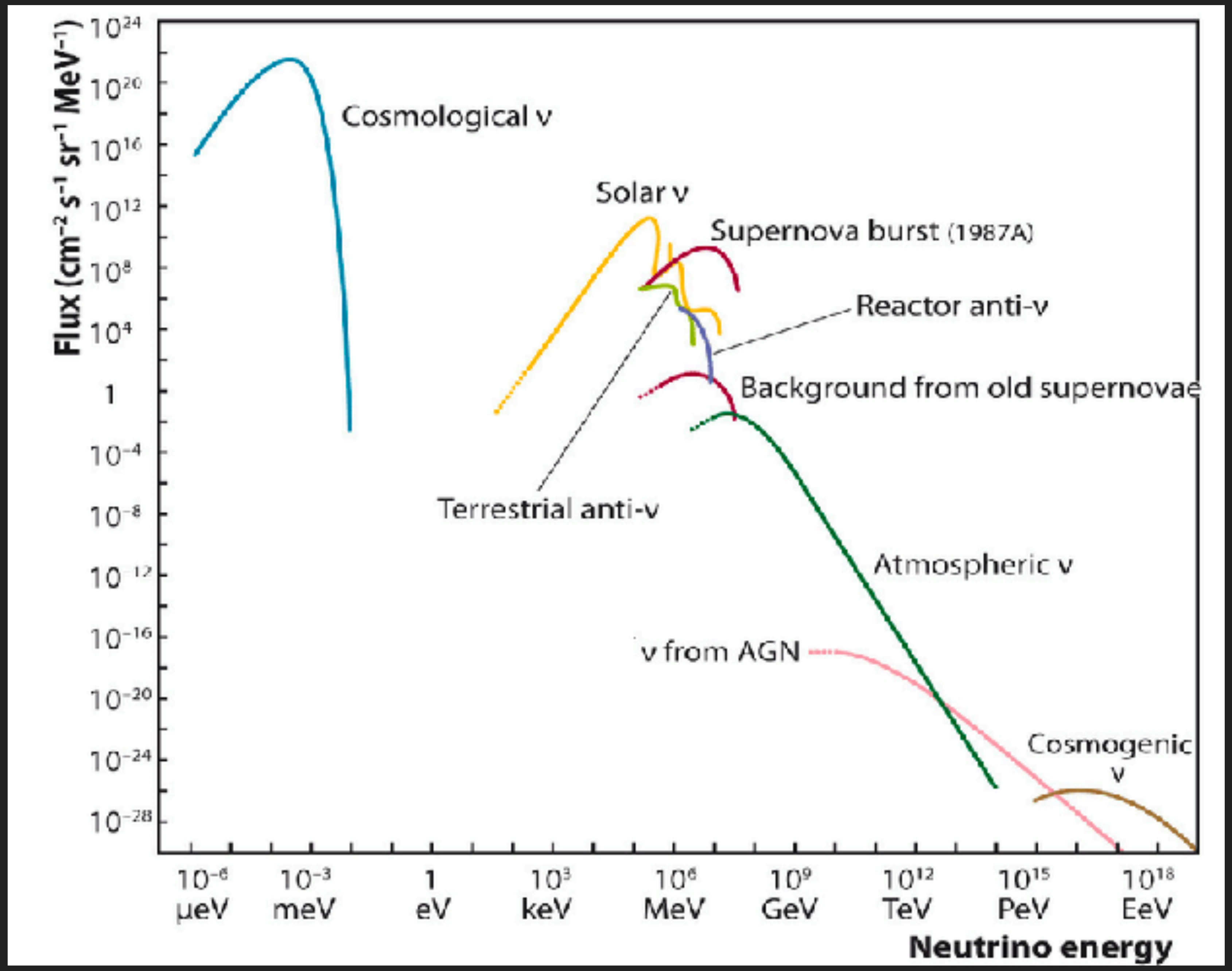


R. PLESTID | UKY & FNAL

IN COLLABORATION WITH V. BRDAR & N. ROCCO

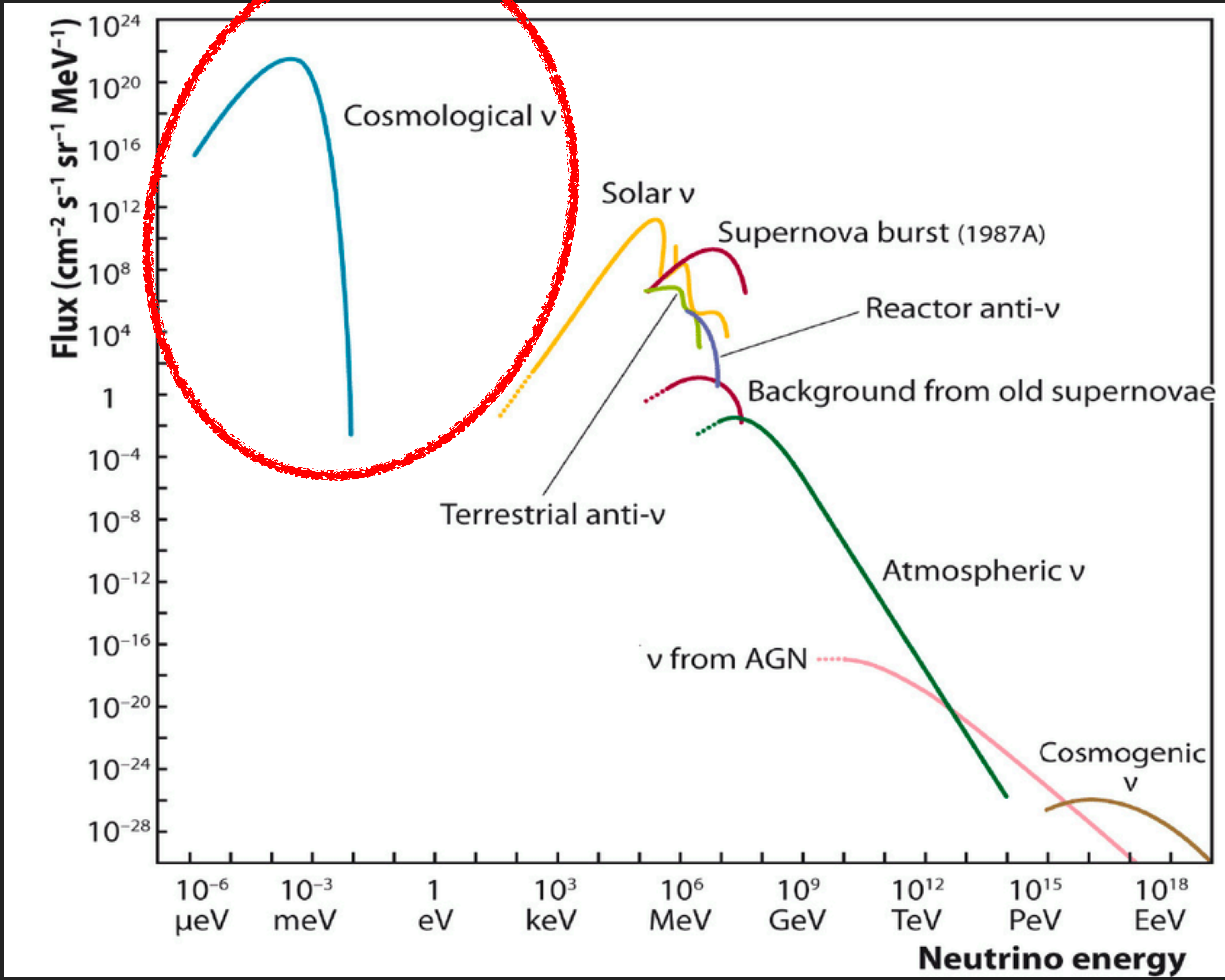
RELIC NEUTRINO CAPTURE WITH HEAVY NUCLEI & THE ξ-APPROXIMATION

COSMIC NEUTRINO BACKGROUND

- Much like the cosmic microwave background.
- $T \sim 2\ K \implies p_\nu \ll m_\nu$.
- Large flux at very low energy.

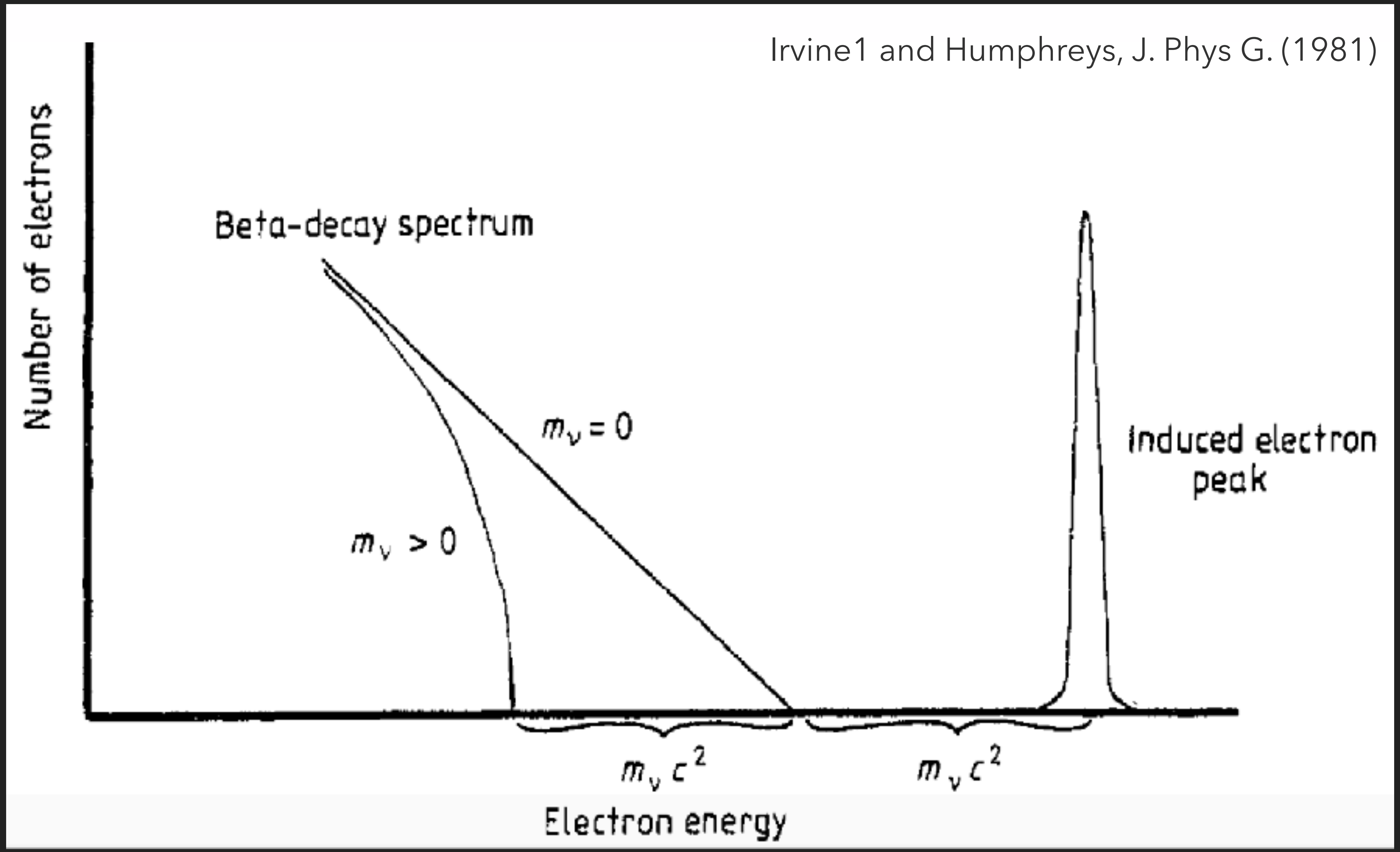
$$\nu\ A_1 \to e^- A_2^+$$

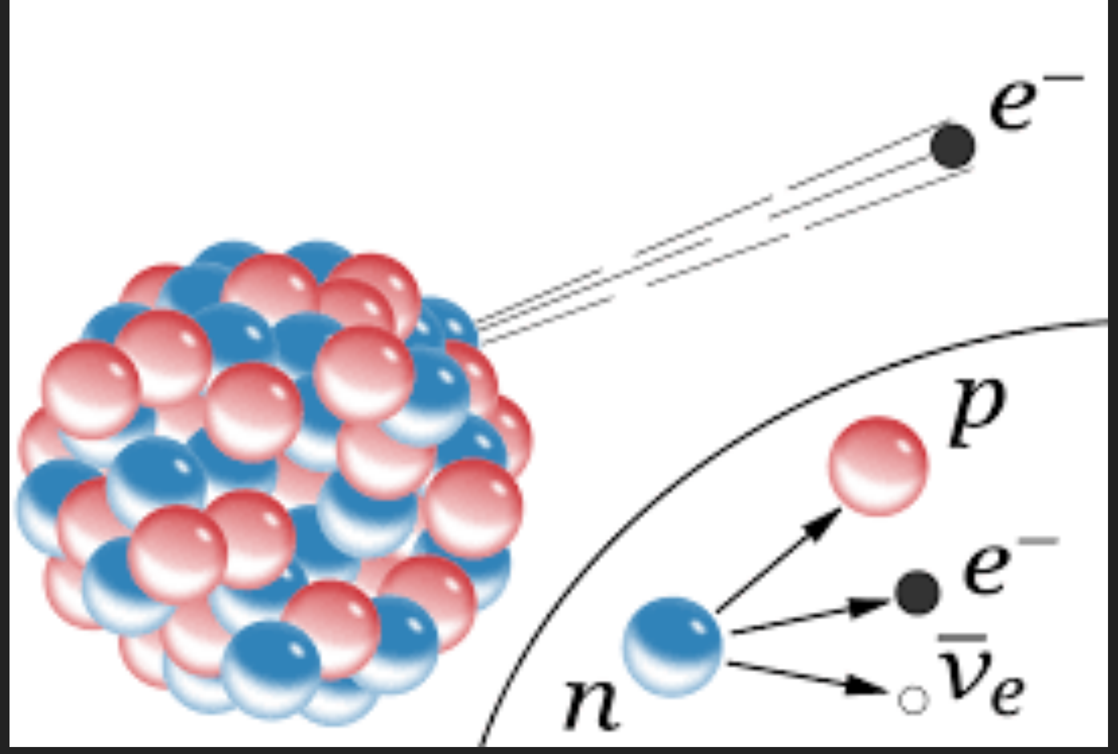
$$M_1 - M_2 \lesssim 200\ \text{keV}$$



WHAT IS THIS TALK ABOUT?

- Search for events beyond the end point of the beta decay spectrum.
- How often does a $C\nu B$ neutrino capture on a nucleus?
- This drives experimental design, e.g. how much material do we need?





BASED ON RECENT WORK

FERMILAB-PUB-22-026-T
NUHEP-TH/22-01

Empirical capture cross sections for cosmic neutrino detection with ^{151}Sm and ^{171}Tm

Vedran Brdar,[1,2,*] Ryan Plestid,[3,2,†] and Noemi Rocco[2,‡]

[1] *Department of Physics and Astronomy, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, USA*
[2] *Theoretical Physics Department, Fermilab, Batavia, IL 60510,USA*
[3] *Department of Physics and Astronomy, University of Kentucky Lexington, KY 40506, USA*

- Heavy nuclei with low-Q values have “simple” nuclear matrix elements.
- Analysis is driven by simple power counting arguments.
- Ideas are not new [see Behrens & Buhring Nucl.Phys.A 162 (1971)].

MAIN CONCLUSIONS

FERMILAB-PUB-22-026-T
NUHEP-TH/22-01

Empirical capture cross sections for cosmic neutrino detection with ^{151}Sm and ^{171}Tm

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- For low Q-value nuclei the shape is predictable **even when decay is first forbidden non-unique**.
- There are no theoretical obstacles that prevent the use of heavy nuclei for CνB detection.

CROSSING SYMMETRY:
RELATING CAPTURE TO DECAY

EXACT S-MATRIX FOR REACTION

$$S_{\text{cap}}(\mathbf{p}, \mathbf{k}) := {}_{\text{out}}\langle A_2\ e(\mathbf{p}) \,|\, A_1\ \nu(\mathbf{k})\rangle_{\text{in}}$$

$$S_{\beta}(\mathbf{p}, \mathbf{k}) := {}_{\text{out}}\langle A_2\ e(\mathbf{p})\ \bar{\nu}(\mathbf{k}) \,|\, A_1\rangle_{\text{in}}$$

$$S_{\beta}(\mathbf{p}, \mathbf{k}) = S_{\text{cap}}(\mathbf{p}, -\mathbf{k}) \implies S_{\beta}(\mathbf{p},0) = S_{\text{cap}}(\mathbf{p},0)$$

CAPTURE CROSS SECTION SETS NORMALIZATION OF $\beta -$ SPECTRUM AT ENDPOINT

$$G(W_e^{\mathrm{max}}) = 1$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}W_e} = \frac{E_\nu p_\nu}{\pi^2} \times (\sigma v)_\nu \times G(W_e)$$

PHASE SPACE

END-POINT
MATRIX ELEMENT

ENERGY DEPENDENCE
OF MATRIX ELEMENT

CAPTURE CROSS SECTION SETS NORMALIZATION OF $\beta-$ SPECTRUM AT ENDPOINT

$$G(W_e^{\mathrm{max}}) = 1$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}W_e} = \frac{E_\nu p_\nu}{\pi^2} \times (\sigma v)_\nu \times G(W_e)$$

- If we can compute $G(W_e)$ then we can extract $(\sigma v)_\nu$.

THEORY OF BETA DECAY WITH A COULOMB FIELD

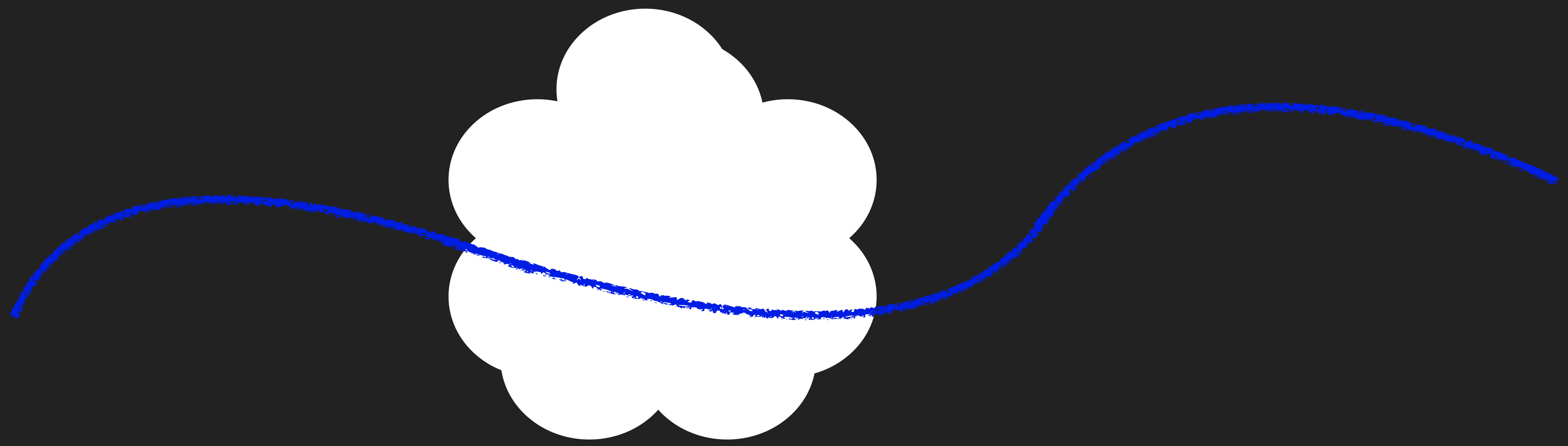
MULTIPOLE EXPANSION WITHOUT A COULOMB FIELD

$$\mathscr{M} \sim \int \mathrm{d}^3x \, \langle A_2 | J_\mu(x) | A_1 \rangle \, \overline{\mathscr{U}}_p(x) \gamma^\mu u_k \mathrm{e}^{\mathbf{ik}\cdot\mathbf{x}}$$

0 for $r \gg R$

Taylor expand

$Q \ll 1/R$



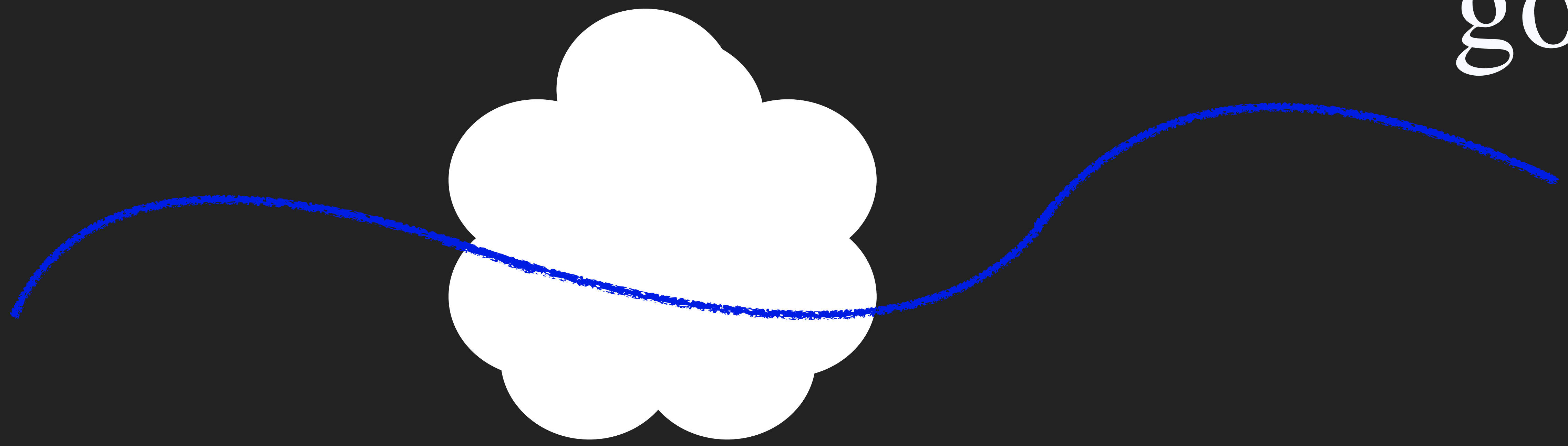
Expand in spherical harmonics

MULTIPOLE EXPANSION WITHOUT A COULOMB FIELD

$$\mathscr{M} \sim \int \mathrm{d}^3x \, \langle A_2 | J_\mu(x) | A_1 \rangle \, \overline{\mathscr{U}}_p(x) \gamma^\mu u_k \mathrm{e}^{\mathbf{ik \cdot x}}$$

J and π
good quantum numbers

$$Q \ll 1/R$$



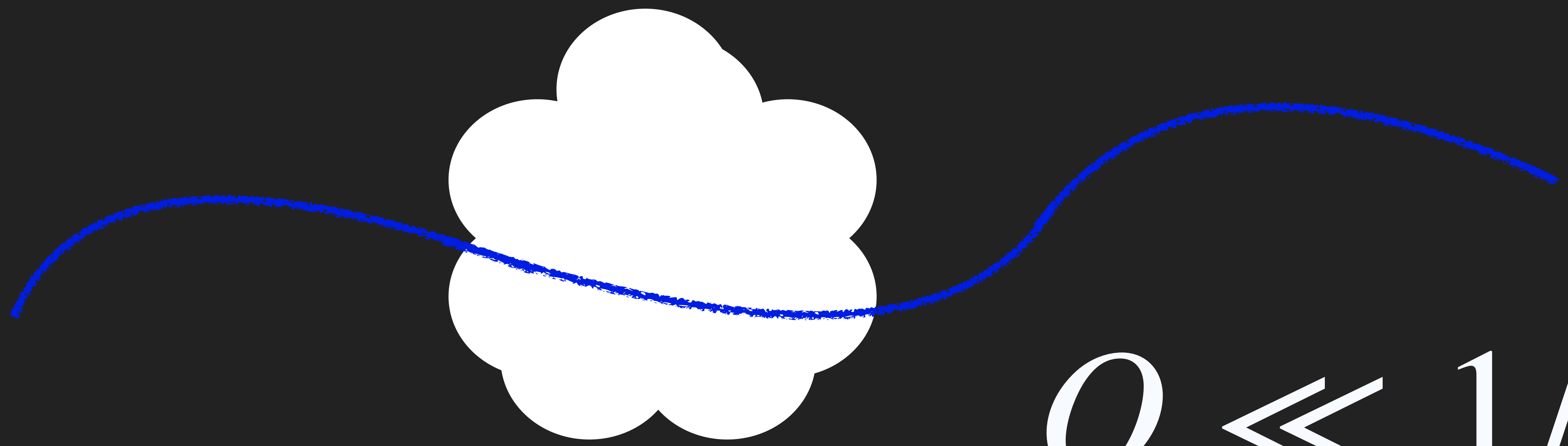
Expand in spherical harmonics

MULTIPOLE EXPANSION WITHOUT A COULOMB FIELD

Expand in spherical harmonics

$$\mathscr{M} = \sum_{K,\kappa_e,\kappa_\nu} M_K^2(\kappa_e, \kappa_\nu) + m_K^2(\kappa_e, \kappa_\nu) + \text{interference}$$

$$|J_i - J_f| \leq K \leq J_i + J_f$$



$$Q \ll 1/R$$

FIRST FORBIDDEN NON-UNIQUE

Type of Transition	Selection Rules	$L_{e\nu}$	$\Delta\pi$?
superallowed	$\Delta I = 0, \pm 1^*$	0	no
allowed	$\Delta I = 0, \pm 1$	0	no
1^{st} forbidden	$\Delta I = 0, \pm 1$	1	yes
unique** 1^{st} forbidden	$\Delta I = \pm 2$	1	yes
2^{nd} forbidden	$\Delta I = \pm 1^{***}, \pm 2$	2	no
unique 2^{nd} forbidden	$\Delta I = \pm 3$	2	no
3^{rd} forbidden	$\Delta I = \pm 2^{***}, \pm 3$	3	yes
unique 3^{rd} forbidden	$\Delta I = \pm 4$	3	yes
4^{th} forbidden	$\Delta I = \pm 3^{***}, \pm 4$	4	no
unique 4^{th} forbidden	$\Delta I = \pm 5$	4	no

Credit: Alex Bielajew

- Spins of nuclei determine angular momenta of lepton.
- Non-unique transitions can receive contributions from multiple operators.

FIRST FORBIDDEN NON-UNIQUE SHAPE

$$\left\{M_1^2(1,1),\ m_1^2(1,1),\ M_1^2(1,2),\ M_1^2(2,1)\right\}$$

$$G(W_e) = \frac{(1 + aW_e + b/W_e + cW_e^2)}{(1 + aW_e + b/W_e + cW_e^2)_{W_e = W_e^{\max}}}$$

- Generally shape depends on 4 different matrix elements.
- Can have non-trivial energy dependence.

FIRST FORBIDDEN NON-UNIQUE SHAPE

$$\left\{M_1^2(1,1),\ m_1^2(1,1),\ M_1^2(1,2),\ M_1^2(2,1)\right\}$$

$$G(W_e) \sim (1 + aW_e + b/W_e + cW_e^2)$$

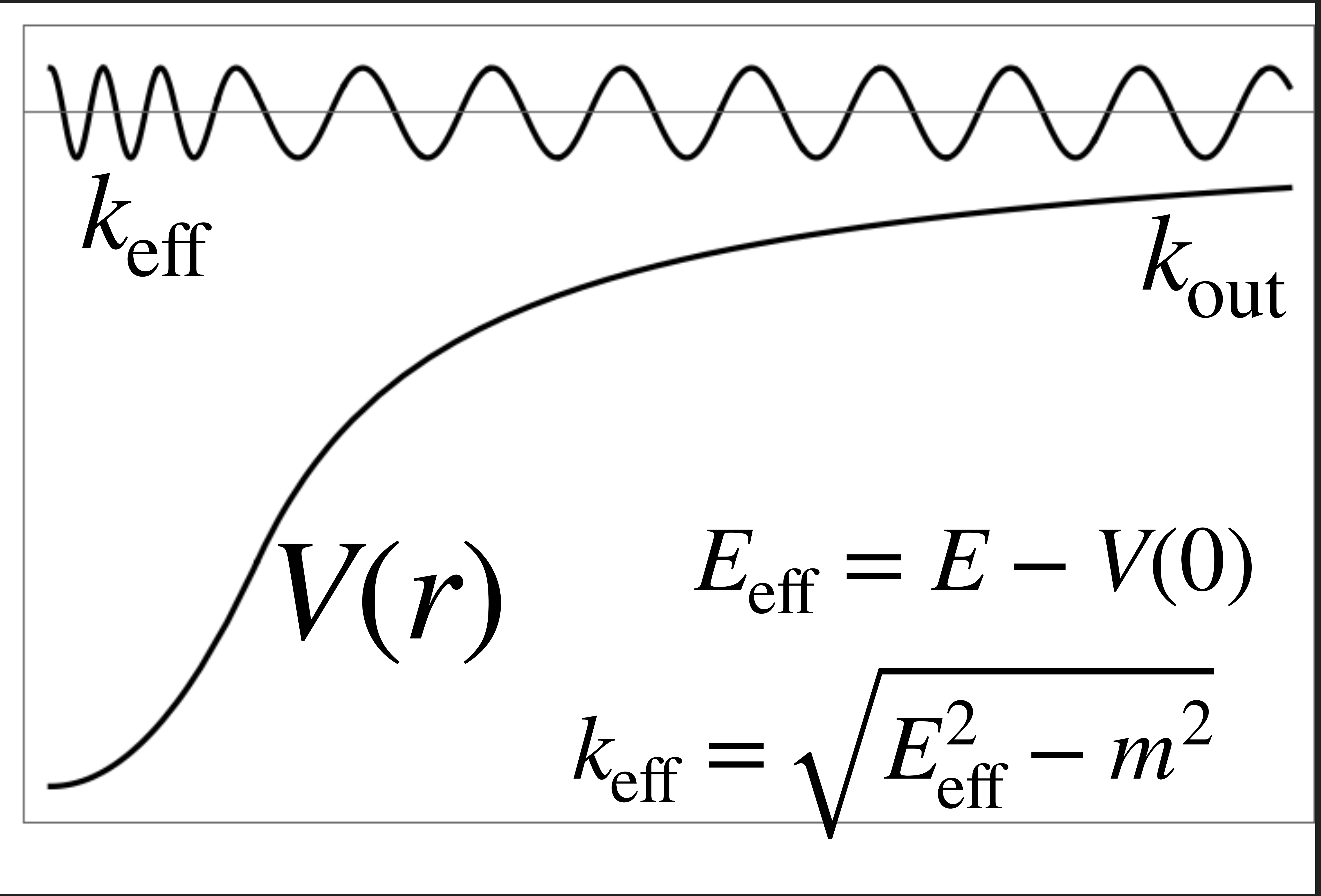
Supressed by $1/\xi = W_e R / Z\alpha$

NEW SCALE IN THE PROBLEM

- Potential energy introduces new scale in the problem.
- Will generically have

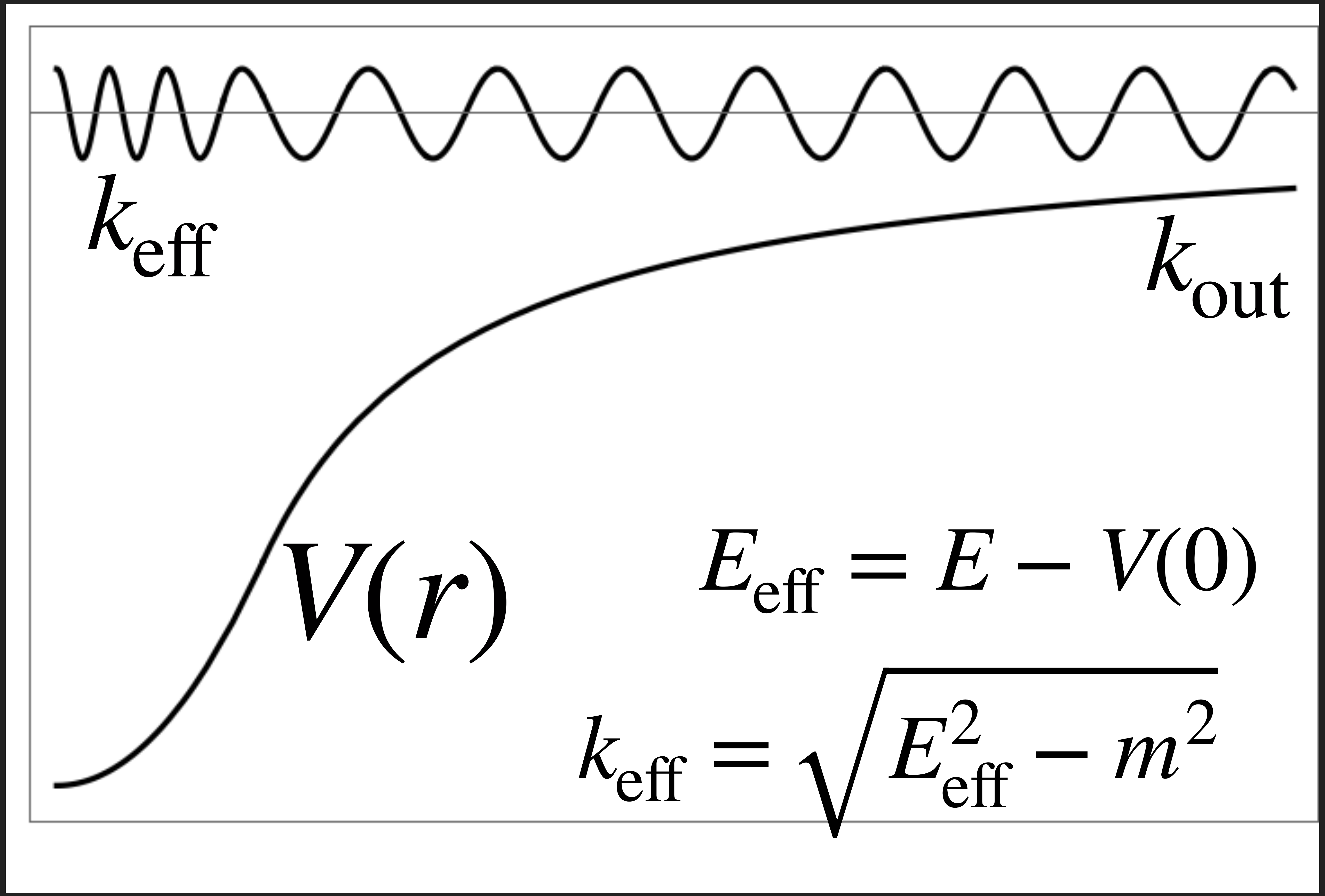
$$Q \ll V(0) \ll 1/R$$

- Heuristically, wave near origin has large "kinetic energy".



CONCLUSION

- With large Coulomb field matrix elements become energy independent for $W_e \ll Z\alpha/R$.
- Shape is the same as an allowed decay.



SHAPE OF THE ALLOWED SPECTRUM

EXTRACTING CROSS SECTION

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}W_e} = \frac{E_\nu p_\nu}{\pi^2} \times (\sigma v)_\nu \times G(W_e)$$

METHOD 1: USE HALF-LIFE

$$t_{1/2} = \ln 2 \Big/ \int_0^{W_e^{\max}} \mathrm{d}W_e \, \frac{\mathrm{d}\Gamma}{\mathrm{d}W_e}$$

METHOD 2: MEASURE ENDPOINT

$$\lim_{W_e \to W_e^{\max}} \frac{\mathrm{d}\Gamma}{\mathrm{d}W_e}$$

SEE BOYARSKY ET. AL. ARXIV:2111.09292

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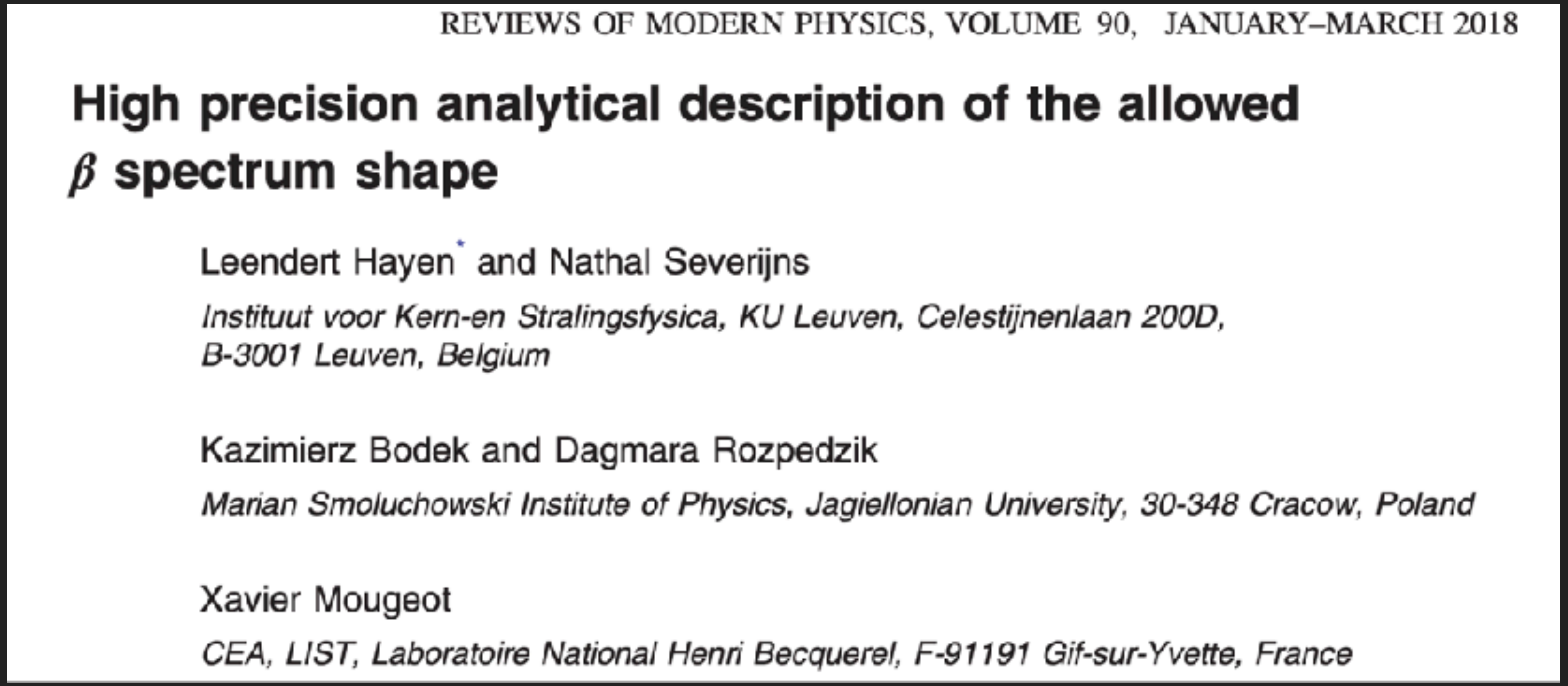
$$G(W_e) = \frac{F_0(Z, W_e) \times L_0(W_e) \times R(W_e) \times X(W_e) \times r(W_e) \times W_e p_e}{F_0(Z, W_e) L_0(W_e) R(W_e) X(W_e) r(W_e) W_e p_e \;/\; .\; W_e \to W_e^{\max}}$$



REVIEWS OF MODERN PHYSICS, VOLUME 90, JANUARY–MARCH 2018

High precision analytical description of the allowed β spectrum shape

Leendert Hayen* and Nathal Severijns

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Kazimierz Bodek and Dagmara Rozpedzik

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Xavier Mougeot

CEA, LIST, Laboratoire National Henri Becquerel, F-91191 Gif-sur-Yvette, France

- Need spectrum over full energy range.
- Includes low energy region. Atomic effects, exchange etc.

EXTRACTING CROSS SECTION

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}W_e} = \frac{E_\nu p_\nu}{\pi^2} \times (\sigma v)_\nu \times G(W_e)$$

$$G(W_e) = \frac{F_0(Z, W_e) \times L_0(W_e) \times R(W_e) \times X(W_e) \times r(W_e) \times W_e p_e}{F_0(Z, W_e)L_0(W_e)R(W_e)X(W_e)r(W_e)W_e p_e \,/\, .\, W_e \to W_e^{\max}} + O(1/\xi)$$

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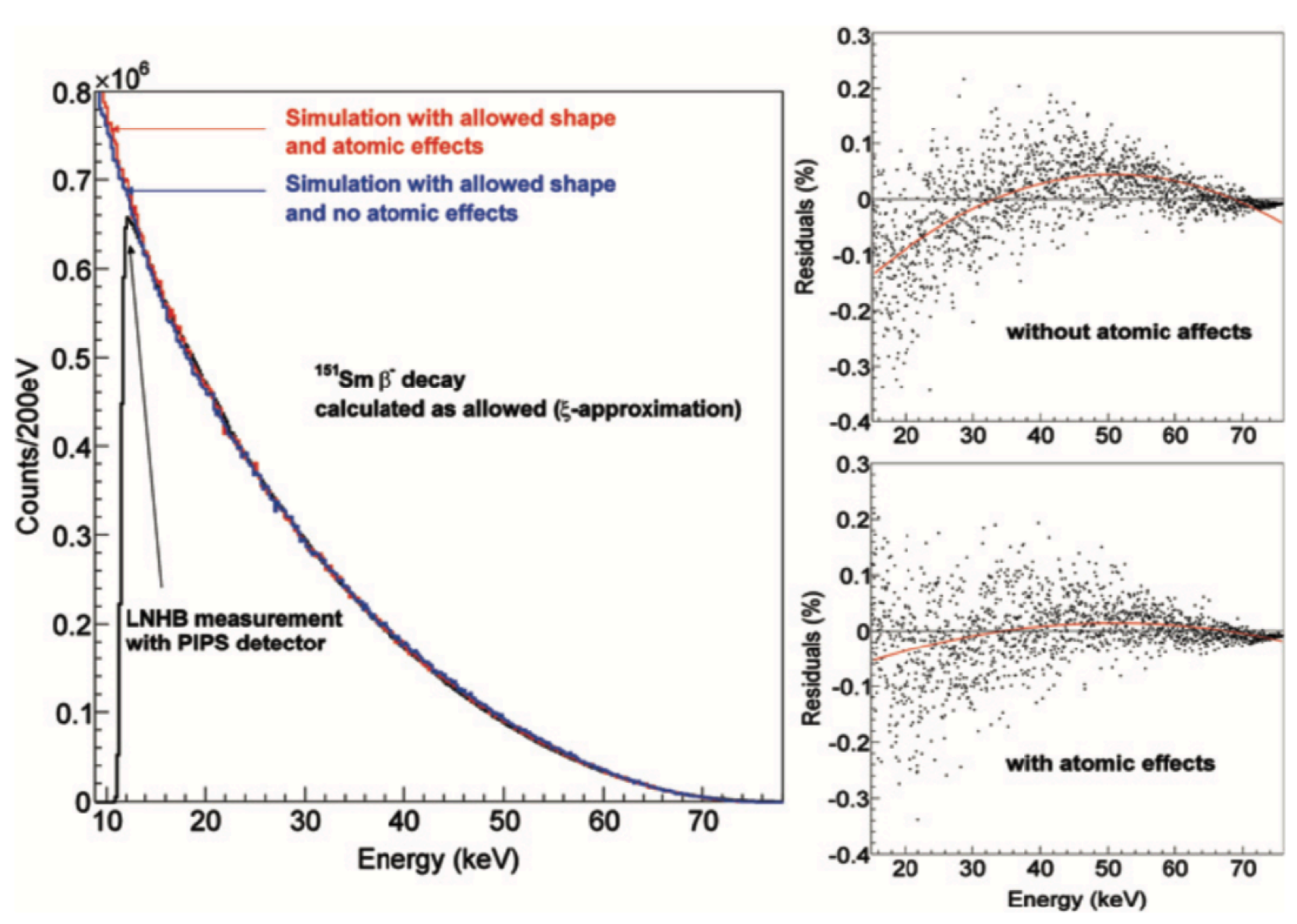
- Includes low energy region. Atomic effects, exchange etc.

STRONG EVIDENCE FOR 151-SM

- Shape effects are under control for allowed decays at the ~0.1% level.
- Residuals near endpoint are ~0.3% .
- $\xi = Z\alpha / W_e R \approx 150$.

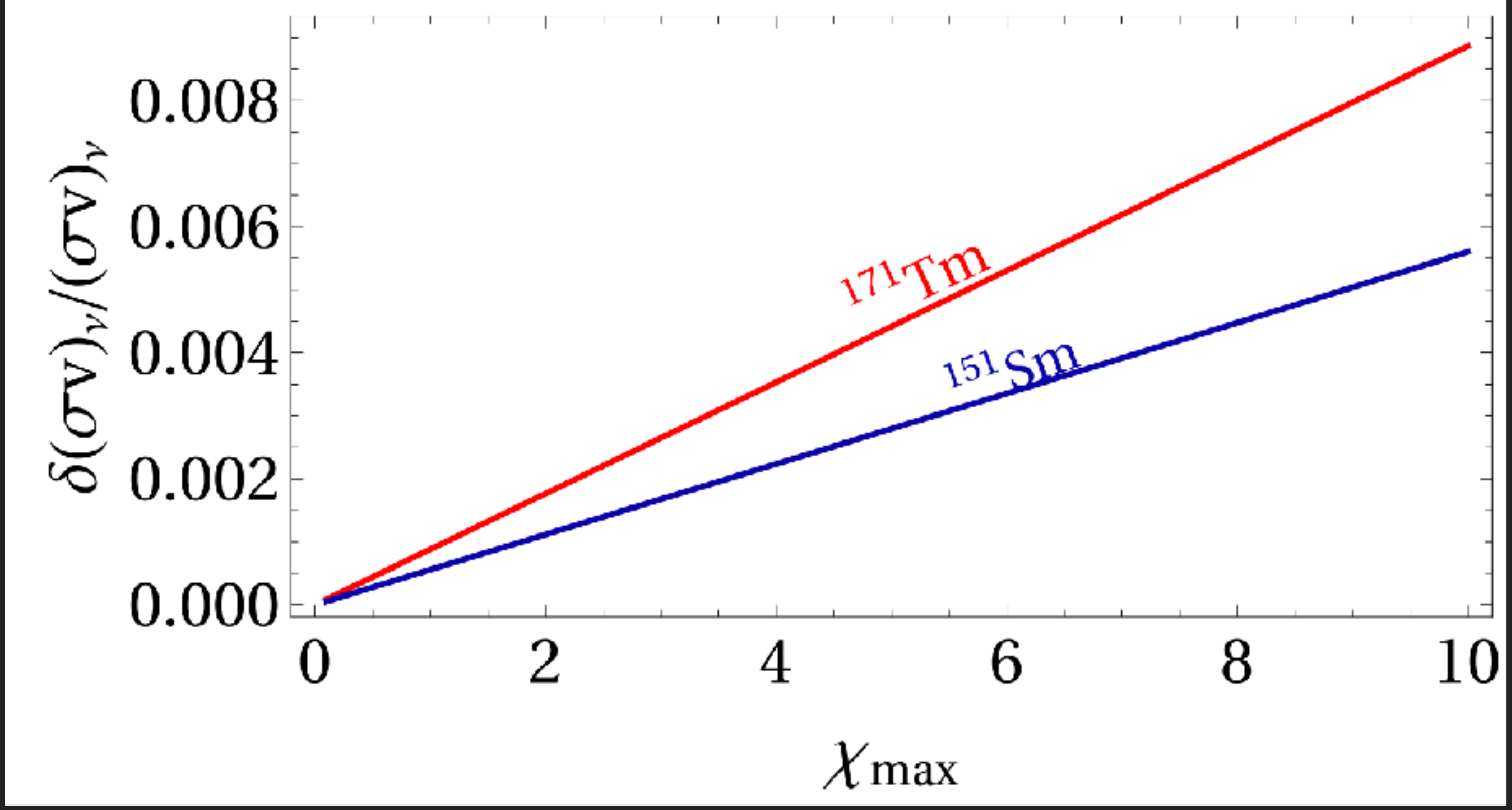
Determination of the ^{151}Sm half-life

Radiochim. Acta 2015; 103(9): 619–626



QUANTIFYING UNCERTAINTY DUE TO ξ— APPROXIMATION

$$G(W_e) = G_0(W_e) \times \left(1 + \chi \cdot \frac{W_e/m_e}{\xi} \right)$$



Vary dimensionless quantity between $[-\chi_{max}, \chi_{max}]$.

Even allowing for a order of magnitude fine-tuning, cross section is only sensitive at ~1% level.

LESSON: BIGGER FISH TO FRY

EFFECTS AT THE ~1% LEVEL.

- Shake-off: Lowers neutrino yield: $\nu\, A \to A^{2+}\, e^- e^-$

- Atomic corrections at very low energies: modify half-life $\Longrightarrow$ alters $(\sigma v)_\nu$.

Recent communication with X. Mougeot suggests atomic corrections alter our conclusions by ~1%

PRACTICAL CONSIDERATIONS

WHAT LIMITS AN EXPERIMENT LIKE PTOLEMY

- Tritium procurement is not a major bottleneck.
- Main issue comes down to ***packing.***
- Provided heavy nuclei are cost effective, and pack efficiently, the total material should not be an issue.
- In this sense comparing cross sections is not helpful.

CONCLUSIONS

SUMMARY

- Heavy nuclei offer certain practical advantages and certain practical disadvantages.
- There are ***no major theoretical hurdles*** that inhibit their use as a target.
- Cross sections are smaller than tritium, but quantity is not the bottleneck (could be packing efficiency and/or cost).