Shedding Light on Heavy-Quark Hadron Decays

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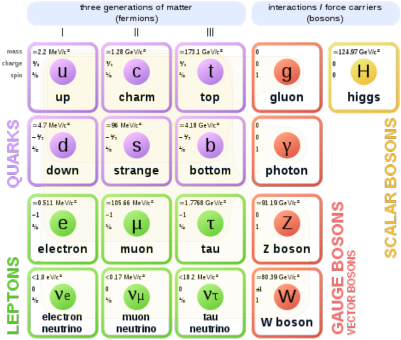
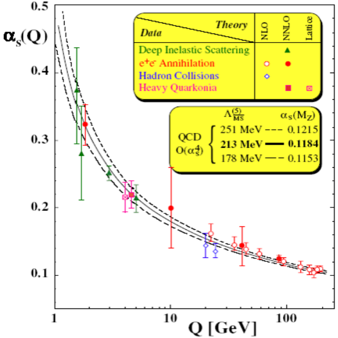
The Standard Model of Particle Physics

- Elementary Particles:



- Asymptotic Freedom:



- The Millennium Prize Problem: Yang-Mills existence and mass gap.
  - *Prove that for any compact simple gauge group G, a non-trivial quantum Yang-Mills theory exists on* $\mathbb{R}^4$ *and has a mass gap* $\Delta > 0$.

Why precision calculations?

- Understanding the general properties of power expansion in EFTs (HQET, SCET, NRQCD).

- Interesting to understand the strong interaction dynamics of heavy quark decays.
  - Factorization properties of the subleading-power amplitudes.
  - Renormalization and asymptotic properties of the higher-twist B-meson DAs.
  - Interplay of different QCD techniques.

- Precision determinations of the CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$.
  Power corrections, QED corrections, BSM physics.

- Crucial to understand the CP violation in B-meson decays.
  Strong phase of $\mathscr{A}(B \to M_1 M_2)$ @ m_b scale in the leading power.

- Indispensable for understanding the flavour puzzles (continuously updated).
  - P_5' and $R_{K^{(*)}}$ anomalies in $B \to K^{(*)} \ell^+ \ell^-$.
  - $R_{D^{(*)}}$ anomalies in $B \to D^{(*)} \ell \bar{\nu}_\ell$.
  - Color suppressed hadronic B-meson decays.
  - Polarization fractions of penguin dominated $B_{(s)} \to VV$ decays.

Theory tools for precision flavor physics

New Physics: $\mathscr{L}_{NP}$

$\downarrow$

EW scale (m_W): $\mathscr{L}_{SM} + \mathscr{L}_{D>4}$

$\downarrow$

Heavy-quark scale (m_b): $\mathscr{L}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_i C_i Q_i + \mathscr{L}_{eff,D>6}$

$\downarrow$

QCD scale ($\Lambda_{\rm QCD}$)

- Aim: $\langle f | Q_i | \bar{B} \rangle = ?$
- QCD factorization [Diagrammatic approach].
- SCET factorization [Operator formalism].
- TMD factorization.
- (Light-cone) QCD sum rules.
- Lattice QCD.

- Key concepts: Factorization, Resummation, Evolution.

Factorization in Classical Physics

- Galileo's Leaning Tower of Pisa Experiment:

$$\mathscr{L}_{\rm eff} = \frac{1}{2}\, m\, \dot{h}^2 - m\, g\, h\,.$$

Symmetry of the effective Lagrangian: $h \to h + a$.
Dynamical interpretation: The force acting on the ball, $F = m\, g$, independent of h.

- Newton's Gravity Theory:

$$V_{\rm full}(h) = -G\, \frac{M\, m}{r} = -G\, \frac{M\, m}{R+h}\,.$$

  - Power expansion of the full potential energy:

$$V_{\rm eff}(h) = C_1(R)\, m\, (h/R) + C_2(R)\, m\, (h/R)^2 + \ldots\,.$$

  The general form of the effective potential can be written without knowing $V_{\rm full}$.

  - Matching the full theory and the effective theory:

$$C_1(R) = -C_2(R) = \frac{G\, M}{R}\,, \qquad V_{\rm eff}(h) = m\, g\, h - \frac{m\, g}{R}\, h^2 + \ldots\,.$$

- Symmetry of the effective Lagrangian broken in the full theory.

$$g(r) = \frac{G\, M}{r^2}\,, \qquad r\, \frac{\partial}{\partial r} g(r) = \gamma_g\, g(r)\,.$$

This differential equation is actually a renormalization group equation.

Factorization in Quantum Physics

- Wigner-Eckart theorem:

$$\langle \tau' j' m' | T^k_q | \tau j m \rangle = \langle j k j' | m q m' \rangle \, \langle \tau' j' || T^k || \tau j \rangle \, .$$

  Separation of geometry and dynamics.

- Generalized Wigner-Eckart theorem in Lie algebra.
  An example from SU(3): u, v and W are all 8s.

$$\langle u | W | v \rangle = \lambda_1 \, \mathrm{Tr}[\bar{u} \, W \, v] + \lambda_2 \, \mathrm{Tr}[\bar{u} \, v \, W] \, .$$

  Notice that $8^3 = 512$ matrix elements expressed in terms of only two parameters.

- Factorization for strong interaction physics.
  An example from $B \to \gamma \ell \nu_\ell$:

$$F_{V,\mathrm{LP}}(n \cdot p) = \frac{Q_u \, m_B}{n \cdot p} \tilde{f}_B(\mu) \, C_\perp(n \cdot p, \mu) \int_0^\infty d\omega \, \frac{\phi_B^+(\omega, \mu)}{\omega} \, J_\perp(n \cdot p, \omega, \mu) \, .$$

  - Separation of hard, hard-collinear and soft fluctuations.
  - Key input: B-meson light-cone distribution amplitude $\phi_B^+(\omega, \mu)$.

B-meson distribution amplitudes

- The light-ray HQET matrix element [Grozin, Neubert, 1997]:

$$\langle 0|\bar{q}_\beta(z)\,[z,0]\,h_{v\,\alpha}(0)|\bar{B}(v)\rangle = -\frac{i\tilde{f}_B m_B}{4}\left[\frac{1+\not{v}}{2}\left\{2\,\tilde{\phi}_B^+(t,\mu)+\frac{\tilde{\phi}_B^-(t,\mu)-\tilde{\phi}_B^+(t,\mu)}{t}\,\not{z}\right\}\gamma_5\right]_{\alpha\beta}.$$

Important EOM constraints in HQET [Kawamura, Kodaira, Qiao, Tanaka, 2001, 2003].

- Evolution equation at one loop [Lange, Neubert, 2003]:

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu}{\omega}+\gamma_+(\alpha_s)\right]\phi_B^+(\omega,\mu)-\omega\int_0^\infty d\eta\,\Gamma_+(\omega,\eta,\alpha_s)\,\phi_B^+(\eta,\mu)\,.$$

This is an integro-differential equation!

- (Relatively) complicated solution [Lee, Neubert, 2005]:

$$\begin{aligned}
\phi_B^+(\omega,\mu) &= e^{V-2\gamma_E g}\,\frac{\Gamma(2-g)}{\Gamma(g)}\int_0^\infty\frac{d\eta}{\eta}\,\phi_B^+(\eta,\mu_0)\left(\frac{\max(\omega,\eta)}{\mu_0}\right)^g \\
&\quad\times\frac{\min(\omega,\eta)}{\max(\omega,\eta)}\,{}_2F_1\left(1-g,2-g,2,\frac{\min(\omega,\eta)}{\max(\omega,\eta)}\right), \\
g(\mu,\mu_0) &= \int\limits_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}\approx\frac{2C_F}{\beta_0}\,\ln\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\,.
\end{aligned}$$

Making the QCD resummation for enhanced logarithms complicated!

B-meson distribution amplitudes

- Better understanding from the RGE in coordinate space [Braun, Ivanov, Korchemsky, 2004]:

$$\frac{d\tilde{\phi}_B^+(t,\mu)}{d\ln\mu} = -\Gamma_{\rm cusp}(\alpha_s)\left\{\left[\ln(i\,\tilde{\mu}\,t) - \frac{1}{4}\right]\tilde{\phi}_B^+(t,\mu) + \int_0^1 du\,\frac{\bar{u}}{u}\,\left[\tilde{\phi}_B^+(t,\mu) - \tilde{\phi}_B^+(\bar{u}\,t,\mu)\right]\right\}.$$

  - Absence of the local operator product expansion due to the non-analytical term!
  - $\tilde{\phi}_B^+(t,\mu)$ only mixes into $\tilde{\phi}_B^+(\bar{u}\,t,\mu)$ with $0 \leq \bar{u} \leq 1$ under renormalization.

- Renormalization of $[\bar{q}_s(t\bar{n})\,\Gamma\,b_v(0)]$ does not commute with the short-distance expansion.

$$[(\bar{q}_s\,Y_s)(t\,\bar{n})\not{n}\,\Gamma\,(Y_s^\dagger\,b_v)(0)]_R \neq \sum_{p=0} \frac{t^p}{p!}\left[\bar{q}_s(0)\,(n\cdot\overleftarrow{D})^p\,\not{n}\,\Gamma\,b_v(0)\right]_R.$$

  - Many other examples in QCD physics (e.g., Light-cone projection and renormalization)!
  - Non-negative moments of the B-meson distribution amplitude ill defined.
  - Non-trivial generalization of the QCD equations of motion beyond the tree level.

B-meson distribution amplitudes

- Fourier/Mellin transformation:

$$\varphi_B^+(\theta,\mu) = \int_0^\infty \frac{d\omega}{\omega} \left(\frac{\omega}{\mu}\right)^{-i\theta} \phi_B^+(\omega,\mu) \quad \Leftrightarrow \quad \phi_B^+(\omega,\mu) = \int_{-\infty}^\infty \frac{d\theta}{2\pi} \left(\frac{\omega}{\mu}\right)^{i\theta} \varphi_B^+(\theta,\mu).$$

- Solution to the RGE in Mellin space:

$$\varphi_B^+(\theta,\mu) = e^{V-2\gamma_E g} \left(\frac{\mu}{\mu_0}\right)^{i\theta} \frac{\Gamma(1-i\theta)}{\Gamma(1+i\theta)} \cdot \frac{\Gamma(1+i(\theta+ig))}{\Gamma(1-i(\theta+ig))} \varphi_B^+(\theta+ig,\mu_0).$$

  Already very symmetric solution in Mellin space.

- Yet simper solution to the integral-differential equation exists?

  ⇔ Eigenfunctions of the Lange-Neubert kernel [Bell, Feldmann, YMW and Yip, 2013].

$$\begin{aligned}\varphi_B^+(\theta,\mu) &:= \frac{\Gamma(1-i\theta)}{\Gamma(1+i\theta)} \bar{\rho}_B^+(\theta,\mu) \\ &= \frac{\Gamma(1-i\theta)}{\Gamma(1+i\theta)} \int_0^\infty \frac{d\omega'}{\omega'} \rho_B^+(\omega',\mu) \left(\frac{\omega'}{\mu}\right)^{-i\theta}.\end{aligned}$$

B-meson distribution amplitudes

- Linear differential equation [Bell, Feldmann, YMW and Yip, 2013]:

$$\frac{d\rho_B^+(\omega',\mu)}{d\ln\mu} = -\left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{\mu}{\hat{\omega}'} + \gamma_+(\alpha_s)\right]\rho_B^+(\omega',\mu)\,.$$

  Local evolution in the dual space!

- Integral transformation:

$$\begin{aligned}\phi_B^+(\omega,\mu) &= \int_0^\infty \frac{d\omega'}{\omega'}\sqrt{\frac{\omega}{\omega'}}\,J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\rho_B^+(\omega',\mu)\,,\\ \rho_B^+(\omega',\mu) &= \int_0^\infty \frac{d\omega}{\omega}\sqrt{\frac{\omega}{\omega'}}\,J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\phi_B^+(\omega,\mu)\,.\end{aligned}$$

  Eigenfunction of the Lange-Neubert kernel at one-loop is the Bessel function!

- Interesting transformation from the coordinate space to the dual space:

$$\rho_B^+(\omega',\mu) = \int \frac{dt}{2\pi}\left(1-\exp\left[-\frac{i}{\omega'\,t}\right]\right)\tilde{\phi}_B^+(t,\mu)\,.$$

  $\rho_B^+(\omega',\mu)$ cannot be constructed from the local OPE of $\tilde{\phi}_B^+(t,\mu)$.

B-meson distribution amplitudes

- Solution to the RGE in dual space [Bell, Feldmann, YMW, Yip, 2013]:

$$\rho_B^+(\omega',\mu) \quad = e^V \left(\frac{\mu_0}{\hat{\omega}'}\right)^{-g} \rho_B^+(\omega',\mu_0)\,.$$

Very compact expression in a full analytical form!

- Solution to the RGE in momentum space:

$$\phi_B^+(\omega,\mu) = e^V \int_0^\infty \frac{d\omega'}{\omega'} \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \left(\frac{\mu_0}{\hat{\omega}'}\right)^{-g} \rho_B^+(\omega',\mu_0)\,.$$

Still a beautiful expression!

- The logarithmic inverse moments of LCDA and spectral function:

$$\int_0^\infty \frac{d\omega}{\omega} \ln^n\left(\frac{\omega}{\mu}\right) \phi_B^+(\omega,\mu) \overset{n=0,1,2}{=} \int_0^\infty \frac{d\omega'}{\omega'} \ln^n\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega',\mu) \equiv L_n(\mu)\,.$$

  - Non-trivial mixing of $L_n(\mu)$ in dual space under renormalization.

$$\frac{dL_n(\mu)}{d\ln\mu} \quad = \Gamma_{\rm cusp}(\alpha_s)\, L_{n+1}(\mu) - \gamma_+(\alpha_s) L_n(\mu) - n\, L_{n-1}(\mu)\,.$$

  - More complicated RGE for the logarithmic inverse moments in momentum space.

B-meson distribution amplitudes

- Collinear conformal symmetry for the Lange-Neubert kernel [Braun, Manashov, 2014]:

$$\left(\frac{d}{d\ln\mu} + \mathscr{H}_{\rm LN}\right) O_+(z,\mu) = 0\,.$$

$\mathscr{H}_{\rm LN}$ is almost determined by the commutation relations completely [Knoedlseder, Offen, 2011]

$$[S_+, \mathscr{H}_{\rm LN}] = 0, \qquad [S_0, \mathscr{H}_{\rm LN}] = 1\,.$$

The beautiful solution in terms of S_+:

$$\mathscr{H}_{\rm LN} = \ln(i\,\mu\, S^+) - \psi(1) - \frac{5}{4}\,.$$

- Generators of the collinear conformal group:

$$S_+ = z^2\,\partial_z + 2jz\,, \qquad S_0 = z\,\partial_z + j\,, \qquad S_- = -\partial_z\,.$$

Eigenfunctions of S_+ [Braun, Manashov, 2014]:

$$i S_+\, Q_s(z) = s\, Q_s(z)\,, \qquad Q_s(z) = -\frac{1}{z^2}\, e^{is/z}\,.$$

$$\langle e^{-i\omega z} | Q_s(z)\rangle = \frac{1}{\sqrt{\omega s}}\, J_1(2\sqrt{\omega s})\,.$$

Wide applications of the conformal symmetry in high energy physics!

B-meson distribution amplitudes

- RG evolution of $\phi_B^+(\omega,\mu)$ at two loops [Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]:

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = \left[\Gamma_{\rm cusp}(\alpha_s)\ln\frac{\omega}{\mu} - \gamma_\eta(\alpha_s)\right]\phi_B^+(\omega,\mu) + \Gamma_{\rm cusp}(\alpha_s)\int_0^\infty dx\,\Gamma(1,x)\,\phi_B^+(\omega/x,\mu) + \left(\frac{\alpha_s}{2\pi}\right)^2 C_F\int_0^1\frac{dx}{1-x}\,h(x)\,\phi_B^+(\omega/x,\mu)\,.$$

The last missing element for the NLL predictions of exclusive B-meson decay observables!

- The two-loop eigenfunctions depend on the strong coupling α_s [Braun, Ji, Manashov, 2019].

- Applying the Laplace transformation of the LCDA [Galda, Neubert, 2020]

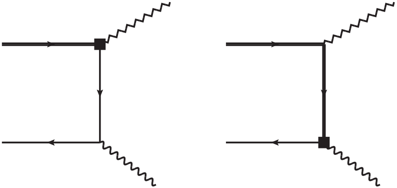
$$\tilde\phi_B^+(\eta,\mu) = \int_0^\infty\frac{d\omega}{\omega}\,\phi_B^+(\omega,\mu)\left(\frac{\omega}{\bar\omega}\right)^{-\eta},$$

$\Rightarrow$ the general solution to the two-loop RGE of $\phi_B^+(\omega,\mu)$

$$\begin{aligned}\tilde\phi_B^+(\eta,\mu) &= \exp\left[S(\mu_0,\mu)+a_\gamma(\mu_0,\mu)+2\,\gamma_E\,a_\Gamma(\mu_0,\mu)\right]\left(\frac{\bar\omega}{\mu_0}\right)^{-a_\Gamma(\mu_0,\mu)}\\ &\times\frac{\Gamma(1+\eta+a_\Gamma(\mu_0,\mu))\,\Gamma(1-\eta)}{\Gamma(1-\eta-a_\Gamma(\mu_0,\mu))\,\Gamma(1+\eta)}\exp\left[\int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)}\frac{d\alpha}{\beta(\alpha)}\,\mathscr{G}(\eta+a_\Gamma(\mu_\alpha,\mu),\alpha)\right]\\ &\times\tilde\phi_B^+(\eta+a_\Gamma(\mu_0,\mu),\mu_0)\,.\end{aligned}$$

Radiative leptonic B meson decays in QCD

- Tree diagrams:



Kinematics:

$$p_B \equiv p + q = m_B\, v\,, \qquad p = \frac{n \cdot p}{2}\,\bar{n}\,, \qquad q = \frac{n \cdot q}{2}\,\bar{n} + \frac{\bar{n} \cdot q}{2}\, n\,.$$

- Decay amplitude:

$$\mathcal{M}(B^- \to \gamma \ell\, \nu) = \frac{G_F\, V_{ub}}{\sqrt{2}}\, (i\, g_{em}\, \varepsilon_\nu^*) \left\{ T^{\nu\mu}(p,q)\, \bar{\ell}\, \gamma_\mu\, (1-\gamma_5)\nu + Q_\ell\, f_B\, \bar{\ell}\, \gamma^\nu\, (1-\gamma_5)\nu \right\}.$$

Hadronic tensor:

$$\begin{aligned} T_{\nu\mu}(p,q) &\equiv \int d^4x\, e^{ip\cdot x}\, \langle 0|\mathrm{T}\{j_{\nu,em}(x), \left[\bar{u}\gamma_\mu(1-\gamma_5)b\right](0)\}|B^-(p+q)\rangle\,, \\ &= v\cdot p \left[-i\,\varepsilon_{\mu\nu\rho\sigma}\, n^\rho\, v^\sigma\, F_V(n\cdot p) + g_{\mu\nu}\, \hat{F}_A(n\cdot p)\right] + v_\nu\, p_\mu\, F_1(n\cdot p) \\ &\quad + v_\mu\, p_\nu\, F_2(n\cdot p) + v\cdot p\, v_\mu\, v_\nu\, F_3(n\cdot p) + \frac{p_\mu\, p_\nu}{v\cdot p}\, F_4(n\cdot p)\,. \end{aligned}$$

General aspects of $B \to \gamma \ell \nu$

- Ward identity [Grinstein and Pirjol, 2000; Khodjamirian and Wyler, 2001]:

$$p_\nu \, T^{\nu\mu}(p,q) = -(Q_b - Q_u) f_B \, p_B^\mu \, .$$

$$\Downarrow$$

$$\hat{F}_A(v\cdot p) = -F_1(v\cdot p)\,, \qquad F_3(v\cdot p) = -\frac{(Q_b - Q_u) f_B \, m_B}{(v\cdot p)^2} \, .$$

- Reduced parametrization:

$$T_{\nu\mu}(p,q) \;=\; -i\, v\cdot p\, \varepsilon_{\mu\nu\rho\sigma}\, n^\rho\, v^\sigma\, F_V(n\cdot p) + \left[g_{\mu\nu}\, v\cdot p - v_\nu\, p_\mu\right] \hat{F}_A(n\cdot p) \underbrace{- \frac{(Q_b - Q_u) f_B\, m_B}{v\cdot p} v_\mu\, v_\nu}_{\text{contact term}} \, .$$

- Absorb the photon emission off the lepton [Beneke and Rohrwild, 2011]:

$$\left[g_{\mu\nu}\, v\cdot p - v_\nu\, p_\mu\right] \hat{F}_A(n\cdot p) \;=\; -Q_\ell f_B\, g_{\mu\nu} + \left[g_{\mu\nu}\, v\cdot p - v_\nu\, p_\mu\right] \underbrace{\left[\hat{F}_A(n\cdot p) + \frac{Q_\ell f_B}{v\cdot p}\right]}_{F_A(n\cdot p)\,.} \underbrace{+ \frac{v_\nu p_\mu}{v\cdot p} Q_\ell f_B}_{\text{irrelevant after the contraction with } \varepsilon_\nu^*}$$

Current status of $B \to \gamma \ell \bar{\nu}_\ell$

- Factorization properties at leading power [Korchemsky, Pirjol, Yan, 2000; Descotes-Genon, Sachrajda, 2002; Lunghi, Pirjol, Wyler, 2003; Bosch, Hill, Lange, Neubert, 2003].

- Leading power contributions at NLL and (partial)-subleading power corrections at tree level [Beneke, Rohrwild, 2011].

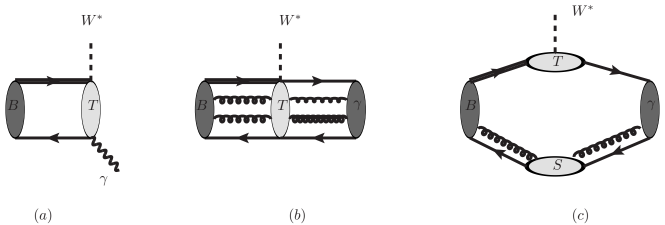
- Subleading power corrections from the dispersion technique:
  - Soft two-particle correction at tree level [Braun, Khodjamirian, 2013].
  - Soft two-particle correction at one loop [YMW, 2016].
  - Three-particle B-meson DA's contribution at tree level [YMW, 2016; Beneke et al, 2018].
  - Subleading effective current and twist-5 and 6 corrections at tree level. [Beneke et al, 2018].

- Subleading power corrections from the direct QCD approach:
  - Hadronic photon corrections at tree level up to the twist-4 accuracy [Khodjamirian, Stoll, Wyler, 1995; Ali, Braun, 1995; Eilam, Halperin, Mendel, 1995].
  - Hadronic photon corrections of twist-two at one loop and of higher-twist at tree level [Ball, Kou, 2003; YMW, Shen, 2018].

- Leading power contributions at NNLL and the updated NLP corrections:
  - Two-loop RG evolution of $\phi_B^+(\omega, \mu)$ derived in [Braun, Ji, Manashov, 2019].
  - Two-loop jet function obtained in [Liu, Neubert, 2020].
  - Further improvement on $B \to \gamma \ell \bar{\nu}_\ell$ will appear soon [Cui, Shen, Wang, YMW, Wei, 2021].
  - Interesting generalization to $B_u^- \to \ell' \bar{\ell}' \ell \bar{\nu}_\ell$ in [Wang, YMW, Wei, 2021].

Radiative leptonic B meson decays in QCD

- Schematic structure of the distinct mechanisms:



A: hard subgraph that includes both photon and W^* vertices $\left(\frac{\Lambda}{m_b}\right)^{1/2} + \left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

B: real photon emission at large distances $\left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

C: Feynman mechanism: soft quark spectator $\left(\frac{\Lambda}{m_b}\right)^{3/2} + \ldots$

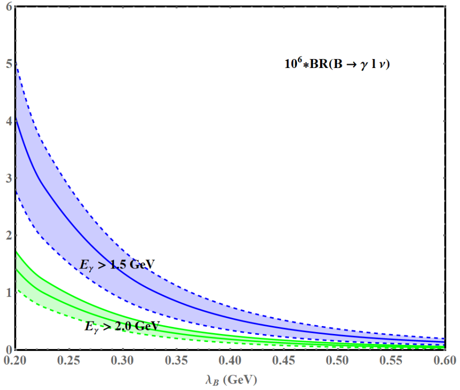
- Operator definitions of different terms needed for an unambiguous classification.

Theory predictions for $B \to \gamma \ell \bar{\nu}_\ell$

- Integrated decay rate $\Delta BR(E_{\rm cut})$:

$$\Delta BR(E_{\rm cut}) = \tau_B \int_{E_{\rm cut}}^{m_B/2} d E_\gamma \, \frac{d\Gamma}{d E_\gamma} \, (B \to \gamma \ell \bar{\nu}_\ell) \, .$$

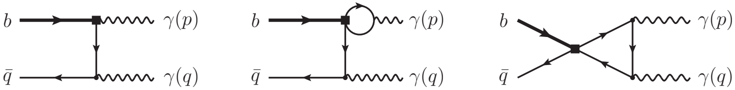
- $\lambda_B(\mu_0)$ dependence of $\Delta BR(E_{\rm cut})$ [YMW, Shen, 2018]:



- Belle 2015 data: $\Delta BR(1\,\text{GeV}) < 3.5 \times 10^{-6}$.
- Belle 2018 data [arXiv:1810.12976]: $\Delta BR(1\,\text{GeV}) = (1.4 \pm 1.0 \pm 0.4) \times 10^{-6}$.
- Expected statistical error for $\Delta BR(1\,\text{GeV})$ with $50\,\text{ab}^{-1}$ of Belle-II data: ${}^{+0.18}_{-0.17} \times 10^{-6}$.
- The photon-energy cut not sufficiently large. Power corrections numerically important for $E_\gamma < 1.5\,\text{GeV}$.

Double Radiative B_q-Meson Decays

- Leading-order contributions at $\mathcal{O}(\alpha_s^0)$:



Kinematics:

$$p_\mu = \frac{n\cdot p}{2}\,\bar{n}_\mu \equiv \frac{m_{B_q}}{2}\,\bar{n}_\mu\,, \qquad q_\mu = \frac{\bar{n}\cdot q}{2}\,n_\mu \equiv \frac{m_{B_q}}{2}\,n_\mu\,.$$

Interplay of the soft and collinear QCD dynamics!

- Decay amplitude:

$$\bar{\mathcal{A}}(\bar{B}_q \to \gamma\gamma) = -\frac{4\,G_F}{\sqrt{2}}\,\frac{\alpha_{\rm em}}{4\pi}\,\varepsilon_1^{*\alpha}(p)\,\varepsilon_2^{*\beta}(q)\sum_{p=u,c} V_{pb}V_{pq}^*\sum_{i=1}^{8} C_i\,T_{i,\alpha\beta}^{(p)}\,.$$

Hadronic tensors:

$$\begin{aligned} T_{7,\alpha\beta} &= 2\,\overline{m}_b(\nu)\int d^4x\,e^{iq\cdot x}\,\langle 0|\mathrm{T}\left\{j_\beta^{\rm em}(x),\bar{q}_L(0)\sigma_{\mu\alpha}p^\mu b_R(0)\right\}|\bar{B}_q(p+q)\rangle \\ &\quad +[p\leftrightarrow q,\alpha\leftrightarrow\beta]\,, \\ T_{i,\alpha\beta}^{(p)} &= -(4\,\pi)^2\int d^4x\int d^4y\,e^{ip\cdot x}\,e^{iq\cdot y}\,\langle 0|\mathrm{T}\left\{j_\alpha^{\rm em}(x),j_\beta^{\rm em}(y),P_i^{(p)}(0)\right\}|\bar{B}_q(p+q)\rangle\,, \\ &\quad (i=1,\ldots 6,\,8)\,. \end{aligned}$$

Main task: Construct the SCET factorization formulae beyond the leading power.

Current status of $B_q \to \gamma\gamma$

- QCD factorization at leading power in Λ/m_b and at NLO in α_s [Descotes-Genon, Sachrajda, 2003].
  - No collinear strong interaction dynamics at LP.
  - The two-loop $b \to q\gamma$ matrix elements of QCD penguin operators NOT included.
    $\Rightarrow$ A complete factorization-scale independence at NLO is absent!

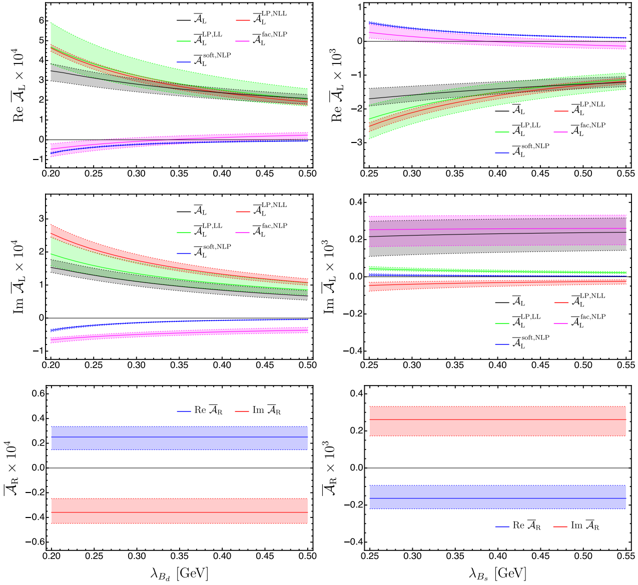
- Subleading power contributions from the weak annihilation diagrams [Bosch, Buchalla, 2002].
  - Complex perturbative hard functions evaluated at one loop.
  - Diagrammatic factorization established at two loops.

- Previous model-dependent calculations introduce additional systematic uncertainties.

- The new (technical)-ingredients from [Shen, YMW, Wei, 2020]:
  - A complete NLL calculation for the LP contribution $\Rightarrow$ 2-loop evolution of ϕ_B^+.
  - The NLP factorization for the energetic photon radiation off the light quark.
    The so-called "soft form factor" defined in [Beneke, Rohrwild, 2011] is factorizable!
  - The NLP factorization for the light-quark mass effect.
  - The NLP factorization for the SCET current $J^{(A2)} \supset (\bar{\xi}_{\rm hc} W_{\rm hc})\, \gamma_\alpha^\perp \, P_L \left(\frac{i \overleftarrow{\not{D}}_\top}{2 m_b} \right) h_v$.
  - The NLP factorization for the subleading twist B-meson LCDAs.
  - The resolved photon contribution with the dispersion technique.

The helicity amplitudes of $B_q \to \gamma\gamma$

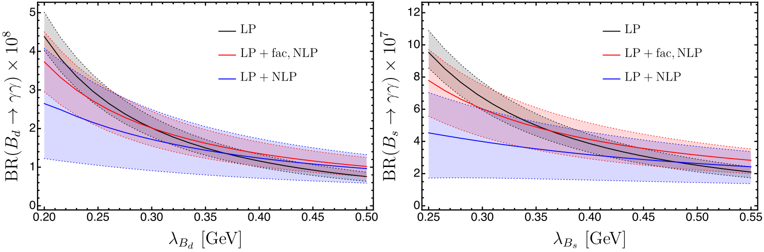
- Breakdown of the various QCD mechanisms [Shen, YMW, Wei, 2020]:



- NLL effects stabilize the factorization-scale dependence.
- Factorizable NLP effects around $\mathcal{O}(30\%)$.
- Destructive effects from the NLP soft corrections.
- Strong phase from the 2-loop matrix element of P_2^c and the weak annihilation.

Phenomenological observables for $B_q \to \gamma\gamma$

- Time-integrated branching fraction [Shen, YMW, Wei, 2020]:



  - The yielding theory predictions

$$\mathcal{BR}(B_d \to \gamma\gamma) = \left(1.44^{+1.35}_{-0.80}\right) \times 10^{-8}, \qquad \mathcal{BR}(B_s \to \gamma\gamma) = \left(3.17^{+1.96}_{-1.74}\right) \times 10^{-7}.$$

  with the dominant uncertainties from λ_{B_q}, $\widehat{\sigma}^{(1)}_{B_q}$, $\widehat{\sigma}^{(2)}_{B_q}$ and the QCD renormalization scale ν.

  - The ratio of the two branching ratios for $B_{d,s} \to \gamma\gamma$

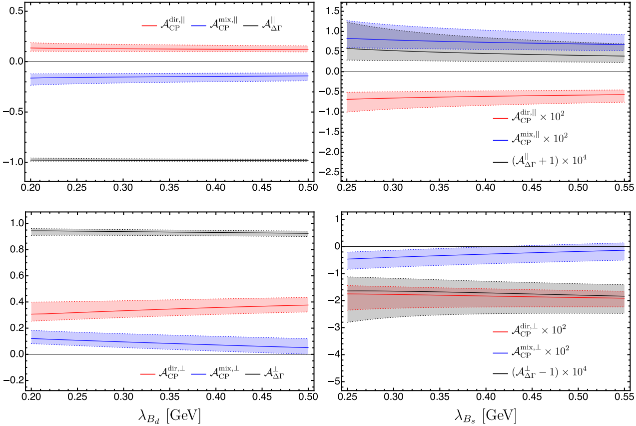
$$\frac{\mathcal{BR}(B_s \to \gamma\gamma)}{\mathcal{BR}(B_d \to \gamma\gamma)} = 33.80 \left(\frac{\lambda_{B_d}}{\lambda_{B_s}}\right)^2 + \mathcal{O}\left(\frac{\Lambda}{m_b}, \alpha_s\right).$$

  The λ_{B_q}-scaling violation effect due to the NLP contributions approximately $(10-20)\,\%$.

Phenomenological observables for $B_q \to \gamma\gamma$

- Time-dependent CP asymmetries [Shen, YMW, Wei, 2020]:

$$A_{\rm CP}^{\chi}(t) = \frac{\bar{\Gamma}^{\chi}(\bar{B}_q(t) \to \gamma\gamma) - \Gamma^{\chi}(B_q(t) \to \gamma\gamma)}{\bar{\Gamma}^{\chi}(\bar{B}_q(t) \to \gamma\gamma) + \Gamma^{\chi}(B_q(t) \to \gamma\gamma)} = -\frac{\mathscr{A}_{\rm CP}^{\rm dir,\,\chi} \cos(\Delta m_q t) + \mathscr{A}_{\rm CP}^{\rm mix,\,\chi} \sin(\Delta m_q t)}{\cosh(\Delta\Gamma_q t/2) + \mathscr{A}_{\Delta\Gamma}^{\chi} \sinh(\Delta\Gamma_q t/2)}\,.$$

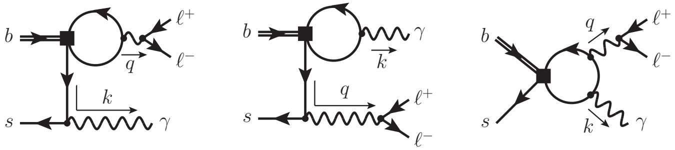


- $|\mathscr{A}_{\rm CP}^{\rm dir,\,\chi}|$ and $|\mathscr{A}_{\rm CP}^{\rm mix,\,\chi}|$ around $(10-40)\%$ for $B_d \to \gamma\gamma$.
- Tiny CP asymmetries for $B_s \to \gamma\gamma$ as expected.
- The difference between $\mathscr{A}_{\rm CP}^{\rm dir,\,\parallel}$ and $\mathscr{A}_{\rm CP}^{\rm dir,\,\perp}$ due to the NLP corrections.

Radiative leptonic $B_q \to \gamma \ell \bar{\ell}$ decays

- Systematic calculations for the helicity form factors [Beneke, Bobeth and YMW, 2020]:
  - SCET factorization for the LP contribution at $\mathcal{O}(\alpha_s)$.
  - The NLP corrections from the real or virtual photon radiation off the b-quark at $\mathcal{O}(\alpha_s^0)$.
  - The NLP corrections from the hard-collinear light-quark propagator at $\mathcal{O}(\alpha_s^0)$.
  - The NLP corrections from the weak-annihilation diagrams at $\mathcal{O}(\alpha_s^0)$.

- Three possible contractions of the four-quark operators:

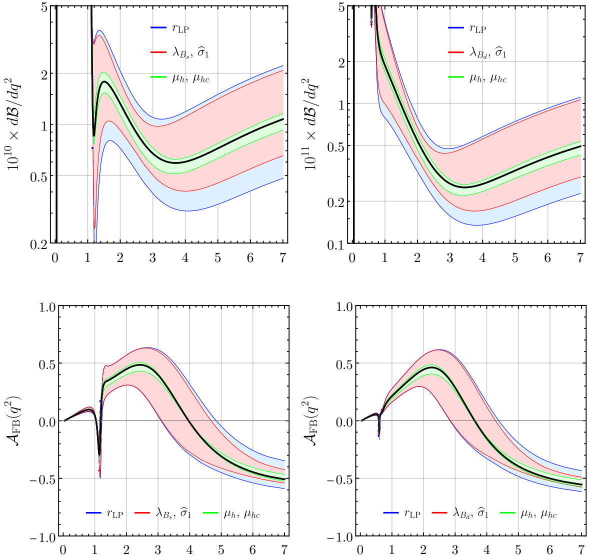


  - The left and middle diagrams are defined as the A and B-type insertions (also for P_7).
  - Need the more general $B \to \gamma^{(*)}$ form factors [YMW, 2016].
  - Factorization works for $q^2 \leq 6\,\mathrm{GeV}^2$ (excluding the narrow light-meson resonance region).
  - Strong duality violation of $B_s \to \gamma \ell \bar{\ell}$ due to the narrow width of $\phi(1020)$.

- Previous investigations with different techniques/methods [Guadagnoli, Reboud and Zwicky, 2017; Albrecht, Stamou, Ziegler, Zwicky, 2019; Kozachuk, Melikhov, Nikitin, 2018; etc].

Phenomenological observables for $B_q \to \gamma \ell \bar{\ell}$

- Theory predictions for $d\mathscr{B}/dq^2$ and $\mathscr{A}_{\rm FB}(q^2)$ [Beneke, Bobeth and YMW, 2020].



- Adding successively in quadrature the uncertainties due to $\mu_{h,hc}$ [green], $(\lambda_{B_q}, \widehat{\sigma}^{(1)}_{B_q})$ [red], $r_{\rm LP}$ [blue].

- The largest uncertainty due to the LCDA parameters: about $^{+70}_{-30}\%$ for $\mathscr{B}(B_s \to \gamma \ell \bar{\ell})$ in the q^2-bin $[2.0, 6.0]{\rm GeV}^2$.

Heavy-to-light B-meson decay form factors

- QCD/SCET factorization formulae [BBNS, BPRS, and many others].

$$F_i^{B\to M}(E) = C_i^{(\mathrm{A0})}(E)\,\xi_a(E) + \int_0^\infty \frac{d\omega}{\omega}\int_0^1 dv\, \underbrace{T_i(E;\ln\omega, v)}_{C_i^{(\mathrm{B1})} * J_i}\,\phi_B^+(\omega)\phi_M(v)\,.$$

Need the hadronic matrix elements of both the LP and NLP SCET currents!

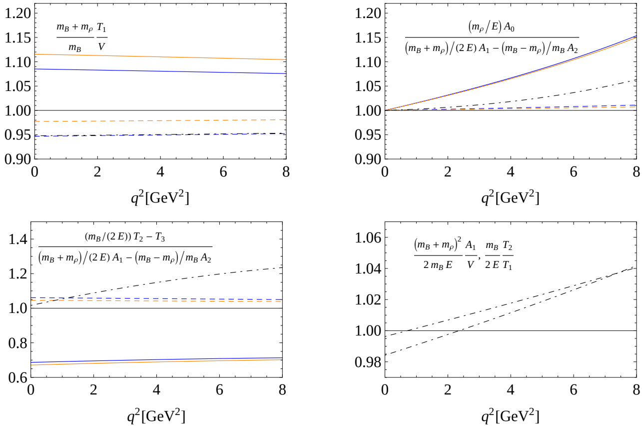
- Diagrammatic factorization for heavy-to-light form factors at one loop [Beneke, Feldmann, 2001].

- Perturbative calculations of the hard matching coefficients $C_i^{(\mathrm{A0})}(E)$:
  - One-loop SCET computations in [Bauer, Fleming, Pirjol, Stewart, 2001; Beneke, Kiyo, Yang, 2004].
  - Two-loop SCET computations in [Bonciani, Ferroglia, 2008; Asatrian, Greub, Pecjak, 2008; Beneke, Huber, Li, 2009; Bell, 2009; Bell, Beneke, Huber, Li, 2011].

- Perturbative calculations of the hard matching coefficients $C_i^{(\mathrm{B1})}$:
  - Infrared subtractions complicated by the appearance of evanescent operators and the D-dimensional Fierz transformation.
  - One-loop SCET computations in [Becher, Hill, 2004; Hill, Becher, Lee, Neubert, 2004; Beneke, Yang, 2006].

Semileptonic $B \to V$ form factors in QCD

- A long-standing puzzle [Bell, Beneke, Huber, Li, 2011]:

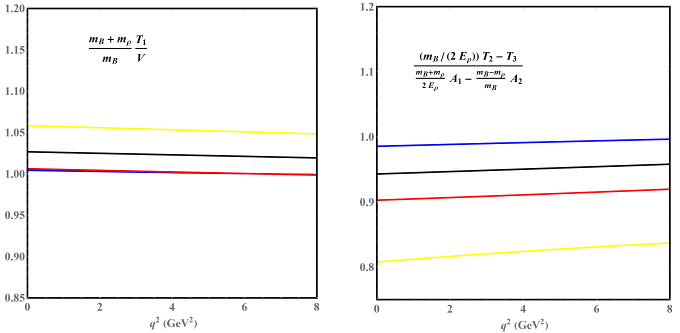


Discrepancies between the SCET predictions (blue solid) and the LCSR results with light vector meson DAs (black dashed-dotted) [Ball, Zwicky, 2005].

- Subleading power correction in heavy quark expansion, systematic uncertainties?

Semileptonic $B \to V$ form factors in QCD

- QCD factorization (yellow) versus SCET sum rules (red) [Gao, Lü, Shen, YMW, Wei, 2019]:



- The underlying mechanism responsible for the discrepancies:

$$\mathscr{R}_{1,\mathrm{LCSR}} = 1 + (-0.049)\big|_{C_i^{(\mathrm{A0})}} + (+0.054)\big|_{C_i^{(\mathrm{B1})}} + (-3.5 \times 10^{-5})\big|_{\mathrm{3PHT}},$$

$$\mathscr{R}_{1,\mathrm{QCDF}} = 1 + (-0.023)\big|_{C_i^{(\mathrm{A0})}} + (+0.086)\left[1 + \mathscr{O}(\alpha_s)\right]\big|_{C_i^{(\mathrm{B1})}}.$$

(a) No perturbative expansion for the A0-type $\mathrm{SCET_I}$ form factor in QCDF.

(b) Almost a factor of two smaller prediction of the B1-type $\mathrm{SCET_I}$ form factor in LCSR.

QCD/SCET sum rules with heavy-hadron LCDA

- LO QCD calculations of $B \to M$ form factors [Khodjamirian, Offen, Mannel, 2006; Faller, Khodjamirian, Klein, Mannel, 2008; Gubernari, Kokulu, van Dyk, 2018].
- NLO QCD calculations of $B \to \pi$ form factors at (two-particle) twist-3 [De Fazio, Feldmann, Hurth, 2006, 2008; YMW, Shen, 2015].
- NLO QCD calculations of $B \to \pi, K$ form factors at (two-particle) twist-3 and LO QCD calculations of higher-twsit corrections up to twist-6 [Lü, Shen, YMW, Wei, 2018].
- NLO QCD calculations of $B \to D$ form factors at (two-particle) twist-3 and LO QCD calculations of higher-twsit corrections up to twist-6 [YMW, Wei, Shen, Lü, 2017].
  - NLO twist-3 jet function complicated by two distinct hard-collinear variables.
  - Power-enhanced charm-quark mass effect.
  - Updated subleading power corrections at twist-6 [Gao, Huber, Ji, Wang, YMW, Wei, 2021].
- NLO QCD calculations of $B \to V$ form factors at (two-particle) twist-3 and LO QCD calculation of higher-twsit corrections up to twist-6 [Gao, Lü, Shen, YMW, Wei, 2019].
  - Rigorous perturbative matching with the evanescent-operator approach.
  - First SCET computation of the light-quark mass effect.
  - Three-particle higher-twist corrections compatible with the EOM constraints.
- NLO QCD calculations of $\Lambda_b \to \Lambda$ form factors at twist-four accuracy [YMW, Shen, 2016].
  - Incomplete NLO calculation of the jet function [Feldmann, Yip, 2015].
  - Confirmed by the lattice QCD calculations [Detmold, Meinel, 2016].
  - SCET factorization at leading power in Λ/m_b [Wei Wang, 2011].

Light-cone QCD sum rules with light-hadron LCDA

- Semileptonic $B \to P$ form factors:
  - LO QCD calculations of $B \to P$ form factors [Belyaev, Khodjamirian, Rückl, 1993].
  - NLO QCD calculations of $B \to P$ form factors at twist-2 accuracy [Khodjamirian, Rückl, Weinzierl, Yakovlev, 1997; Bagan, Ball, Braun, 1997].
  - NLO QCD calculations of $B \to P$ form factors at twist-3 accuracy [Ball, Zwicky, 2005; Duplancic, Khodjamirian, Mannel, Melic, Offen, 2008].
  - Improved NLO QCD calculations of $B \to P$ form factors at twist-3 accuracy [Khodjamirian, Mannel, Offen, YMW, 2011].
  - (Partial) NNLO QCD calculations of $B \to P$ form factors at twist-2 accuracy [Bharucha, 2012].
  - Twist-5 and -6 corrections of $B \to P$ form factors in the factorization limit [Rusov, 2017].

- Semileptonic $B \to V(\to P_1 P_2)$ form factors:
  - LO QCD calculations of $B \to V$ form factors [Ali, Braun, Simma, 1994; Ball and Braun, 1997].
  - NLO QCD calculations of $B \to V$ form factors at twist-2 accuracy [Ball, Braun, 1998].
  - NLO QCD calculations of $B \to V$ form factors at twist-3 accuracy [Ball, Zwicky, 2005].
  - Updated analysis of $B \to V$ form factors including NLO QCD calculations at twist-3 and LO QCD calculations at (partial)-twist-5 [Bharucha, Straub, Zwicky, 2015].
  - LO QCD calculations of $B \to \pi\pi$ form factors [Hambrock, Khodjamirian, 2016; Cheng, Khodjamirian, Virto, 2017].

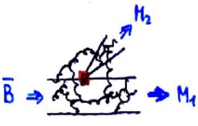
- Semileptonic Λ_b-baryon form factors:
  - LO QCD calculation of $\Lambda_b \to \Lambda$ form factors [YMW, Li, Lü, 2008].
  - LO QCD calculation of $\Lambda_b \to p$ form factors [Khodjamirian, Mannel, Klein, YMW, 2011].

The charmless hadronic B-meson decays

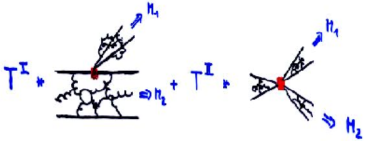
- QCD/SCET factorization [BBNS, BPRS, Chay, Kim, and many others].

  Heavy quark limit: $m_b \gg \Lambda_{\text{QCD}}$
  Large-energy limit: $E_M \approx m_b/2 \gg \Lambda_{\text{QCD}}$
  Scales: m_b, $\sqrt{m_b \Lambda_{\text{QCD}}}$, Λ_{QCD}, $(M_{\text{EW}}, \Lambda_{\text{NP}})$

$\mu \approx m_b$



$\mu \approx 1\,\text{GeV}$

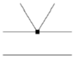
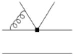
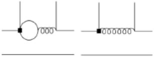
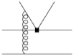
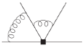
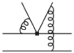
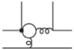


- Reduces $\langle M_1 M_2|\mathcal{O}|B\rangle$ to simpler $\langle M|\mathcal{O}|B\rangle$ (form factors), $\langle 0|\mathcal{O}|B\rangle$, $\langle M|\mathcal{O}|0\rangle$ (decay constants and distribution amplitudes).
- Calculation from first principles, but limited accuracy by Λ_{QCD}/m_b corrections.

Status of the NNLO QCD calculations

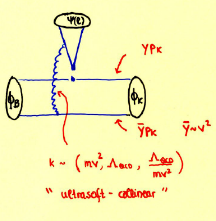
$$\langle M_1 M_2 | C_i O_i | \bar{B} \rangle_{\mathcal{L}_{\rm eff}} = \sum_{\rm terms} C(\mu_h) \times \Big\{ F_{B\to M_1} \times \underbrace{T^{\rm I}(\mu_h, \mu_s)}_{1+\alpha_s+\ldots} \star f_{M_2} \Phi_{M_2}(\mu_s)$$

$$+ f_B \Phi_B(\mu_s) \star \Big[\underbrace{T^{\rm II}(\mu_h, \mu_I)}_{1+\ldots} \star \underbrace{J^{\rm II}(\mu_I, \mu_s)}_{\alpha_s+\ldots} \Big] \star f_{M_1} \Phi_{M_1}(\mu_s) \star f_{M_2} \Phi_{M_2}(\mu_s) \Big\}$$

	T^I, tree	T^I, penguin	T^{II}, tree	T^{II}, penguin
LO: $\mathcal{O}(1)$				
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'04				
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07,'09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11, Bell Beneke, Huber, Li '15 Bell, Beneke, Huber, Li '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

- Strong cancellation of the NNLO corrections to the topological tree amplitudes between the vertex term and the spectator-scattering mechanism.
- Strong cancellation of the NNLO corrections to the QCD penguin amplitudes between the current-current operators $Q_{1,2}^p$ and the penguin operators $Q_{3-6,8g}$.

Factorization of $B \to \chi_{cJ} K$ [Beneke, Vernazza, 2008]



$$A(B\to HK)_{\rm spect} = \frac{G_F}{\sqrt{2}} V_{cb}V_{cs}^* \cdot 2C_1 \cdot \frac{\pi\alpha_s C_F}{N_c^2}\, \langle 0|[n]|H[n]\rangle \cdot \frac{1}{\lambda_B}\cdot M_B$$

$$\int_0^1 dy\, f_K \phi_K(y) \left\{ \left[\frac{e^{B,\rm fin}[n]}{\bar y} + \frac{B[n]}{\bar y^2} \right] \theta(1-\mu-y) \right.$$

hard / P-wave colour singlet

$$\left. + \; B[n] \frac{1}{\left(b+\sqrt{-(\bar y + a)}\right)^4}\, \theta(y-(1-\mu)) \right\}$$

ultrasoft / S-wave colour octet

$$b = \frac{\gamma}{m_b\sqrt{1-z}}, \qquad a = \frac{4 m_c E_B}{m_b^2(1-z)}$$

Endpoint div. in hard spectator-scattering
(Song et al., 2002; Meng et al., 2005)

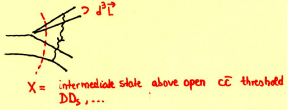
NEW

$$\int_0^1 dy\, \phi_K(y) \{\;\} = e^{B,\rm fin}[n] \int_0^1 dy \frac{\phi_K(y)}{\bar y} + B[n] \int_0^1 dy \frac{\phi_K(y) + \bar y \phi_K'(1)}{\bar y^2} + B[n]\,\phi_K'(1) \ln\mu$$

$$- B[n]\,\phi_K'(1) \left\{ \ln\mu + \ln\frac{m_b^2(1-z)}{\gamma^2} - i\pi - 2\ln(1+A) + 1 + \frac{2}{3}\frac{4+A}{(1+A)^2} \right\}$$

"large log" $\ln \frac{m_b^2}{m_c^2 v^2}$: endpoint log

$$A = \sqrt{-\frac{4(E_B + i\epsilon)}{\gamma^2/m_c}} = \mathcal{O}(1)$$



Large rescattering phase from endpoint contribution, none from hard scattering.

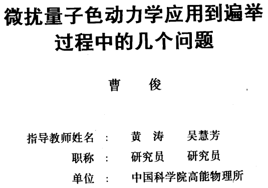
- The first field-theoretical treatment of rapidity divergences in the heavy-quark limit $m_c v^2 \gg \Lambda_{\rm QCD}$.

PQCD factorization for hard exclusive reactions

- Pioneer works on hard exclusive reactions:
  - Hard scattering approach formulated in [Lepage, Brodsky, 1979, 1980].
  - The hadronic wave function in QCD [Brodsky, Lepage, Huang, 1980].
  - Sudakov effects in hadron-hadron elastic (Landshoff) scattering [Botts, Sterman, 1989].
  - Sudakov resummation for the pion electromagnetic form factor [Li, Sterman, 1992].
    Key observation: Perturbative QCD formalism to hard exclusive processes applicable for $\sqrt{Q^2} \sim 20\,\Lambda_{\rm QCD}$.
    Saving us from the strong doubts raised in [Isgur, Llewellyn Smith, 1988; Radyushkin, 1984].
  - More references can be found in the Bible by John Collins.

Important pieces of work accomplished in China:

- Applicability of PQCD factorization for the pion electromagnetic form factor [Huang, Shen, 1990].
- Sudakov suppression for hard exclusive reactions [PhD thesis by Jun Cao].
- Many more interesting papers to be discussed.


微扰量子色动力学应用到遍举
过程中的几个问题

曹　俊

指导教师姓名 ：　黄　涛　　吴慧芳
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PQCD factorization for heavy hadron decays

- Birth of the modern PQCD approach for charmless hadronic B-meson decays:
  - Penguin enhancement for $B \to \pi K$ [Keum, Li, Sanda, 2001].
  - Tree-dominated processes $B \to \pi \pi$ [Lü, Ukai, Yang, 2001].

- Hundreds of papers on the tree-level PQCD calculations [Groups led by Li, Lü, Xiao, etc].

- PQCD calculations at NLO complicated by the appearance of multi-scales:
  - The pion-photon form factor at large momentum transfer [Nandi, Li, 2007].
  - The pion electromagnetic form factor at large Q^2 [Li, Shen, YMW, Zou, 2011].
  - $B \to \pi$ form factors at large recoil [Li, Shen, YMW, 2012].
  - Increasing complexities mainly due to the infrared subtractions!

- Intensive NLO PQCD calculations subsequently:
  - NLO twist-3 correction to the pion electromagnetic form factor [Cheng, Fan, Xiao, 2014].
  - NLO twist-3 correction to $B \to \pi$ form factors [Cheng, Fan, Yu, Lü, Xiao, 2014].
  - NLO correction to $B \to \rho$ form factors [Hua, Zhang, Xiao, 2018].
  - (Partial)-NLO correction to $B \to \pi\pi$ [Cheng, Xiao, Zhang, 2014].
  - (Partial)-NLO correction to $B_s \to PP$ [Yan, Liu, Xiao, 2019].
  - (Partial)-NLO correction to $B_c \to J/\psi$ form factors [Liu, Li, Xiao, 2020].
  - Review on the NLO PQCD computations [Cheng, Xiao, 2020].
  - Many more papers on the NLO PQCD calculations.

Conceptual framework of PQCD factorization

- Formal developments of the PQCD approach:
  - Rapidity resummation for the TMD B-meson wavefunction in Mellin space [Li, Shen, YMW, 2013].
  - Joint resummation for the threshold and Sudakov logarithms for the pion-photon form factor [Li, Shen, YMW, 2014].
  - Factorization-compatible definitions of the TMD pion wavefunction [Li, YMW, 2015]

- Naïve definition of the TMD pion wavefunction:

$$\phi_\pi^{\rm naive}(x, \vec{k}_T, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} \, e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$
$$\times \langle 0 | \bar{q}(0) W_{n_-}^\dagger(+\infty, 0) \, \not{n}_- \, \gamma_5 \, [\text{tr. link}] \, W_{n_-}(+\infty, z) \, q(z) | \pi^+(p) \rangle \, .$$

- Rapidity divergence in the infrared subtraction:

$$\phi_\pi^{(1)} \otimes H^{(0)} \supset \int [dl] \frac{1}{[(k+l)^2 + i0][l_+ + i0][l^2 + i0]}$$
$$\times \left[H^{(0)}(x + l_+/p_+, \vec{k}_T + \vec{l}_T) - H^{(0)}(x, \vec{k}_T) \right] .$$

  - Rapidity divergence due to the eikonal propagator.
  - Key difference: both the longitudinal and transverse components of the partonic momentum changed in TMD factorization!

Light-cone singularity

- Regularization of the rapidity divergence [Collins, 2003].
  - Rotating the gauge links away from the light-cone ($u = (u_+, u_-, \vec{0}_T)$):

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} \, e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$
$$\times \langle 0 | \bar{q}(0) W_u^\dagger(+\infty, 0) \, \not{n}_- \, \gamma_5 \, [\text{tr. link}] \, W_u(+\infty, z) \, q(z) | \pi^+(p) \rangle \, .$$

  - Introducing soft subtractions:

$$\phi_\pi(x, \vec{k}_T, y_u, \mu) \stackrel{?}{=} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} \, e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)}$$
$$\times \frac{\langle 0 | \bar{q}(0) W_{n_-}^\dagger(+\infty, 0) \, \not{n}_- \, \gamma_5 \, [\text{tr. link}] \, W_{n_-}(+\infty, z) \, q(z) | \pi^+(p) \rangle}{\langle 0 | W_{n_-}^\dagger(+\infty, 0) W_u(+\infty, 0) \, [\text{tr. link}] \, W_{n_-}(+\infty, z) \, W_u^\dagger(+\infty, z) | 0 \rangle} \, .$$

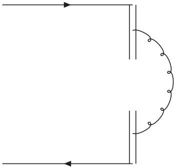
- Are these definitions compatible with the factorization theorems?

- Can employ multiple non-light-like Wilson lines at the price of introducing the soft function in the factorization formulae and using another parameter ρ beyond the scale parameters of CSS [Ji, Ma, Yuan, 2004].

Pinch singularity

- Singularity from Wilson-line self energies [Bacchetta, Boer, Diehl, Mulders, 2008].
  - Pinch singularity only appears in a TMD parton density with $u^2 < 0$.
  - Pinch singularity appears in the TMD wave functions for any off-light-cone u.

$$\phi_\pi \supset \int [dl] \frac{u^2}{[l+i0)][u\cdot l+i0][u\cdot l-i0]} \times \delta(x'-x+l_+/p_+)\delta^{(2)}(\vec{k}_T'-\vec{k}_T+\vec{l}_T)\,.$$



  - Pinch singularity corresponds to the linear divergence in the length of the Wilson line in the coordinate space.

- Off-light-cone Wilson lines regularize rapidity divergence, at the price of introducing unwanted pinch singularity.

- How to achieve factorization-compatible definitions of TMD wavefunctions?

Collins modification

- New definition without pinch singularity [Collins, 2011]:

$$\begin{aligned}\phi_\pi^{\rm C}(x,\vec{k}_T,y_2,\mu) \;=\;& \lim_{\substack{y_1\to+\infty \\ y_u\to-\infty}} \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2}\, e^{i(xp_+ z_- - \vec{k}_T\cdot\vec{z}_T)} \\ & \times \langle 0|\bar{q}(0)\, W_u^\dagger(+\infty,0)\, \not{n}_-\, \gamma_5\, [\text{tr. link}]\, W_u(+\infty,z)\, q(z)|\pi^+(p)\rangle \\ & \times \sqrt{\frac{S(z_T;y_1,y_2)}{S(z_T;y_1,y_u)\, S(z_T;y_2,y_u)}}\,.\end{aligned}$$

↑

rapidity of the gauge vector $n_2 = (e^{y_2}, e^{-y_2}, \vec{0}_T)$

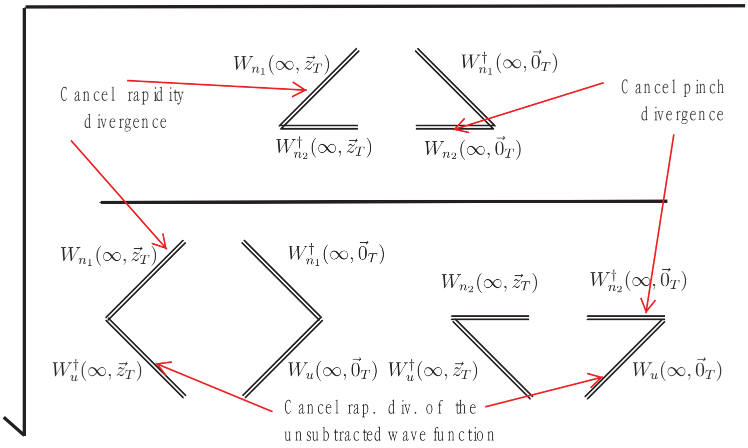
  - Soft function:

$$S(z_T;y_A,y_B) = \frac{1}{N_c}\langle 0|W_{n_B}^\dagger(\infty,\vec{z}_T)_{ca}\, W_{n_A}(\infty,\vec{z}_T)_{ad}\, W_{n_B}(\infty,0)_{bc}\, W_{n_A}^\dagger(\infty,0)_{db}|0\rangle\,.$$

- General properties of the new definition:
  - The unsubtracted wave function only involves light-cone Wilson lines.
  - Each soft factor has at most one off-light-cone Wilson line.
  - No rapidity divergences and no pinch singularities.
  - A detailed comparison with many other definitions [Collins, arXiv:1409.5408].

Why the new definition works?

- Cancellation mechanism ($y_1 \to +\infty$, $y_u \to -\infty$):



Simplified definitions of TMDs possible?

- Treatment of rapidity and pinch singularities:

Light-cone Wilson lines

$\downarrow$ TMD hard function

Rapidity divergence

$\downarrow$

Off-light-cone Wilson lines

$\downarrow$ Dipolar structure

Pinch singularity

$\swarrow$ $\searrow$

Nontrivial soft subtraction | Non-dipolar Wilson lines

$\downarrow$

Simper soft subtraction

TMDs with non-dipolar Wilson lines

- Orthogonal Wilson lines ($n_2 \cdot v = 0$) [Li, Wang, 2015]:

$$\phi_\pi^{\mathrm{I}}(x, \vec{k}_T, y_2, \mu) = \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \langle 0 | \bar{q}(0) W_{n_2}^\dagger(+\infty, 0) \not{n}_- \gamma_5 [\mathrm{links}@\infty] W_v(+\infty, z) q(z) | \pi^+(p) \rangle .$$

$$\uparrow \qquad\qquad \uparrow$$

$$n_2 = (e^{y_2}, e^{-y_2}, \vec{0}_T), \qquad v = (-e^{y_2}, e^{-y_2}, \vec{0}_T).$$

  - Wilson-line self energies vanish in Feynman gauge.
    $\Rightarrow$ Soft subtraction not needed.
  - $\phi_\pi^{\mathrm{I}} \otimes H^{(0)}$ reproduces the collinear logarithm of QCD diagrams:

$$\phi_\pi^{\mathrm{I}} \otimes H^{(0)} = -\frac{\alpha_s C_F}{4\pi} [2 \ln x + 3] \ln \left(\frac{k_T^2}{Q^2} \right) H^{(0)}(x, k_T) + \cdots .$$

- Antiparallel Wilson lines [Li, Wang, 2015]:

$$\phi_\pi^{\mathrm{II}}(x, \vec{k}_T, y_2, \mu) = \int \frac{dz_-}{2\pi} \int \frac{d^2 z_T}{(2\pi)^2} e^{i(xp_+ z_- - \vec{k}_T \cdot \vec{z}_T)} \times \frac{\langle 0 | \bar{q}(0) W_{n_2}^\dagger(+\infty, 0) \not{n}_- \gamma_5 [\mathrm{links}@\infty] W_{n_2}(-\infty, z) q(z) | \pi^+(p) \rangle}{[\mathrm{color}] \langle 0 | W_{n_2}^\dagger(+\infty, 0) [\mathrm{links}@\infty] W_{n_2}(-\infty, 0) | 0 \rangle} .$$

Theoretical wishlist

- Systematic understanding of the (high-twist) B-meson distribution amplitudes.
  - Renormalization properties beyond the one-loop approximation [conformal symmetry].
  - Perturbative constraints at large ω_i [OPE technique].
  - Renormalon analysis and the renormalization-scheme dependence.
  - Precision determinations of the inverse moment λ_B.

- QCD factorization for the subleading power corrections.
  - SCET analysis for the pion-photon form factor as the first step [operator structures, symmetry constraints, etc].
  - General treatment of the rapidity divergences in the (naïve)-factorization formulae.
  - Rigorous factorization proof taking into account the Glauber gluons.
  - Novel resummation techniques for enhanced logarithms.

- Technical issues for future improvements.
  - Factorization techniques for the electromagnetic corrections.
  - NNLO QCD computations for $B \to V\gamma$ and $B \to V\ell\ell$.
  - QCD factorization for the radiative and electroweak penguin decays of the Λ_b-baryon.
  - Improved understanding of the parton-hadron duality violation.

- Very promising future for QCD aspects of heavy-quark physics!