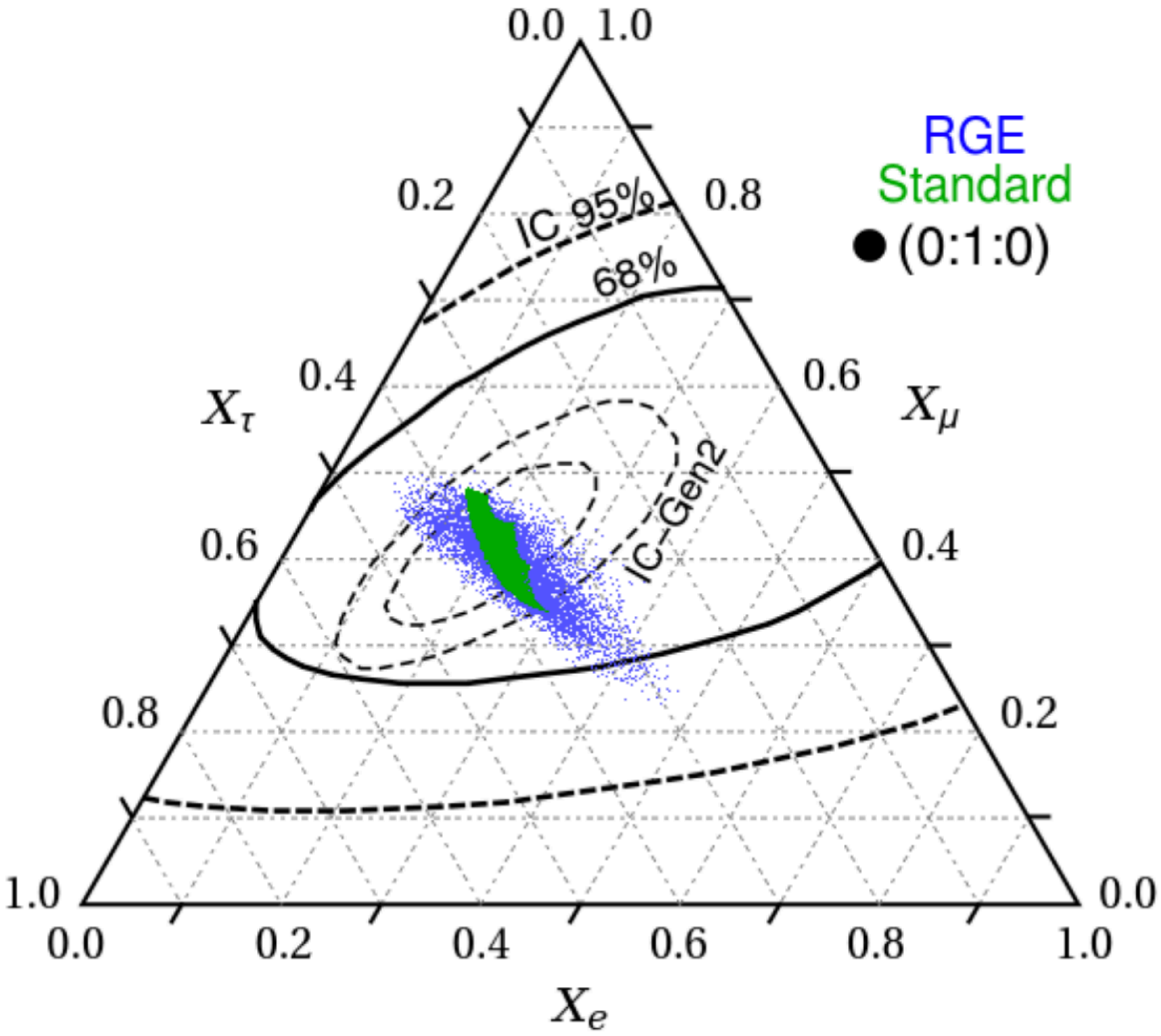

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